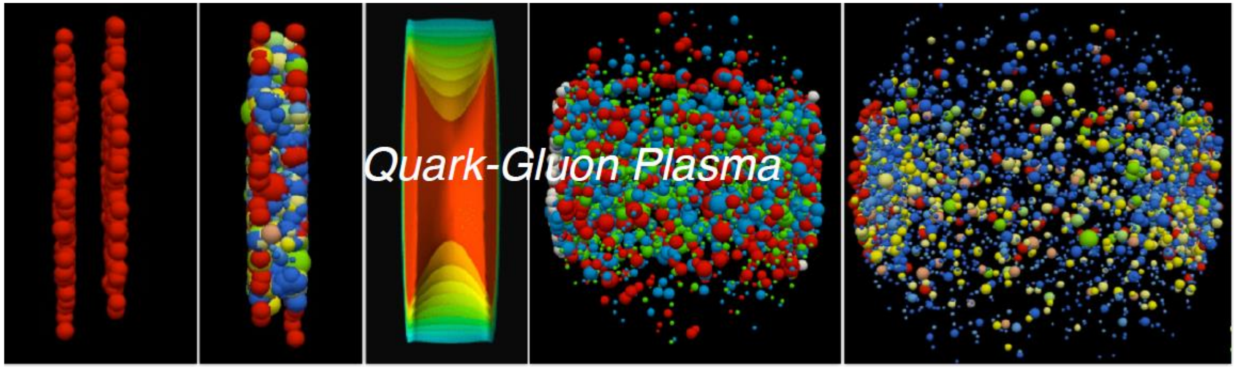
QCD Phase Transitions
& Relativistic Heavy Ion Physics



I. Why Heavy Ion Collisions — QCD matter
II. Quark-gluon plasma at RHIC and LHC
III. Rare particles ($J/\psi, \Upsilon, B_c, \Omega_{ccc}, cc\bar{c}\bar{c}, \mu^+\mu^-$)

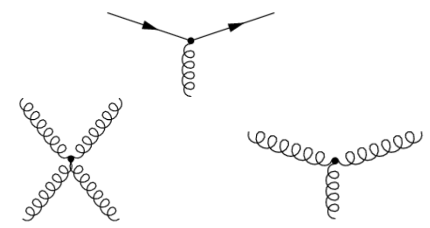
I. Why heavy ion collisions – QCD matter

QCD

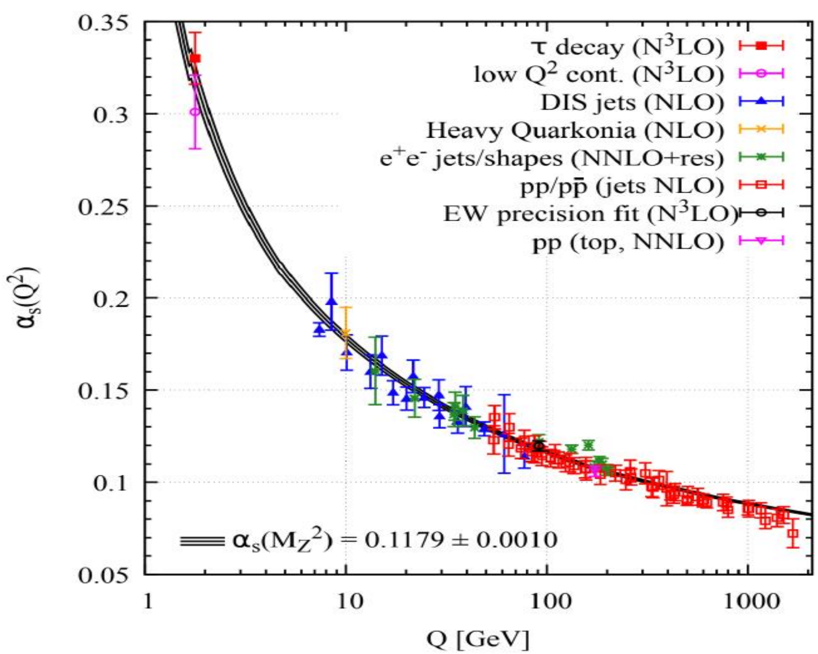
$$standard\ model \begin{cases} Electromagnetic\ initeraction \\ Weak\ interaction \\ Strong\ interaction, Quantum\ chromodynamics\ (QCD) \end{cases}$$

$$\mathcal{L}_{QCD} = \bar{\psi}_i \left(i\gamma_\mu D^\mu_{ij} - m\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$D^\mu_{ij} = \delta_{ij}\partial^\mu + i\, g\, A^\mu_a T^a_{ij} \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\, f^{abc} A^b_\mu A^c_\nu$$



Non-Abelian interaction → QCD running coupling



Short range: weak coupling, asymptotic freedom, pQCD

2004 诺贝尔物理奖

David J. Gross

H. David Politzer

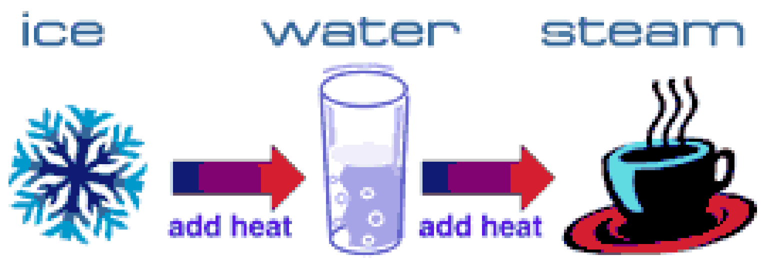
Frank Wilczek

Long range: strong coupling, color confinement, non-perturbative properties !

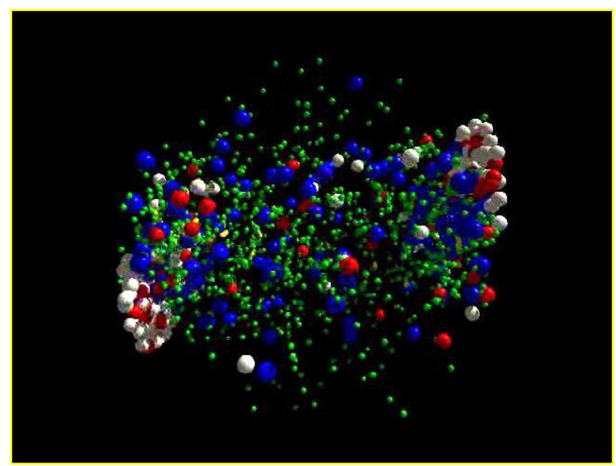
物质的状态

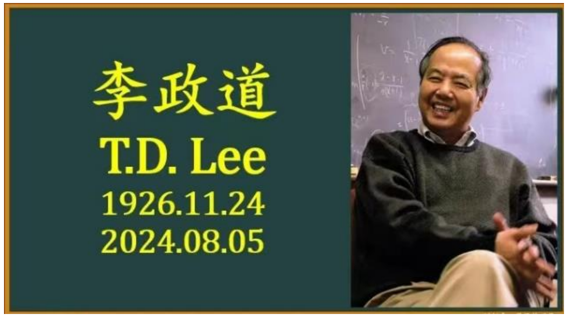
问题：看不见的夸克，质量起源，真空结构等非微扰效应。
方法：真空不知道，动动真空怎么样（例如量子力学中的*Casimir*效应）？
物质结构 → （相对于真空的）物质状态

熟悉的物质三态：固态，液态，气态



第四态：原子弹爆炸后产生大范围的电磁等离子体
猜测: 是否存在由夸克和胶子组成的*QCD*等离子体(*QGP*)?





李先生是相对论重离子碰撞物理的开创者！

统一研究物质的结构与状态

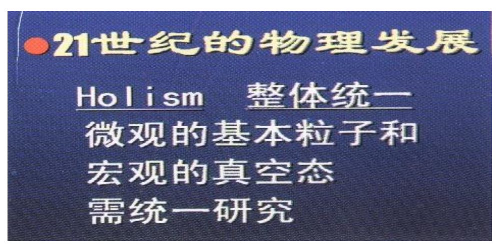
李政道【*Rev. Mod. Phys. 47, 267(1975)*】

In order to study the question of ‘vacuum’, we must turn to a different direction: we should investigate some ‘bulk’ phenomena by distributing high energy over a relatively large volume.”

李政道【布鲁克海文报告, *2006*】

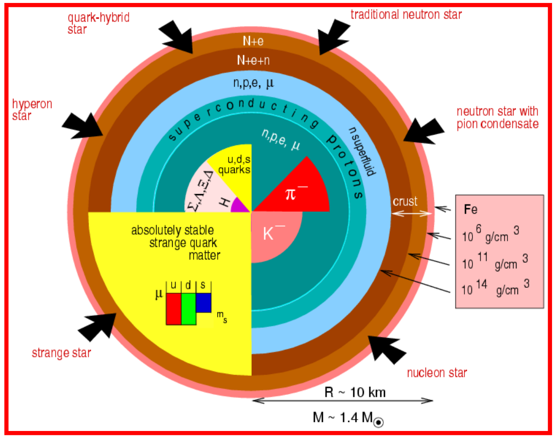
from 1897 to present: to comprehend the largest, we need only understand the smallest.
from present to 21st century: to know the smallest, we need also the largest!

李政道【人民大会堂报告, *2012*】

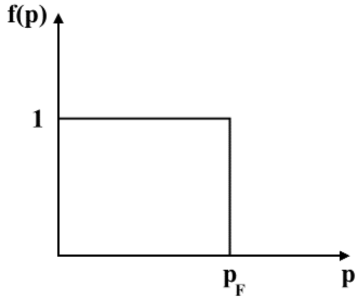


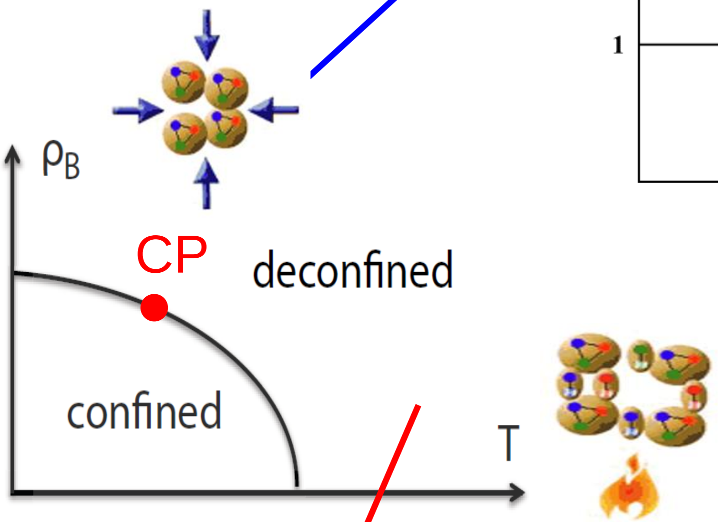
Vacuum Excitation VS Matter Compression

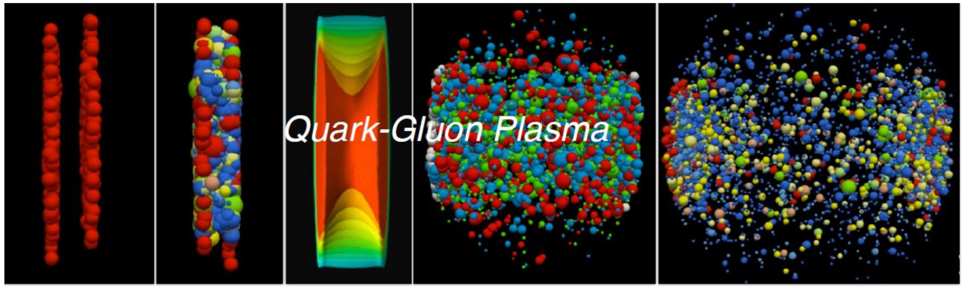
- *ρ_B-induced QUANTUM Phase Transition*
- *Poor Theories*



Matter compression !







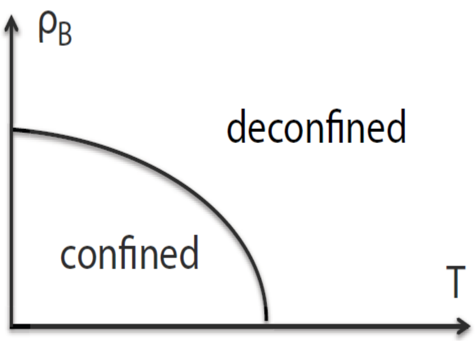
Vacuum excitation !

- *T-induced CLASSICAL Phase Transition (Landau)*
- *Lattice QCD, (resumed)pQCD, FRG, DSE, Models*

QCD Phase Diagrams

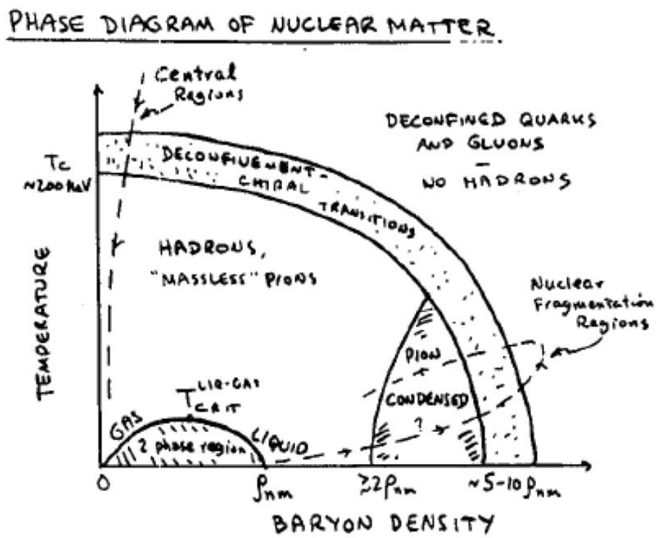
Deconfinement

Cabbibo and Parisi, PLB59, 67(1975)

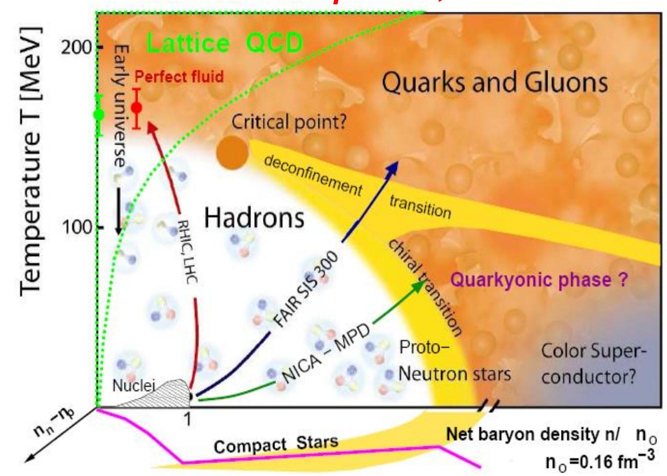


+ Chiral restoration

Baym, NSAC Long Rang Plan, 1983



+ CSC, quarkyonic phase and critical point, ……



有哪些QCD相变？

相变是体系对称性质的改变

Deconfinement：	*Z(3) symmetry*
Chiral restoration:	*SU(3) chiral symmetry*
Color superconductivity:	*SU(3) color symmetry*
Pion superfluidity:	*SU(2) isospin symmetry*

研究量子物质的方法：量子统计力学

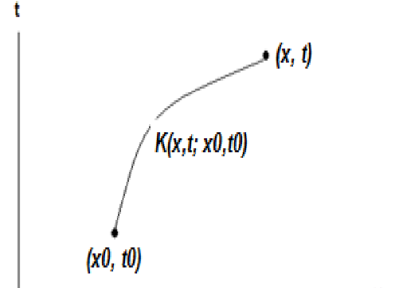
波函数的时间演化：$|\psi,t\rangle = \hat{U}(t,0)|\psi,0\rangle$

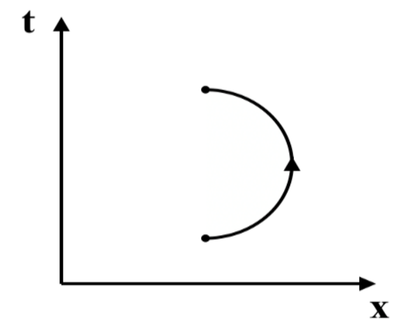
坐标表象： $\psi(x,t) = \langle x|\psi,t\rangle = \langle x|\hat{U}(t,0)|\psi,0\rangle$

$$= \int dx_0 \langle x|\hat{U}(t,0)|x_0\rangle\langle x_0|\psi,0\rangle = \int dx_0 K(x,t;x_0,0)\psi(x_0,0)$$

时空中波函数的传播子$K(x,t;x_0,0)$

空间封闭传播子$K(x,t;x,0)$





考虑所有的封闭传播子的集合：

$$\int dx\, K(x,t;x,0) = \int dx\, \langle x|e^{-\frac{i}{\hbar}\hat{H}t}|x\rangle = \sum_E \int dx\, \langle x|E\rangle\langle E|e^{-\frac{i}{\hbar}\hat{H}t}|x\rangle$$

$$= \sum_E \int dx\, \langle E|x\rangle\langle x|E\rangle\, e^{-\frac{i}{\hbar}Et} = \sum_E e^{-\frac{i}{\hbar}Et} \xrightarrow{it/\hbar\to 1/T} \sum_E e^{-\frac{E}{T}} = Z$$

封闭传播子就是配分函数。用虚时动力学方法计算粒子的热力学。

极高温极高密：*pQCD Thermodynamics*

Free QGP (封闭自由传播子):

$$\Omega_0(T,\mu) = -2N_cN_f\left[\frac{7\pi^2}{360}T^4 + \frac{\mu^2T^2}{12} + \frac{\mu^4}{24\pi^2}\right] - 2(N_c^2-1)\frac{\pi^2}{90}T^4$$

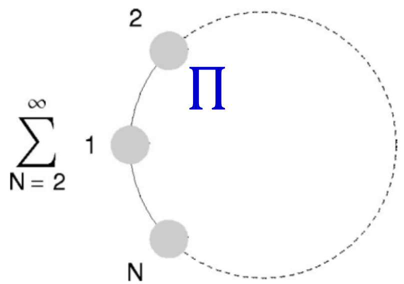
$\sim g$:

$$\Omega_1(T,\mu) = 0$$

$\sim g^2$:

$$\Omega_2^q(T,\mu) = \frac{N_f(N_c^2-1)}{8\pi}\alpha_s\left[\frac{5\pi^2}{18}T^4 + \mu^2T^2 + \frac{\mu^4}{2\pi^2}\right]$$

Summation over ring diagrams:



HTL:

$$V_{HTL}(r) = -\tilde{\alpha}_s\left[m_D + \frac{e^{-m_Dr}}{r} + iT\phi(m_Dr)\right] + \mathcal{O}(g^4)$$

$$\phi(x) = 2\int_0^\infty \mathrm{d}z\,\frac{z}{(z^2+1)^2}\left(1 - \frac{\sin(xz)}{xz}\right)$$

袋模型估计相变温度

将*QGP*看成一个大口袋，袋内微扰真空，是弱耦合*QGP*，袋外物理真空。两种真空之差表现为压强作用在口袋上，平衡袋内的压强，形成稳定的口袋。

设袋外压强常数B。体系的压强为

$$P = P_{QGP} - B$$
$$= 2N_c N_f \left[\frac{7\pi^2}{360} T^4 + \frac{\mu^2 T^2}{12} + \frac{\mu^4}{24\pi^2}\right] + 2(N_c^2 - 1)\frac{\pi^2}{90} T^4 - B$$

如果袋内为强子气体，只考虑π介子时的压强，

$$P = \frac{3\pi^2}{90} T^4$$

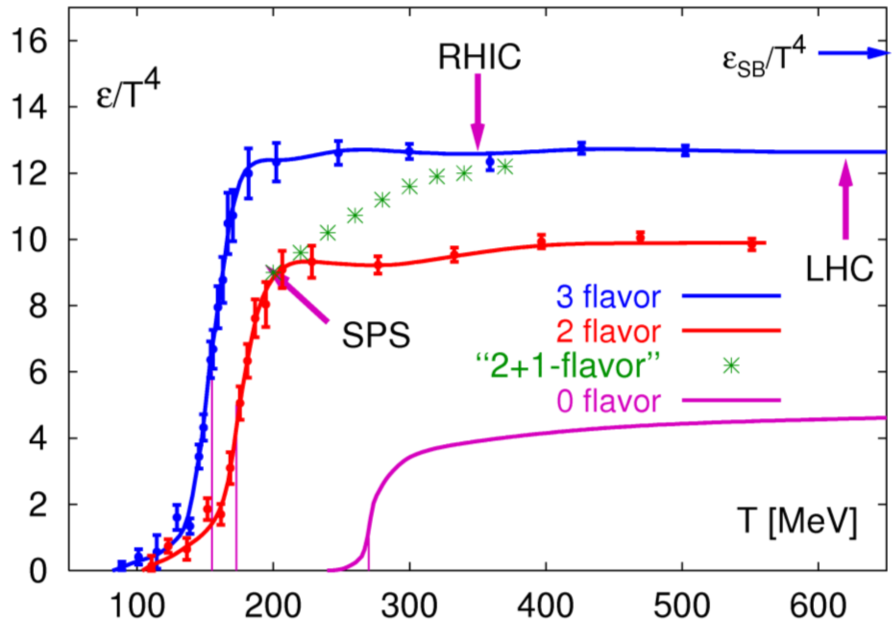
设体系在T_c时发生相变，由*Gibbs*相平衡条件，

$$2N_c N_f \left[\frac{7\pi^2}{360} T_c^4 + \frac{\mu^2 T_c^2}{12} + \frac{\mu^4}{24\pi^2}\right] + 2(N_c^2 - 1)\frac{\pi^2}{90} T_c^4 - B = \frac{3\pi^2}{90} T_c^4$$

取$B^{1/4} = 200$ MeV，有

$$T_c = 144 \text{ MeV}$$

有限温度低密: Lattice QCD

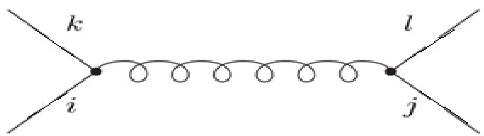


- *Not a phase transition but a crossover around* $T_c \sim 160$ MeV
- *Still interacting QGP at* $T/T_c \sim 4$

低温高密：色超导

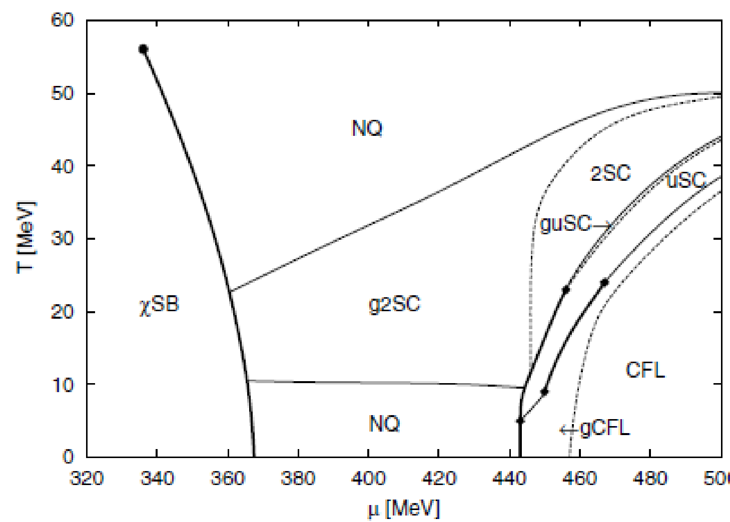
由*BCS*理论，在*QED*凝聚态中，两个电子通过交换光子是排斥相互作用，吸引相互作用是通过交换集体激发模式—声子来实现的(低温超导)。*Cooper*对的形成自发破缺了电磁规范对称性，光子获得了质量。

Frank Wilczek(2004诺贝尔奖): 色超导研究的开创者之一



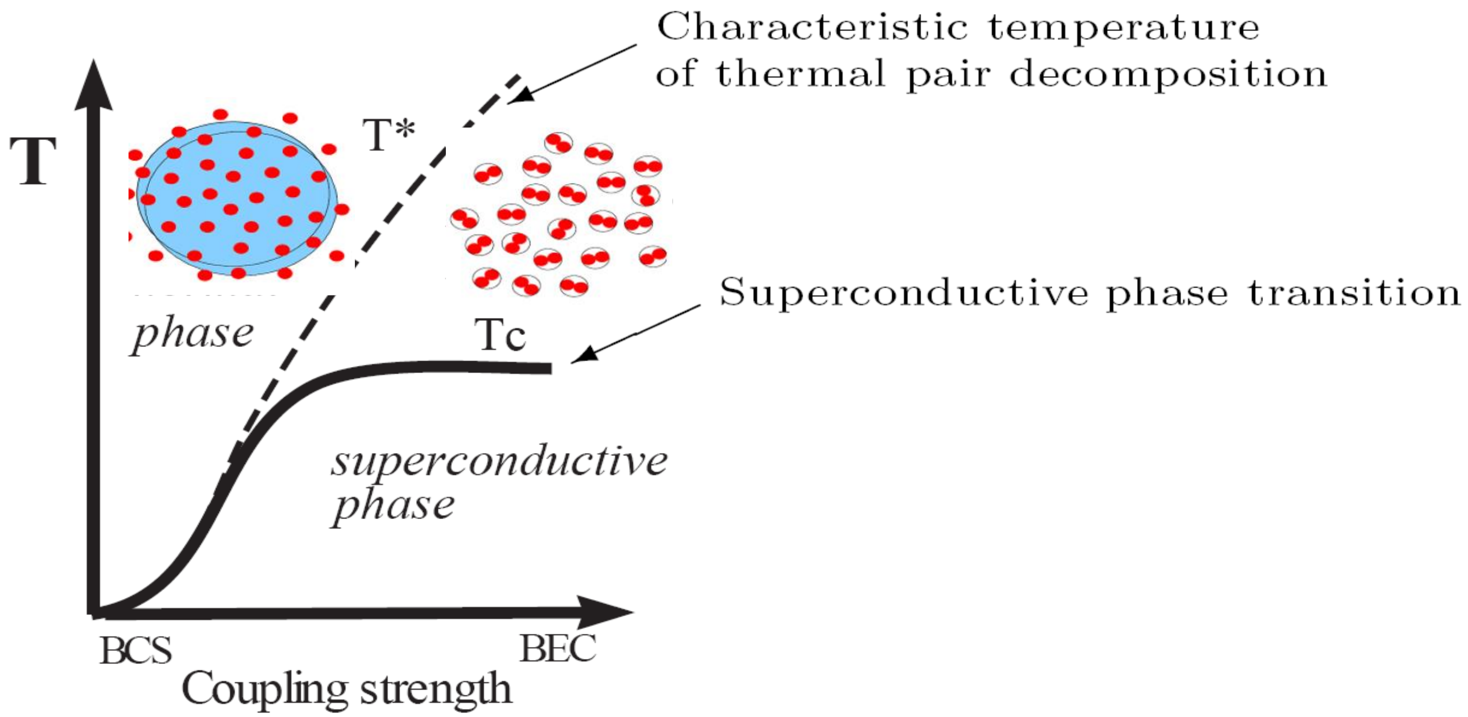
$$\sim (T_a)_{ki}(T_a)_{lj} = -\frac{N_c+1}{4N_c}\left(\delta_{jk}\delta_{il} - \delta_{ik}\delta_{jl}\right) + \frac{N_c-1}{4N_c}\left(\delta_{jk}\delta_{il} + \delta_{ik}\delta_{jl}\right)$$

第一项在交换初态或末态的两个夸克色指标时是反对称的，吸引相互作用。在单胶子交换的层次就使得两个夸克可以配对，*Diquark*凝聚，形成色超导 (高温超导)。由于色对称性自发破缺，胶子获得质量。



夸克的色味结构导致丰富的色超导相图

BCS-BEC Crossover



in BCS, T_c is determined by thermal excitation of fermions,
in BEC, T_c is controlled by thermal excitation of collective Bosons.

BCS-BEC crossover is probably a way to understand deconfinement in QCD !

Quantum Phase Transitions

Symmetry of $\mathcal{L}$
(may be explicitly broken like the term $m\bar{\psi}\psi$ in QCD)

$T=0$: *spontaneously broken at classical level (mean field level), like chiral symmetry*

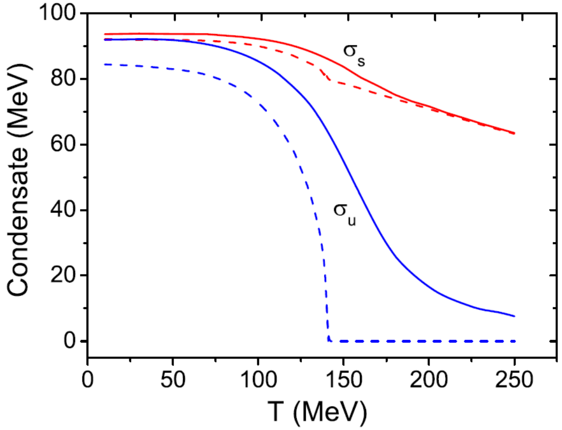
$T\neq 0$: *classical phase transition, like chiral restoration*

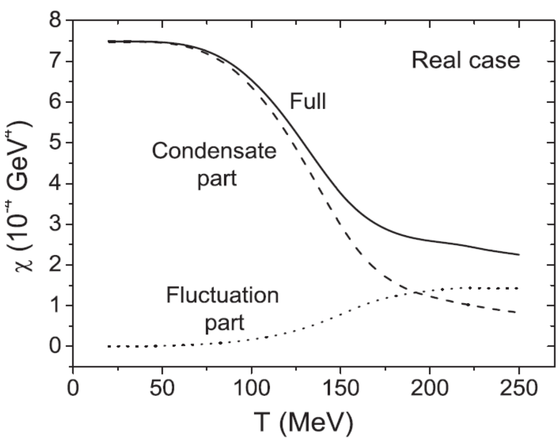
anomalously broken at quantum level (loop level), like $U_A(1)$ symmetry

quantum phase transition, like $U_A(1)$ restoration

- *$U_A(1)$ symmetry*

$$\chi=\int d^4x\langle T\big(Q(x)Q(0)\big)\rangle_{connected}\begin{cases}\neq 0 & U_A(1)\ breaking\\ =0 & U_A(1)\ symmetry\end{cases}\qquad Q(x)=\frac{g^2}{32\pi^2}F^a_{\mu\nu}(x)\tilde{F}^{\mu\nu}_a(x)$$



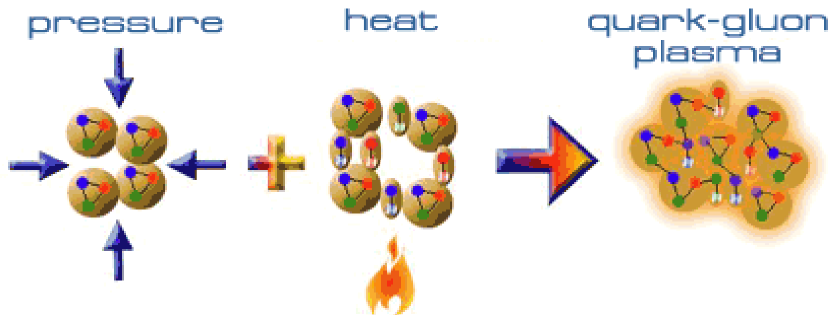


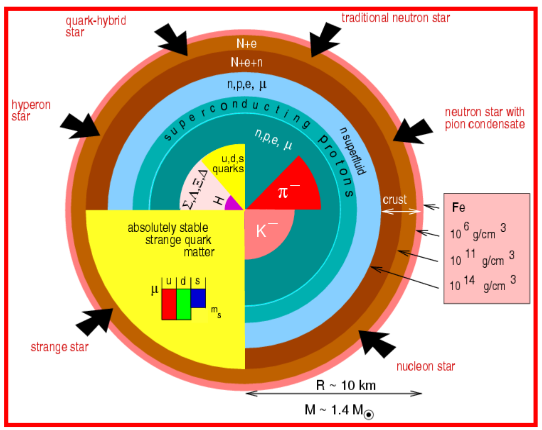
- *Chiral symmetry is restored at $T_c \sim 150$ MeV, but $U_A(1)$ symmetry is still broken above T_c !*

Lattice QCD:
HotQCD, PRD86, 094503(2012)
Effective models:
Jiang, Xia and PZ, PRD93, 074006(2016)
Li, Gui, Zhuang, CPC, 2023

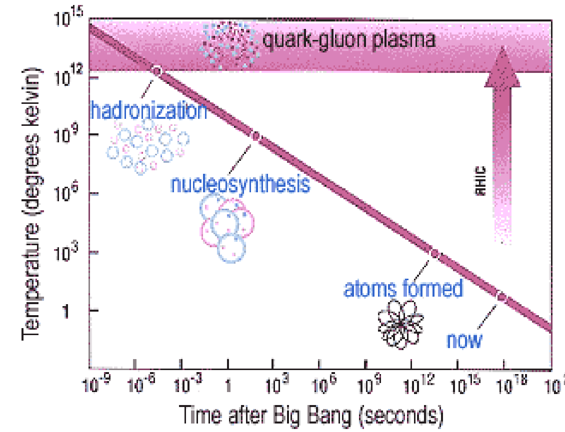
如何实现QGP

产生QGP的条件:

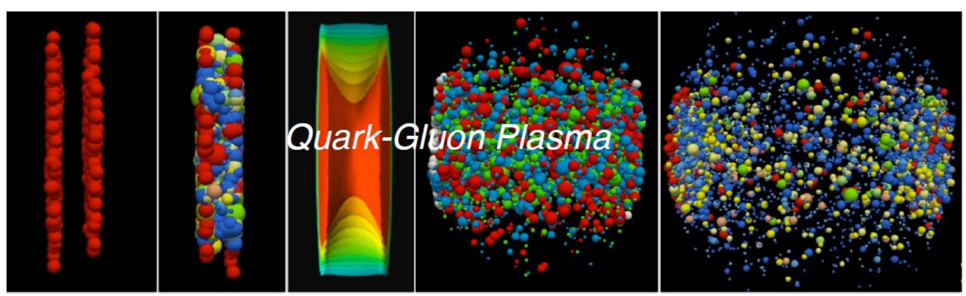




致密星体



早期宇宙

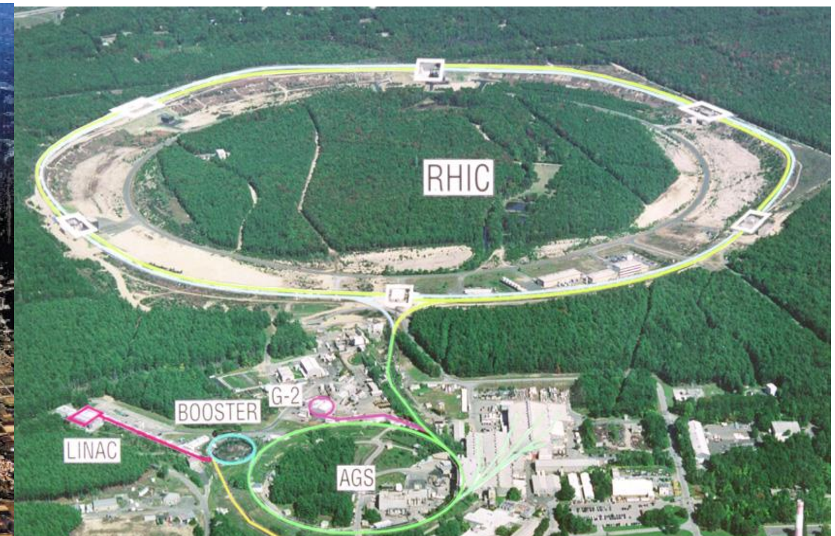


相对论重离子碰撞是在实验室产生QGP的唯一可能手段！

相对论重离子碰撞是在实验室实现QGP的唯一可能方式

LHC, Pb+Pb@√s = 5500 GeV, ALICE,ATLAS,CMS,LHCb

RHIC, Au+Au@√s = 200 GeV, STAR, PHENIX



FAIR, U+U@Elab = 40A GeV, CBM

CSR, U+U@Elab = 0.6A GeV, CEE

常用物理量

核几何

碰撞参数$\vec{b}$：两个碰撞核中心的相对位置，矢量。$\vec{b}=0$,中心碰撞。

参与碰撞核子数N_{part}：软过程数$\sim N_{part}$

参与碰撞的核子对数N_{coll}：硬过程数$\sim N_{coll}$

纵向快度

（沿碰撞方向）$y=\frac{1}{2}ln\frac{E+p_z}{E-p_z}$, $\begin{cases} E=m_\perp Cosh(y) \\ p_z=m_\perp Sinh(y) \end{cases}$

优点：Lorentz变换时，快度相加 $y'=y+Y$

E_{lab}与$\sqrt{s}$的转换

$$\sqrt{s}=\sqrt{(E_1+E_2)^2-(\vec{p}_1+\vec{p}_2)^2}=\sqrt{(E_{lab}+m_N)^2-\vec{p}_1^2}=\sqrt{2m_N(m_M+E_{lab})}$$

正常核物质的能量密度

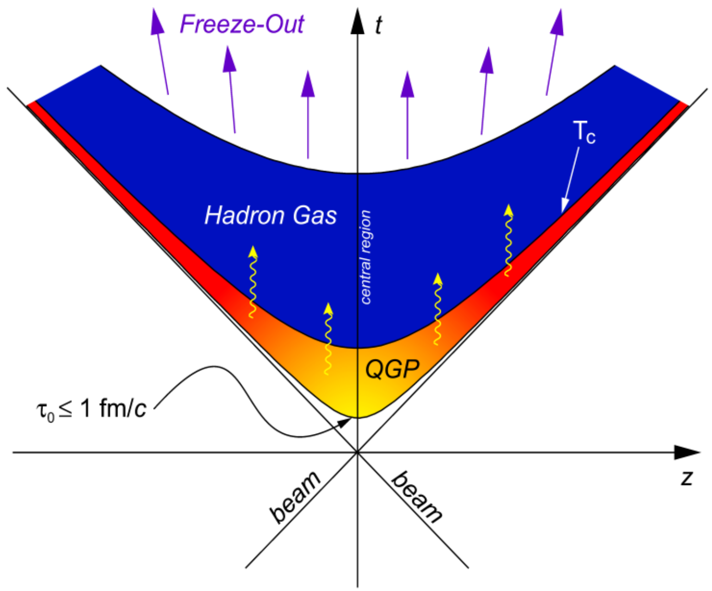
核内核子数密度 $n_0=0.17/fm^3$，核内能量密度$\varepsilon_0=m_N n_0\sim 0.17\text{GeV}/fm^3$

产生QGP的最低能量密度

即核子内的能量密度 $\varepsilon_{min}=\frac{m_N}{\frac{4}{3}\pi r_N^3}\xrightarrow{r_N=0.8\,fm}0.5\text{ GeV}/fm^3$

QGP的信号

QGP只是重离子碰撞的中间态，即使在RHIC和LHC产生了QGP，也不能在末态直接观测到，只能通过携带QGP信号的末态粒子来判断。



希望看到什么？

1）与pp的不同 *(off-diagonal interactions)*，有新的物理

核修正因子：$R_{AA}=\frac{\sigma_{AA}}{N_{coll}\sigma_{pp}}$

2）与强子气体的差别，夸克层次的新物理，QGP的产生！

热密物质的时空演化—流体力学

Landau: 用流体力学描述高能碰撞后至衰变前体系的时空演化。

忽略粘滞效应的理想流体力学方程

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 & (\text{能动量守恒}) \\ \partial_\mu n^\mu = 0 & (\text{重子数守恒}) \end{cases}$$

$$T^{\mu\nu} = -Pg^{\mu\nu} + (\varepsilon + P)u^\mu u^\nu, \qquad n^\mu = n_B u^\mu, \qquad u_\mu u^\mu = 1$$

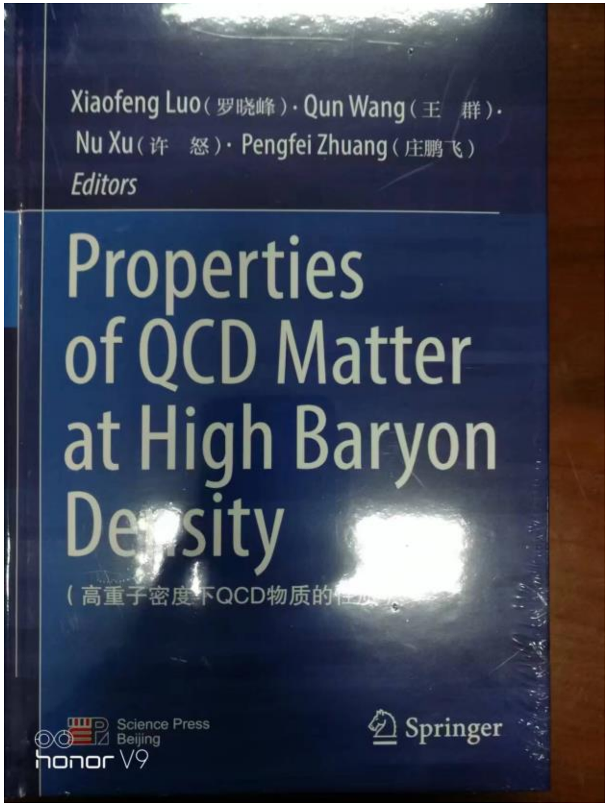
+ *Equation of state* $\epsilon(P)$ （*QGP or Hadron gas from QCD thermodynamics*）

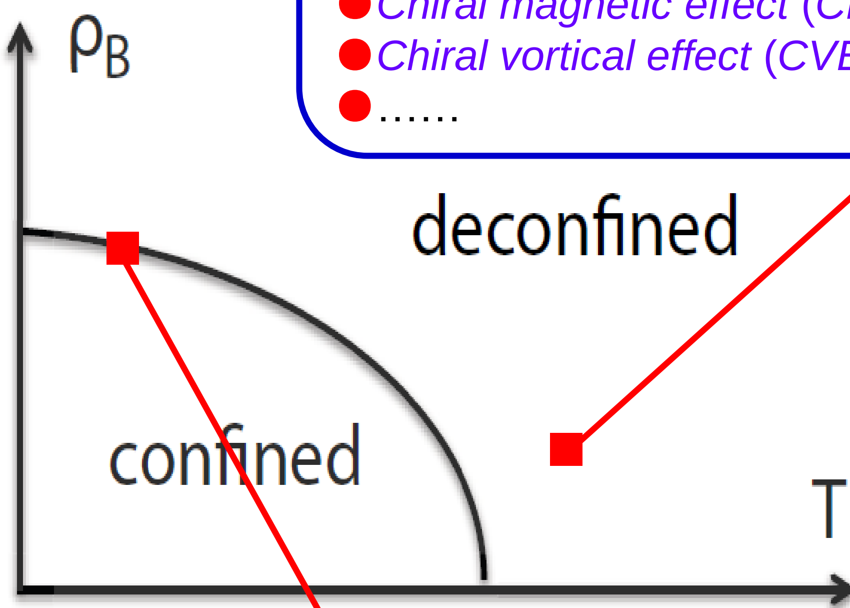
$\rightarrow\ u_\mu(x), T(x), \mu_B(x), P(x)$

一维膨胀→快度中心区的能量密度(*Bjorken*估计)

$$\begin{cases} \varepsilon(\tau) = \epsilon(\tau_0)\left(\dfrac{\tau_0}{\tau}\right)^{1+c_s^2} \\ n(\tau) = n(\tau_0)\dfrac{\tau_0}{\tau} \\ T(\tau) = T(\tau_0)\left(\dfrac{\tau_0}{\tau}\right)^{c_s^2} \end{cases}$$

Probes of QCD Phases





- *Jet quenching*
- *Quarkonium suppression ,*
- *Dileptons*
- *Strangeness enhancement……*
- *Chiral magnetic effect (CME)*
- *Chiral vortical effect (CVE)*
- ……

Probes of the chiral critical point:
Dynamical fluctuations of conserved charges.