Relativistic (anomalous) magneto-hydrodynamics and spin hydrodynamics
相对论（反常）磁流体力学和自旋流体力学

浦 实
中国科学技术大学

“极端等离子体：从夸克-胶子到聚变能”研讨会

2025.08.13

Outline

- **Introduction**

  Strongest magnetic fields

- **Relativistic magneto-hydrodynamics**
  - Basic equations
  - Analytic solutions
  - Numerical results from (kinetic theory)

2015 - 2022

New Insight

Last month

- **Spin hydrodynamics**

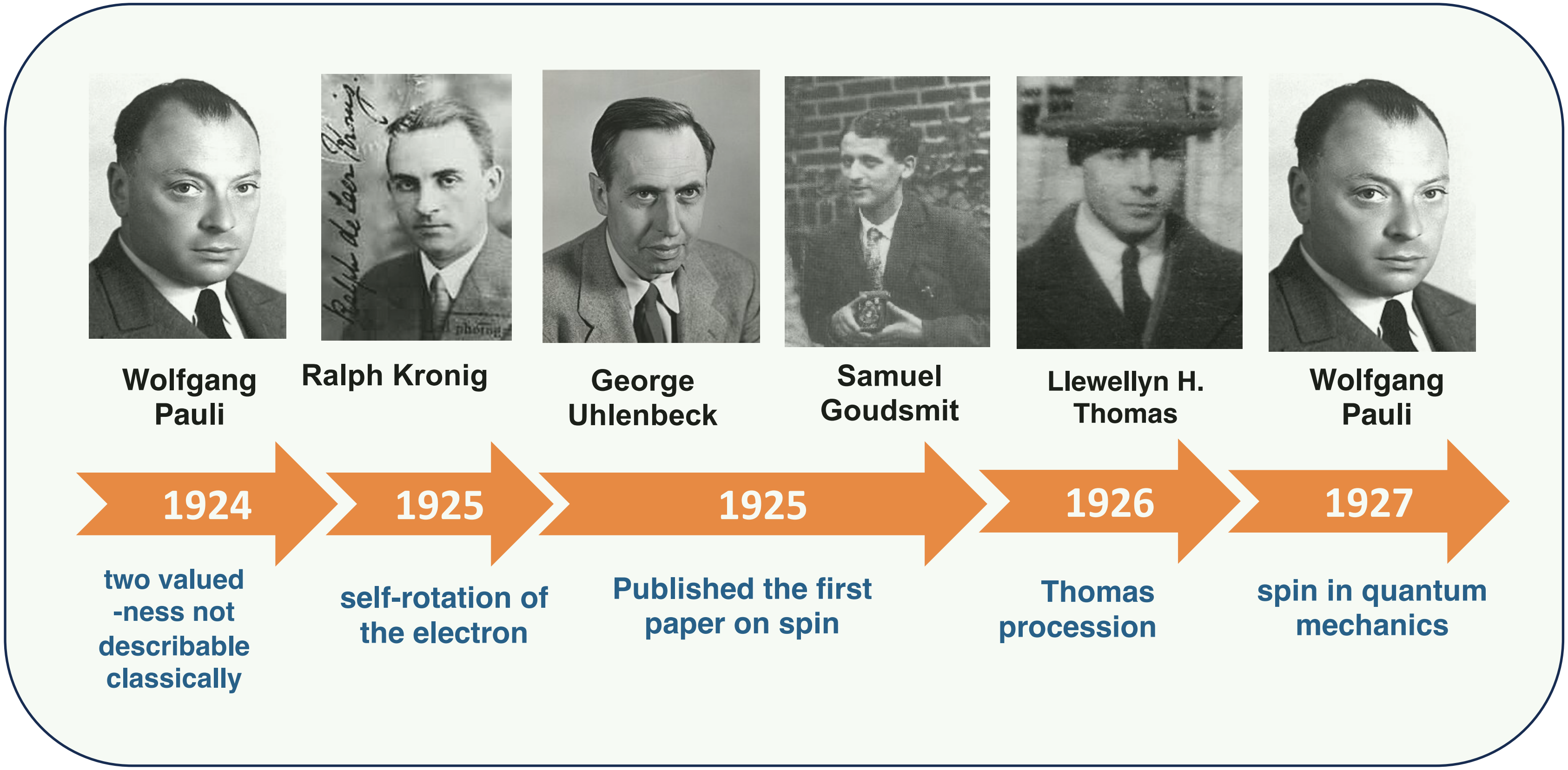
  Extension of Bargmann-Michel-Telegdi equation

- **Summary**

Introduction

Discovery of spin in physics

This year marks the 100th anniversary of the discovery of spin and the 20th anniversary of the proposal for spin polarization in relativistic heavy-ion collisions.



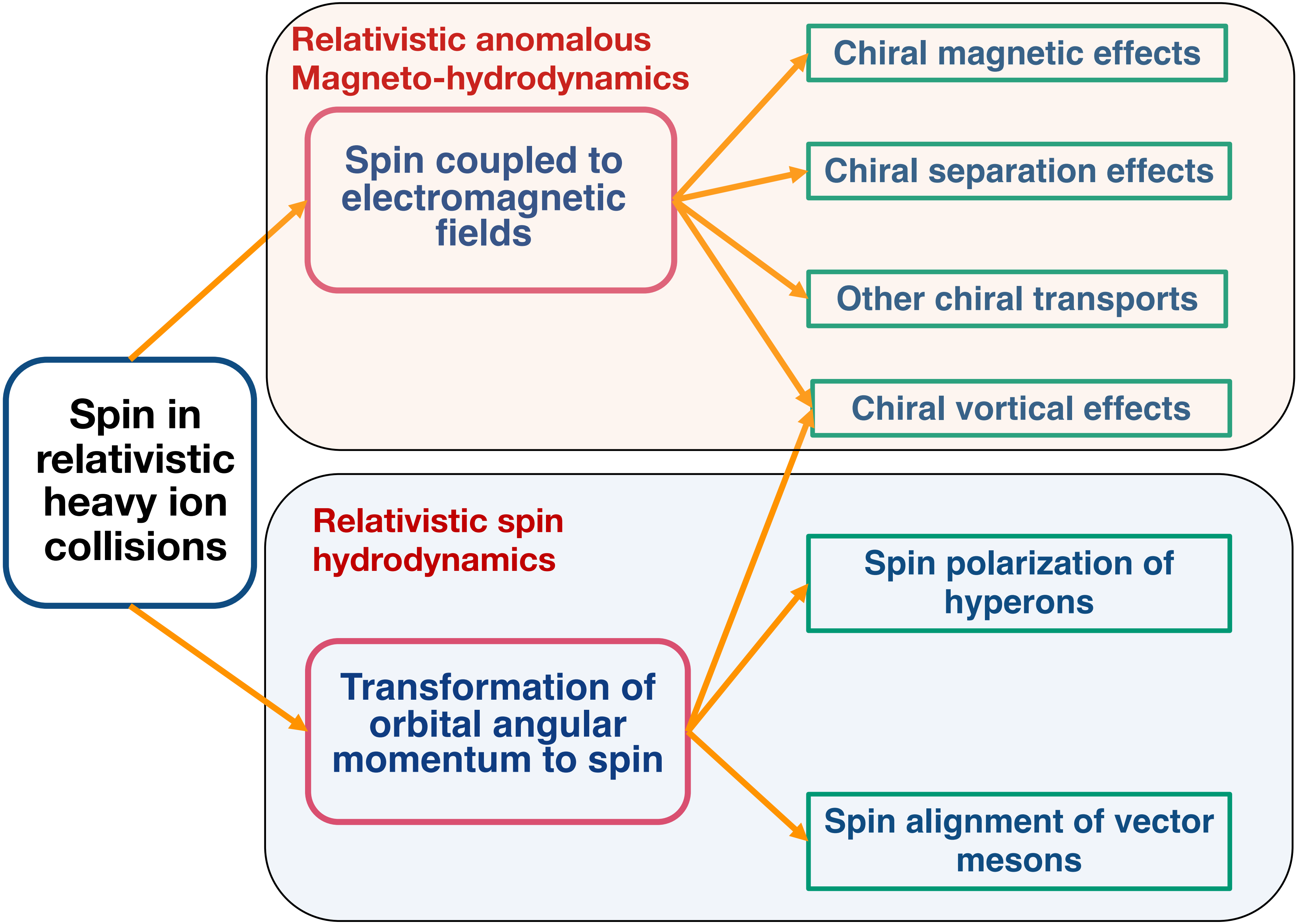
Wolfgang Pauli
1924
two valued-ness not describable classically

Ralph Kronig
1925
self-rotation of the electron

George Uhlenbeck
Samuel Goudsmit
1925
Published the first paper on spin

Llewellyn H. Thomas
1926
Thomas procession

Wolfgang Pauli
1927
spin in quantum mechanics



Spin in relativistic heavy ion collisions

Relativistic anomalous Magneto-hydrodynamics

Spin coupled to electromagnetic fields

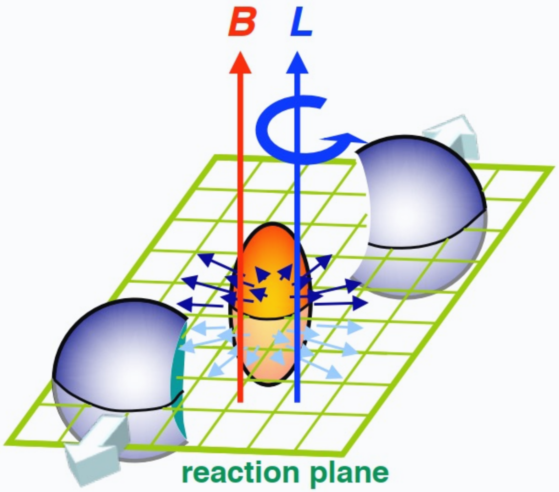
- Chiral magnetic effects
- Chiral separation effects
- Other chiral transports
- Chiral vortical effects

Relativistic spin hydrodynamics

Transformation of orbital angular momentum to spin

- Chiral vortical effects
- Spin polarization of hyperons
- Spin alignment of vector mesons

Strongest electromagnetic fields

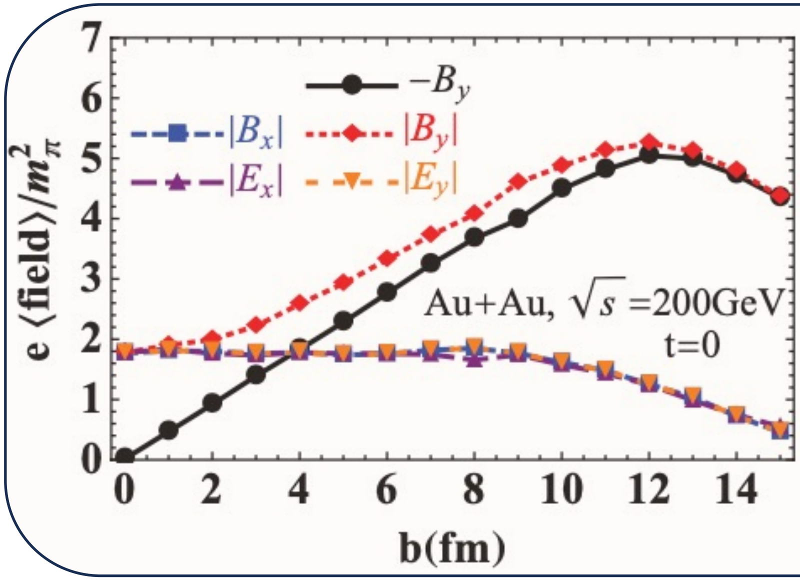


- Two charged nuclei moving along the z-direction generate the E and B fields, which can be computed using the Lienard-Wiechert potential.

$$\vec{E}(\vec{r},t)=\frac{e}{4\pi}\sum_{i=1}^{N_{\mathrm{proton}}} Z_i\frac{\vec{R}_i-R_i\vec{v}_i}{(R_i-\vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2),$$

$$\vec{B}(\vec{r},t)=\frac{e}{4\pi}\sum_{i=1}^{N_{\mathrm{proton}}} Z_i\frac{\vec{v}_i\times\vec{R}_i}{(R_i-\vec{R}_i\cdot\vec{v}_i)^3}(1-v_i^2),$$

Textbook: Jackson, classical electrodynamics

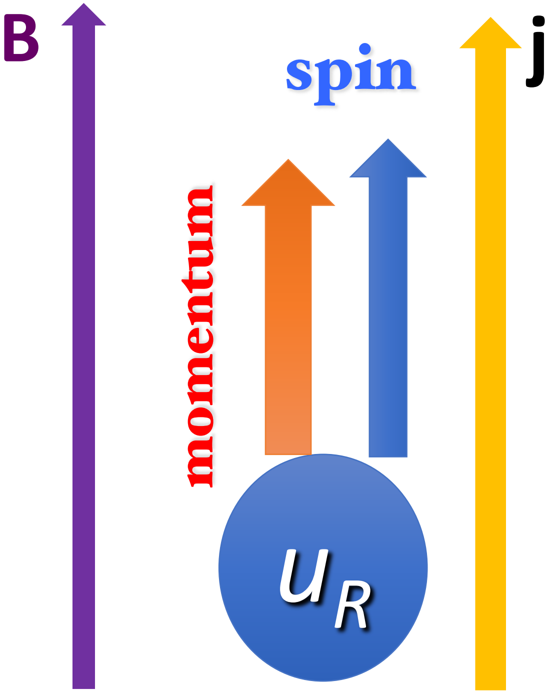


$$eB\sim 10^{17}-10^{18}\mathrm{Gauss}$$

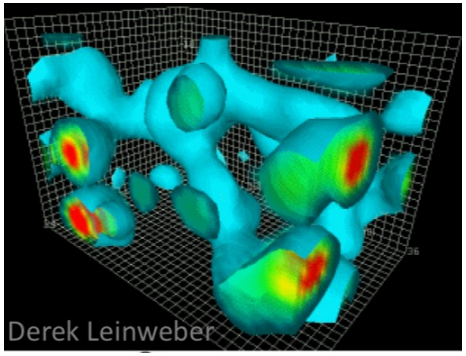
Strongest magnetic fields discovered so far!

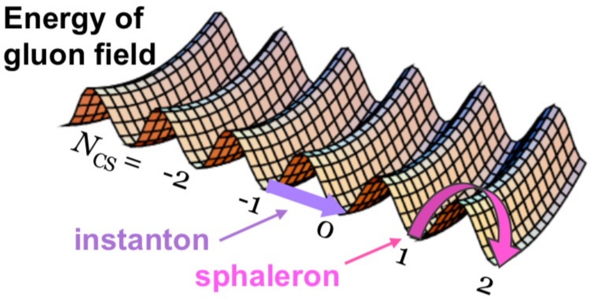
Bzdak, V. Skokov PRC 2012
W.T. Deng, X.G. Huang PRC 2012;
V. Roy, SP, PRC 2015;
H. Li, X.l. Sheng, Q.Wang, 2016; …
Review: K. Tuchin 2013

Chiral Magnetic Effect



- **Magnetic fields**
- **Nonzero axial chemical potential**





- **Chiral magnetic effect:**

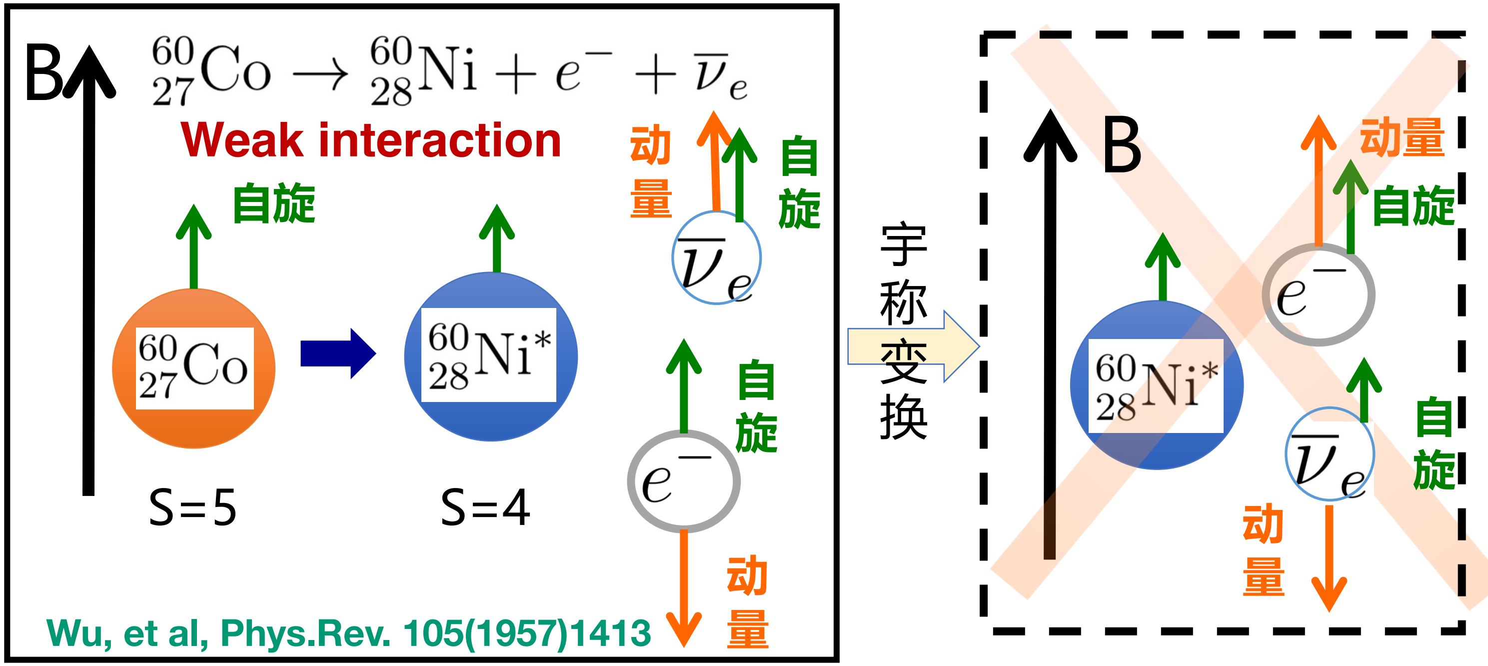
charge separation induced by magnetic fields

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B},$$

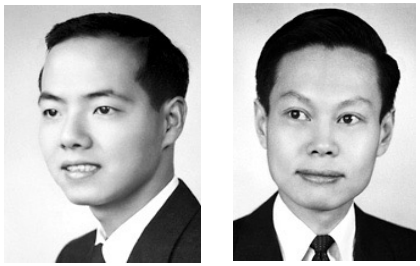
Kharzeev, Fukushima, Warrigna, (08,09), etc. ...

Also see the works done by the groups in PKU, Tsinghua Uni, Fudan Uni., IMP, SINAP, USTC, CCNU…

Parity violation (?)

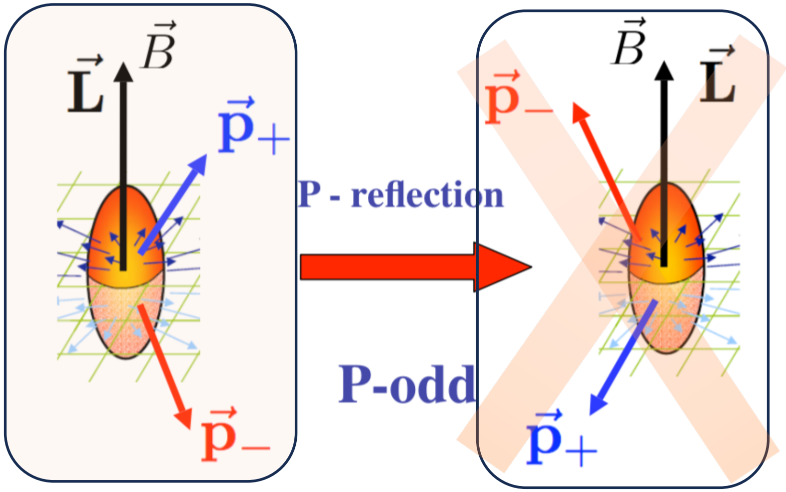


Wu, et al, Phys.Rev. 105(1957)1413



T.D. Lee, C.N. Yang

Wu



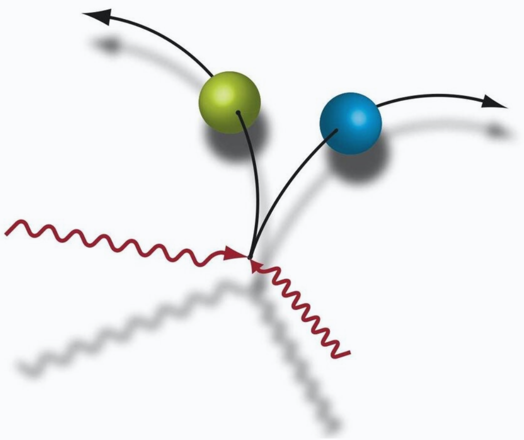
In relativistic heavy ion collisions, the strong interaction domains.

Charge separation induced by CME implies the possible (local) parity violation of strong interaction!

Kharzeev's talk at 26th Winter Workshop on Nuclear Dynamics (2010)

CME VS Schwinger pair production

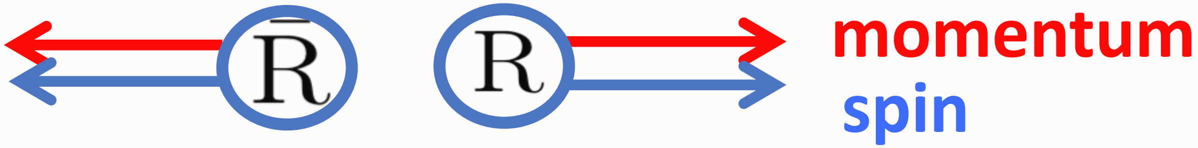
Strong electric fields can generate particle and anti-particle pairs.



Schwinger,
Nobel Prize 1965

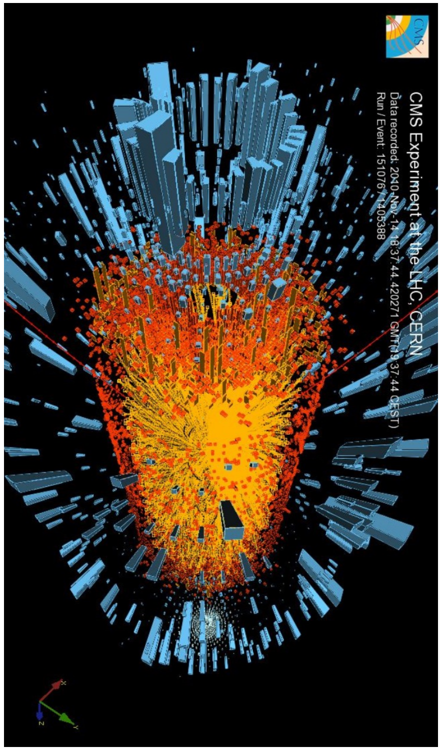
Schwinger pair production at extremely strong magnetic fields is the CME.

Fukushima, et. al, PRL(2010);
Copinger, Fukshima, SP, PRL (2018);
Invited review:
Copinger, SP, Int.J.Mod.Phys.A (2020)



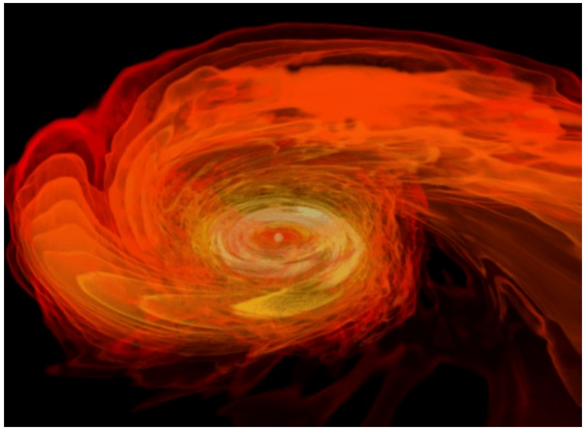
Non-perturbative real time (In-in worldline formulism) in pair creating fields will come soon! Copinger, SP, in preparation

How to investigate these effects?



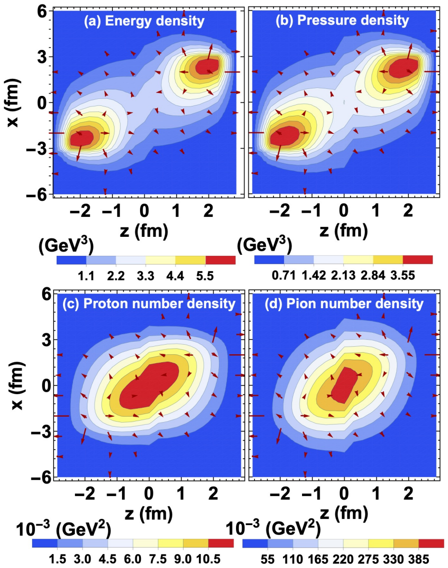
Relativistic Heavy ion collisions

macroscopic



Anomalous magneto-hydrodynamics

microscopic



Boltzmann equations coupled to Maxwell's equation

Strategy for studying Magneto-hydrodynamics (MHD)

- **Hydrodynamics:**

macroscopic effective theory in the long-wavelength and low-frequency

- **Basic idea:**

Focus on the macroscopic conservation equations (or laws) rather than the microscopic detailed interactions between particles

- **Strategy to build the theory:**

Conservation equations + tensor decomposition

- **Strategy to solve these differential equations:**

Imposing symmetries and constrains to reduced variables

Do we need the MHD for HIC in general?

$$\sigma(x,y,\boxed{b}) = \frac{B^2}{2\epsilon}$$

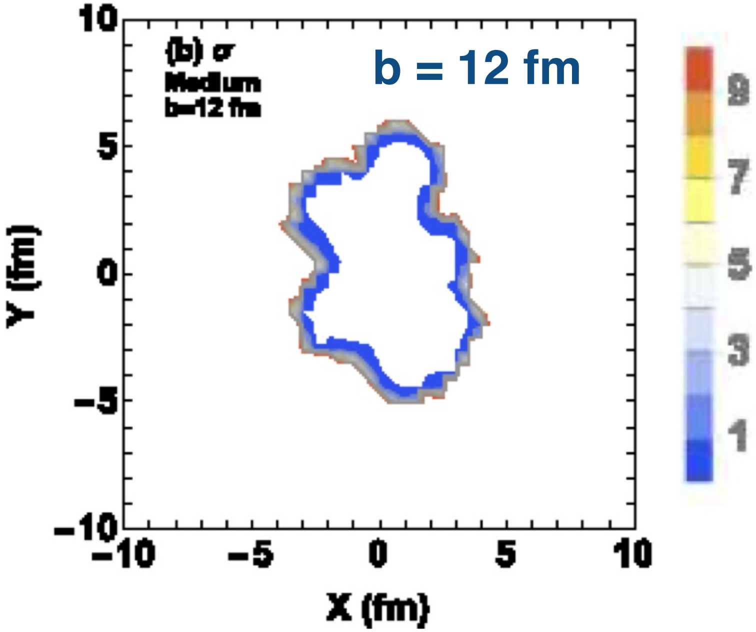
$$\frac{\text{Energy density for initial B field}}{\text{Initial energy density for fluid}}$$

Impact parameter

For central collisions, $\sigma \ll 1$.

For non-central collisions:
$\sigma \ll 1$ for the bulk medium.

We do not need to consider the MHD for most of observables.
But, the MHD is essential for studying those quantum effects.



Roy, SP, Phys. Rev. C 92 (2015) 064902

Studies on MHD from our group

- 1+1 D ideal MHD Bjorken flow

V.Roy, SP, L. Rezzolla, D. Rischke, Phys.Lett. B750, 45

- 1+1 D ideal MHD Bjorken flow with magnetization

SP, V. Roy, L. Rezzolla, D. Rischke, Phys.Rev. D93, 074022

Theoretical framework

- 2+1 D ideal MHD Bjorken flow (perturbative)

SP, Di-Lun Yang, Phys.Rev. D93, 054042

- Background Magnetic field: contribution to v2

V.Roy, SP, L. Rezzolla, D. Rischke, Phy.Rev. C96, 054909

- Anomalous magneto-hydrodynamics in Bjorken flow

Siddique, Ren-jie Wang, SP, Qun Wang, PRD, (2019)

- Temperature dependent electric conductivity

Peng, Wu, Wang, She, SP, PRD (2023)

- Simulations by kinetic theory

Zhang, Sheng, SP, Chen, Peng, Wang, PRR (2022)

Main differential equations

- **Energy-momentum conservation equation**

$$\partial_\mu T^{\mu\nu} = 0$$

- **Charge current conservation equation**

$$\partial_\mu j_e^\mu = 0$$

- **Axial (chiral) current anomalous equation**

$$\partial_\mu j_5^\mu = -e^2 C E \cdot B$$

- **Maxwell's equations**

$$\partial_\mu F^{\mu\nu} = j_e^\nu \qquad \partial_\mu(\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0$$

Maxwell’s equation and “covariant” EM fields

$$\partial_\mu F^{\mu\nu} = j_e^\nu \qquad \partial_\mu(\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}) = 0$$

One can define the “covariant” electric and magnetic fields:

$$F^{\mu\nu} = \boxed{E^\mu u^\nu - E^\nu u^\mu} + \boxed{\epsilon^{\mu\nu\alpha\beta}u_\alpha B_\beta}$$

$$\boxed{E^\mu = F^{\mu\nu}u_\nu,} \quad \boxed{B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}.}$$

boost in x direction

	Lorentz Transform	Fluid dynamics
0	No E^0, B^0	$E'_0 = \gamma v^x E_x, B'_0 = \gamma v^x B_x$
x	$E'_x = E_x,\ B'_x = B_x$	$E'_x = \gamma E_x,\ B'_x = \gamma B_x$
y	$E'_y = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_y$ $B'_y = \gamma(\mathbf{B} - \mathbf{v} \times \mathbf{E})_y$	$E'_y = \gamma(E_y - v^x B_z)$ $B'_y = \gamma(\mathbf{B} - \mathbf{v} \times \mathbf{E})_y$
z	$E'_z = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_z$ $B'_z = \gamma(\mathbf{B} - \mathbf{v} \times \mathbf{E})_z$	$E'_z = \gamma(E_z + v^x B_y)$ $B'_z = \gamma(\mathbf{B} - \mathbf{v} \times \mathbf{E})_z$

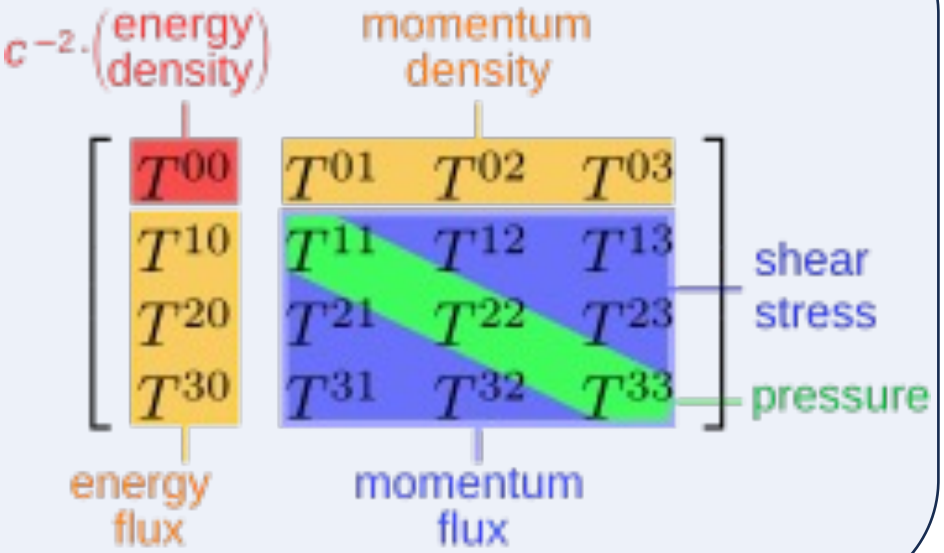
Two different choices for energy-momentum tensor

(1)

$$T^{\mu\nu} = \boxed{T^{\mu\nu}_{\text{fluid}}} + \boxed{T^{\mu\nu}_{\text{EM}},} \quad \partial_\mu T^{\mu\nu} = 0$$

Adopted in this talk

Fluid (matter) part



Electromagnetic fields part

$$\begin{aligned} T^{\mu\nu}_{\text{EM}} &= -F^{\mu\lambda}F^{\nu}_{\lambda} \\ &\quad + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \end{aligned}$$

(2)

$$\partial_\mu T^{\mu\nu}_{\text{Fluid}} = -\partial_\mu T^{\mu\nu}_{\text{EM}} = F^{\nu\lambda} j_{e,\lambda}$$

Adopted by e.g. in Pang, et. al, Phys. Rev. C 93 (2016) 4, 044919

Energy momentum tensor

Energy density of matter

Pressure

Fluid velocity

$$
\begin{aligned}
T^{\mu\nu} \quad = \quad & (\boxed{\epsilon} + \boxed{P} + E^2 + B^2)\boxed{u^\mu} u^\nu \\
& - \left(P + \frac{1}{2}E^2 + \frac{1}{2}B^2 \right) g^{\mu\nu} \\
& - E^\mu E^\nu - B^\mu B^\nu \\
& - u^\mu \epsilon^{\nu\rho\alpha\beta} u_\rho E_\alpha B_\beta - u^\nu \epsilon^{\mu\rho\alpha\beta} u_\rho E_\alpha B_\beta
\end{aligned}
$$

+ dissipative terms (heat flow, viscous tensor, etc.)

Total energy density

$$u_\mu u_\nu T^{\mu\nu} = \epsilon + \frac{1}{2}(E^2 + B^2)$$

Currents

Charge density

Electric conducting flow

Chiral magnetic effect

Charge current

$$j_e^\mu = \boxed{n_e} u^\mu + \boxed{\sigma E^\mu} + \boxed{\xi B^\mu},$$

Axial current (anomalous current)

$$j_5^\mu = \boxed{n_5} u^\mu + \boxed{\sigma_5 E^\mu} + \boxed{\xi_5 B^\mu},$$

Chiral charge density

Chiral Electric conducting flow

Chiral Separation effect

+ Other dissipative terms (heat current, etc.)

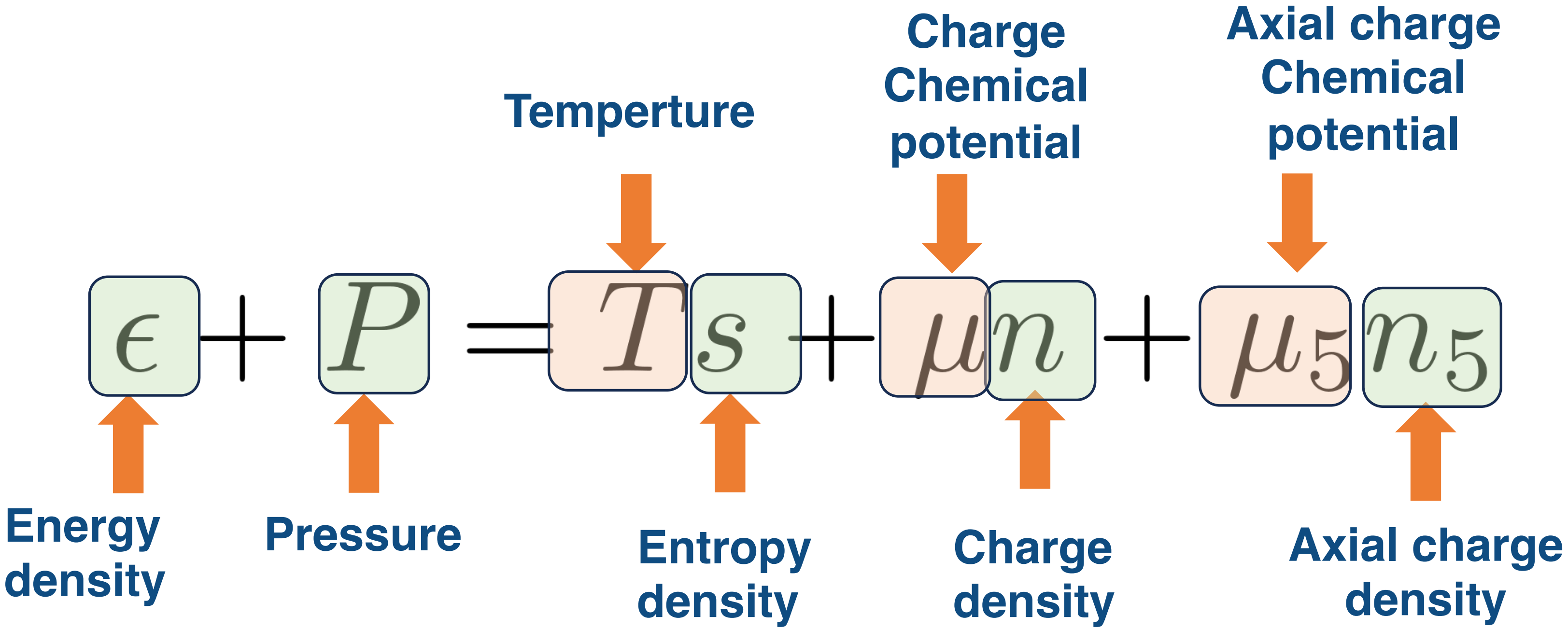
🟩 U(1)/U(1)A density

🟦 Dissipative effects

🟧 Non-dissipative Anomalous related

Thermodynamic relation and equation of states

We neglect the magnetization.



$$\epsilon + P = Ts + \mu n + \mu_5 n_5$$

Ideal MHD limit

Ideal MHD limit:

electric conductivity $\sigma \to \infty$ (prefect conducting fluid)

Must be finite

$$\mathbf{j} = \boxed{\sigma}\left(\boxed{\mathbf{E} + \mathbf{v} \times \mathbf{B}}\right)$$

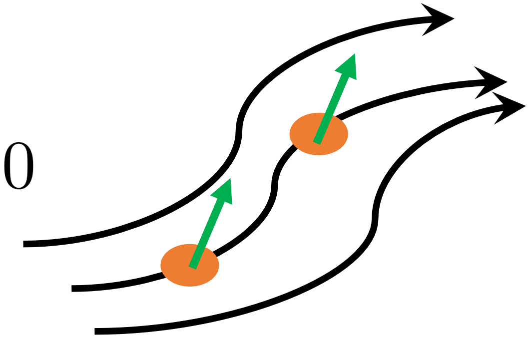
Divergent Must be vanishing!

Maxwell's equation reduce to one single equation!

$$\boxed{\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = -\nabla \times (\mathbf{v} \times \mathbf{B})}$$

If the total entropy is also conserved (ideal fluid),

$$\frac{d}{dt}\left(\frac{\mathbf{B}}{s}\right) = 0$$



Frozen-flux theorem for ideal MHD:
The magnetic field will move with the (each degree of freedom of) fluid cells.

Beyond ideal MHD limit

However, the chiral effects need to consider the system beyond MHD limit.

Ideal MHD

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = -\nabla \times (\mathbf{v} \times \mathbf{B})$$

These two equations are consistent with each other only in ideal MHD limit.

$$j_e^\mu = \xi B^\mu + \dots$$

NO space for the chiral magnetic current

$$\nabla \times \mathbf{B} = \mathbf{j} + \partial_t \mathbf{E}$$

Our strategy

(1) Simplify the equations:

reduce the independent variables; neglect some terms

$$n_e = 0$$

Assuming the fluid is charge neutral.
Electric field will not accelerate the fluid.

(2) Find a simple equations of states for thermodynamic variables

High temperature limit
QGP ~ trillion degree C

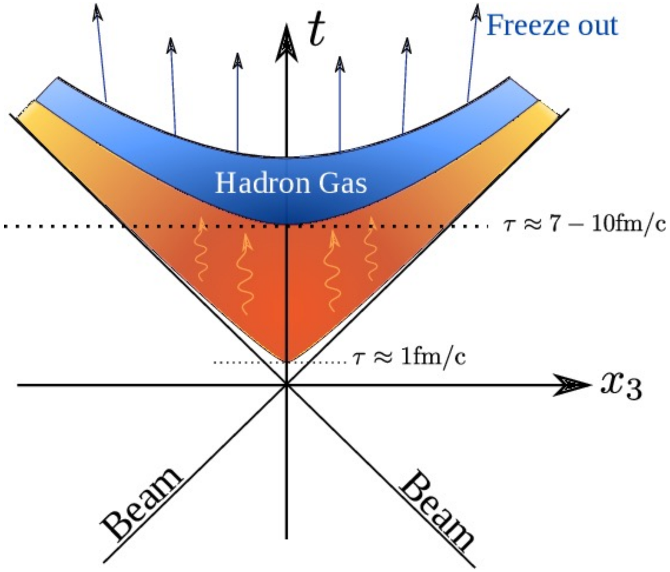
$$\begin{aligned} \epsilon &= c_s^2 p = p/3, \\ n_5 &= a\mu_5 T^2 \end{aligned}$$

(3) Find a well-known analytic solution for conventional hydrodynamics and extend it for MHD.

Profound Bjokren flow

Introduction to Bjorken flow

- **Bjorken flow:** **A well-known 0+1 dimensional analytic solution for the relativistic hydrodynamics with longitudinal boost invariance**



- **Physical picture:**
  - **All the particles are generated at the same time in the rest frame of the fluid cell.**
  - **The system has longitudinal boost invariance, all thermodynamic variables depend on proper time only.**

The solutions in (t, x, y, z) coordinates:

Scaling law for Energy density

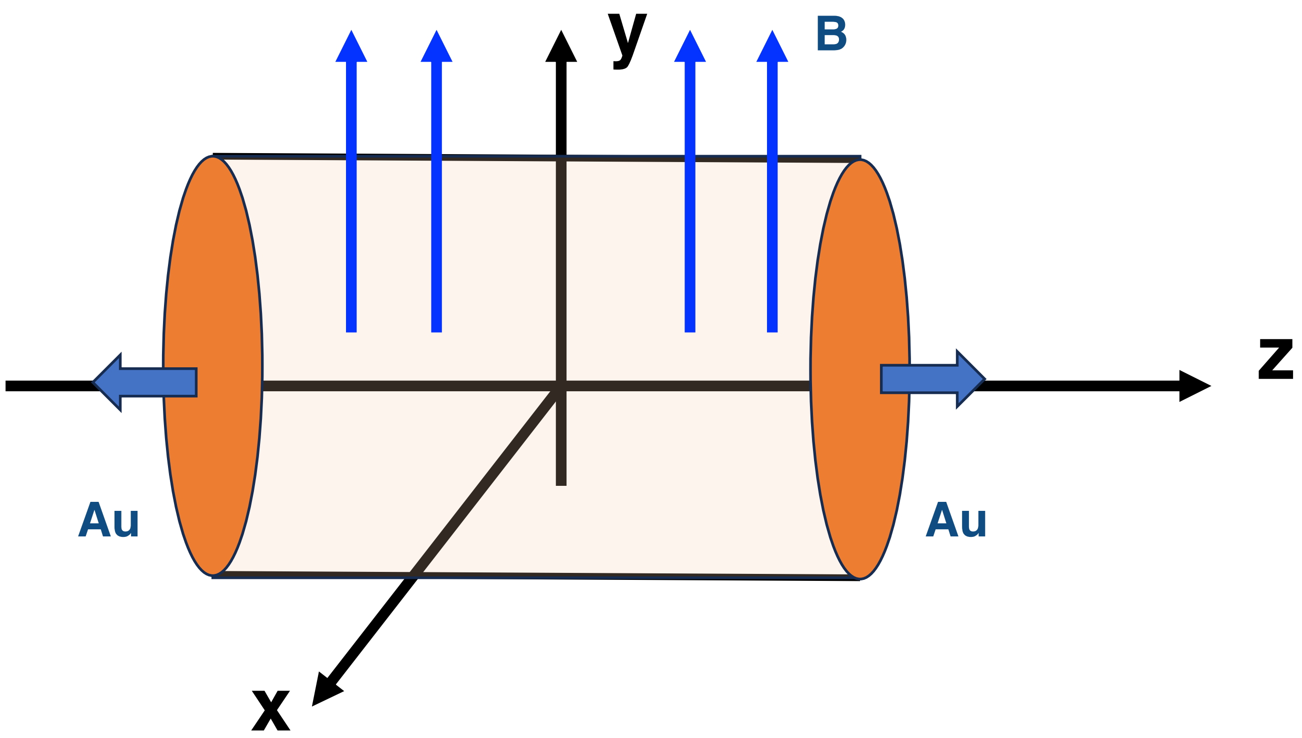
$$\frac{\epsilon(\tau)}{\epsilon(\tau=0)} = \frac{1}{\tau^{4/3}}$$

τ: proper time

Fluid velocity

$$u^{\mu} = \left(\frac{t}{\tau}, 0, 0, \frac{z}{\tau}\right)$$

Setup for initial electromagnetic fields



$$E_0^\mu = (0, 0, \chi E(\tau), 0),\ B_0^\mu = (0, 0, B(\tau), 0)$$

Differential equations

Energy-momentum tensor conservation equation

$$\partial_\mu T^{\mu\nu} = 0$$

Energy conservation equation:

Too complicated…
But!

$$\begin{aligned} 0 &= u_\mu \partial_\nu T^{\mu\nu} \\ &= (u\cdot\partial)(\epsilon + \frac{1}{2}E^2 + \frac{1}{2}B^2) + (\epsilon + p + E^2 + B^2)(\partial\cdot u) \\ & \quad u_\mu (E\cdot\partial)E^\mu + u_\mu(B\cdot\partial)B^\mu + \epsilon^{\nu\lambda\alpha\beta}\partial_\nu(E_\lambda B_\alpha u_\beta) \\ & \quad + u_\mu (u\cdot\partial)\epsilon^{\mu\lambda\alpha\beta}E_\lambda B_\alpha u_\beta . \end{aligned}$$

Fluid acceleration equation:

$$\begin{aligned} (u\cdot\partial)u_\alpha &= \frac{1}{(\varepsilon + p + E^2 + B^2)}[\Delta^\nu_\mu \partial_\nu(p + \frac{1}{2}E^2 + \frac{1}{2}B^2) + \Delta_{\mu\alpha}(E\cdot\partial)E^\mu + E_\alpha(\partial\cdot E) \\ & + \Delta_{\mu\alpha}(B\cdot\partial)B^\mu + B_\alpha(\partial\cdot B) + \epsilon^{\nu\lambda\rho\sigma}E_\lambda B_\rho u_\sigma(\partial_\nu u_\alpha) + (\partial\cdot u)\epsilon_{\alpha\lambda\rho\sigma}E^\lambda B^\rho u^\sigma \\ & + \Delta_{\mu\alpha}(u\cdot\partial)\epsilon^{\mu\lambda\rho\sigma}E_\lambda B_\rho u_\sigma]. \end{aligned}$$

If all initial thermodynamic quantities depend only on proper time and the initial velocity is Bjorken type, then we observe that both the initial profile of the EM fields and the fluid velocity will retain their initial values forever.

Main equations in this case

- **Energy conservation equation:**

$$(u\cdot\partial)(\varepsilon+\frac{1}{2}E^{2}+\frac{1}{2}B^{2})+(\varepsilon+p+E^{2}+B^{2})(\partial\cdot u)=0$$

- **Maxwell's equations:**

$$\begin{aligned}\frac{d}{d\tau}E+\frac{1}{\tau}E+\sigma E+\chi\xi B &= 0\\ \frac{d}{d\tau}B+\frac{B}{\tau}&=0\end{aligned}$$

$$\begin{aligned}E &= \sqrt{-E^{\mu}E_{\mu}}\\ B &= \sqrt{-B^{\mu}B_{\mu}}\end{aligned}$$

- **Axial current anomalous equation:**

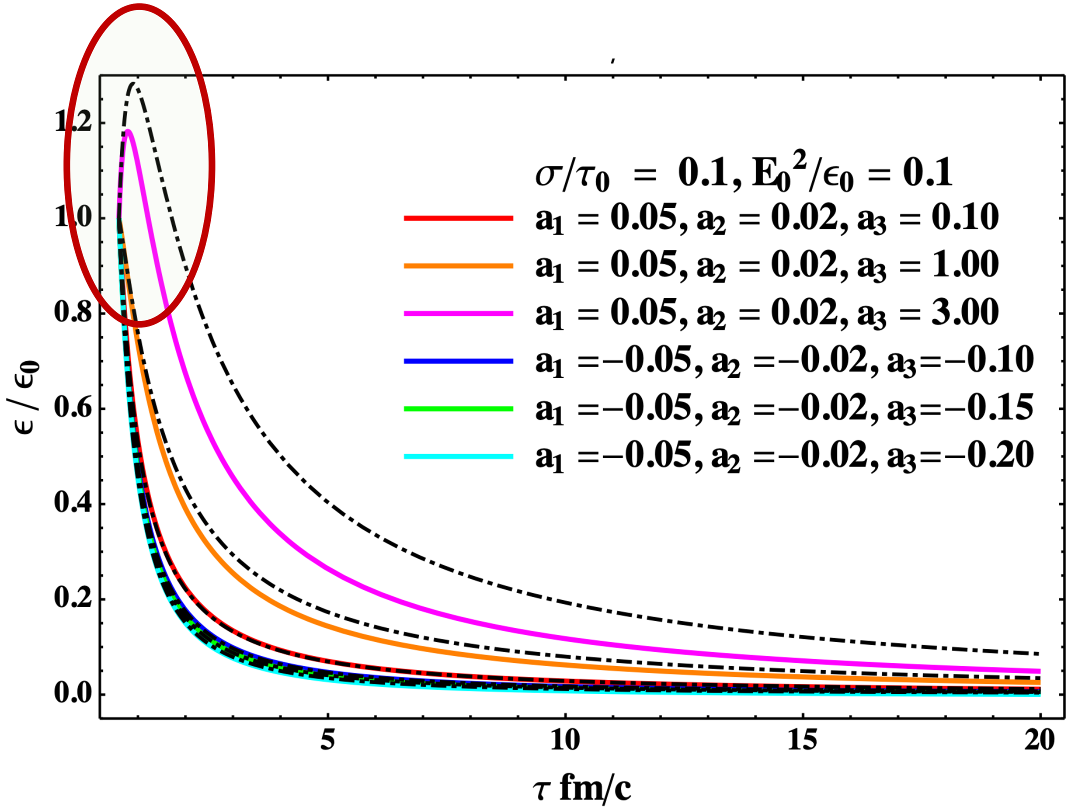
$$\frac{d}{d\tau}n_{5}+\frac{n_{5}}{\tau} = e^{2}C\chi EB.$$

Analytic solutions

Energy density

$$\varepsilon(\tau) = \epsilon_0 \left(\frac{\tau_0}{\tau}\right)^{1+c_s^2} \left\{1 + \sigma \frac{E_0^2}{\varepsilon_0} e^{2\sigma\tau_0}\left[\tau_0 \mathrm{E}_{1-c_s^2}(2\sigma\tau_0) - \tau\left(\frac{\tau}{\tau_0}\right)^{c_s^2-1} \mathrm{E}_{1-c_s^2}(2\sigma\tau')\right] + \frac{a_3}{\tau_0} e^{\sigma\tau_0}\left[\tau_0 \mathrm{E}_{2-3c_s^2}(\sigma\tau_0) - \tau\left(\frac{\tau_0}{\tau}\right)^{2-3c_s^2} \mathrm{E}_{2-3c_s^2}(\sigma\tau)\right]\right\}.$$

Re-heating behavior in the early stage



Dashed lines: Analytic solutions

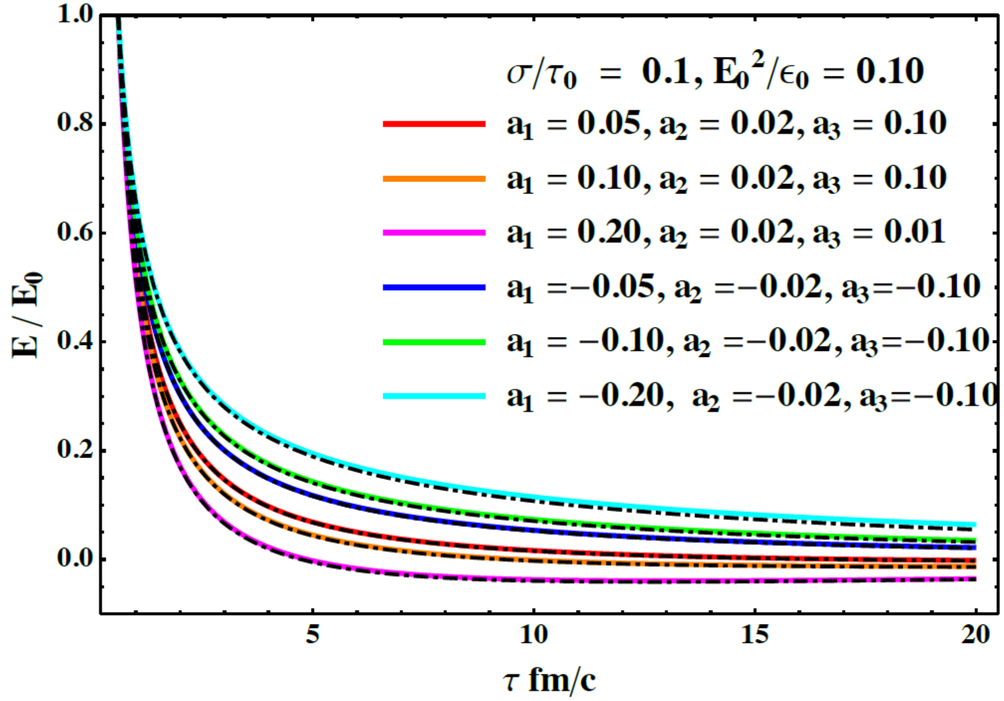
Solid lines: Numerical results for this simplified case

a1,a2,a3: related to the anomalous coefficients

Electromagnetic fields in the Lab frame

$$\mathbf{E}_L = (\gamma v^z B(\tau),\ \chi\gamma E(\tau),\ 0),$$
$$\mathbf{B}_L = (-\gamma v^z \chi E(\tau),\ \gamma B(\tau),\ 0),$$

$$E(\tau) = E_0\left(\frac{\tau_0}{\tau}\right)\left\{e^{-\sigma(\tau-\tau_0)} - a_1 e^{-\sigma\tau}\left[\mathrm{E}_{1-2c_s^2}(-\sigma\tau_0) - \left(\frac{\tau}{\tau_0}\right)^{2c_s^2}\mathrm{E}_{1-2c_s^2}(-\sigma\tau)\right]\right\},$$

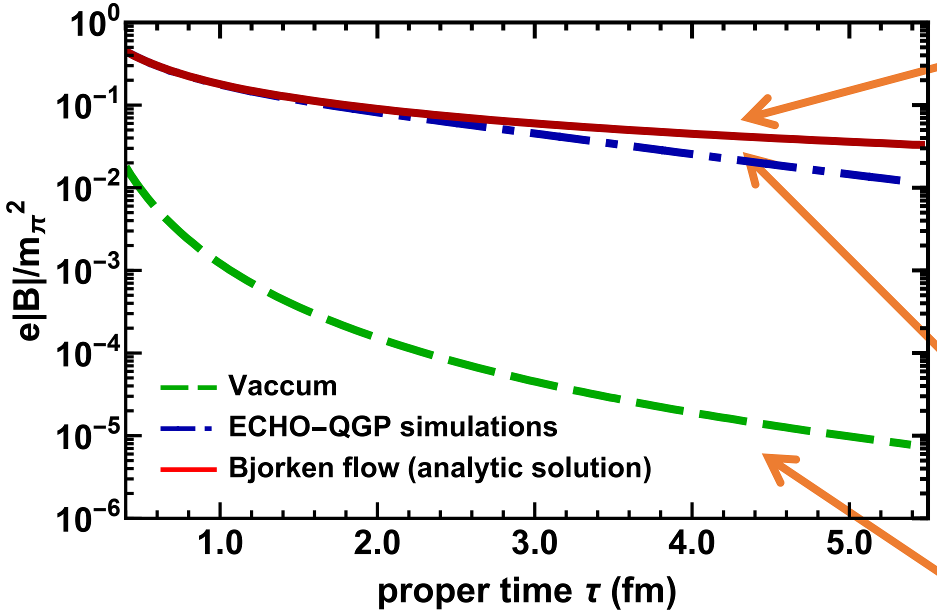


$$B(\tau) = B_0\frac{\tau_0}{\tau},$$

The slowest decaying mode for the magnetic fields

Siddique, Wang, SP, Wang, PRD, (2019)

Evolution of electromagnetic fields



Ideal Bjorken MHD

Pu, Roy, Rezzolla, Rischke, PRD(2015)

MHD + CME + Chiral anomaly

Siddique, Wang, Pu, Wang, PRD (2019)

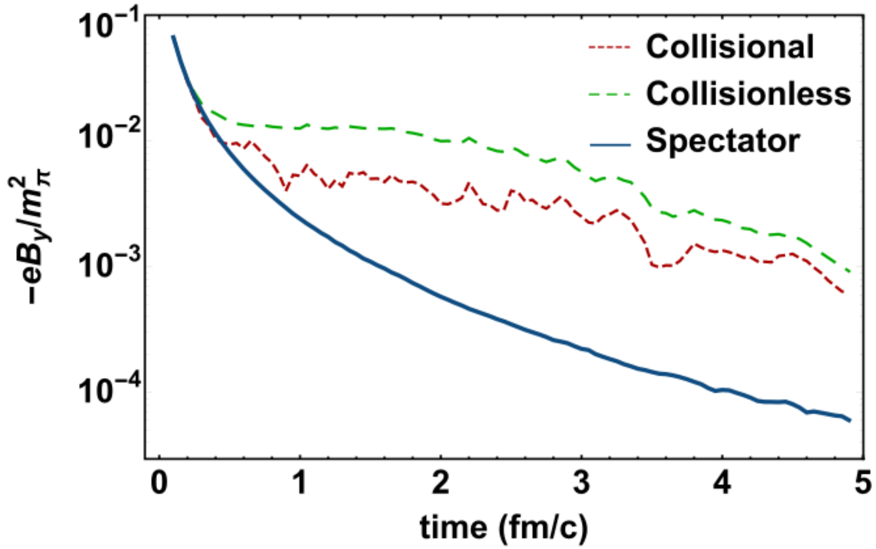
Peng, Wu, Wang, She, Pu, PRD(2023)

ECHO-QGP

Inghirami, Zanna, Moghaddam,Becattini, Bleicher, EPJC(2016)

Vacuum

Kharzeev, McLerran,Warringa,NPA(2008)



Relativistic kinetic theory coupled with Maxwell's equations + 2->2 Leading-Log pQCD scattering

Zhang, Sheng, Pu, Chen, Peng, Wang, Wang, PRR (2022)

RBG-Maxwell equations and v1, dv1 problem

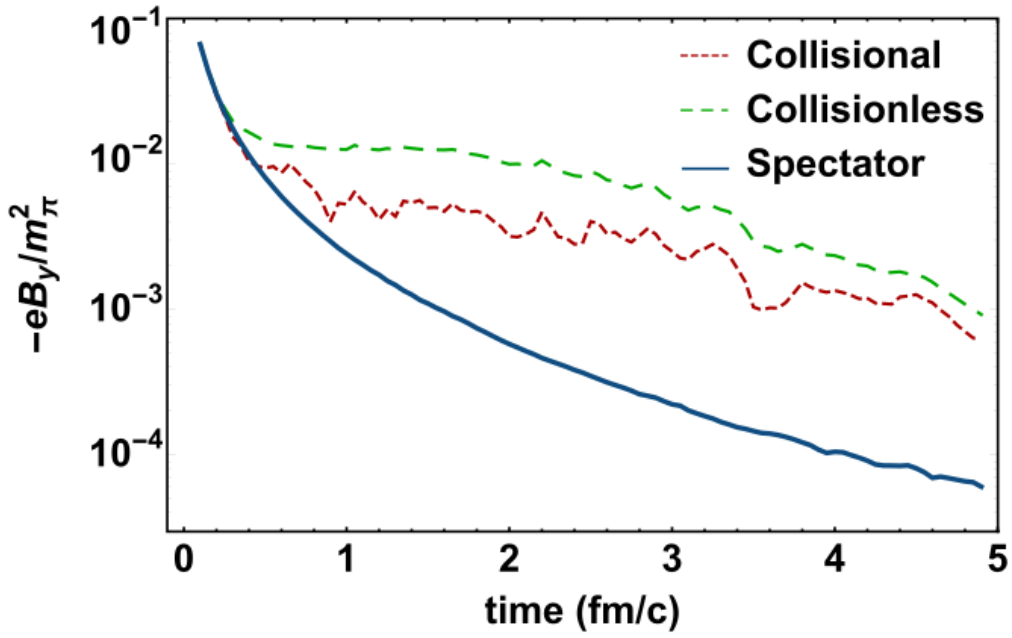
- **We solve the Boltzmann equation coupled to Maxwell equations.**

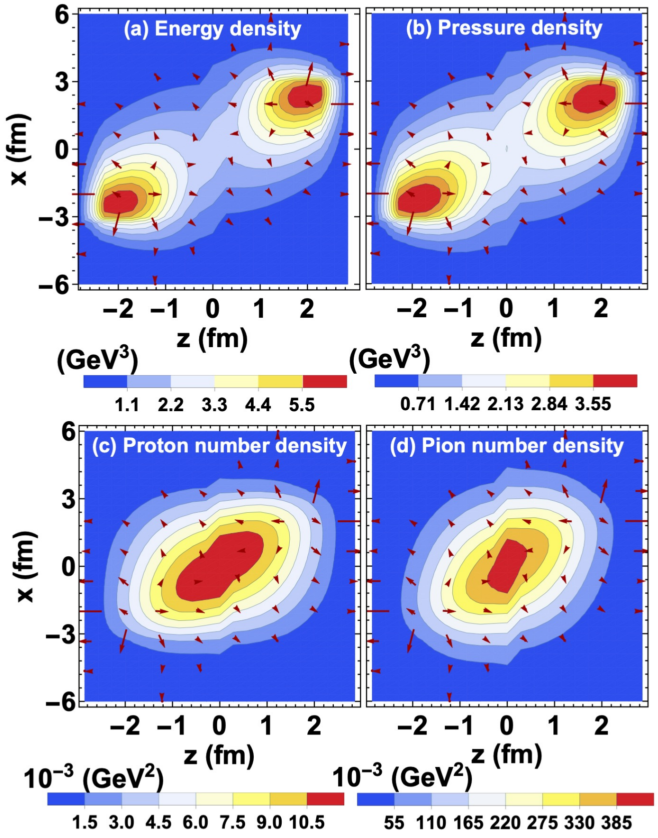
$$[p^\mu \partial_\mu + Q_a p_\mu F^{\mu\nu} \partial_{p^\nu}] f_a(t, \mathbf{x}, \mathbf{p}) = \mathcal{C}[f_a],$$

$$\partial_\mu F^{\mu\nu} = j^\nu_{\text{ext}} + j^\nu_{\text{med}},$$ **(Spectators + Participant)**

QCD 2->2 scattering

Leading-log order

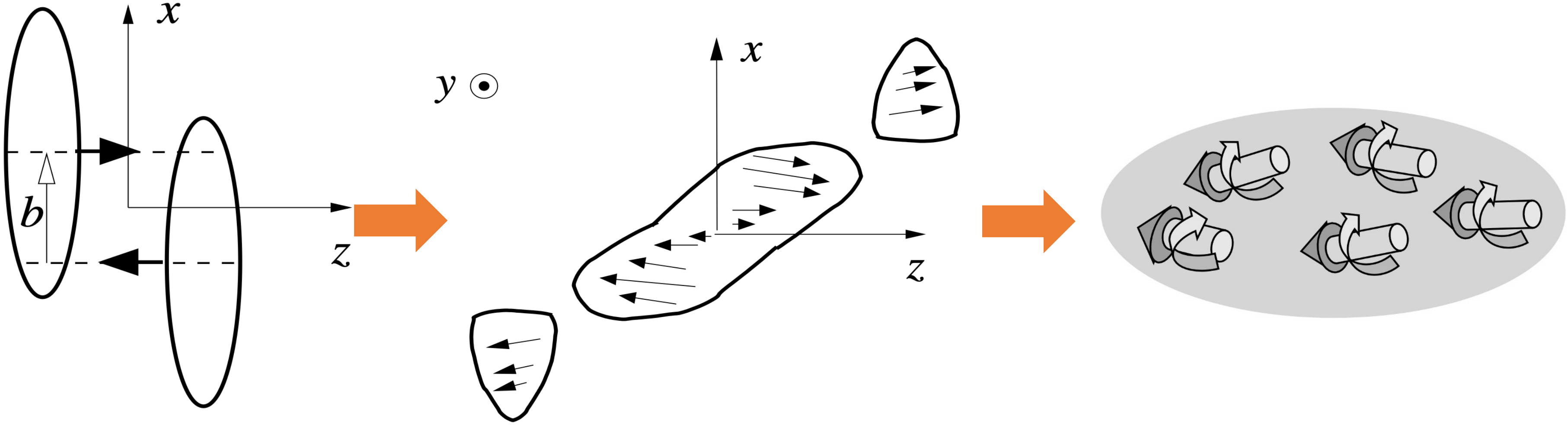


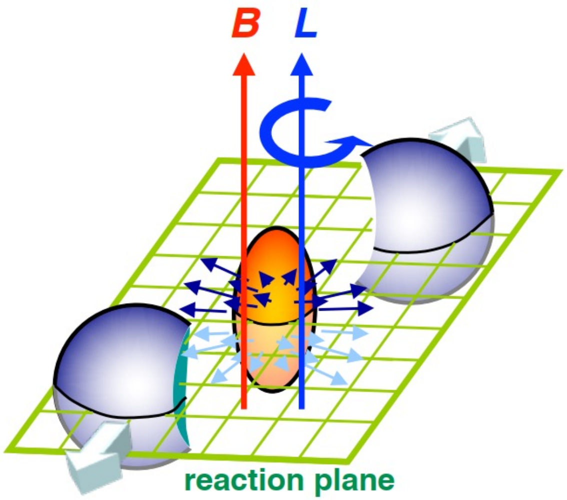


Zhang, Sheng, Pu, Chen, Peng, Wang, Wang, PRR (2022)

Spin hydrodynamics

OAM to spin polarization in HIC





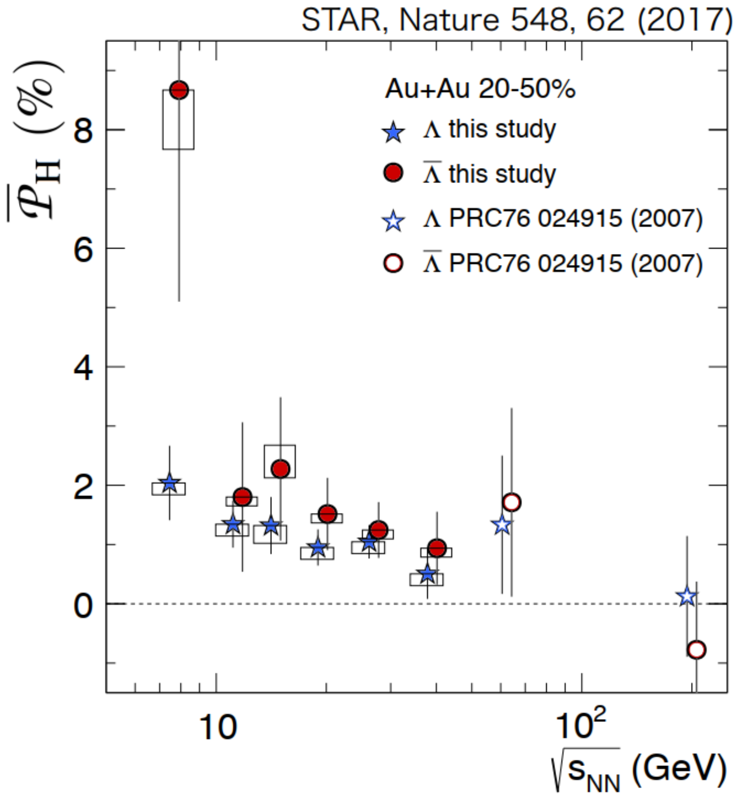
- **Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.**

- **Global orbital angular momentum leads to the polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbital coupling.**

Liang, Wang, PRL (2005); PLB (2005);
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Polarization effects

Global polarization of hyperons

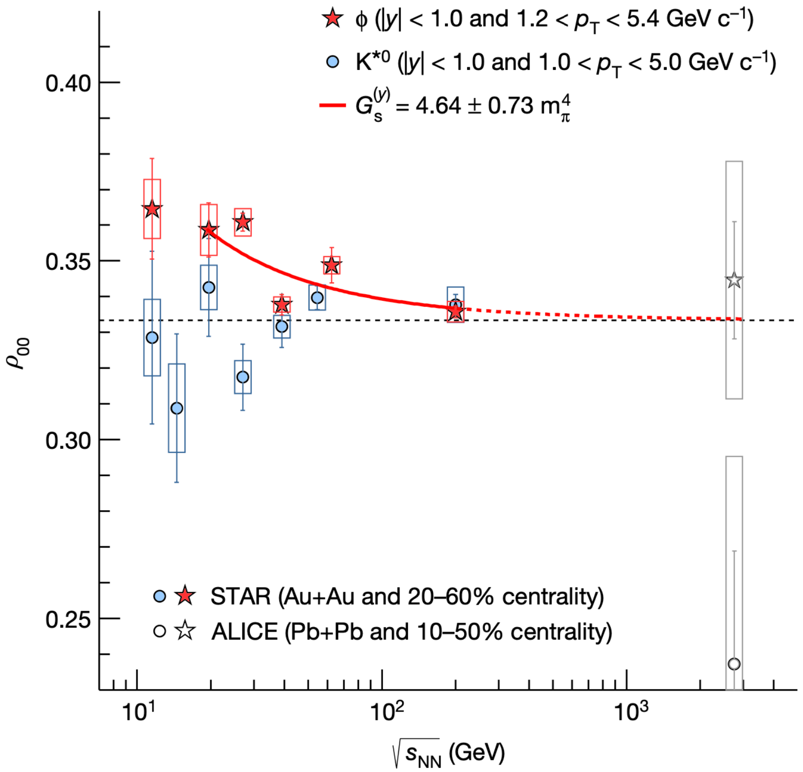


STAR, Nature 548, 62 (2017)

ω = (9 ± 1)x10^{21}/s, greater than previously observed in any system.

QGP is most vortical fluid so far.

Spin alignment of vector mesons



STAR, Nature 614, 244 (2023)

A key question:

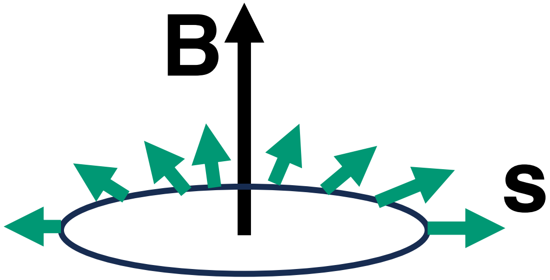
What is the evolution equation for spin?

How is it connected to well-known spin phenomena?

Extension of the Bargmann-Michel-Telegdi (BMT) equation

S. Fang, Kenji Fukushima, SP, D. L. Wang, arXiv:2506.20698

Original BMT equation



The key to get the correct relativistic correction for hydrogen

$$\dot{s}^{\mu} = \boxed{\gamma\Delta^{\mu\rho}F_{\rho\nu}s^{\nu}} - \boxed{u^{\mu}s^{\nu}\dot{u}_{\nu}}$$

Spin-EM fields coupling

Thomas procession

+ possible relaxation (dissipative) effects
Landau-Lifshitz-Gilbert equation for spin

$$\dot{a} = da/dt \qquad \Delta^{\mu\rho} = g^{\mu\rho} - u^{\mu}u^{\rho}$$

Textbook: Jackson, classical electrodynamics

Spin evolution in relativistic heavy ion collisions

What is the BMT equation for a relativistic many-body system in the presence of rotation (vorticity)?

Our strategy:

Conservation equations

Energy momentum

$$\partial_\mu \Theta^{\mu\nu} = 0$$

Charge number

$$\partial_\mu j^\mu = 0$$

Total angular momentum

$$\partial_\lambda J^{\lambda\mu\nu} = 0$$

\+

Second law of thermodynamics

$$\partial_\mu \mathcal{S}^\mu \geq 0$$

Spin tensor

$$s^{\mu} \xrightarrow{\text{Rest frame}} (0, \boldsymbol{s})$$

Two different choices for spin tensor operators:

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\gamma^{\lambda}[\gamma^{\mu},\gamma^{\nu}]\psi$$

Anti-symmetric on $\mu\nu$,
NOT Hermitian
d.o.f for spin tensor is 6.

Commonly used in our field

Hermitian,
Also can be derived by using EoS for feilds

$$\Sigma^{\lambda\mu\nu} = \frac{i}{8}\bar{\psi}\{\gamma^{\lambda},[\gamma^{\mu},\gamma^{\nu}]\}\psi$$

Total anti-symmetric,
Hermitian
d.o.f for spin tensor is 3.

Commonly used in many other fields

See cosmology textbook by Weinberger

Extension of BMT equation

Original BMT equation for EM fields

$$\dot{s}^{\mu} = \gamma\Delta^{\mu\rho}F_{\rho\nu}s^{\nu} - u^{\mu}s^{\nu}\dot{u}_{\nu}$$

+ other dissipative effects

Thomas procession

Spin-vorticial fields coupling

Killing condition

Extension of BMT equation for vorticity

$$\dot{s}^{\mu} = -u^{\mu}s^{\nu}\dot{u}_{\nu} + \left(\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho} - 2\beta\gamma_{\phi}g^{\mu\sigma}\right)\left(2\omega_{\sigma} - \mathfrak{w}_{\sigma}\right) - s_{\nu}\partial^{<\mu}u^{\nu>} - \left(\frac{1}{3} + 2v_{n}^{2}\right)s^{\mu}(\partial\cdot u),$$

Spin coupled to shear tensor, bulk pressure and other dissipative effects

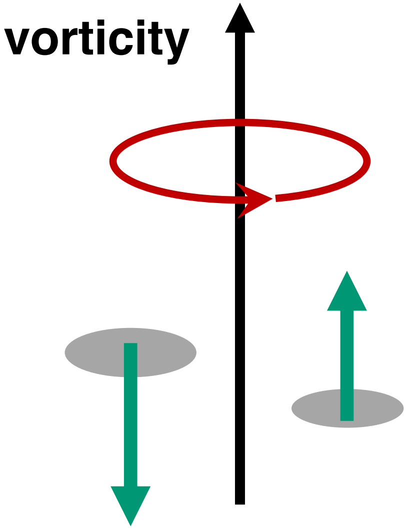
Equilibrium

In global equilibrium,

$$\begin{aligned}\dot{s}^{\mu} \quad = \quad & -u^{\mu}s^{\nu}\dot{u}_{\nu} \\ & +(\varepsilon^{\mu\nu\rho\sigma}s_{\nu}u_{\rho}-2\beta\gamma_{\phi}g^{\mu\sigma})\boxed{\cancel{(2\omega_{\sigma}-\mathfrak{w}_{\sigma})}} \\ & -s_{\nu}\boxed{\cancel{\partial^{<\mu}u^{\nu>}}}+\left(\frac{1}{3}+2v_{n}^{2}\right)s^{\mu}\boxed{\cancel{(\partial\cdot u)}},\end{aligned}$$

⇒

$$\mathfrak{w}^{\mu}s^{\nu}-\mathfrak{w}^{\nu}s^{\mu}=0$$



Spin is parallel to vorticity in equilibrium.

Spin magneto hydrodynamics

What is spin-magneto hydrodynamics?

One key point which is commonly overlooked in our field:

Total spin polarization of a fluid cell is magnetization!

1+1 D ideal MHD Bjorken flow with magnetization
SP, V. Roy, L. Rezzolla, D. Rischke, Phys.Rev. D93, 074022 (2016)

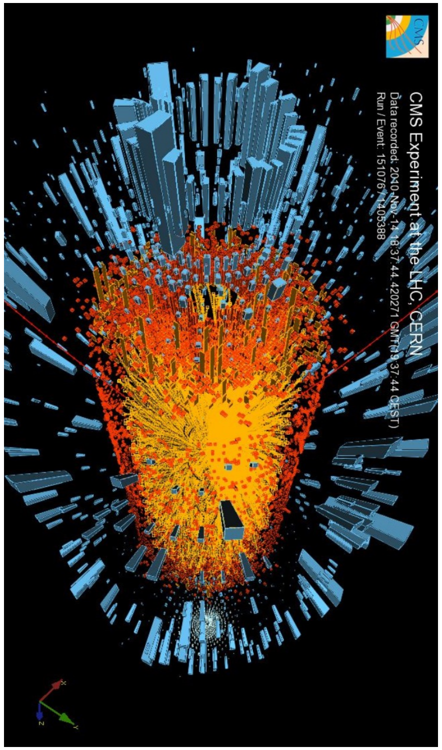
Theoretical framework

When spin effects are involved, they reveal more physics that we did not notice 10 years ago.

Will come soon!

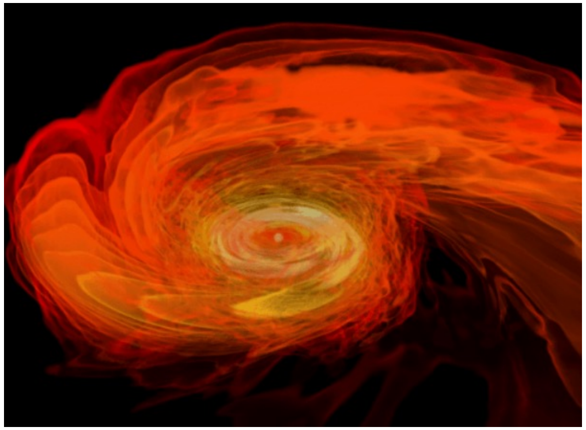
Summary

Summary (I)



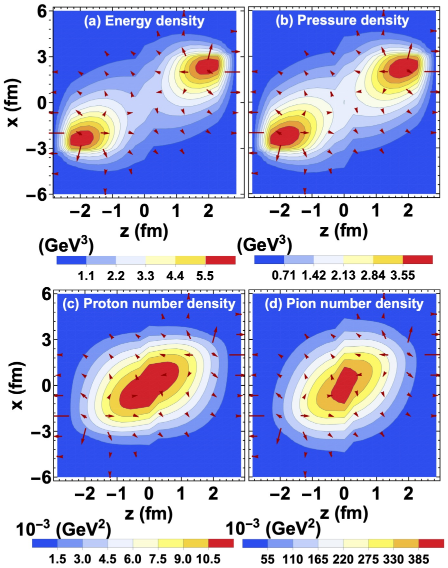
Relativistic Heavy ion collisions

macroscopic



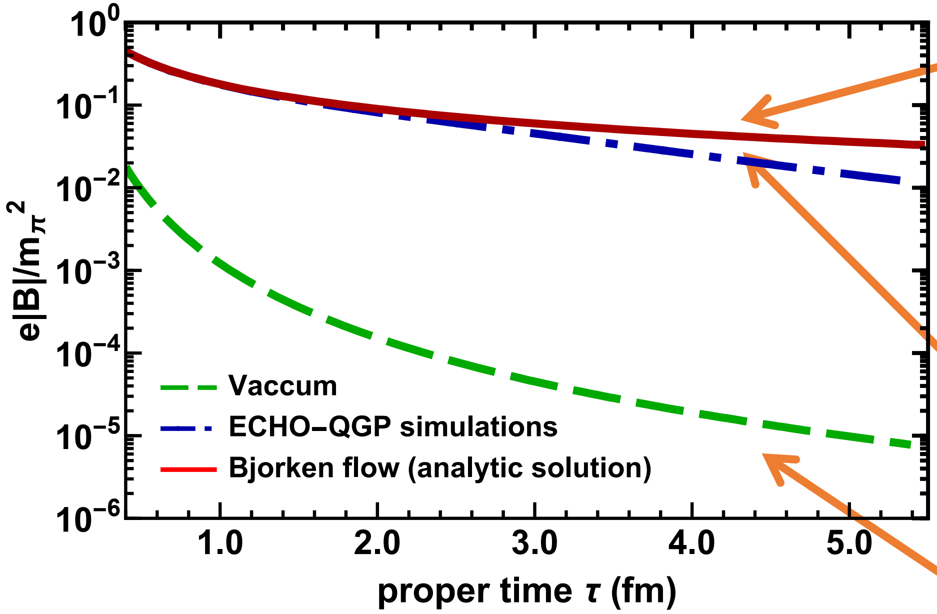
Anomalous magneto-hydrodynamics

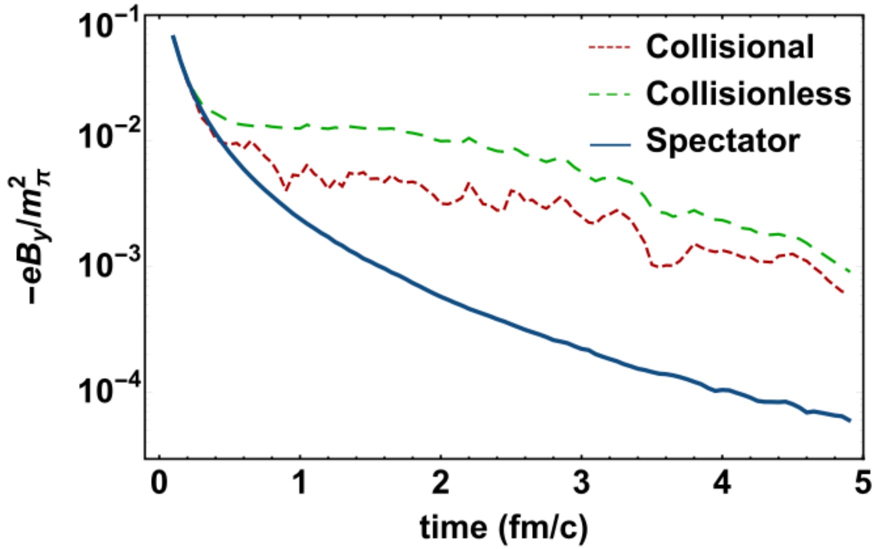
microscopic



Boltzmann equations coupled to Maxwell's equation

Summary (I)





Ideal Bjorken MHD

Pu, Roy, Rezzolla, Rischke, PRD(2015)

MHD + CME + Chiral anomaly

Siddique, Wang, Pu, Wang, PRD (2019)

Peng, Wu, Wang, She, Pu, PRD(2023)

ECHO-QGP

Inghirami, Zanna, Moghaddam,Becattini, Bleicher, EPJC(2016)

Vacuum

Kharzeev, McLerran,Warringa,NPA(2008)

Relativistic kinetic theory coupled with Maxwell's equations + 2->2 Leading-Log pQCD scattering

Zhang, Sheng, Pu, Chen, Peng, Wang, Wang, PRR (2022)

Summary (II)

Extension of BMT equation:

Thomas procession

Spin-vorticial fields coupling

Killing condition

$$\dot{s}^{\mu} = \boxed{-u^{\mu} s^{\nu} \dot{u}_{\nu}} + \left(\boxed{\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho}} - \boxed{2\beta\gamma_{\phi} g^{\mu\sigma}}\right)\boxed{\left(2\omega_{\sigma} - \boxed{\mathfrak{w}_{\sigma}}\right)} \boxed{- s_{\nu}\partial^{<\mu} u^{\nu>}} \boxed{- \left(\frac{1}{3} + 2v_n^2\right) s^{\mu} (\partial \cdot u)} \, ,$$

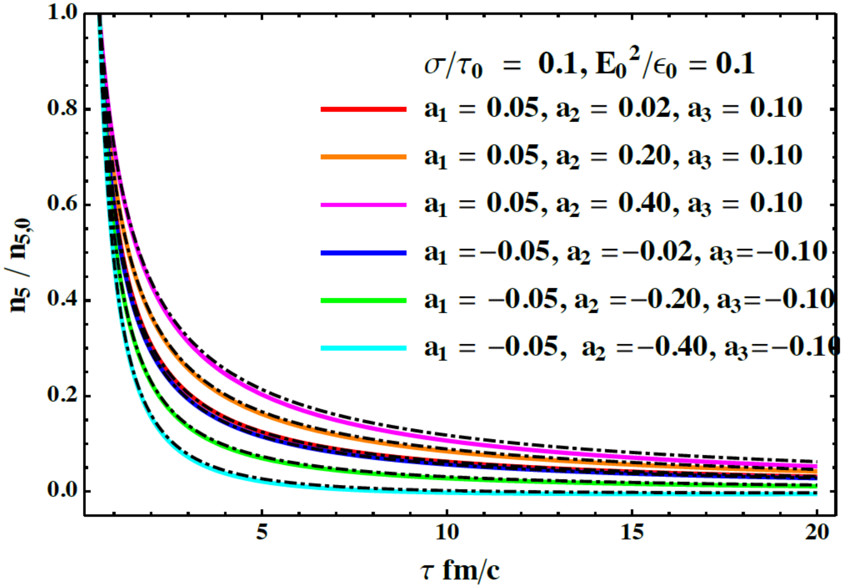
Spin coupled to shear tensor, bulk pressure and other dissipative effects

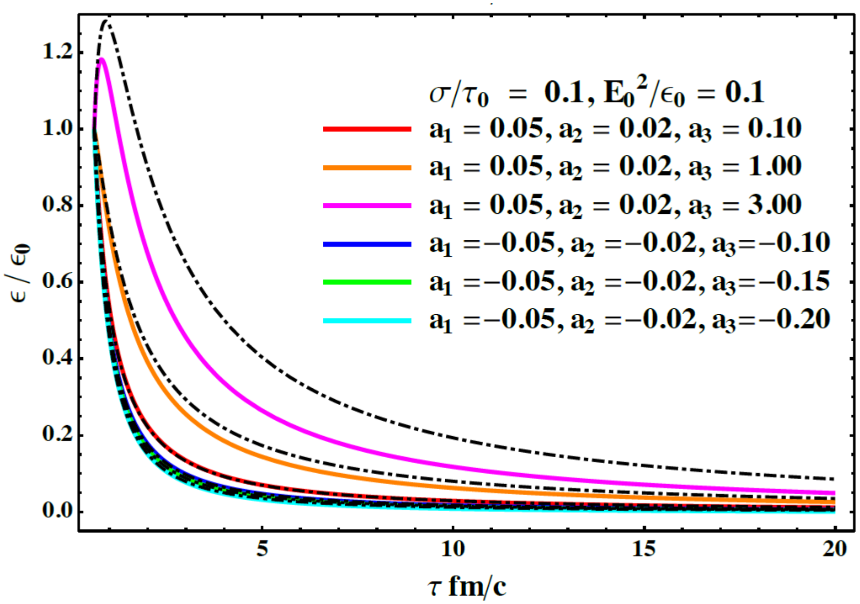
Thank you for your time!

欢迎批评指正！

Backup

Results for Bjorken MHD with CME





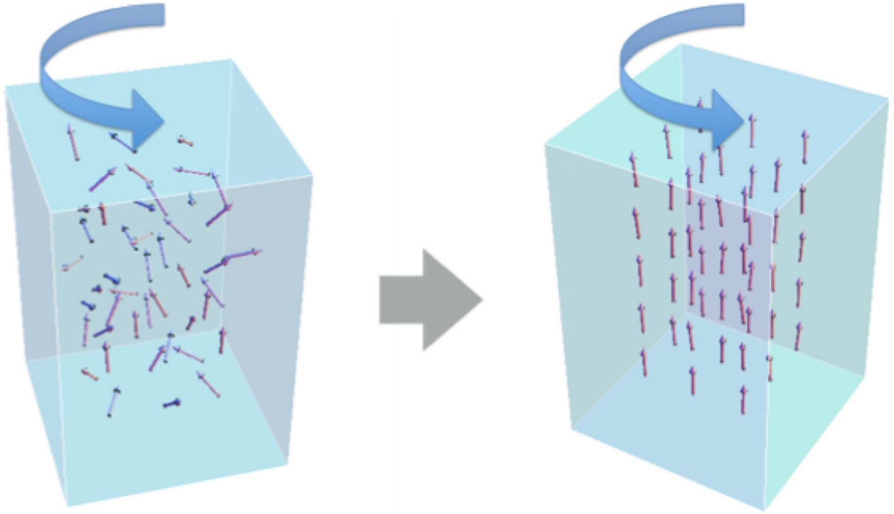
- **CME will affect the decay behaviors of chiral density, energy density, electromagnetic fields, etc.**

$$a_1 = eC\chi\frac{B_0 n_{5,0}}{aT_0^2 E_0}\tau_0, \quad a_2 = \frac{e^2 C\chi E_0 B_0}{n_{5,0}}\tau_0, \quad a_3 = \frac{eC\chi}{a}\frac{n_{5,0}E_0 B_0}{\varepsilon_0 T_0^2}\tau_0.$$

a1,a2,a3 are related to the initial conditions and C is chiral anomaly coefficient

Irfan Siddique, Ren-jie Wang, SP, and Qun Wang, PRD, arXiv: 1904.01807

Barnet and Einstein-de Hass effects



Barnett effect:

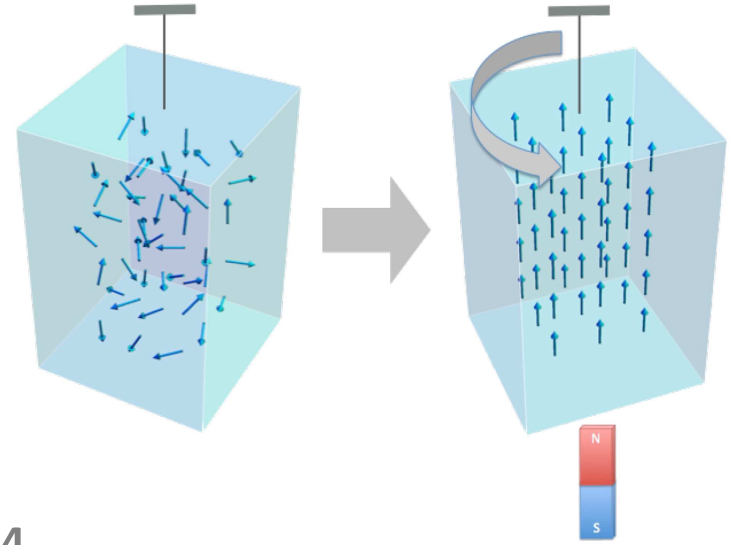
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

Magnetization $\Longrightarrow$ Rotation

Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: from paper doi: 10.3389/fphy.2015.00054

Spin dynamics in classical electrodynamics

Let us consider a spin-1/2 particle moving in a electrodynamics,

$$H = -\gamma \boldsymbol{B} \cdot \boldsymbol{s}$$

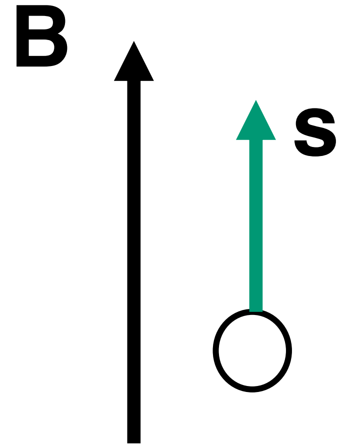
γ:gyromagnetic ratio

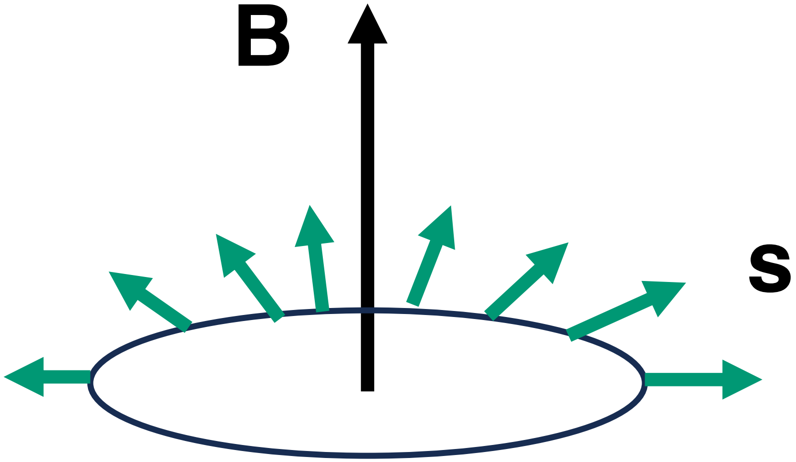
$$\Rightarrow \quad \frac{\partial \boldsymbol{s}}{\partial t} = -\gamma \boldsymbol{B} \times \boldsymbol{s}$$

The spin will not be parallel to B field with some initial conditions.

Relaxation for spin is essential!

We expect:





Landau-Lifshitz-Gilbert equation for spin

$$\begin{aligned}\frac{\partial \boldsymbol{s}}{\partial t} &= -\gamma \boldsymbol{B}\times \boldsymbol{s} + \frac{\alpha}{s}\gamma[\boldsymbol{s}\times(\boldsymbol{s}\times \boldsymbol{B})] \\ &\simeq -\gamma \boldsymbol{B}\times \boldsymbol{s} + \boxed{\frac{\alpha}{s}\gamma\left(\boldsymbol{s}\times\frac{\partial \boldsymbol{s}}{\partial t}\right)}\end{aligned}$$

Effective relaxation for spin

Lesson: relaxation (dissipative) effects are crucial for spin dynamics!

L. D. Landau and E. M. Lifshitz, Statistical physics. 2: Theory of the condensed state

Relativistic extension for spin

$$s^\mu \xrightarrow{\text{Rest frame}} (0, \boldsymbol{s})$$

$$s \cdot u = 0$$

$$\frac{\partial \boldsymbol{s}}{\partial t} = -\gamma \boldsymbol{B} \times \boldsymbol{s}$$

$$\longrightarrow \frac{ds^\mu}{dt} = \gamma F^{\mu\nu} s_\nu + \underline{\kappa u^\mu,}$$

Can be easily derived by contracting this equation with u

Total anti-symmetry spin tensor

We introduce the total anti-symmetric spin tensor in spin hydrodynamics:

$$\Sigma^{\lambda\mu\nu} = u^{\lambda}\boxed{S^{\mu\nu}} + u^{\mu}S^{\nu\lambda} + u^{\nu}S^{\lambda\mu} + \mathcal{O}(\partial^{1})$$

$$S^{\mu\nu} = -S^{\nu\mu}$$

Spin density (tensor)

$$S^{\mu\nu}u_{\nu} = 0$$

Frenkel-Mathisson-Pirani condition (1926)

$$s^{\mu} := -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}u_{\nu}S_{\rho\sigma}$$

Spin density (vector)

$$\xrightarrow{\text{Rest frame}} (0, \boldsymbol{s})$$

Entropy production rate

$$\begin{aligned}\partial_\mu \mathcal{S}^\mu &= (h^\mu - \mathcal{H}\nu^\mu)(\partial_\mu\beta+\beta\dot{u}_\mu)\\ &+\beta\pi^{\mu\nu}\partial_{<\mu}u_{\nu>} + \phi^{\mu\nu}(2\beta\omega_{\mu\nu}+\partial_{[\mu}\beta u_{\nu]})\\ &\boxed{+2\beta\omega_{\mu\nu}S^{\lambda\mu}\partial_\lambda u^\nu + q^\mu(\partial_\mu\beta-\beta\dot{u}_\mu)}\\ &+\mathcal{O}(\partial^3)\end{aligned}$$

Most of the terms can easily be written as squared terms, but …

It is challenging to ensure that the entropy increases with the total antisymmetric spin tensor!

Hongo, Huang, Kaminski, Stephanov, Yee, JHEP 2022

Cao, Hattori, Hongo, Huang, Taya, PRD 2022

…..

Spin correction

$$\begin{aligned}\partial_\mu(\mathcal{S}^\mu+\delta\mathcal{S}^\mu) \quad = \quad & (h^\mu-\mathcal{H}\nu^\mu+h_s^\mu)(\partial_\mu\beta+\beta\dot{u}_\mu)\\ & +\beta(\pi^{\mu\nu}+\pi_s^{\mu\nu})\partial_{(\mu}u_{\nu)}\\ & +(\phi^{\mu\nu}+\phi_s^{\mu\nu})(2\beta\omega_{\mu\nu}+\partial_{[\mu}\beta u_{\nu]})+\mathcal{O}(\partial^3)\end{aligned}$$

Second law of thermodynamics

New spin corrections

Heat flow

$$h^\mu-\mathcal{H}\nu^\mu+h_s^\mu \quad = \quad -\sigma\Delta^{\mu\nu}(\partial_\nu\beta+\beta\dot{u}_\nu)\,,$$

Viscous tensor

$$\pi^{\mu\nu}+\pi_s^{\mu\nu} \quad = \quad \zeta\Delta^{\mu\nu}(\partial\cdot u)+\eta\partial^{<\mu}u^{\nu>}\,,$$

Anti-symmetric part of energy Momentum tensor

$$\phi^{\mu\nu}+\phi_s^{\mu\nu} \quad = \quad \gamma_\phi\Delta^{\mu\rho}\Delta^{\nu\sigma}(2\beta\omega_{\rho\sigma}-\Omega_{\rho\sigma})\,,$$

Non-relativistic limit

In non-relativistic limit,

$$\boldsymbol{j} = \boxed{-\frac{1}{2}(\nabla \times \boldsymbol{s})}$$

inverse spin Hall effect

Sinova, Valenzuela, Wunderlich, Back, Jungwirth, Rev. Mod. Phys. 87, 1213 (2015)

Relativistic extension

$$+\frac{1}{2}(\dot{\boldsymbol{s}} \times \boldsymbol{v}) + (1+\xi)(\boldsymbol{s} \times \dot{\boldsymbol{v}})$$

$$\boxed{+\frac{\xi}{T}(\boldsymbol{s} \times \nabla T)}$$

anomalous Hall effect

Nagaosa, Sinova, Onoda, MacDonald, Ong, Rev. Mod. Phys. 82, 1539 (2010).

Relativistic extension

$$+\frac{\xi}{T}\dot{T}(\boldsymbol{s} \times \boldsymbol{v})$$

$$+\mathcal{O}(v^2)$$

Extension of BMT equations (I)

Original BMT equation:

$$\dot{s}^{\mu} = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu} \boxed{- u^{\mu} s^{\nu} \dot{u}_{\nu}}$$

Thomas procession

+ possible relaxation (dissipative) terms

Extension of BMT equation:

$$\begin{aligned} \dot{s}^{\mu} &= \boxed{-u^{\mu} s^{\nu} \dot{u}_{\nu}} \\ &\quad +(\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho} - 2\beta\gamma_{\phi} g^{\mu\sigma})(2\omega_{\sigma} - \mathfrak{w}_{\sigma}) \\ &\quad -s_{\nu} \partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_{n}^{2}\right) s^{\mu} (\partial \cdot u) \,, \end{aligned}$$

Thomas procession

Extension of BMT equations (II)

Original BMT equation:

$$\dot{s}^{\mu} = \boxed{\gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu}} - u^{\mu} s^{\nu} \dot{u}_{\nu}$$

$$H = -\gamma \boldsymbol{B} \cdot \boldsymbol{s}$$

$$\frac{\partial \boldsymbol{s}}{\partial t} = -\gamma \boldsymbol{B} \times \boldsymbol{s}$$

Spin-EM fields coupling

+ possible relaxation (dissipative) terms

Extended BMT equation:

Spin-vorticial fields coupling

$$H_{\omega} = -\boldsymbol{\omega} \cdot \boldsymbol{s}$$

$$\begin{aligned} \dot{s}^{\mu} &= -u^{\mu} s^{\nu} \dot{u}_{\nu} \\ &\quad + \left(\boxed{\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho}} - 2\beta\gamma_{\phi} g^{\mu\sigma}\right)\left(2\omega_{\sigma} - \boxed{\mathfrak{w}_{\sigma}}\right) \\ &\quad - s_{\nu} \partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_n^2\right) s^{\mu} (\partial \cdot u) , \end{aligned}$$

Vorticity vector

Extension of BMT equations (IV)

Original BMT equation:

$$\dot{s}^{\mu} = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu} - u^{\mu} s^{\nu} \dot{u}_{\nu}$$

+ possible relaxation (dissipative) terms

Extension of BMT equation:

Thermal vorticity combined with spin chemical potential (Killing condition)

$$\begin{aligned} \dot{s}^{\mu} \quad = \quad & -u^{\mu} s^{\nu} \dot{u}_{\nu} \\ & +(\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho} - 2\beta\gamma_{\phi} g^{\mu\sigma}) \boxed{(2\omega_{\sigma} - \mathfrak{w}_{\sigma})} \\ & -s_{\nu} \partial^{<\mu} u^{\nu>} - \left(\frac{1}{3} + 2v_{n}^{2}\right) s^{\mu} (\partial \cdot u) \, , \end{aligned}$$

Extension of BMT equations (V)

Original BMT equation:

$$\dot{s}^{\mu} = \gamma \Delta^{\mu\rho} F_{\rho\nu} s^{\nu} - u^{\mu} s^{\nu} \dot{u}_{\nu}$$

+ possible relaxation (dissipative) terms

Extension of BMT equation:

$$\begin{aligned}
\dot{s}^{\mu} \quad = \quad & -u^{\mu} s^{\nu} \dot{u}_{\nu} \\
& +(\varepsilon^{\mu\nu\rho\sigma} s_{\nu} u_{\rho} \boxed{- 2\beta\gamma_{\phi} g^{\mu\sigma})(2\omega_{\sigma} - \mathfrak{w}_{\sigma})} \\
& \boxed{-s_{\nu}\partial^{<\mu}u^{\nu>}} \boxed{- \left(\frac{1}{3} + 2v_{n}^{2}\right) s^{\mu}(\partial \cdot u)} ,
\end{aligned}$$

Spin coupled to shear tensor, bulk pressure and other dissipative effects