Quantum thermalization of quark-gluon plasma

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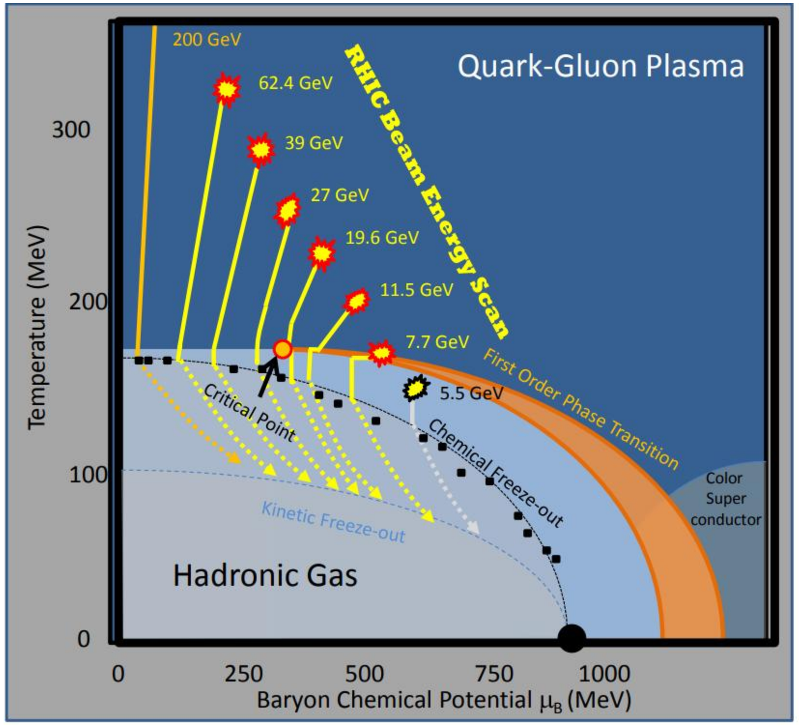
based on 2412.00662, 2501.02777

Outline

- Motivation
- Classical QCD thermalization: extract speed of sound
- Quantum thermalization: ETH
- Toy model of QCD: Schwinger model (1+1Dimemsional QED)

Thermalization of a QCD matter is crucial for (almost) all current studies

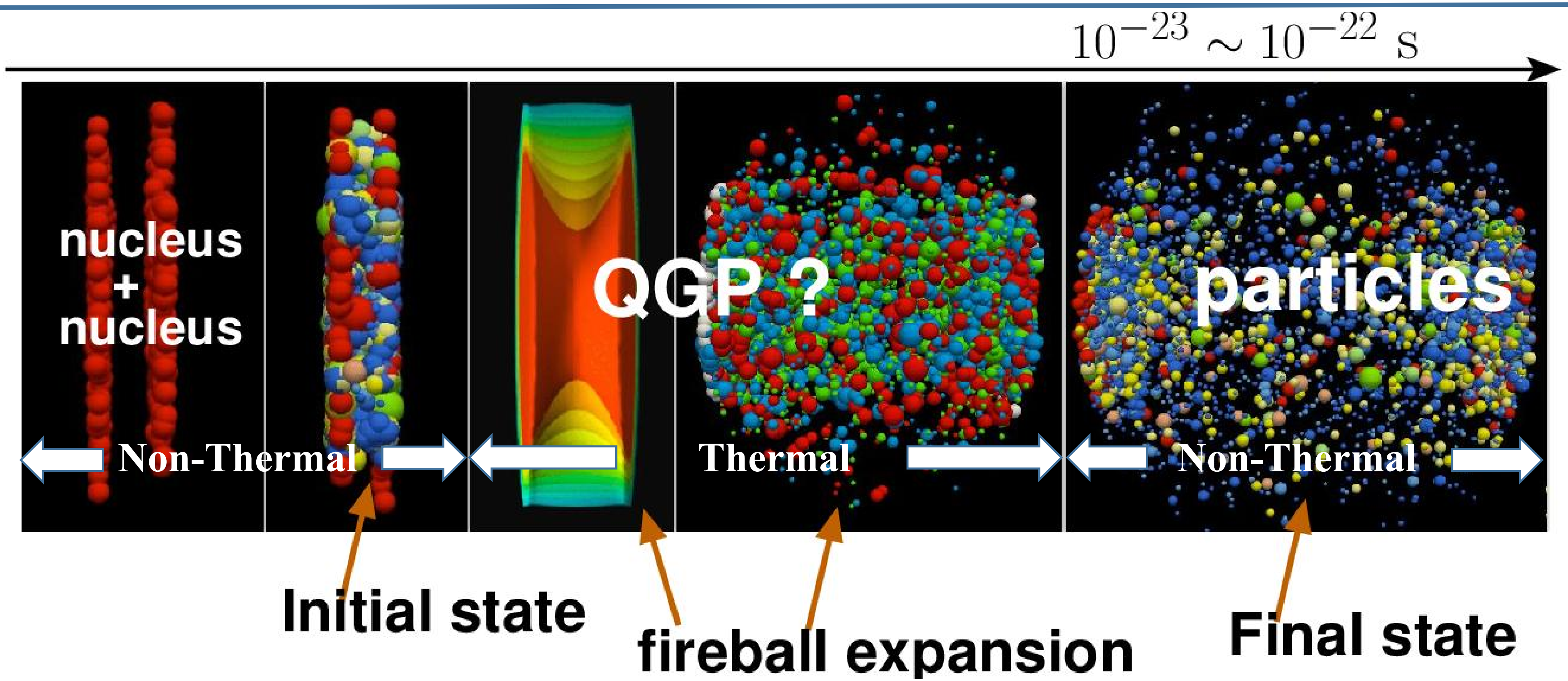


[STAR collaboration]

- QCD phases: QGP, EoS and critical point.
- QCD transport phenomena: eta/s, conductivity, etc.
- Topological and EM QCD effect: CME
- ...

- Does a strongly interacting quantum system thermalize? (QGP, cold atom, condensed matter, ...)
- Any direct probes of QCD thermalization in realistic heavy-ion collisions?

The standard modeling of heavy-ion collisions

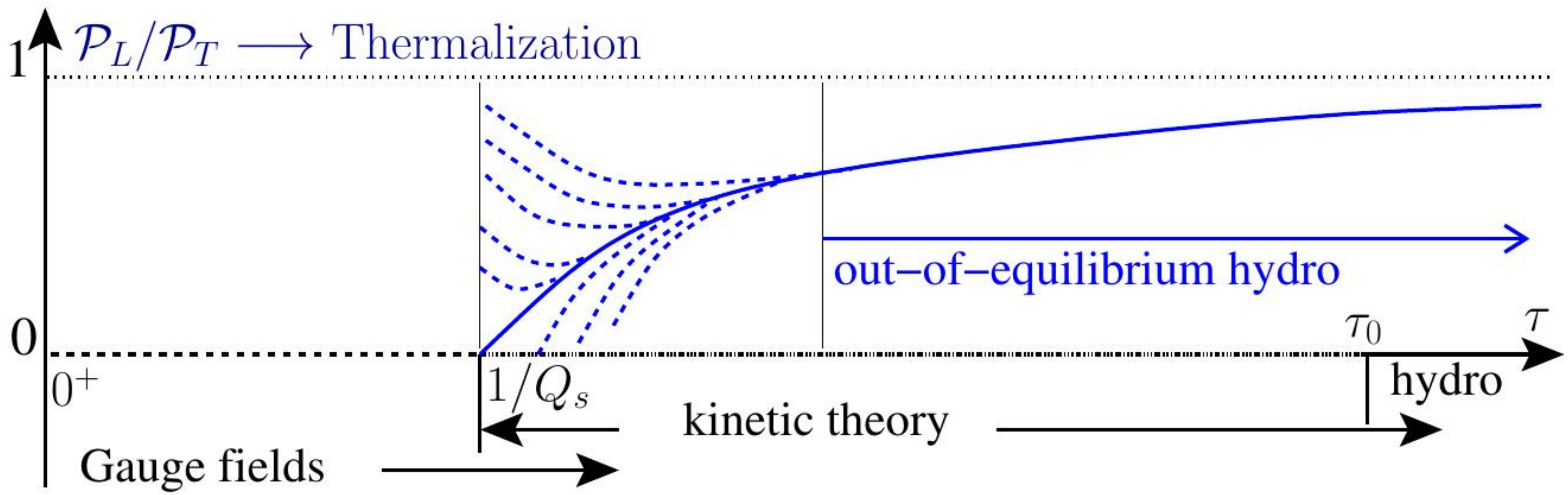


- Indirect signature of (transient) thermalization: collective flow in particle spectrum

$$\text{Hydro response:} \quad V_n \propto \kappa(\mathrm{EoS}, \eta/s, \ldots)\mathcal{E}_n$$

Thermalization of a QCD matter: classical/semi-classical theory

- The emergence of hydro attractor: hydrodynamics, kinetic theory [PRL115,072501(2015)]



- E.g.: thermalization through evolution of phase-space distribution function

$$\frac{d}{dt} f_p = -\mathcal{C}[f_p] \quad \rightarrow \quad \partial_\mu T^{\mu\nu} = 0\,, \quad \text{with} \;\; T^{\mu\nu} \propto \int_p p^\mu p^\nu f_p$$

QCD classical thermalization: Measurement of QCD speed of sound

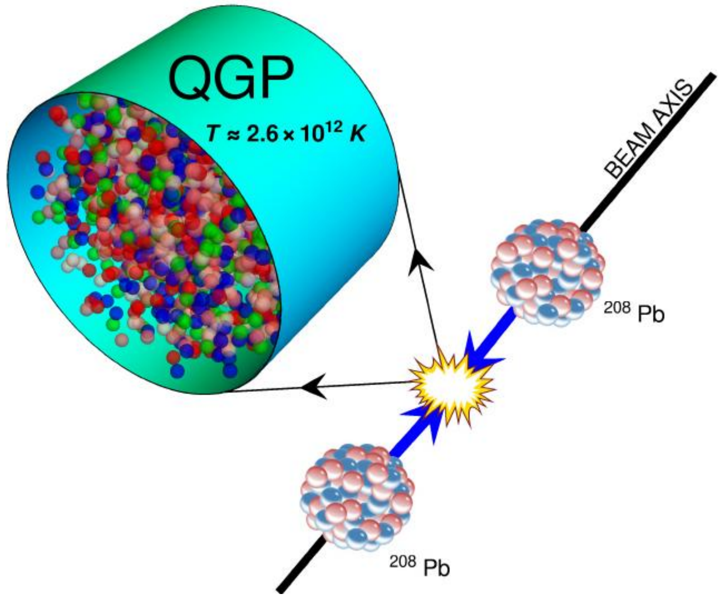
System thermalization $\Longleftrightarrow$ Equation of State

$dP = sdT, \quad de = Tds$ constant volume

$$c_s^2 \equiv \left.\frac{\partial P}{\partial e}\right|_{\text{Adiabatic}} = \left.\frac{d\ln T}{d\ln s}\right|_{\text{Adiabatic}} = \left.\frac{d\ln T}{d\ln S}\right|_{\text{Adiabatic}}$$

Realistic QGP in Heavy-ion collisions:

1. Volume saturates in ultra-central collisions (UCC).
2. Entropy increases due to QM fluctuations (e.g., nucleon scatttering).
3. Non-homogeneous QGP with fixed volume? How to measure temperature and entropy from particles ? Effect of quantum fluctuations?



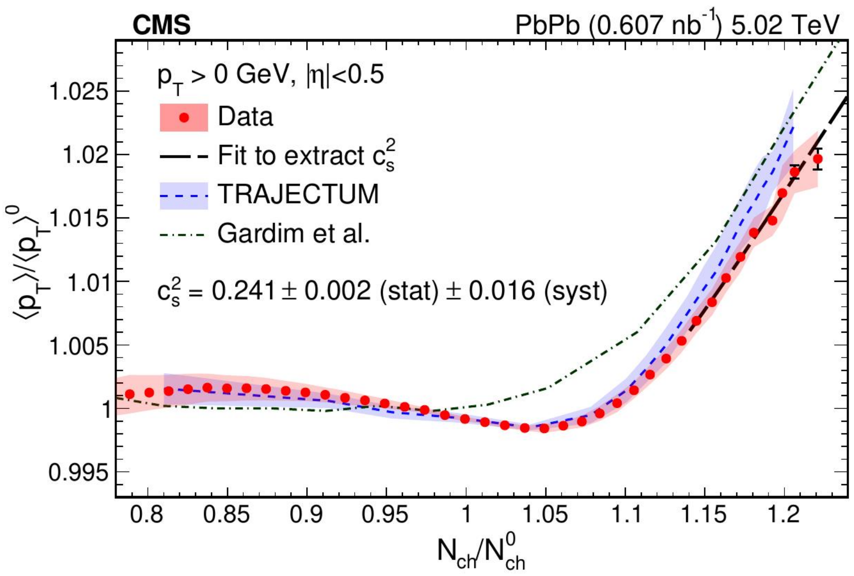
$T_{\text{eff}} \propto \langle p_T \rangle \qquad S \propto dN_{\text{ch}}/d\eta$

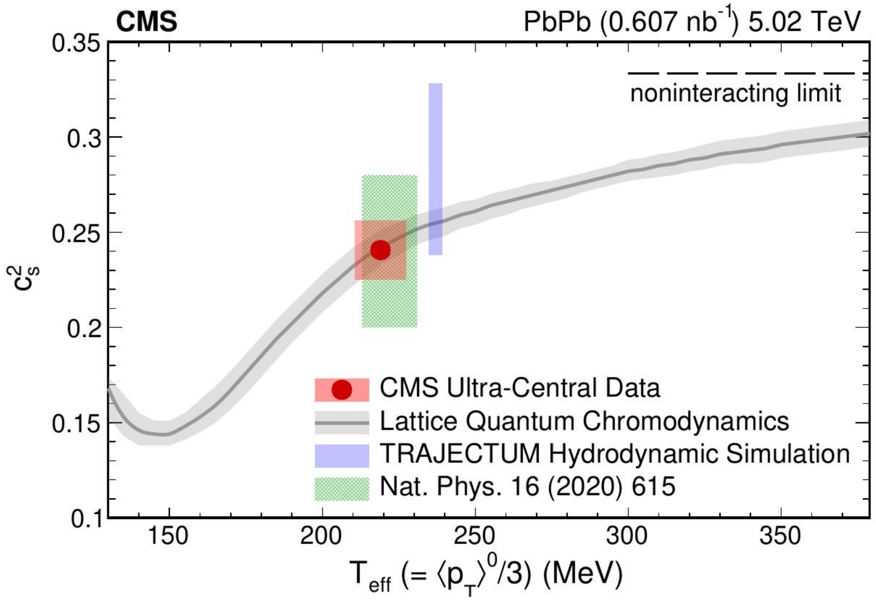
[F. Gardim et al , 1908.09728, Nature Physics]

c_s^2 from UCC experiments in mid-rapidity

$$\begin{cases} \langle p_T \rangle \propto T_{\text{eff}} \\ dN_{\text{ch}}/d\eta \propto S \end{cases} \quad + \quad c_s^2 = \frac{d\ln T}{d\ln S} \Rightarrow \qquad \frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N}{N_0} \quad \text{with} \begin{cases} \Delta_p \equiv \langle p_T \rangle - \langle p_T \rangle_0 \\ \Delta_N \equiv N_{ch} - N_0 \end{cases}$$

- QCD speed of sound implies a linear response relation: thermodynamic and deterministic.
- Extract c_s from sub-bin measurements: $\frac{\{\Delta_p\}_I}{\langle p_T \rangle_0} = c_s^2 \frac{\{\Delta_N\}_I}{N_0}$, with I labels sub-bin in central events

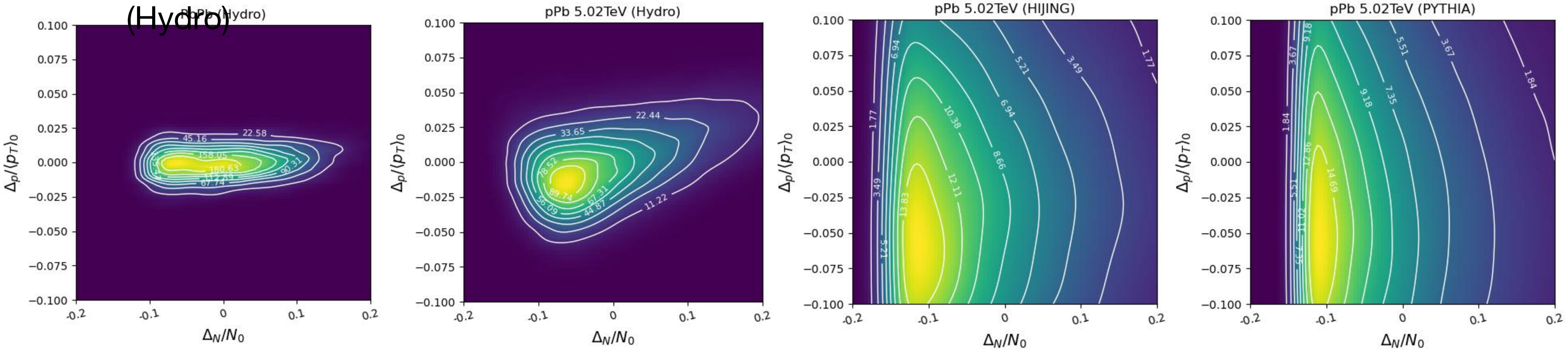




[CMS collaboration, 2401.06896]

Realistic HIC w/ fluctuations: two-dimensional joint probability $\mathcal{P}(\Delta_p, \Delta_N)$

PbPb (Hydro) | pPb (Hydro) | pPb (HIJING) | pPb (PYTHIA)



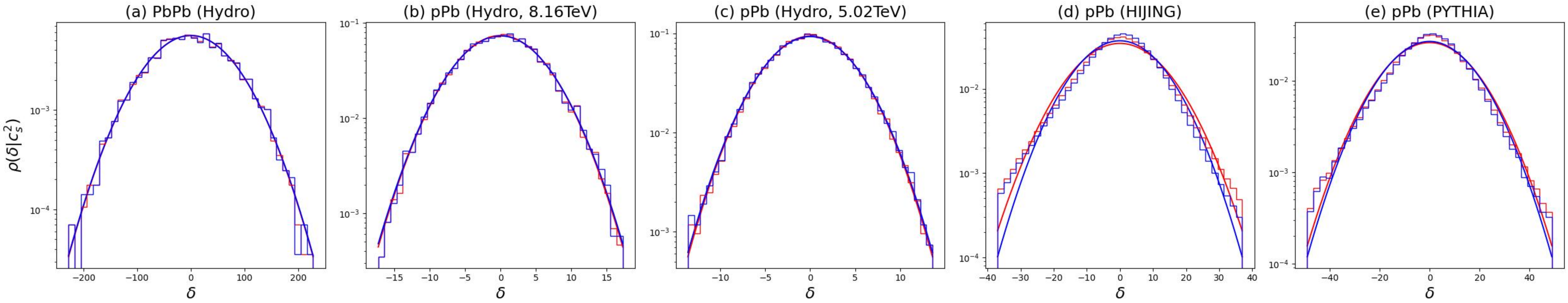
- Hydro: thermodynamic response and quantum noise

$$\frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \quad \longleftrightarrow \quad \text{thermodynamic resp.} + \text{quantum noise}$$

- Non-thermal models: Quantum response relation (e.g., multi-parton scatterings) and quantum noise

$$\frac{\Delta_p}{\langle p_T\rangle_0} = \kappa \frac{\Delta_N + \delta}{N_0} \quad \longleftrightarrow \quad \text{quantum resp.} + \text{quantum noise}$$

Disentangle quantum fluctuations from thermodynamic response: Gaussianity



[Yu-Shan Mu, Jing-An Sun, LY, X-G. Huang]

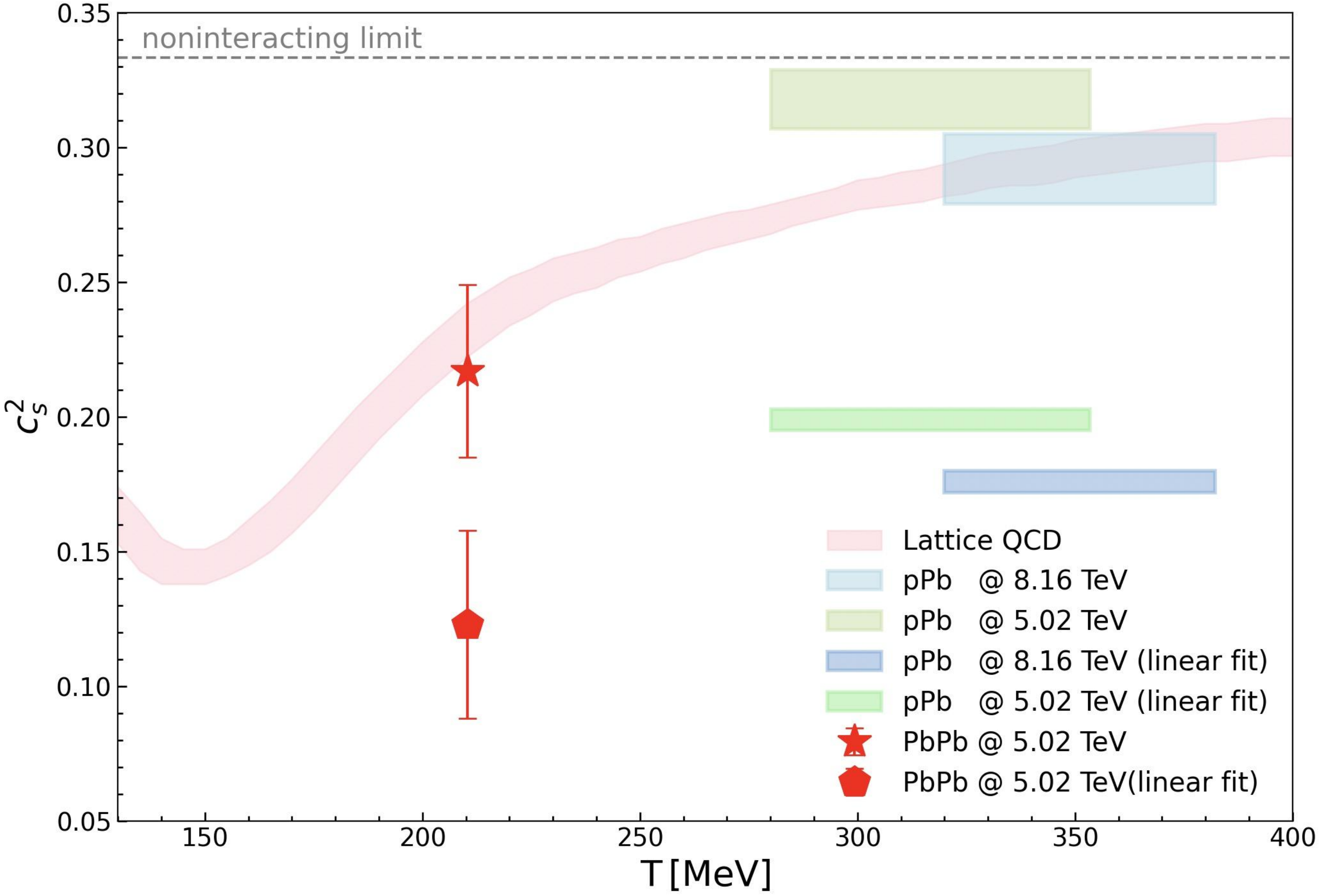
- Gaussianity leads to the zero skewness condition: solve c_s^2 as the root of equation

$$\{\delta^3\}_c = \{\delta^3\} = 0 \quad \rightarrow \quad (c_s^2)^3 \frac{\{\Delta_N^3\}}{N_0^3} - 3(c_s^2)^2 \frac{\{\Delta_N^2 \Delta_p\}}{N_0^2 \langle p_T \rangle_0} + 3c_s^2 \frac{\{\Delta_N \Delta_p^2\}}{N_0 \langle p_T \rangle_0^2} - \frac{\{\Delta_p^3\}}{\langle p_T \rangle_0^3} = 0\,,$$

Exp. measurables: mixed skewness of transverse momentum and charged multiplicity

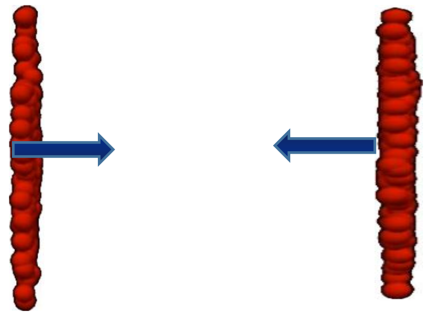
Note that mean of quantum fluctuations vanishes by construction: $\{\delta\} \equiv \frac{1}{N_{\mathrm{eve}}} \sum_{\mathrm{event\ i}} \delta_i = 0$

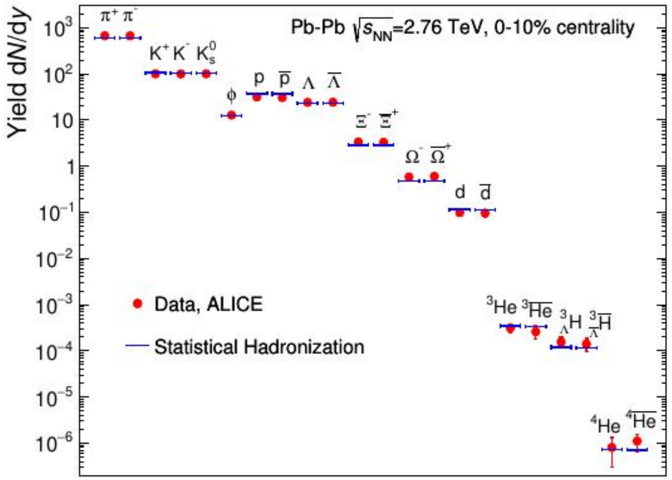
Extract QCD speed of sound



Thermalization of a QCD matter: quantum thermalization

[Nature 561, 321(2018)]





$$\lim_{t\to\infty} \langle \Psi(t)|\hat{O}|\Psi(t)\rangle \propto \mathrm{Tr}[e^{-\hat{H}/T}\hat{O}]$$

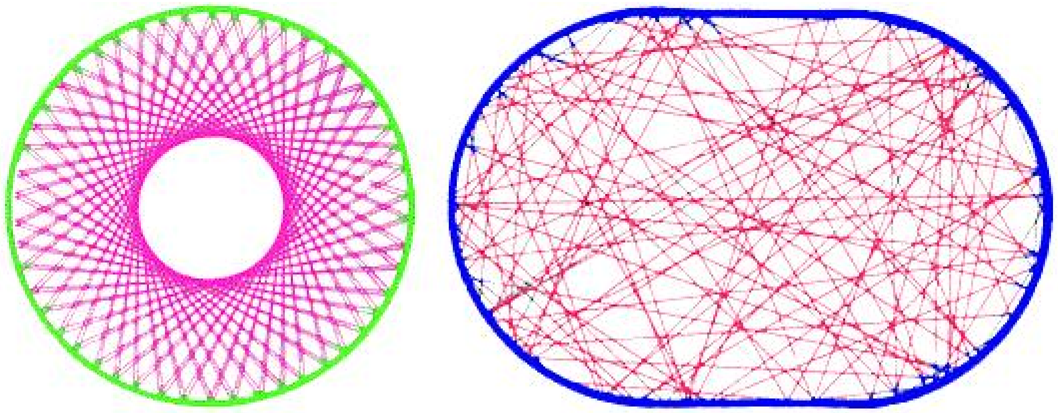
similarly for pp, electron-positron

How thermal ensemble emerges from pure Quantum states, under unitarty interaction?

- QGP is a high-energy (closed) quantum system obeying non-Abelian gauge theory.
- QGP thermalization is beyond perturbative QCD characterization.
- Beyond current lattice QCD simulation: time evolution requires diagonalization of Hamiltonian.

Classical thermalization (closed system)

$$H(p,q) \Rightarrow \dot{O}(t) = \{O(t), H(p,q)\}$$



- Classical thermalization is defined through ensemble average: microcanonical, canonical, etc.
- Ergodicity in phase space (e.g., chaos in non-integrable system) is equivalent to ensemble average

$$\{O(E)\}_{\mathrm{En}} = \langle \hat{O}(E) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} d\tau O(\tau) = \frac{\int_s d\Gamma O(\Gamma)}{\int_s d\Gamma}$$

- Initial information lost during classical thermalization

[1509.06411]

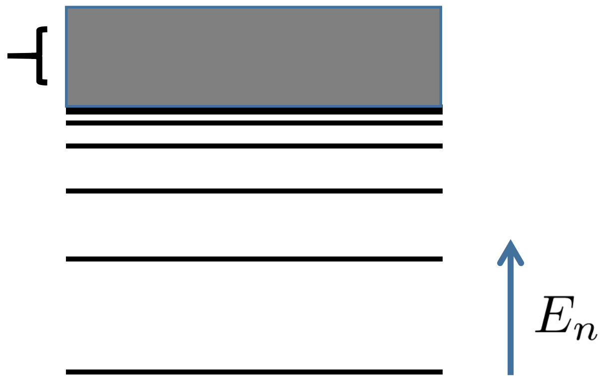
Quantum thermalization: Eigenstate Thermalization Hypothesis (ETH)

- Phase space or phase-space trajectory does not apply to quantum systems,

$$
\begin{aligned}
\{O(E)\}_{\mathrm{En}} \stackrel{?}{=} \langle \hat{O} \rangle &= \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} d\tau \langle \Psi(\tau)|\hat{O}|\Psi(\tau)\rangle \\
&= \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} d\tau \left(\sum_n |c_n|^2 O_n + \sum_{n\neq m} c_n c_m^* e^{i(m-n)t/\hbar} O_{nm} \right)
\end{aligned}
$$

- ETH: one may effectively treat energy eigenstates as thermal states within $\in [E-\delta E, E+\delta E]$

$$O_{nm} = O(\bar{E})\delta_{nm} + \Omega(\bar{E}, E_n - E_m) r_{nm} \to \langle \hat{O} \rangle_{t\to\infty} = \{O\}_{\mathrm{En}}$$



smooth and continuous function of

$$\bar{E} = \frac{E_n + E_m}{2}$$

$$\propto e^{-S(\bar{E})/2}$$

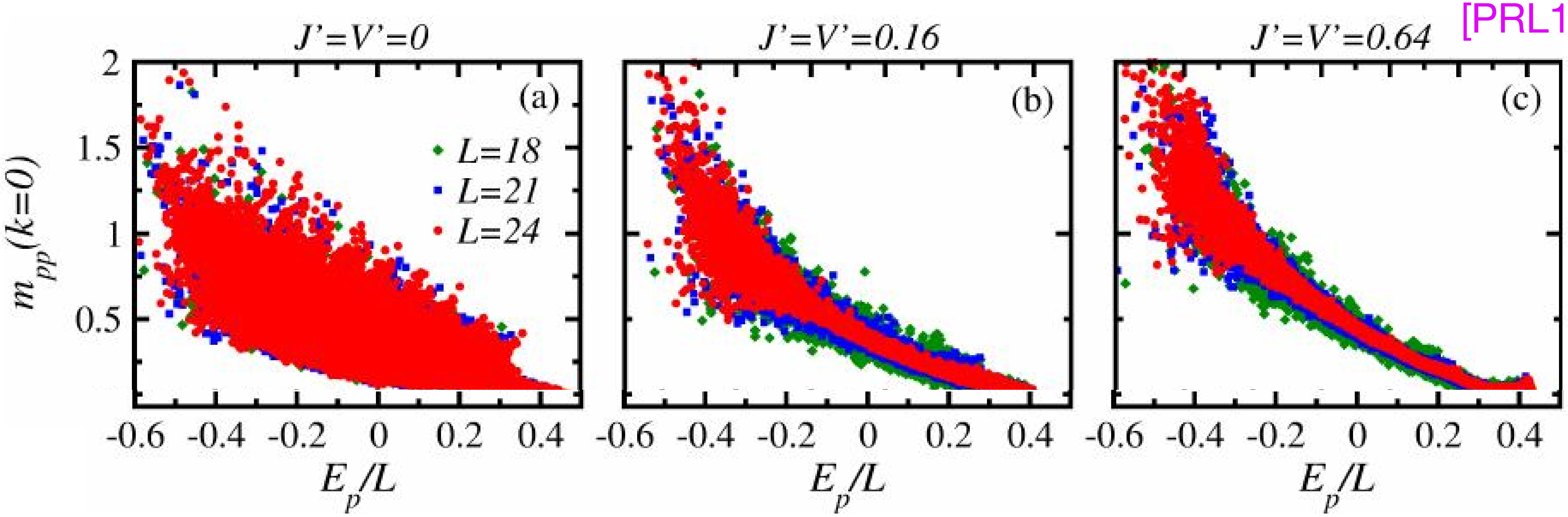
quantum noise of unit variance

[PRA43, 2046 (1991); PRE50.888 (1994)]

ETH: QGP is a high-energy quantum system

- Hypothesis: not proved but has been verified in many (low-energy) quantum systems.
- Valid in general for highly-excited states, few-body operators and non-integrable systems.
- Exceptions: integrable system, quantum many-body localization, quantum many-body scar
- Study/identify quantum thermalization by analyzing diagonal and off-diagonal elements:

[PRL103(2009)]



integrable → non-integrable

Schwinger model: QED 1+1D (QED$_2$)

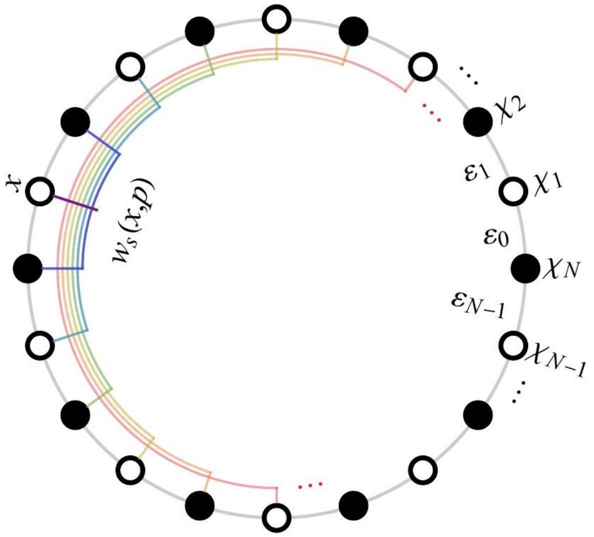
$$\hat{H} = \int \left(\bar{\psi}(\gamma^1(-i\partial_z - g\,A_1) + m\,e^{i\theta\gamma^5})\psi + \frac{1}{2}\mathcal{E}^2 \right) \mathrm{d}z$$

- Resembles QCD in 1+1 D: quark confinement, vacuum topology, anomaly, chiral symmetry breaking
- Analytically solvable for massless theory
- Realization on 1D lattice and quantum computation (classical/quantum device): 1D spin model

$$\chi_{2n} = a^{\frac{1}{2}}\psi_{\uparrow}(z_{2n})$$

$$\chi_{2n+1} = a^{\frac{1}{2}}\psi_{\downarrow}(z_{2n+1})$$

$$|\Psi(t)\rangle \to |e\rangle \otimes |s_1, s_2 \ldots\rangle : \qquad (2\Lambda + 1) \times 2^N$$



Wigner function thermalization

[Shile Chen, Shuzhe Shi, LY]

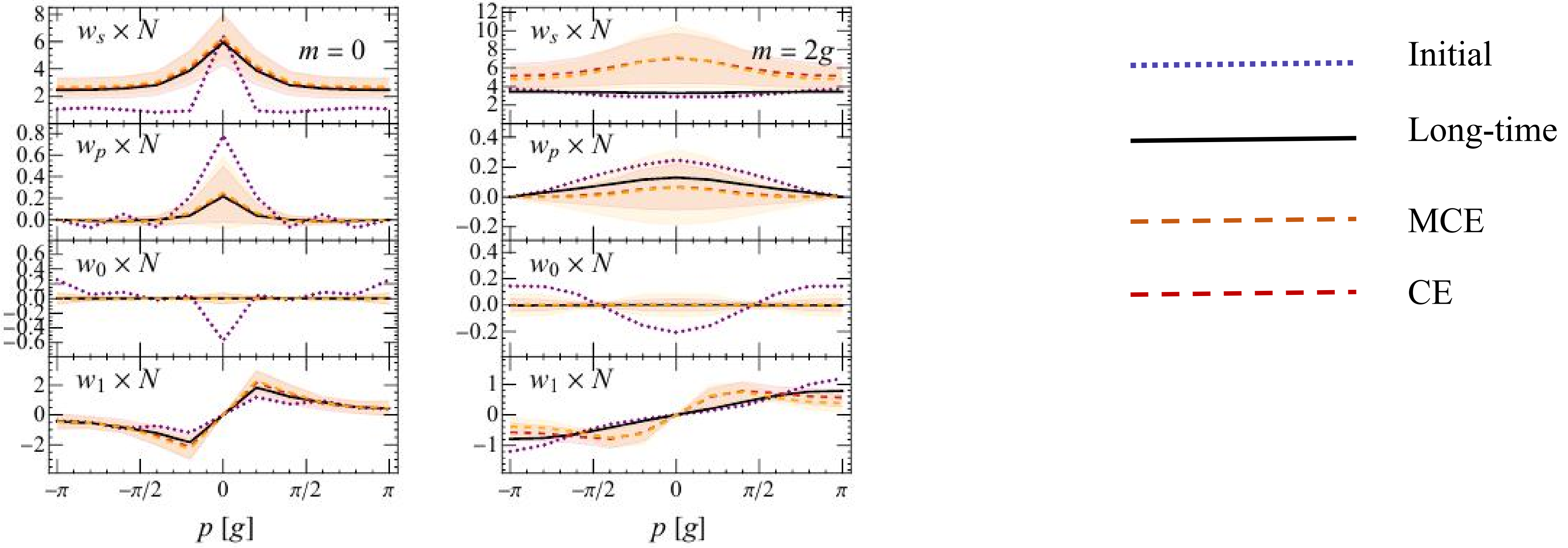
- Quark Wigner function: non-local two-body operators (align with ETH)
- The quantum analogy of quark distribution function

$$\hat{W}_{\alpha\beta}(t,z,p) = \int \bar{\psi}_\alpha(z_+) U(z_+, z_-) \psi_\beta(z_-)\, e^{i\frac{py}{\hbar}}\, \mathrm{d}y$$

$$= \underbrace{\hat{w}_s}_{\text{scalar}} - i\, \underbrace{\hat{w}_p}_{\text{pseudo-scalar}} \gamma^5 + \underbrace{\hat{w}_0}_{\text{vector charge}} \gamma^0 + \underbrace{\hat{w}_1}_{\text{axial vector charge}} \gamma^1$$

Wigner function thermalization in QED_2

$\theta = 0$



- Quantum thermalization realized in “massless”/strongly-coupled system: $\delta m \sim a g^2$
- Quantum thermalization partly realized in massive/weakly-coupled system: parity dependent!

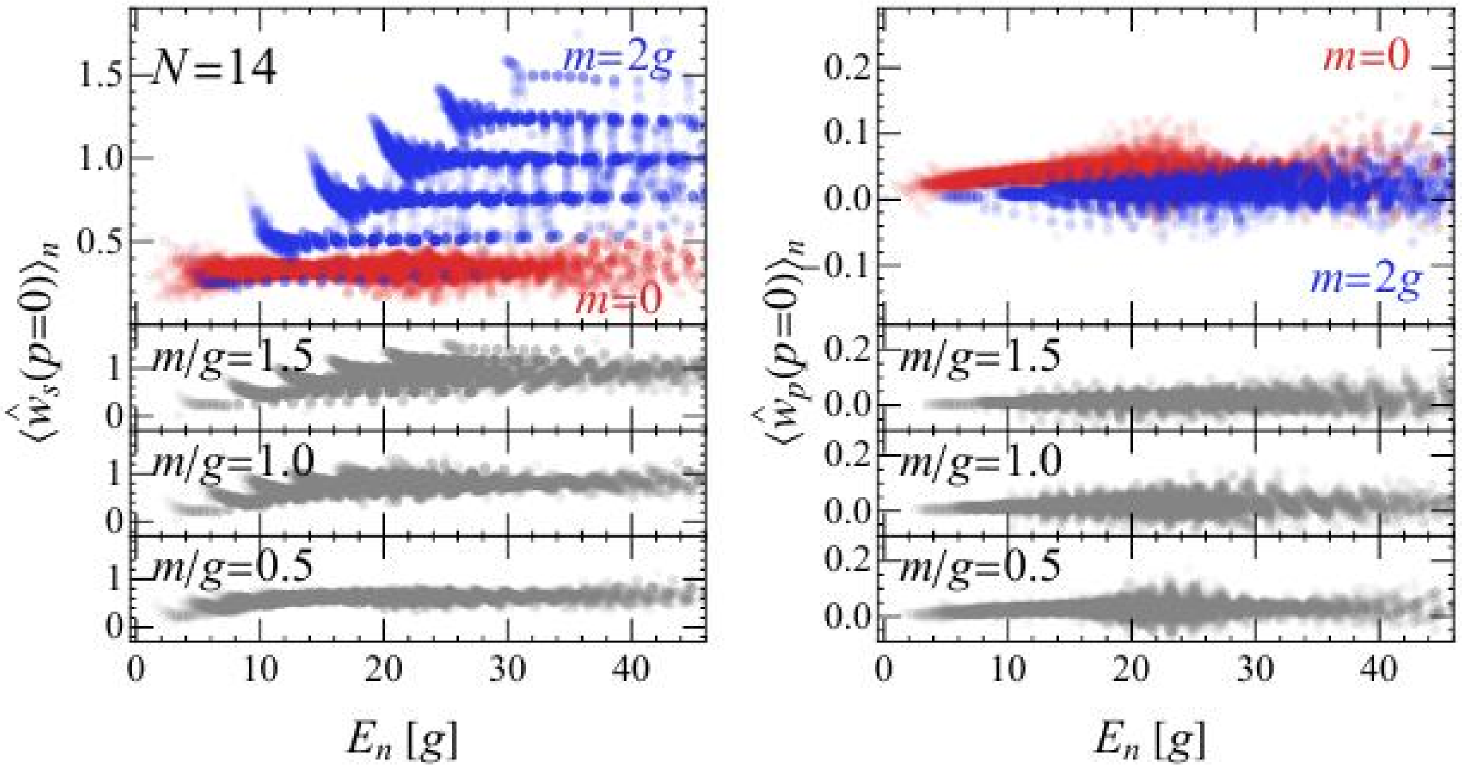
$$\begin{cases} \text{thermalized: pseudo-scalar and vector charge} \\ \text{not thermlized: scalar and axial vector charge} \end{cases}$$

Distribution of diagonal elements of Wigner function

$\theta = 0$

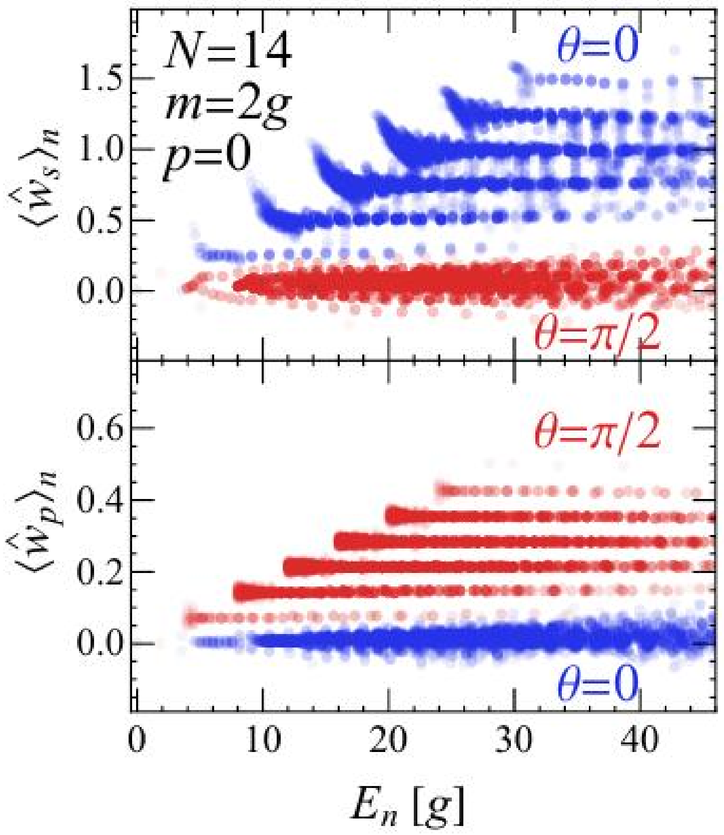
scalar

pseudo-scalar



- Strong coupling: single and narrow band ==> smooth function ==> ETH condition ==> thermalization
- Scalar: The distribution gradually splits as coupling gets weaker: break down of ETH
- Pseudo-scalar: always thermalize

Why ETH depends on parity in weakly coupled QED$_2$?

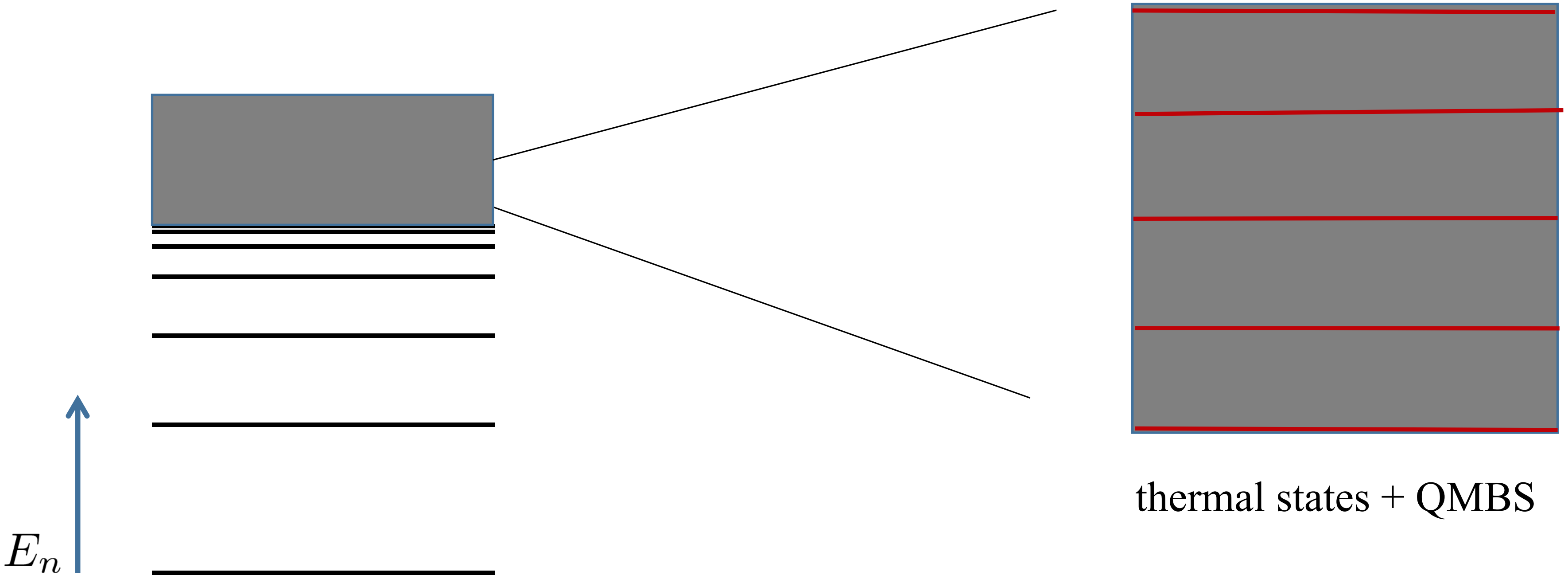


$$
\begin{aligned}
\hat{H}_{\text{lat}} = \frac{i}{2}\sum_{n=1}^{N-1}\left((-1)^n m\sin\theta - \frac{1}{a}\right)\left(\chi_n^\dagger \chi_{n+1} - \chi_{n+1}^\dagger \chi_n\right) \\
+ \frac{i}{2}\left(m\sin\theta - \frac{1}{a}\right)\left(\chi_N^\dagger U^\dagger \chi_1 - \chi_1^\dagger U \chi_N\right) \\
+ \sum_{n=1}^{N}\left((-1)^n m\cos\theta\, \chi_n^\dagger \chi_n + \frac{a\, g^2}{2}\varepsilon_n^2\right).
\end{aligned}
$$

- Swap parity via vacuum theta angle (vacuum topology in QED$_2$)
- Properties of ETH swap accordingly: P-odd not thermalized while P-even thermalized.

Emergence of quantum many-body scar states (QMBS)

- QMBS: $|Q_\alpha\rangle$ is P-even(odd) for $\theta = 0(\pi/2)$



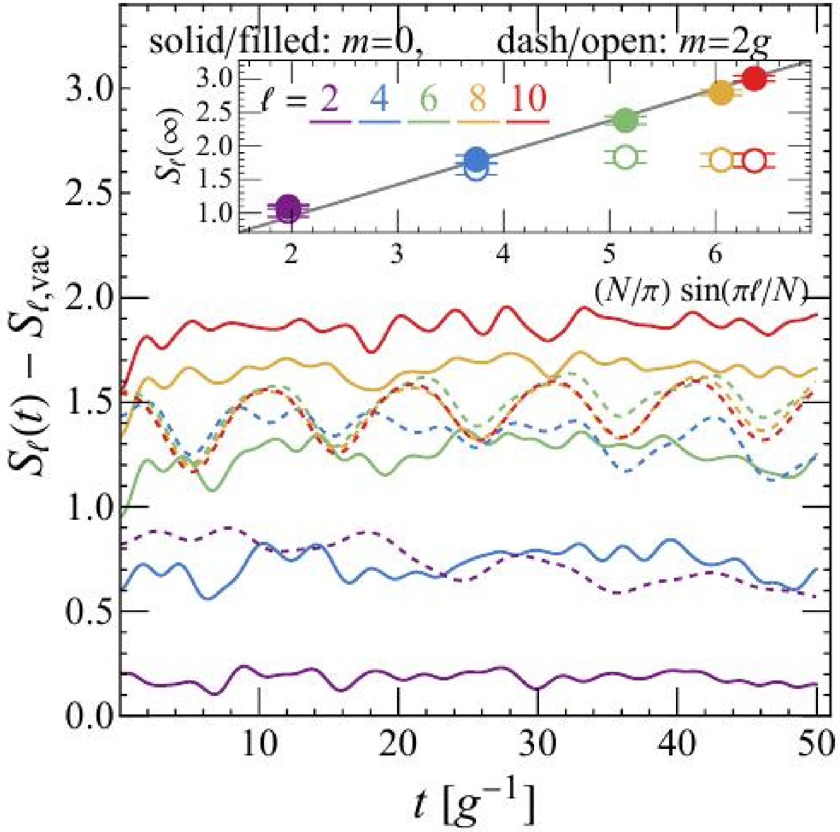
$$\hat{Q}^\dagger_\alpha \equiv \frac{1}{N_{\text{perm.}}} \sum_{\substack{\text{perm.}\\ |\mathcal{A}|+|\mathcal{B}|=\alpha}} \left(\prod_{j\in\mathcal{A}} \chi^\dagger_j \otimes \prod_{k\in\mathcal{B}} \chi_k \otimes \prod_{i\in\overline{\mathcal{A}\cup\mathcal{B}}} I_i \right) \to \begin{cases} |Q_\alpha\rangle = \hat{Q}^\dagger_\alpha |1010\ldots\rangle \\ \hat{H}_{\text{fermion}}|Q_\alpha\rangle = (E_0 + \alpha m)|Q_\alpha\rangle \end{cases}$$

Summary

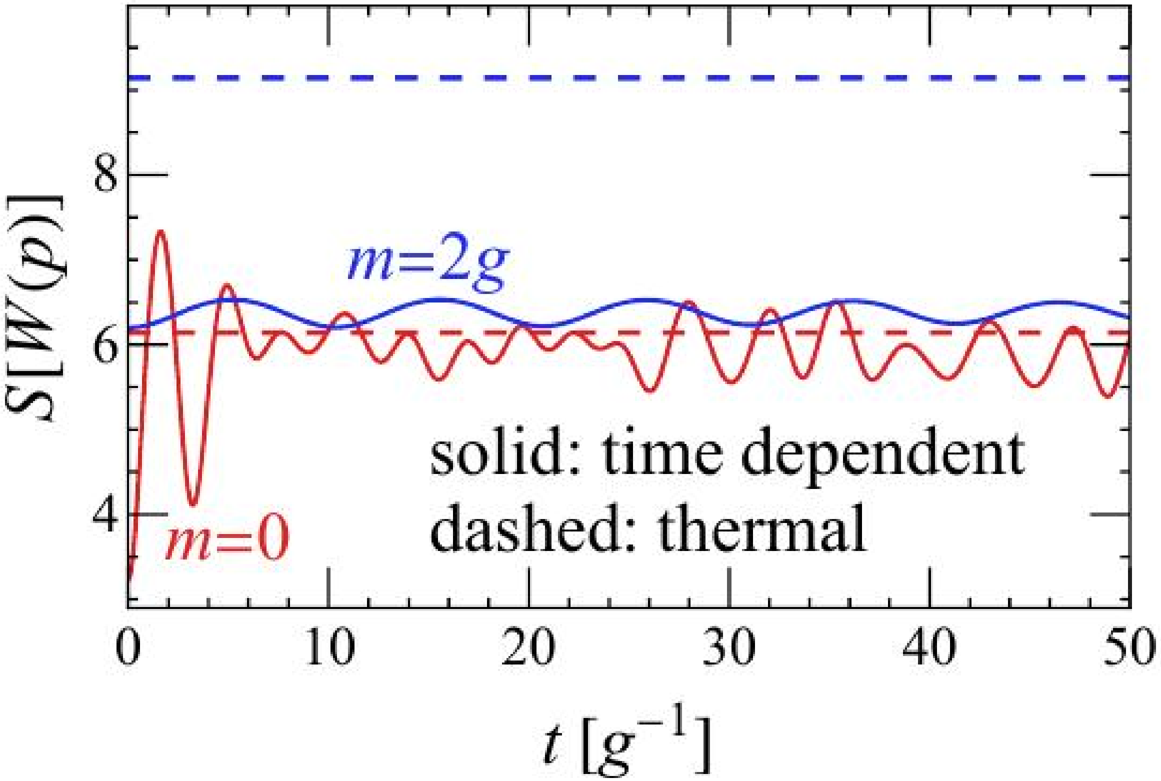
- Classical QCD thermalization can be studies through the extracting speed of sound
- Quantum thermalization of QED_2 (toy model of QCD)

1. Quantum thermalization can be realized, depending of coupling and parity
2. Role of vacuum topology and the emergence of QMBS

Quantum thermalization in terms of entropy in QED_2

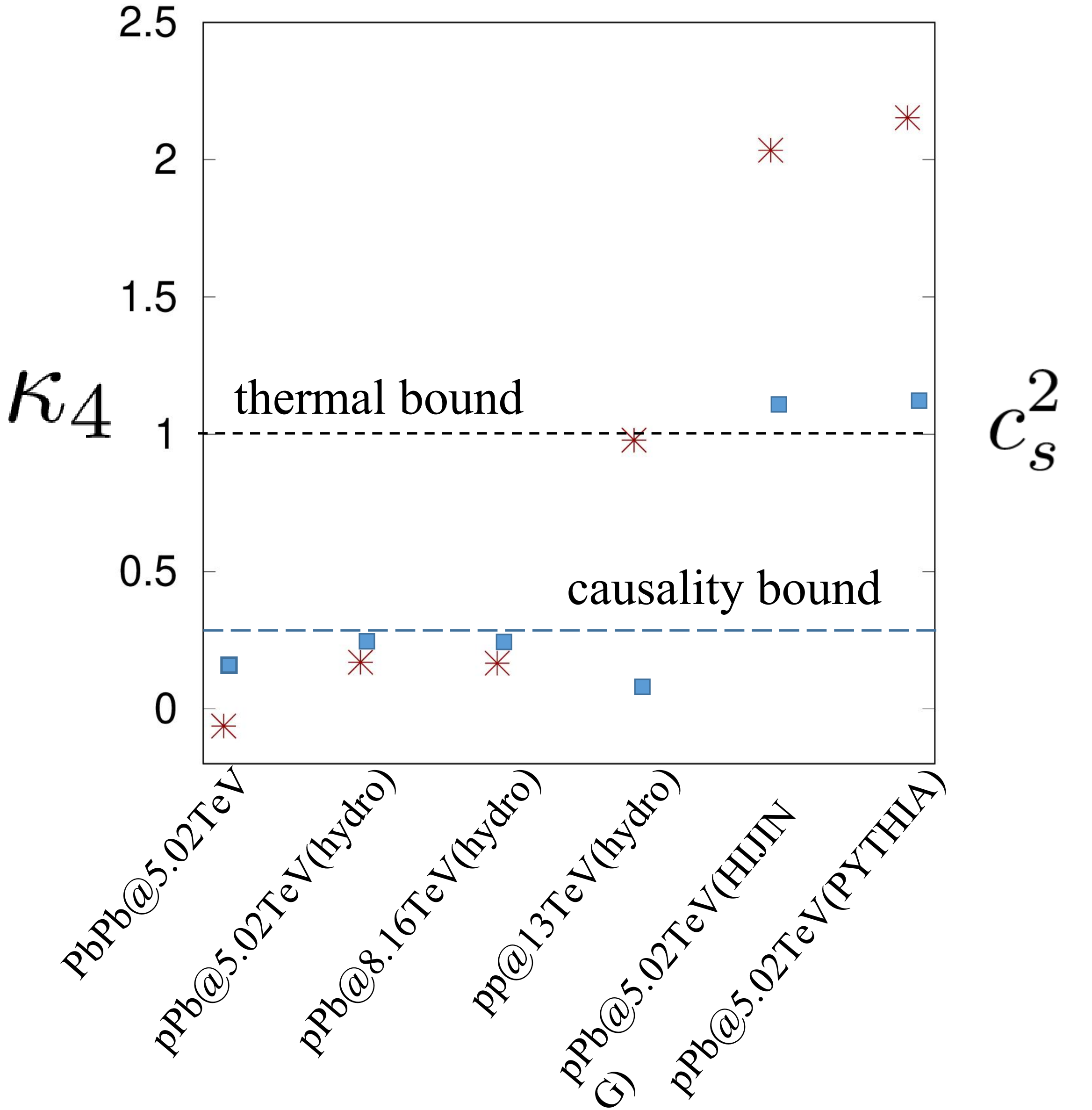


Entanglement Entropy



Boltzmann Entropy

Quantify realistic QCD thermalization: hydro vs. non-hydro



- Even from hydro modeling, the system only achieves partial thermalization, due to, e.g., hadron scatterings.
- From larger (PbPb) to small (pp) systems, the system becomes less thermalized.
- HIJING and PYTIHA do NOT generate thermalized system, as expected.