Uncertainty evaluation of peak energy of giant dipole resonance propagated from uncertainties of Skyrme parameters

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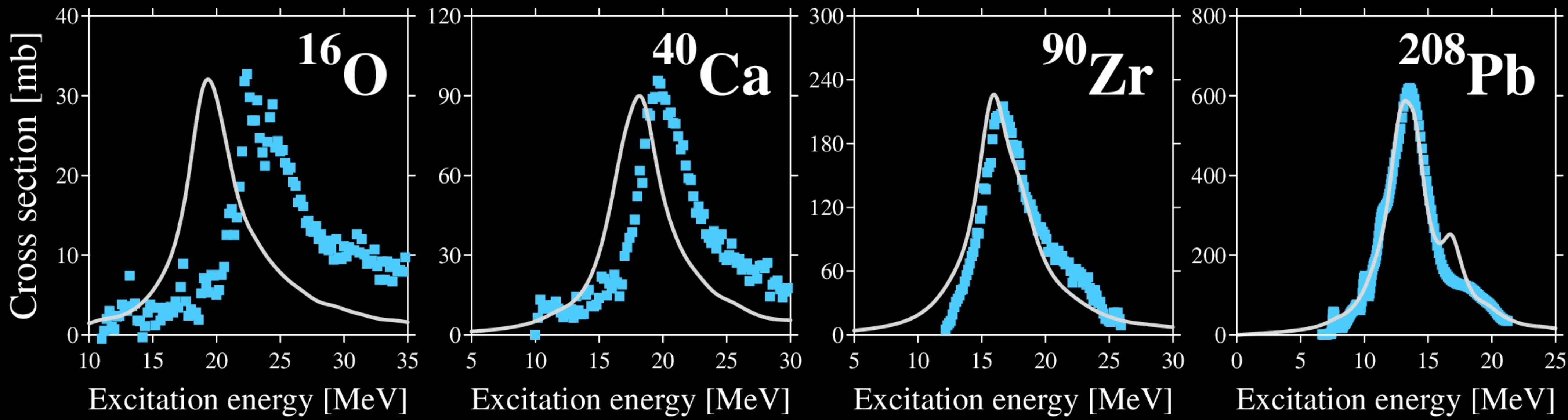
Underestimation of GDR peak energy

- GDR is systematically calculated by Skyrme-RPA / Skyrme-QRPA.
- Underestimated by 2-4 MeV in light nuclei.
- Adjust parameters to reproduce GDR peak energy.

⇒ Evaluating uncertainty of GDR peak energy calculated with Skyrme-RPA using Monte Carlo calculation.



Skyrme interaction

$$
\begin{aligned}
V_{\text{Skyrme}} = & \, t_0(1+x_0P_\sigma)\delta(r_i-r_j) && \text{... Attractive} \\
& + \frac{t_1}{2}(1+x_1P_\sigma)\left[\delta(r_i-r_j)k^2 + k'^2\delta(r_i-r_j)\right] && \text{... Non-local} \\
& + t_2(1+x_2P_\sigma)k'\cdot\delta(r_i-r_j)k && \text{... Non-local} \\
& + \frac{t_3}{2}(1+x_3P_\sigma)\rho^\alpha\left(\frac{r_i+r_j}{2}\right)\delta(r_i-r_j) && \text{... Density-dep repulsive} \\
& + iW_0\,(\sigma_1+\sigma_2)\cdot\left[k'\times\delta(r_i-r_j)\cdot k\right] && \text{... Spin-orbit}
\end{aligned}
$$

$$
P_\sigma = \frac{1+\sigma_i\cdot\sigma_j}{2},\ k = \frac{1}{2i}\left(\overrightarrow{\nabla}_i - \overrightarrow{\nabla}_j\right),\ k' = \frac{-1}{2i}\left(\overleftarrow{\nabla}_i - \overleftarrow{\nabla}_j\right)
$$

10 parameters $t_\#, x_\#, \alpha, W_0$ are fitted to reproduce experimental data such as binding energy, radius, and so on.

Parameter set: SkM*, SLy4, UNEDF1, ….

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10 parameters $t_\#, x_\#, \alpha, W_0$ are fitted to reproduce experimental data such as binding energy, radius, and so on.
Parameter set: SkM*, SLy4, UNEDF1, ….

⇒ Each parameter has best-fit value & uncertainty, and is correlated with other parameters.

Skyrme parameters and their uncertainties

Best-fit values (mean value)

Uncertainties (standard deviation)

SLy5-min

p	p_0		$\sqrt{\mathcal{E}_{ii}}$	units
t_0	-2475.408	$\pm$	149.455	MeV fm^3
t_1	482.842	$\pm$	58.537	MeV fm^5
t_2	-559.374	$\pm$	144.534	MeV fm^5
t_3	13697.07	$\pm$	1672.93	MeV fm$^{3+3\alpha}$
x_0	0.741185	$\pm$	0.189191	
x_1	-0.146374	$\pm$	0.468173	
x_2	-1		fixed	
x_3	1.162688	$\pm$	0.340537	
W_0	126		fixed	MeV fm^5
W_0'	126		fixed	MeV fm^5
α	1/6		fixed	

Correlation Matrix
====================
Cij = Eij/sqrt(Eii Ejj)

Covariance (correlation matrix)

t_0	t_1	t_2	t_3	x_0	x_1	x_3
1.0000	0.9837	0.9854	-0.9997	-0.6766	0.8110	-0.6158
0.9837	1.0000	0.9575	-0.9870	-0.7066	0.8489	-0.6553
0.9854	0.9575	1.0000	-0.9863	-0.6601	0.7843	-0.5964
-0.9997	-0.9870	-0.9863	1.0000	0.6798	-0.8154	0.6197
-0.6766	-0.7066	-0.6601	0.6798	1.0000	-0.9327	0.9928
0.8110	0.8489	0.7843	-0.8154	-0.9327	1.0000	-0.9311
-0.6158	-0.6553	-0.5964	0.6197	0.9928	-0.9311	1.0000

- Almost all Skyrme interactions :
  Only **best-fit values**
  (SkM*, SLy series, ….)
- Some recently-designed Skyrme interactions :
  best-fit values and **uncertainties**
  (SAMi series, SV- series, ….)
- Only a few Skyrme interactions :
  best-fit values, **uncertainties** and **covariances.**
  (SLy5-min, UNEDF-series)

SLy5-min: Roca-Maza, Paar, and Colo,
J. Phys. G: Nucl. Part. Phys. 42 034033 (2015)

Skyrme parameters and their uncertainties

Best-fit values (mean value) **Uncertainties (standard deviation)**

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  best-fit values, **uncertainties** and **covariances.**
  (SLy5-min, UNEDF-series)

t0: strength of attraction,
t3: strength of repulsion
correlation: -0.9997
⇒ strengthening attraction (t0) causes
automatically strengthening repulsion (t3).

Uncertainty evaluation for GDR peak energy

Monte Carlo calculation evaluating uncertainty of GDR peak energy propagated from uncertainties of Skyrme parameters.

SLy5-min parameter

SLy5-min				
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1. **Generating uncorrelated random numbers following normal distribution $N(0,1)$, $X = (x_1, x_2, \ldots)^T$**
2. **Creating correlated random numbers by acting Q, which is obtained from correlation matrix C by singular value decomposition ($C = Q^T Q$), QX**
3. **Using SLy5-min values p_0 and its standard deviations Δp ($= \sqrt{E_{ii}}$), generating correlated randomized parameter set, $p_0 + (QX)\Delta p$**
4. **Generating 2500 randomized parameter sets.**
5. **Employing these 2500 parameter sets, performing RPA calculations 2500 times using skyrme_rpa.**
6. **Evaluating uncertainty of GDR peak energy E_{GDR} from the results.**

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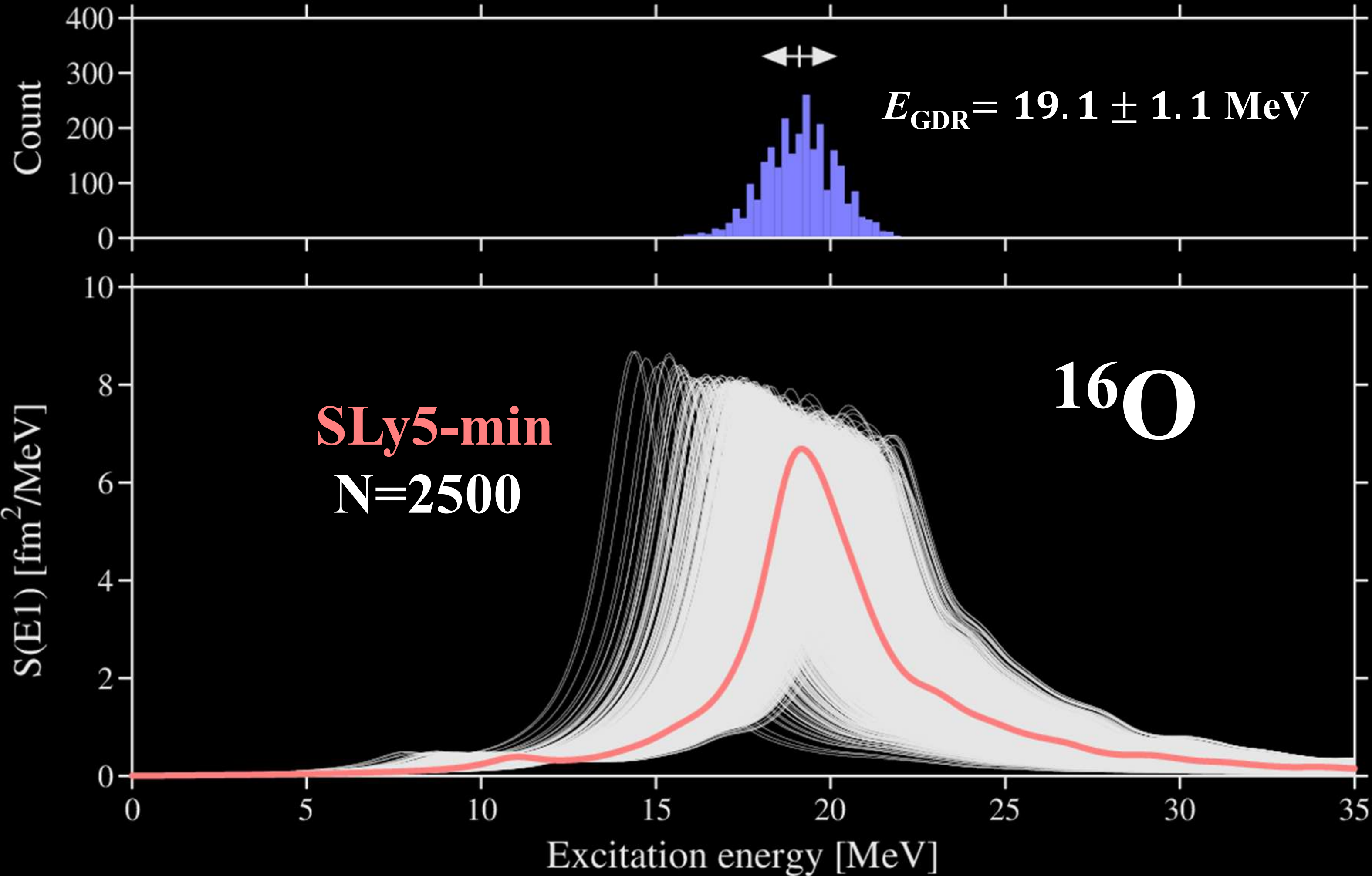
Self-consistent RPA calculations with Skyrme-type interactions: The skyrme_rpa program

Gianluca Colò[a,*], Ligang Cao[b,c,a], Nguyen Van Giai[d], Luigi Capelli[a,1]

Colo, Cao, Giai, Capelli, Comp. Phys. Comm. 184, 142

Randomized GDR in ^{16}O



Correlations between GDR peak energies & Skyrme parameters in ^{16}O

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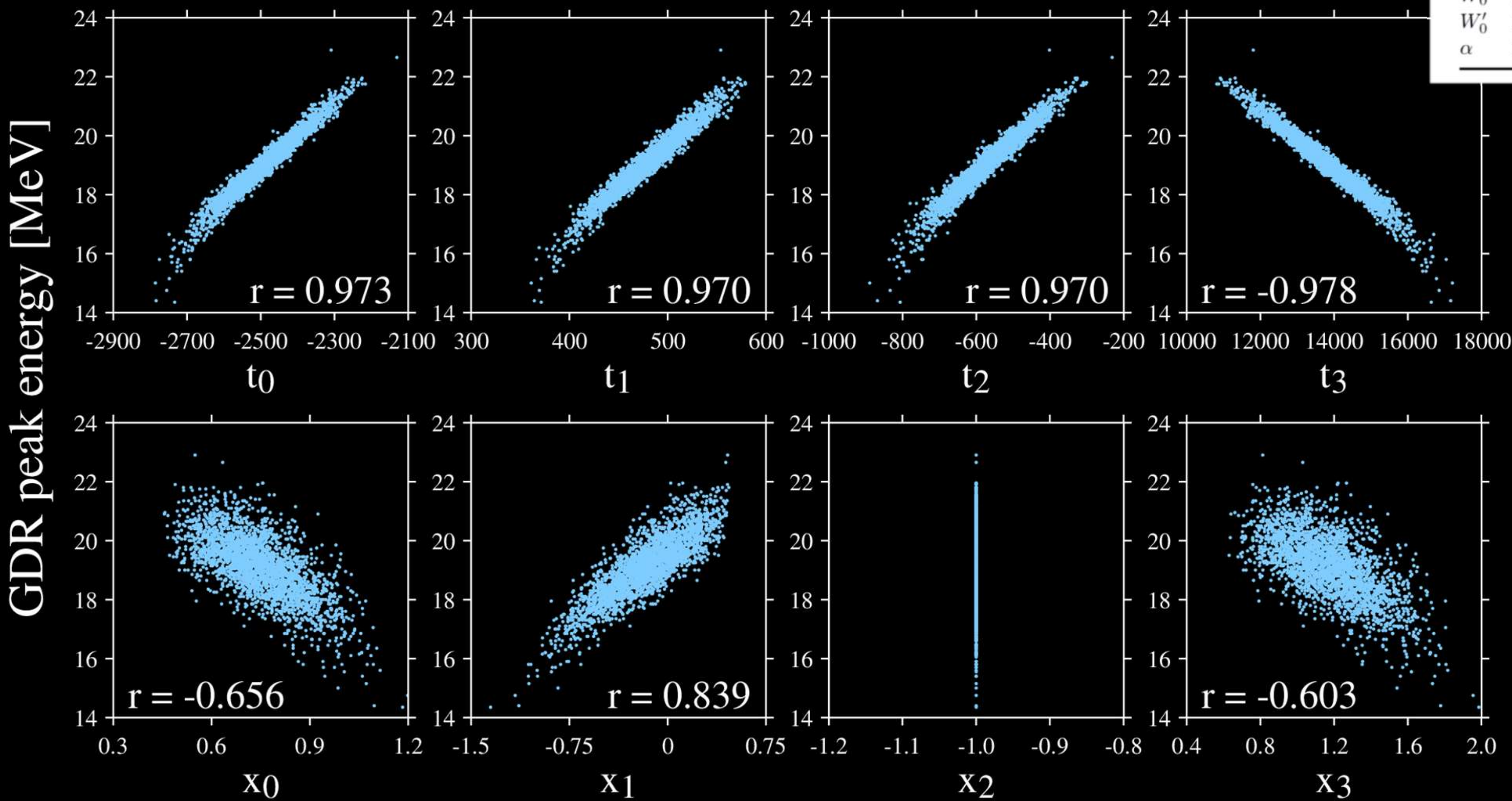
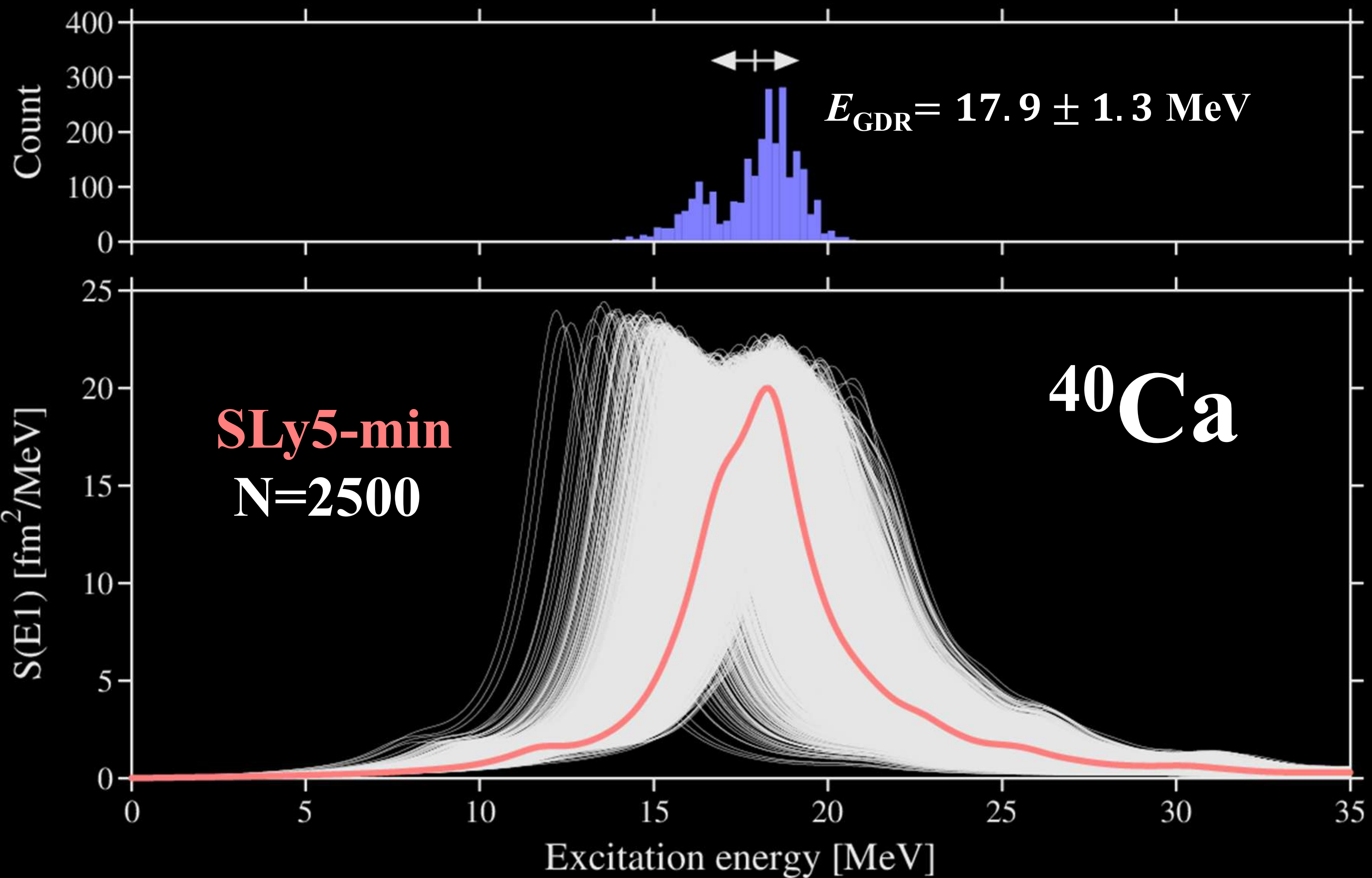
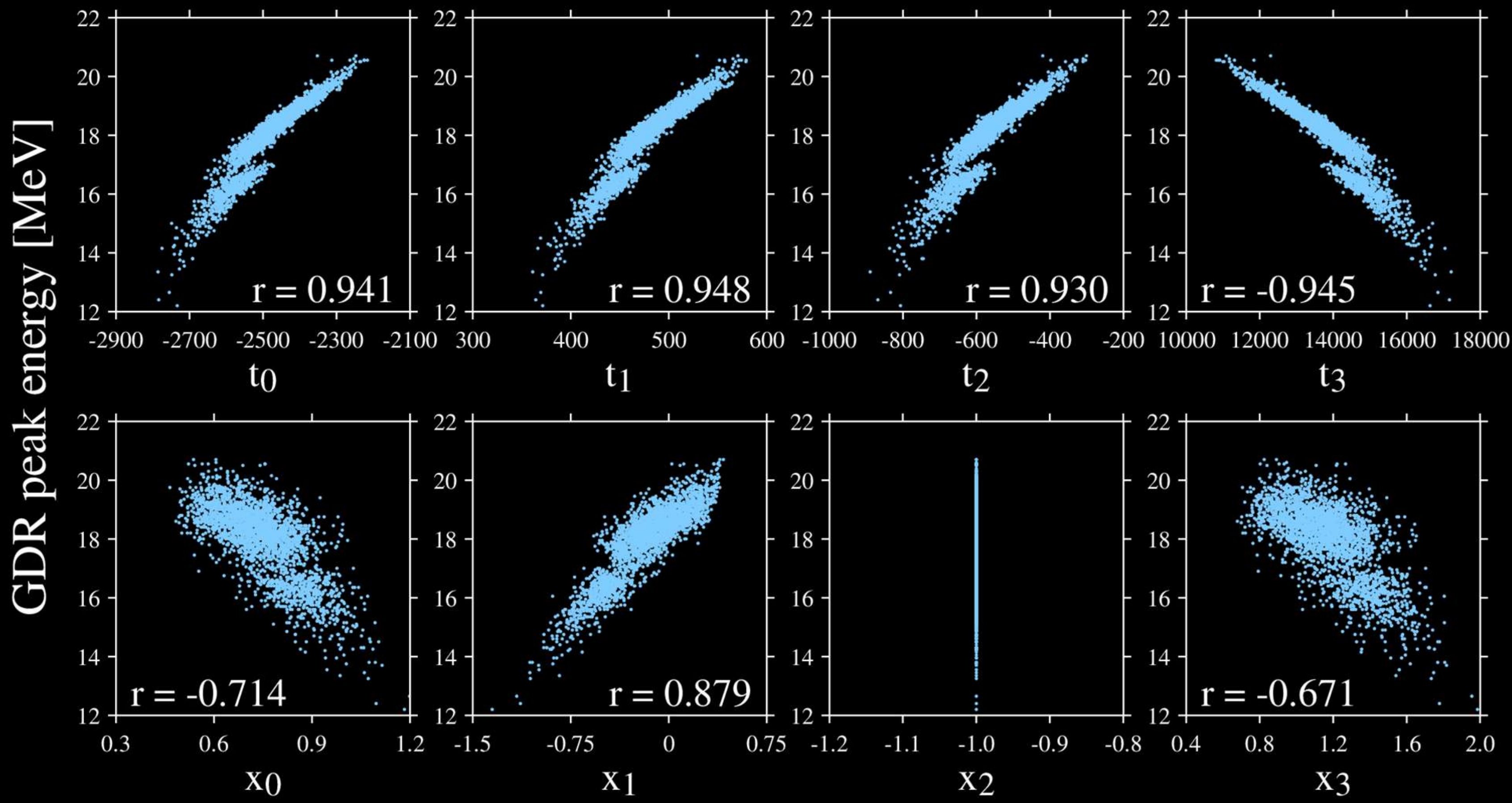


SLy5-min
N=2500

Randomized GDR in ^{40}Ca



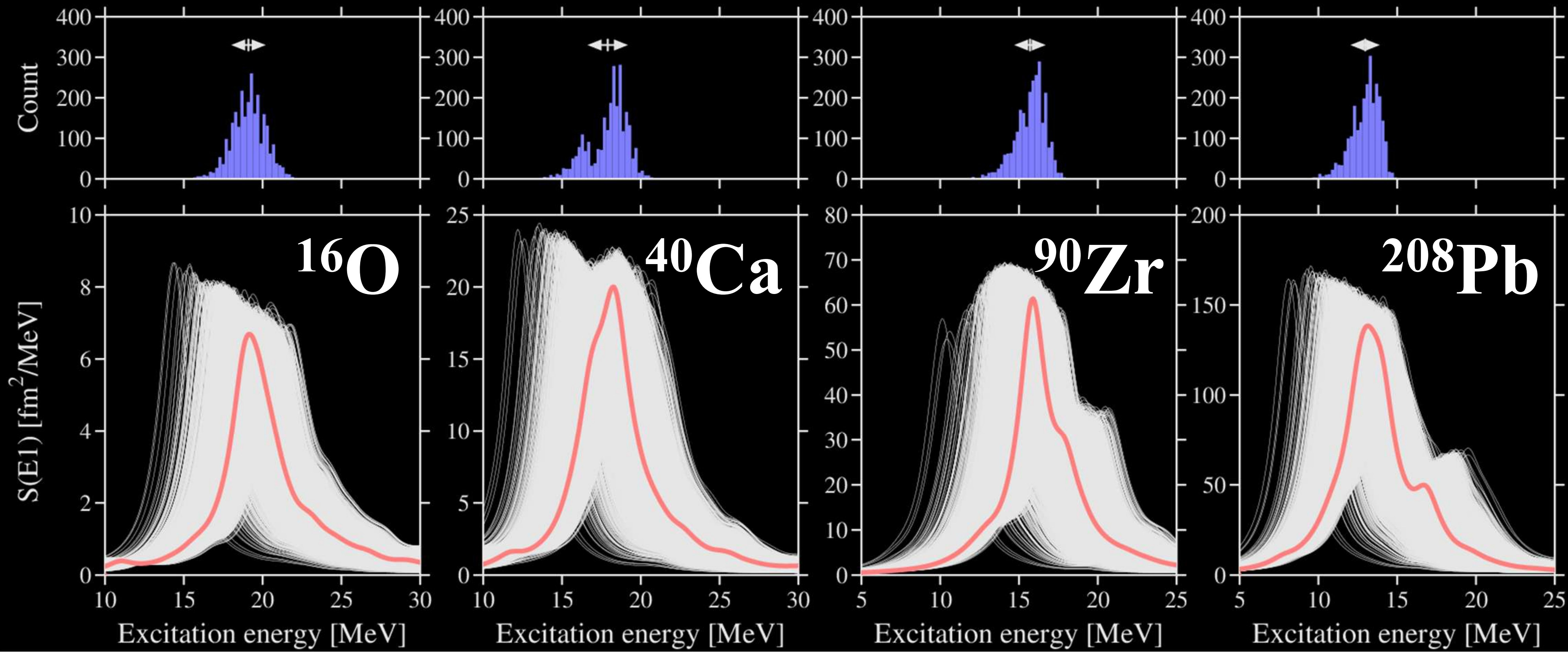
Correlations between GDR peak energies & Skyrme parameters in ^{40}Ca



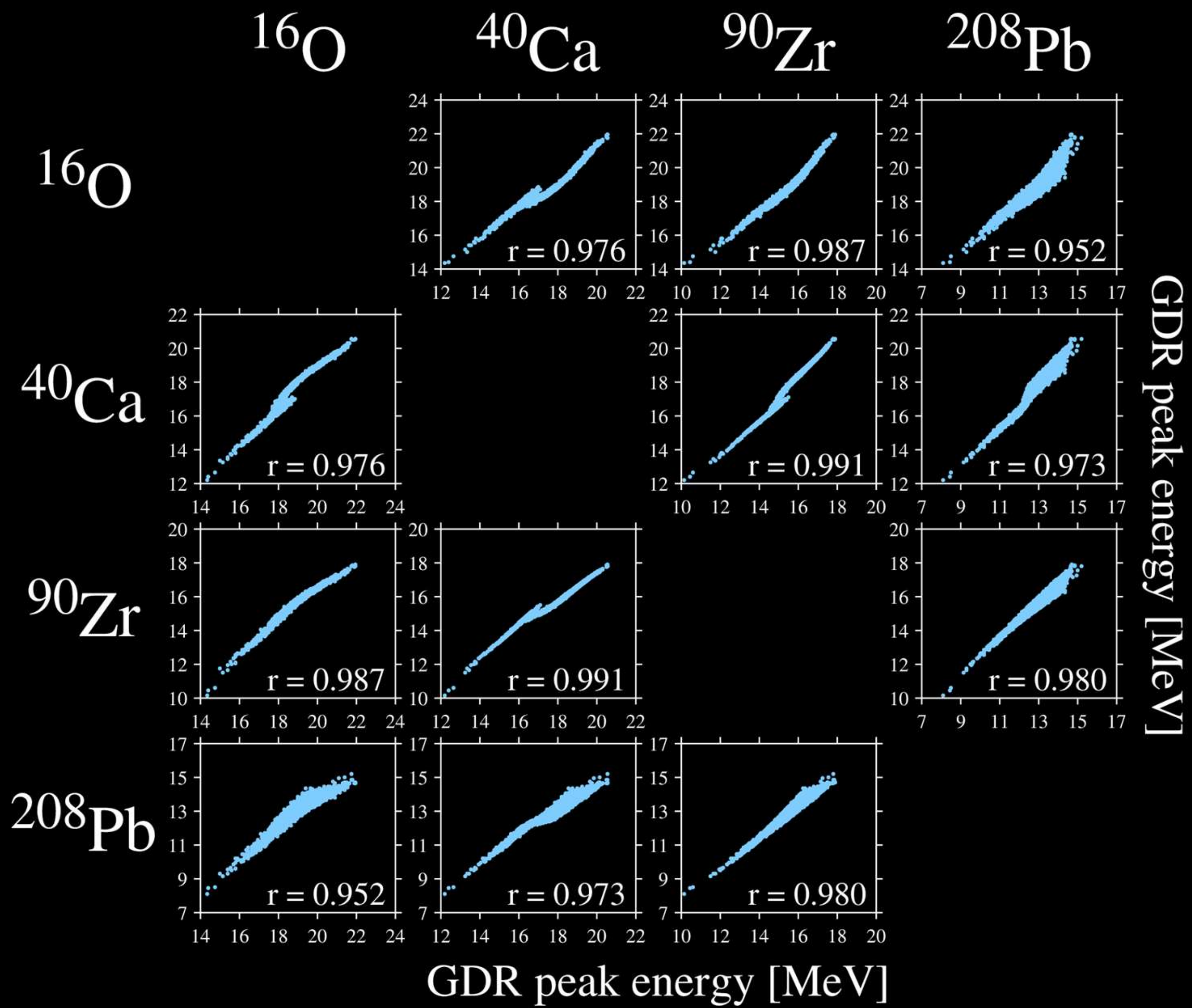
E_{GDR} and uncertainty

	E_{GDR} [MeV]
^{16}O	19.1 ± 1.1
^{40}Ca	17.9 ± 1.3
^{90}Zr	15.7 ± 1.0
^{208}Pb	13.0 ± 0.9

E_{GDR} uncertainty is ~ 1MeV, irrespective of mass.



Correlations between randomized GDR peak energies



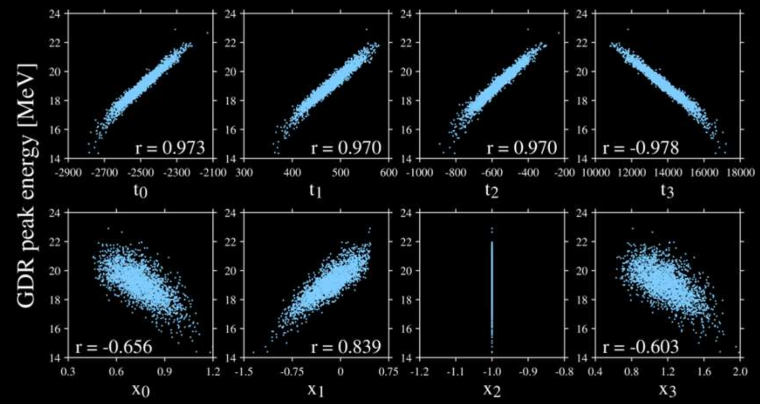
Strong correlations between GDR peak energies in ^{16}O, ^{40}Ca, ^{90}Zr, ^{208}Pb.

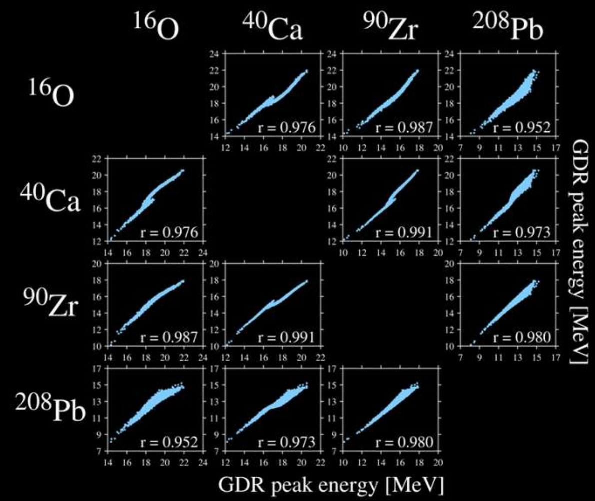
<u>For new parameter set</u>

Short summary

- E_{GDR} can be shifted by adjusting parameters.
- Adjusting parameters shift E_{GDR} simultaneously.
- E_{GDR} uncertainty is almost irrespective of mass.
- Disagreement is larger in light nuclei

⇒ Adjusting parameters
by large shifting in light nuclei
and by small shifting in heavy nuclei.





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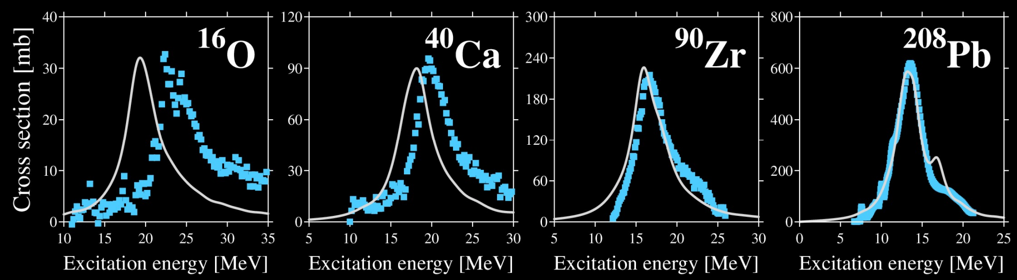

New parameter set

$$p = p_0 + Q\Delta p \cdot f/A$$

p_0 : original SLy5-min parameter set (best-fit values)
Δp : standard deviations of parameters (uncertainties, $\sqrt{\varepsilon_{ii}}$)
Q : factorized correlation matrix, $C = Q^T Q$ (covariance)
f : free parameter

Best-fit values

Uncertainties (standard deviation)

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SLy5-min parameter

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Δp : standard deviations of parameters (uncertainties, $\sqrt{\varepsilon_{ii}}$)

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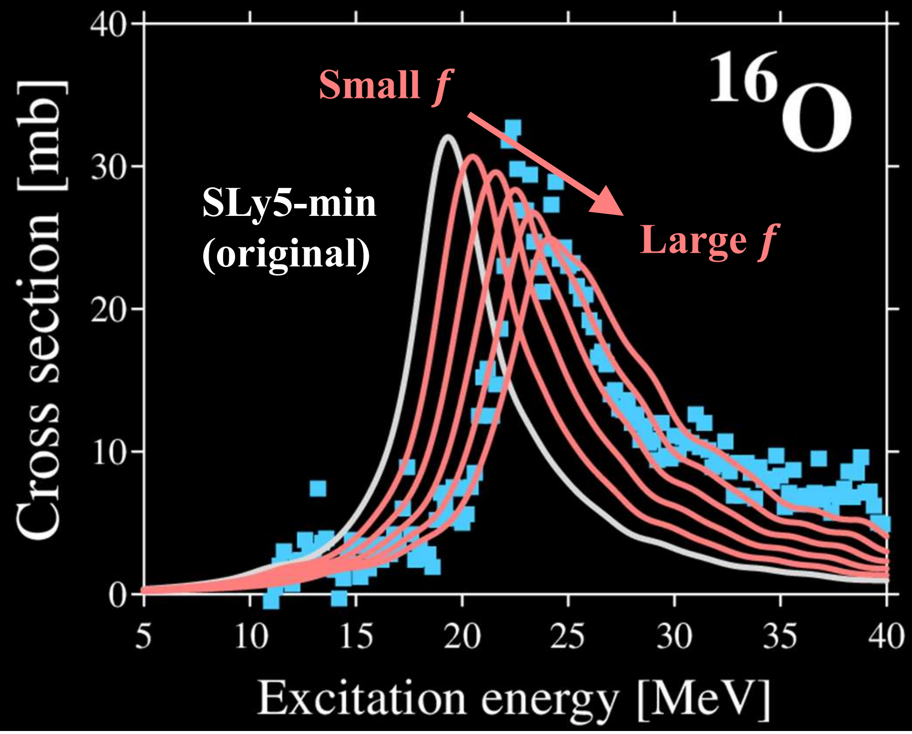
f : **free parameter**



Best-fit values **Uncertainties (standard deviation)**

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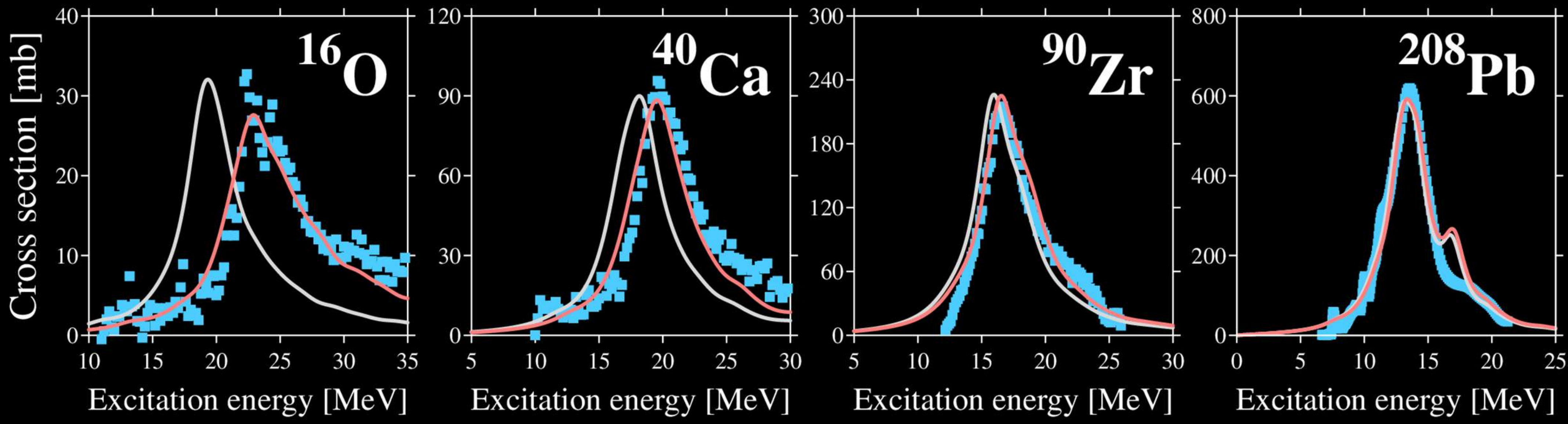
Q : factorized correlation matrix, $C = Q^T Q$ (covariance)
f : free parameter, $f = 35$



Takeaways

- **Uncertainty of GDR peak energy propagated from uncertainty of Skyrme parameters is evaluated using Total Monte Carlo calculations.**
  - **In the case of SLy5-min interaction, uncertainty of GDR peak energy is ~ 1 MeV, irrespective of mass.**
  - **GDR peak energy is strongly correlated with Skyrme parameters when the correlations between Skyrme parameters are considered.**
- **A new Skyrme parameter set which reproduces well GDR peak energy is proposed.**

Perspective

- **Deformed nuclei**