Quark-gluon plasma at RHIC and LHC

Electromagnetic and Rotational Fields at RHIC and LHC

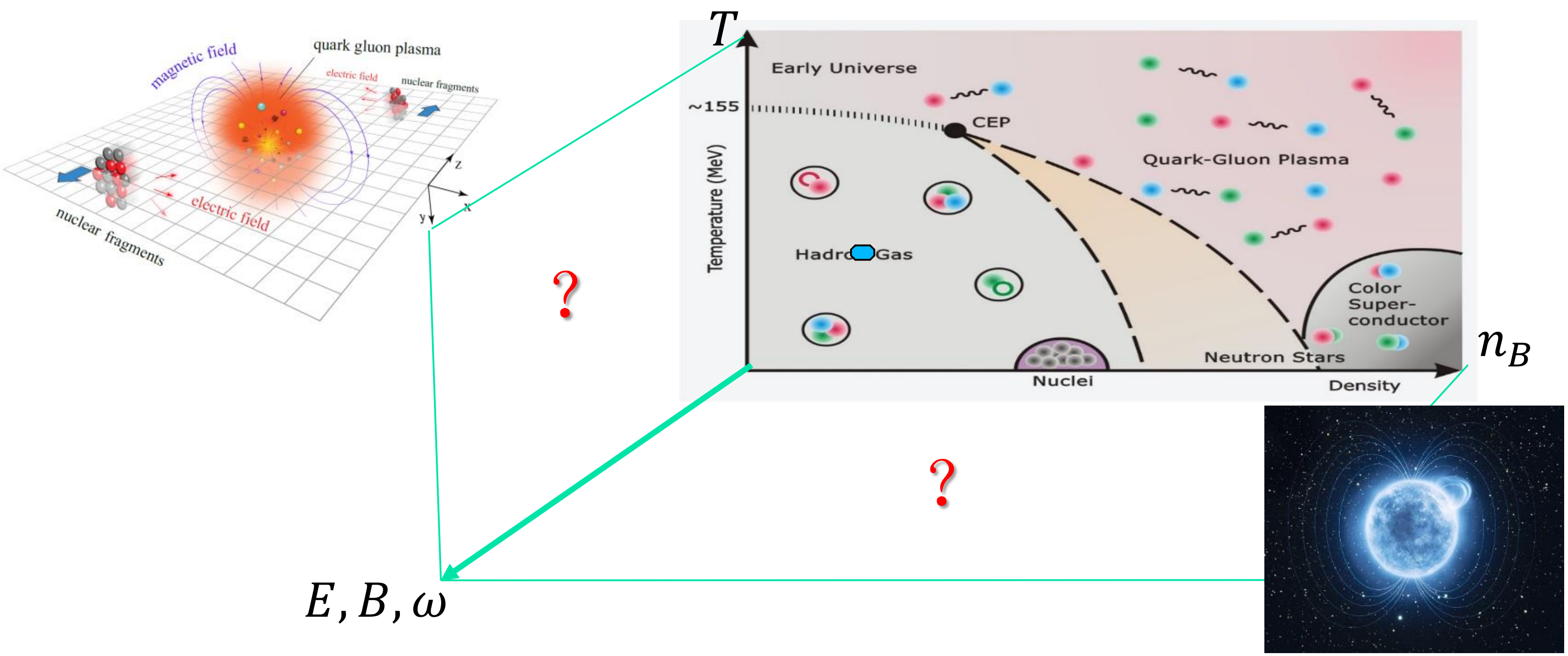
由于电磁相互作用远比强相互作用弱，一般研究QCD物质时不考虑电磁相互作用。

非对心核碰撞使QGP处于自然界最强的电磁场和涡旋场之中：

$|eB| \sim 5m_\pi^2$ *at RHIC and* $70m_\pi^2$ *at LHC !*

$\omega \sim 10^{21}/s$ *at RHIC !*

其强度已经与强相互作用可比拟。



3 Questions

1) Strength:

E & B breaks down the translation invariance, but T will restore the invariance. Only charged quarks join electromagnetic interaction, but all the partons join thermal motion
→ *Is the E and B field strong enough in comparison with the fireball T?*

$$|eB| \sim 10 m_\pi^2, \qquad T \sim 300 - 500 \text{ MeV}$$

2) Time:

The lifetime of external E and B field is very short, we need induced E and B field in QGP
→ *Can the current be completely induced?*

$$\tau_{ind}, \qquad \tau_B \sim 0.1 \text{ fm}$$

3) Clear signals:

If the answers to the above 2 questions are negative, is it possible to have sensitive electromagnetic signals of QCD matter?

Feynman Rules in Magnetic Field

Kostenko and Thompson, Astrophys J. 869, 44(2018), 875, 23(2019).

External lines:

$$[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi = 0$$

$$\psi^\sigma_\mp(x,p) = \begin{cases} e^{-ip\cdot x} u_\sigma(\boldsymbol{x},p) \\ e^{ip\cdot x} v_\sigma(\boldsymbol{x},p) \end{cases}$$

$$u_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z p_n \phi_{n-1} \\ (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \\ -ip_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -p_z(\epsilon_n+m)\phi_n \end{bmatrix}, \qquad v_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -p_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -ip_z(\epsilon_n+m)\phi_n \\ -p_z p_n \phi_{n-1} \\ i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \end{bmatrix},$$

$$u_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ -ip_z p_n \phi_n \\ p_z(\epsilon_n+m)\phi_{n-1} \\ ip_n(\epsilon+\epsilon_n)\phi_n \end{bmatrix} \qquad v_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z(\epsilon_n+m)\phi_{n-1} \\ -p_n(\epsilon+\epsilon_n)\phi_n \\ -i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ p_z p_n \phi_n \end{bmatrix}$$

Quark propagator:

$$G(x'-x) = -i\left(\frac{L}{2\pi\lambda}\right)^2 \int dp_z da \sum_{\sigma,n}\left[\theta(t'-t)u_\sigma(\boldsymbol{x}',p)\bar{u}_\sigma(\boldsymbol{x},p)e^{-ip\cdot(x'-x)} - \theta(t-t')v_\sigma(\boldsymbol{x}',p)\bar{v}_\sigma(\boldsymbol{x},p)e^{ip\cdot(x'-x)}\right]$$

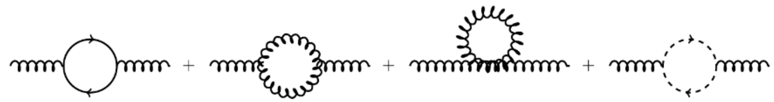
$$G(p) = -\int_0^\infty \frac{dv}{|qB|}\left\{[m+(\gamma\cdot p)_\parallel][1 - isgn(q)\gamma_1\gamma_2\tanh(v)] - \frac{(\gamma\cdot p)_\perp}{cosh^2(v)}\right\} e^{-\frac{v}{|qB|}\left[m^2 - p_\parallel^2 + \frac{\tanh(v)}{v}p_\perp^2\right]}$$

Schwinger propagator, 1951

- no more translation invariance.
- the two Schwinger phases for q and $\bar{q}$ are cancelled to each other in loop calculation.

Gluon Propagator in QCD Matter

Gluon self-energy in magnetic field:



for quark loop

$$\Pi^{||}_{\mu\mu}(T,B) = g^2 T|qB| \sum_{np_z n_1} \frac{(2-\delta_{n_1 0})\,(\delta^{||}_{\mu\mu} + g^{||}_{\mu\mu})(-\omega_n^2 + p_z^2)}{(m^2 + \omega_n^2 + p_z^2 + 2n_1|qB|)^2}$$

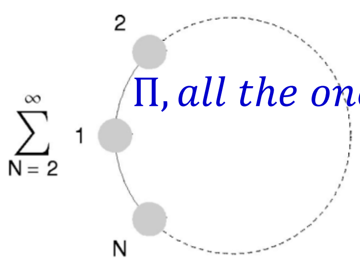
$$\Pi^{\perp}_{\mu\mu}(T,B) = 0$$

Matsubara frequencies $p_0 = i\omega_n = i(2n+1)\pi T$
quark longitudinal momentum p_z
transverse Landau energy $\varepsilon_k = 2n_1|qB|$

for gluon and ghost loops

$$\overline{\Pi}_{\mu\nu}(T,B) = \overline{\Pi}_{\mu\nu}(T,0)$$

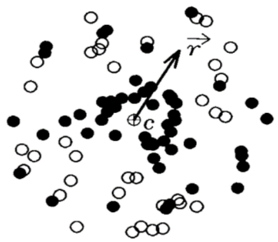
To include non-perturbative effect, we take the summation of ring diagrams
→ gluon propagator and thermodynamic potential:



$\Pi, all\ the\ one-loop\ polarization$

Color Screening in QCD Matter

Debye screening of a pair of charged particles $q\bar{q}$:



$$\frac{1}{r} \rightarrow \frac{1}{r} e^{-m_D r} = \frac{1}{r} e^{-r/r_D}$$

screening mass m_D

screening length r_D

Pole of the propagator at gluon momentum $\left(k_0^2 = 0,\ \vec{k}^2 = -m_D^2\right) \rightarrow$ *screening mass:*

$$m_D^2(T,B) = m_Q^2(T,B) + m_G^2(T),$$

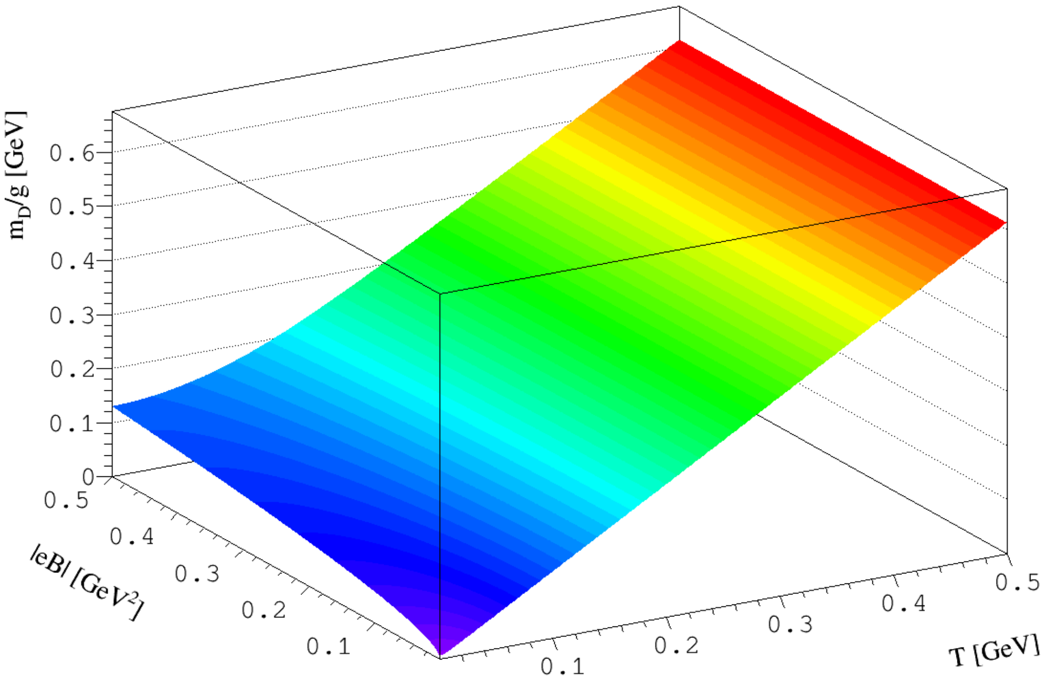
$$m_Q^2(T,B) = -\Pi_{00}^{||}(T,B),$$

$$m_G^2(T) = -\overline{\Pi}_{00}^{||}(T).$$

$$m_Q^2(T,B) = -g^2 T|qB| \sum_{n p_z n_1} \left[(2-\delta_{n_1,0}) \frac{m^2 - \omega_n^2 + p_z^2 + 2n_1|qB|}{(m^2 + \omega_n^2 + p_z^2 + 2n_1|qB|)^2}\right]$$

$$m_G^2(T) = \frac{N_c}{3} g^2 T^2$$

$m_D(T, B)$



Huang, Zhao, Zhuang, PRD107, 114035(2023)

- $m_D(T = 0, B) = 0.13$ GeV *at* $eB = 25m_\pi^2$.
- *the B effect is gradually washed out by thermal motion!*

Two time scales

- ✦ *The induced current needs time* τ_{rel} *to relax from zero to the Ohm's current*

$$\vec{J}_{Ohm} = \sigma_{el}\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- ✦ *The lifetime of the external B-field:* τ_B

A complete electromagnetic response of QGP requires

$$\tau_{rel} < \tau_B$$

For normal materials like conductor and EM plasma,

$$\tau_{rel} \ll \tau_B$$

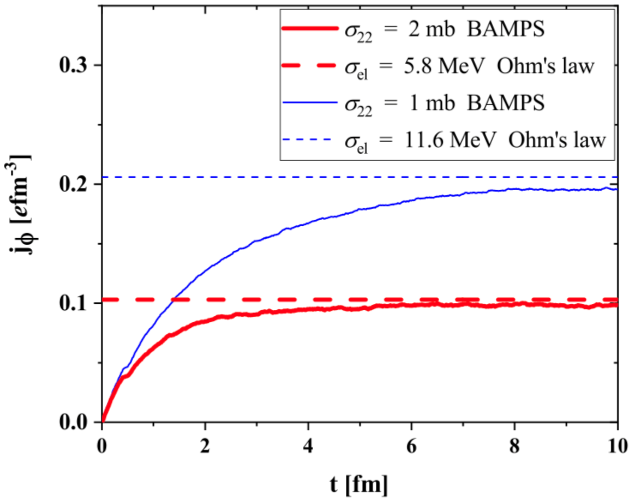
the relaxation time can be neglected.

How is about the relaxation in HIC where τ_B *and* τ_{QGP} *are very short?*

Broken Ohm's Law in HIC

The evolution of the fireball in described by a Boltzmann Approach of MultiParton Scatterings (BAMPS):

$$\left(\partial/\partial t + \vec{p}/E \cdot \vec{\nabla} + \vec{F} \cdot \vec{\nabla}_p\right) f = C_{22} + C_{23}, \qquad Lorentz\ force\ \vec{F} = q\left(\vec{p}/E \times \vec{B} + \vec{E}\right)$$



Wang, Zhao, Greiner, Xu and Zhuang,
PRC105, L041901(2022), Letter, Featured in Physics

$\vec{j} < \vec{j}_{Ohm}$, *Ohm's law is broken!*

The electromagnetic response of the hot QCD matter to the fast decay of the external electromagnetic field is incomplete, which strongly suppresses the induced magnetic field.

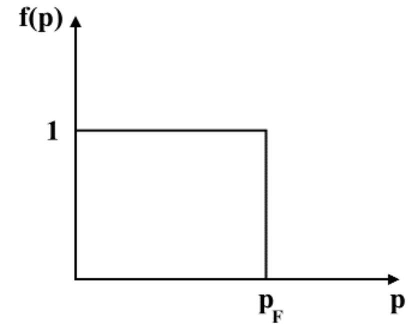
Color Screening in Dense QCD Matter

In the frame of ring diagram summation

$$m_D^2(\mu_f, B) = \frac{g^2}{(2\pi)^2} \sum_{n=0} (2 - \delta_{n0}) \frac{\mu_f |qB|}{\sqrt{\mu_f^2 - 2n|qB|}}$$

For chiral quarks, the distribution

$$f(p) = \theta(\mu_f - p)$$



the Fermi surface is determined by

$$p_z^2 + 2n|qB| = \mu_f^2$$

when μ_f and Landau levels are equal

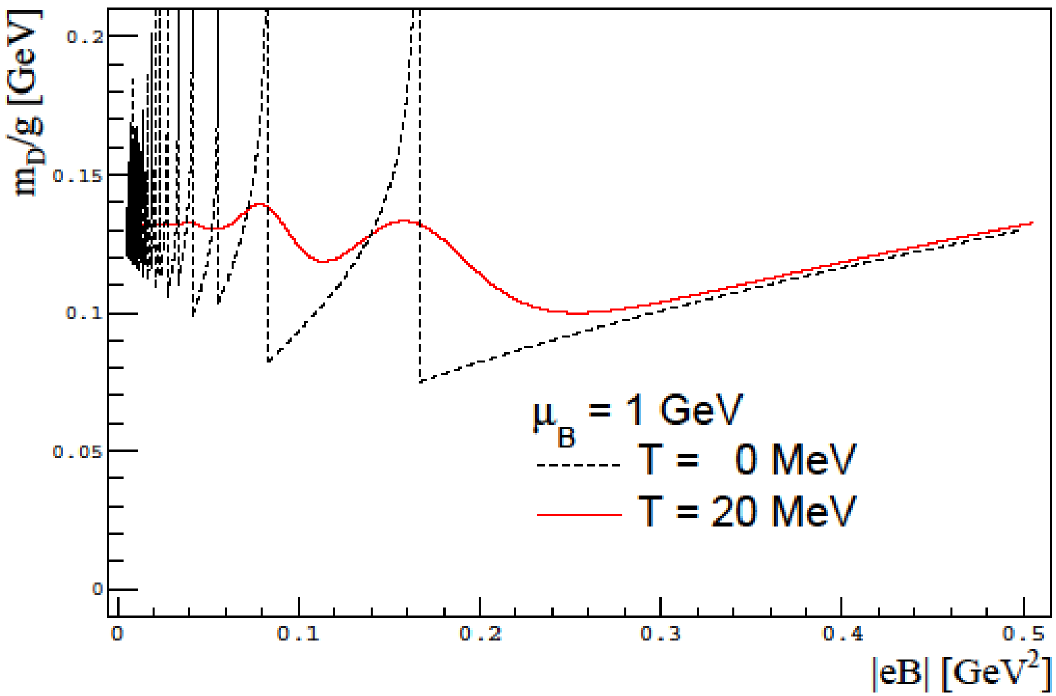
$$\mu_f^2 = 2n|qB|$$

the infrared divergence at the Fermi surface induces a complete screening $m_D \to \infty$, called resonant screening.

The case here is similar to the resonant transmission in Quantum Mechanics.

Resonant Screening

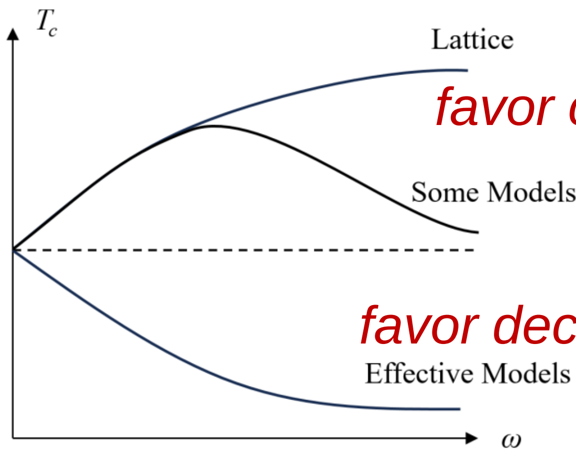
Huang, Zhao and Zhuang, PRD108, L091503(2023), Letter



The color interaction between a pair of quarks is completely screened in dense and magnetized QCD matter, when chemical potential matches the Landau levels $\mu_f^2 - 2n|qB| = 0$.

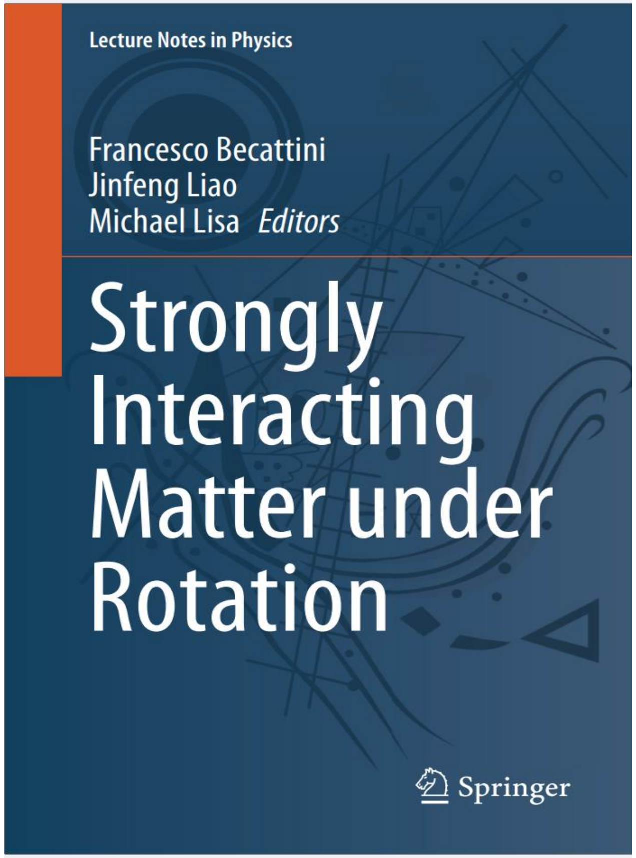
Rotation Puzzle

Does a rotation favor QCD phase transition or not ?



favor confinement

favor deconfinement



We need serious calculations in QCD.

Gluon Propagator in Curved Space ($\omega \neq 0$)

A rotating system is equivalent to a curved space,
it can be described by a transformation between the flat and curved space.

Flat space: $\bar{x}_\mu$ *Curved space:* x_μ

Transformation: $\begin{cases} t = \bar{t} \\ x = \bar{x}cos(\omega\bar{t}) - \bar{y}sin(\omega\bar{t}) \\ y = \bar{x}sin(\omega\bar{t}) + \bar{y}cos(\omega\bar{t}) \\ z = \bar{z} \end{cases}$

Gluon self-energy:

$$\Pi_{\mu\nu}^{ab}(q,q') = \int \frac{dQ_0}{2\pi}\sum_{\vec{Q}} h_\mu^A(Q|q)h_\nu^B(-Q|q')\bar{\Pi}_{AB}^{ab}(Q)$$

curved space *flat space*

$$h_\mu^A(Q|q) = \int d^4x\sqrt{-det\big(g_{\sigma\rho}(x)\big)}\frac{\partial\bar{x}^A}{\partial x^\mu}e^{i(qx-Q\bar{x})}$$

Causality and Boundary Condition

Causality:

$$\omega R < 1$$

For a rotating quark-gluon plasma created in RHIC,

$R \sim 10\ \mathrm{fm}$

$\omega \lesssim 30\ \mathrm{MeV}, \mathit{smaller\ in\ comparison\ with\ normal}\ \mu_f\ \mathit{and}\ T$

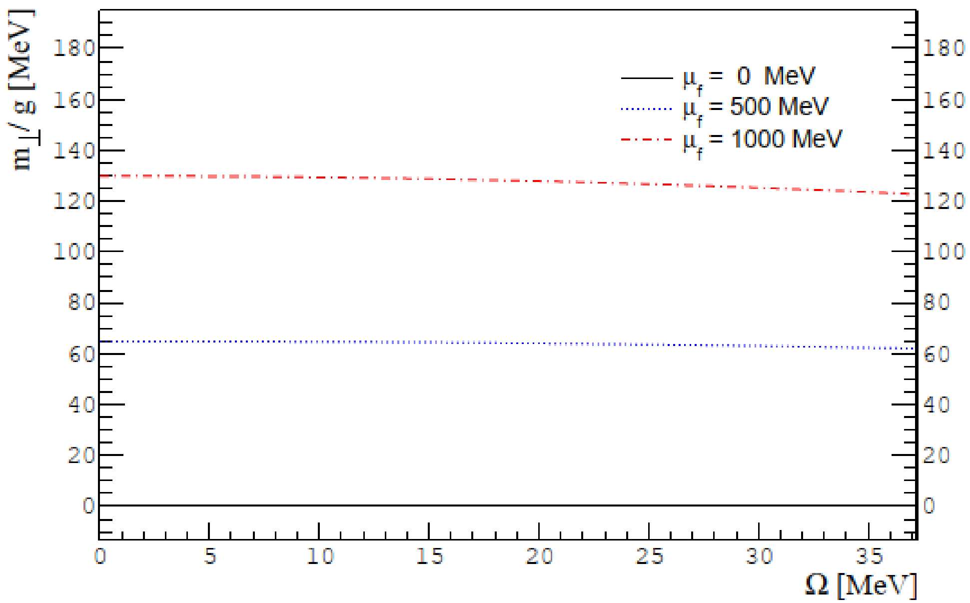
→ *Boundary condition*:

$$\vec{Q} = \frac{2\pi}{L}\vec{N}, \qquad \vec{N} = (N_1, N_2, N_3)$$

$$\vec{q} = \frac{2\pi}{L}\vec{n}, \qquad \vec{n} = (n_1, n_2, n_3)$$

Gluon Mass at Finite μ_f and ω

*Huang, Chen, Jiang, Zhao and Zhuang, arXiv 2408****



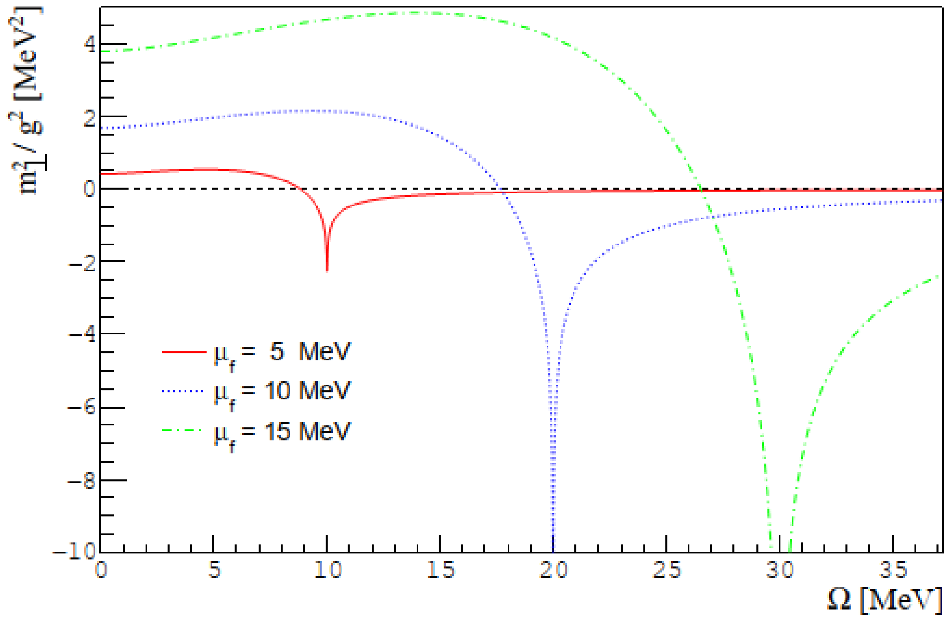
1) *m decreases with ω, it means an antiscreening effect.*
The density induced screening effect is partly cancelled by the rotation.
Rotation favors confinement.

2) *ω ($\lesssim 30\ \mathrm{MeV}$) is much smaller than μ_f ($\sim 500\ \mathrm{MeV}$), the rotation effect is weak, the maximum cancellation is $\sim 5\%$.*

3) *There is no rotation effect in vacuum $(\mu_f = 0)$.*

Rotation Effect at Small μ_f

Huang, Chen, Jiang, Zhao and Zhuang, arXiv 2408***

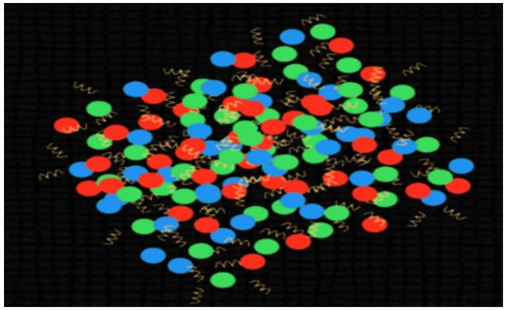


1) At small μ_f, the rotation effect becomes dominant!

2) The density effect is completely cancelled at a critical rotation $\omega_c = 2\mu_f$, and then the imaginary mass means that gluons can no longer be considered as physical particles (confinement), as suggested by Gribov.

3) At finite T, the divergence is expected to become an oscillation.

Classical Boltzmann equation



$$\left(p^\mu \partial_\mu^x + F^\mu \partial_\mu^p\right) f(x,p) = C$$

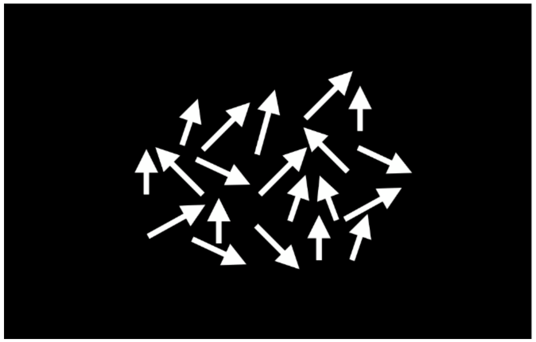
1) A single distribution (particle number) f

2) Quasi-particle (on-shell) approximation:

$$(p^2 - m^2) f(x,p) = 0 \rightarrow\ f(x,\vec{p}|p_0 = \pm\sqrt{\vec{p}^2 + m^2})$$

Spin

Many quantum anomalies in science are induced by spin, for instance CME and CVE.



Schrodinger equation	Dirac equation

Wave function:

scalar $\psi(x)$

spinor $\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$

Probability in coordinate space:

$\psi(x)\psi^*(x)$

$\psi(x)\bar{\psi}(x)$, $\quad a\ 4 \times 4\ matrix$

Probability in phase space (Wigner transformation):

$$f(x,p) = \int d^4 y e^{ipy} \psi(x + y/2)\psi^*(x - y/2)$$

$$W(x,p) = \int d^4 y e^{ipy}\, \psi(x + y/2)\bar{\psi}(x - y/2)$$

$Wigner\ function\ with\ 16\ distributions$

Beyond quasi-particle

Particles in medium are in general not quasi-particles (mean field approximation), especially in high energy physics.

1) On-shell → Off-shell constraint:

$$(p^2 - m^2) f(x,p) = 0 \rightarrow [(p^2 - m^2) + \hbar \mathcal{A}(p)\ ] W(x,p) = 0$$

*2) Physics distributions $W(x,\vec{p})$ are defined in **7D** phase space*
→ Equal-time formalism

$$W(x,p) \rightarrow W(x,\vec{p}) = \int dp_0 W(x,p)$$

Covariant and Equal-time Wigner functions

- Covariant Wigner operator for fermions interacting with a gauge field:

$$\widehat{W}(x,p) = \int d^4y e^{ipy} \hat{\psi}(x+\frac{y}{2}) e^{iq\int_{-1/2}^{1/2} ds \hat{A}(x+sy)y} \hat{\bar{\psi}}(x-\frac{y}{2})$$

gauge link $e^{iq\int_{-1/2}^{1/2} ds \hat{A}(x+sy)y}$ to guarantee gauge invariance

Ensemble average → Wigner function

$$W(x,p) = \langle \widehat{W}(x,p) \rangle \qquad (W = \widehat{W} \; in \; quantum \; mechanics)$$

- Dyson-Schwinger equation for quantum fields $\hat{\psi}$ or Dirac equation for wave function ψ

→ kinetic equations for $W(x,p)$

QED: D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)
QCD: H.-Th.Elze and U.Heinz, Phys. Rep. 183, 81(1989)

- Problem:

Initial $W(x,p)$ is related to the fields $\hat{\psi}(x)$ and $\hat{A}(x)$ at all times (due to $\int_{-\infty}^{\infty} dy_0$)
→ Causality problem!

Physics distributions are defined through Equal-time Wigner function

$$W_0(x,\vec{p}) = \int d^3\vec{y} e^{-i\vec{p}\cdot\vec{y}} \left\langle \hat{\psi}(x+\vec{y}/2) e^{-iq\int_{-1/2}^{1/2} ds \hat{\vec{A}}(x+s\vec{y})\cdot\vec{y}} \hat{\psi}^+(x-\vec{y}/2) \right\rangle$$

Dirac-Heisenberg-Wigner equation

Bialynicki-Birula, Gornicki and Rafelski, PRD44, 1825(1991)

- *Dirac equation in coordinate space:*

$$\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi(x) = 0$$

→ *DHW equation in phase space:*

$$D_t W_0 = -\frac{1}{2}\vec{D}\cdot\{\rho_1\vec{\sigma}, W_0\} - \frac{i}{\hbar}\left[\rho_1\vec{\sigma}\cdot\vec{P} + \rho_3 m, W_0\right]$$

$$D_t = \partial_t + q\int_{-1/2}^{1/2} ds\vec{E}(\vec{x} + is\hbar\vec{\nabla}_p)\cdot\vec{\nabla}_p,$$
$$\vec{D} = \vec{\nabla} + q\int_{-1/2}^{1/2} ds\vec{B}(\vec{x} + is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$
$$\vec{P} = \vec{p} - iq\hbar\int_{-1/2}^{1/2} dss\vec{B}(\vec{x} + is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$

- *However,*

$$W_0(x,\vec{p}) = \int dp_0 W(x,p)\gamma_0$$

is not equivalent to $W(x,p)$. We should consider all the moments

$$W_n(x,\vec{p}) = \int dp_0 p_0^n W(x,p)\gamma_0 \qquad (n = 0,1,2,\dots)$$

Zhuang and Heinz, Ann.Phys.245, 311(1996)

Only for quasi-particles (on-shell, $p^2 - m^2 = 0,\ p_0 = \pm\sqrt{m^2 + \vec{p}^2}$),

$$W_n(x,\vec{p}) = E_p^n W_0(x,\vec{p}),$$

$W_0(x,\vec{p})$ can completely describe the system.

Equal-time hierarchy I

Zhuang and Heinz, PRD57, 6525(1998)

1) Covariant kinetic equations

$$\left(\gamma^\mu K_\mu - m\right)W = 0$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2}D_\mu \qquad \Pi_\mu = p_\mu - iq\hbar\int_{-1/2}^{1/2} dss F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu$$

$$D_\mu = \partial_\mu - q\int_{-\frac{1}{2}}^{\frac{1}{2}} ds F_{\mu\nu}\left(x - i\hbar s\partial_p\right)\partial_p^\nu$$

Constraint (with p_μ) and transport (with ∂_μ) equations

$$\begin{cases} \left[\gamma^\mu\left(K_\mu + K_\mu^\dagger\right) - 2m\right]W(x,p) = 0 \\ \gamma^\mu\left(K_\mu - K_\mu^\dagger\right)W(x,p) = 0 \end{cases}$$

2) Equal-time hierarchy

$$\int dp_0[covariant\ constraint\ and\ transport\ equations] \rightarrow$$

$$\begin{cases} Transport\ equations\ for\ W_0(x,\vec{p}) & \rightarrow\ DHW\ equation \\ Constraint\ equation\ for\ W_1(x,\vec{p}) \end{cases}$$

$$\int dp_0\, p_0 \cdot [covariant\ constraint\ and\ transport\ equations] \rightarrow$$

$$\begin{cases} Transport\ equations\ for\ W_1(x,\vec{p}) \\ Constraint\ equation\ for\ W_2(x,\vec{p}) \end{cases}$$

Equal-time hierarchy II

Zhuang and Heinz, PRD57, 6525(1998)

3) *Spin decomposition*

$$W_0(x,\vec{p}) = \frac{1}{4}[f_0 + \gamma^5 f_1 - i\gamma^0\gamma^5 f_2 + \gamma^0 f_3 + \gamma^5\gamma^0\vec{\gamma}\cdot\vec{g}_0 + \gamma^0\vec{\gamma}\cdot\vec{g}_1 - i\vec{\gamma}\cdot\vec{g}_2 - \gamma^5\vec{\gamma}\cdot\vec{g}_3]$$

D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)

Conservation laws → Physics of the spin components

f_0 : *number density*, $\vec{g}_0$: *spin density*

f_1 : *helicity density* , f_2 : *topologic charge density*, f_3 : *mass density*

$\vec{g}_1$: *number current*, $\vec{g}_3$: *magnetic moment*

4) *Truncating the hierarchy*

spin 1/2 particles: W_0 and W_1 form a closed subgroup

spin 0 particles: W_0, W_1 and W_2 form a closed subgroup

Spin distribution at lowest level

Zhuang and Heinz, PRD53, 2096(1996)

$$W_0(x,\vec{p}) = W_0^{(0)}(x,\vec{p}) + \hbar W_0^{(1)}(x,\vec{p}) + \cdots$$

- *Constraint equations → only 4 independent components:*

  number density f_0 and spin density $\vec{g}_0$

- *Boltzmann equation for number density f_0:*

$$\left(D_t + \frac{\vec{p}}{E_p}\cdot\vec{D}\right) f_0 = 0$$

$$D_t = \partial_t + q\vec{E}\cdot\vec{\nabla}_p, \qquad \vec{D} = \vec{\nabla} + q\vec{B}\times\vec{\nabla}_p$$

- *Bargmann-Michel-Telegdi equation for spin density $\vec{g}_0$:*

$$\left(D_t + \frac{\vec{p}}{E_p}\cdot\vec{D}\right)\vec{g}_0 = \frac{q}{E_p^2}\left[\vec{p}\times\left(\vec{E}\times\vec{g}_0\right) - E_p\vec{B}\times\vec{g}_0\right]$$

the phase-space version of the Bargmann-Michel-Telegdi equation

V.Bargmann, L.Michel and V.Telegdi, PRL2, 435(1959)

Quark Matter in a Rotation Field

Chen and Zhuang, CPC (2022)

Dirac equation:

$$\left(i\gamma^\mu\partial_\mu - m + \gamma_0\vec{\omega}\cdot\hat{\vec{j}}\right)\psi(x) = 0$$

Covariant Wigner function:

$$W(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{ip\cdot y}\left\langle\psi(x+\frac{y}{2})\bar{\psi}(x-\frac{y}{2})\right\rangle$$

Covariant kinetic equation:

$$\left(\gamma^\mu K_\mu + \frac{\hbar}{2}\gamma^5\gamma^\mu\omega_\mu - m\right)W(x,p) = 0$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2}D_\mu, \qquad \omega_\mu = (0,\vec{\omega})$$

$$\Pi_\mu = (p_0+\pi_0,\vec{p}), \qquad \pi_0 = \vec{\omega}\cdot\left(\vec{l} + \frac{\hbar^2}{4}\vec{\nabla}\times\vec{\nabla}_p\right) + \mu_B$$

$$D_\mu = \left(d_t,\vec{\nabla}\right), \qquad d_t = \partial_t - \vec{\omega}\cdot\left(\vec{x}\times\vec{\nabla} + \vec{p}\times\vec{\nabla}_p\right)$$

Classical Transport in a Rotation Field

At order $\hbar^0$, only f_0 (number distribution) and $\vec{g}_0$ (spin distribution) are independent components.

Chen and Zhuang, CPC (2022)

16 transport equations are reduced to

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\nabla - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\nabla_p\right] f_0^{(0)\pm} = 0,$$

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\nabla - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\nabla_p\right] \mathbf{g}_0^{(0)\pm} = -\boldsymbol{\omega}\times\mathbf{g}_0^{(0)\pm}.$$

Coriolis force *spin-rotation coupling*

16 constraint equations are reduced to

$$on-shell\ energy \quad E_p^{\pm} = \pm\varepsilon_p - (\vec{\omega}\cdot\vec{l} + \mu_B), \qquad \varepsilon_p = \sqrt{m^2+p^2}$$

$$f_1^{(0)\pm} = \pm\frac{1}{\epsilon_p}\boldsymbol{p}\cdot\mathbf{g}_0^{(0)\pm},$$

$$f_2^{(0)\pm} = 0,$$

$$f_3^{(0)\pm} = \pm\frac{m}{\epsilon_p} f_0^{(0)\pm},$$

$$\mathbf{g}_1^{(0)\pm} = \pm\frac{\boldsymbol{p}}{\epsilon_p} f_0^{(0)\pm},$$

$$\mathbf{g}_2^{(0)\pm} = \frac{1}{m}\boldsymbol{p}\times\mathbf{g}_0^{(0)\pm},$$

$$\mathbf{g}_3^{(0)\pm} = \pm\frac{1}{m\epsilon_p}\left[\epsilon_p^2\mathbf{g}_0^{(0)\pm} - \boldsymbol{p}(\boldsymbol{p}\cdot\mathbf{g}_0^{(0)\pm})\right].$$

One can systematically calculate the higher order contributions.

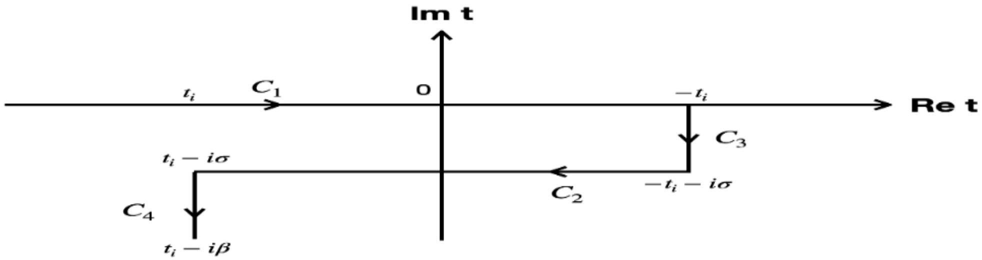
Kadanoff-Baym equations

Dyson-Schwinger equation

$$S(x,y)=S^0(x,y)+\int d^4z d^4w S^0(x,w)\Sigma(w,z)S(z,y)$$

Considering the time order in $W(x,p)=\int d^4y e^{ipy}\,\psi\left(x+\frac{y}{2}\right)\bar{\psi}\left(x-\frac{y}{2}\right)$

→ Schwinger-Keldish time contour



$S^<(x,p)=W(x,p)$, $\qquad S^>(x,p)$

Constraint and transport equations with collision terms

$$\{(\gamma^\mu p_\mu - m), S^<\} + \frac{i\hbar}{2}[\gamma^\mu, \nabla_\mu S^<] = \frac{i\hbar}{2}([\Sigma^<, S^>]_* - [\Sigma^>, S^<]_*)$$

$$[(\gamma^\mu p_\mu - m), S^<] + \frac{i\hbar}{2}\{\gamma^\mu, \nabla_\mu S^<\} = \frac{i\hbar}{2}(\{\Sigma^<, S^>\}_* - \{\Sigma^>, S^<\}_*)$$

$$A * B = AB + \frac{i\hbar}{2}[AB]_{P.B.} + \mathcal{O}(\hbar^2)$$

including spin: Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612

- *Spin decomposition for Wigner function, self-energy and collision term*

General collision terms

Wang, Guo and Zhuang, EPJC, (2021)

- **0th order transport**

描述相对论重离子碰撞的许多模型，例如URQMD, AMPT, BAMPS.

$$\widehat{FG} = \bar{F}G - F\bar{G}$$

$$p\cdot\nabla\mathcal{V}_\mu^{(0)} = m\widehat{\Sigma_S^{(0)}\mathcal{V}_\mu^{(0)}} + p^\nu\widehat{\Sigma_{V\nu}^{(0)}\mathcal{V}_\mu^{(0)}} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma_T^{(0)\alpha\beta}\mathcal{A}^{(0)\lambda}} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma_T^{(0)\alpha\nu}\mathcal{A}^{(0)\lambda}} - p_\mu\widehat{\Sigma_A^{(0)\nu}\mathcal{A}_\nu^{(0)}},$$

$$p\cdot\nabla\mathcal{A}_\mu^{(0)} = m\widehat{\Sigma_S^{(0)}\mathcal{A}_\mu^{(0)}} + p^\nu\widehat{\Sigma_{V\nu}^{(0)}\mathcal{A}_\mu^{(0)}} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma_T^{(0)\alpha\beta}\mathcal{V}^{(0)\lambda}} + \widehat{\Sigma_{A\mu}^{(0)}p^\nu\mathcal{V}_\nu^{(0)}} - p_\mu\widehat{\Sigma_{A\nu}^{(0)}\mathcal{V}^{(0)\nu}},$$

Local collision term
Dynamical effect ,
e.g. diffusion effect

- **1st order transport**

$$\begin{aligned}
p\cdot\nabla\mathcal{V}_\mu^{(1)} =& + m\widehat{\Sigma_S^{(0)}\mathcal{V}_\mu^{(1)}} + p^\nu\widehat{\Sigma_{V\nu}^{(0)}\mathcal{V}_\mu^{(1)}} - p_\mu\widehat{\Sigma_A^{(0)\nu}\mathcal{A}_\nu^{(1)}} - \frac{p_\nu}{m}\epsilon_{\rho\sigma\alpha\mu}p^\rho\widehat{\Sigma_T^{(0)\alpha\nu}\mathcal{A}^{(1)\sigma}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma_T^{(0)\sigma\nu}\mathcal{A}^{(1)\lambda}} \\
& + m\widehat{\Sigma_S^{(1)}\mathcal{V}_\mu^{(0)}} + p^\nu\widehat{\Sigma_{V\nu}^{(1)}\mathcal{V}_\mu^{(0)}} - p_\mu\widehat{\Sigma_A^{(1)\nu}\mathcal{A}_\nu^{(0)}} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma_T^{(1)\alpha\nu}\mathcal{A}^{(0)\lambda}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma_T^{(1)\sigma\nu}\mathcal{A}^{(0)\lambda}} \\
& + \frac{1}{2m}p^\nu[\widehat{\Sigma_{T\mu\nu}^{(0)}(p^\alpha\mathcal{V}_\alpha^{(0)})}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma_{T\mu\nu}^{(0)}\mathcal{V}^{(0)\nu}}]_{\text{P.B.}} + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu[\widehat{\Sigma_A^{(0)\alpha}\mathcal{V}^{(0)\beta}}]_{\text{P.B.}} \\
& - \frac{1}{2m}p_\nu\Sigma_{T\alpha\mu}^{(0)}\widehat{\nabla^{[\alpha}\mathcal{V}^{(0)\nu]}} + \frac{1}{2m}p_\nu\widehat{\Sigma_T^{\alpha\nu(0)}\nabla_{[\alpha}\mathcal{V}_{\mu]}^{(0)}} + \frac{1}{2}\epsilon_{\beta\nu\lambda\mu}\widehat{\Sigma_A^{(0)\beta}\nabla^\nu\mathcal{V}^{(0)\lambda}} \\
& + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\widehat{\nabla^\alpha\Sigma_V^{(0)\nu})\mathcal{A}^{(0)\beta}} - \frac{1}{2m}p_\mu(\widehat{\nabla^\nu\Sigma_P^{(0)})\mathcal{A}_\nu^{(0)}} - \frac{1}{2m}(p^\nu\widehat{\nabla_\nu\Sigma_P^{(0)})\mathcal{A}_\mu^{(0)}} + \frac{p^\nu}{2m}\epsilon_{\mu\nu\alpha\beta}(\widehat{\nabla^\alpha\Sigma_S^{(0)})\mathcal{A}^{(0)\beta}},
\end{aligned}$$

$$\begin{aligned}
p\cdot\nabla\mathcal{A}_\mu^{(1)} =& + m\widehat{\Sigma_S^{(0)}\mathcal{A}_\mu^{(1)}} + p^\nu\widehat{\Sigma_{V\nu}^{(0)}\mathcal{A}_\mu^{(1)}} + p^\nu\widehat{\Sigma_{A\mu}^{(0)}\mathcal{V}_\nu^{(1)}} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma_T^{(0)\alpha\beta}\mathcal{V}^{(1)\lambda}} - p_\mu\widehat{\Sigma_{A\nu}^{(0)}\mathcal{V}^{(1)\nu}} \\
& + m\widehat{\Sigma_S^{(1)}\mathcal{A}_\mu^{(0)}} + p^\nu\widehat{\Sigma_{V\nu}^{(1)}\mathcal{A}_\mu^{(0)}} + p^\nu\widehat{\Sigma_{A\mu}^{(1)}\mathcal{V}_\nu^{(0)}} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma_T^{(1)\alpha\beta}\mathcal{V}^{(0)\lambda}} - p_\mu\widehat{\Sigma_{A\nu}^{(1)}\mathcal{V}^{(0)\nu}} \\
& - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}(\widehat{\nabla^\sigma\Sigma_V^{(0)\nu})\mathcal{V}^{(0)\rho}} - \frac{m}{2}[\widehat{\Sigma_P^{(0)}\mathcal{V}_\mu^{(0)}}]_{\text{P.B.}} + \frac{1}{2m}p_\mu[\widehat{\Sigma_P^{(0)}(p^\nu\mathcal{V}_\nu^{(0)})}]_{\text{P.B.}} \\
& + \frac{1}{2}\epsilon_{\mu\sigma\nu\rho}\nabla^\sigma\widehat{\Sigma_A^{(0)\nu}\mathcal{A}^{(0)\rho}} + \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}[\widehat{\Sigma_A^{(0)\nu}(p^\alpha\mathcal{A}^{(0)\beta})}]_{\text{P.B.}} - \frac{m}{2}[\widehat{\Sigma_{T\mu\nu}^{(0)}\mathcal{A}^{(0)\nu}}]_{\text{P.B.}} + \frac{p_\mu}{2m}[\widehat{\Sigma_{T\rho\nu}^{(0)}(p^\rho\mathcal{A}^{(0)\nu})}]_{\text{P.B}} \\
& - \frac{1}{2m}p_\sigma\nabla^\sigma(\widehat{\Sigma_{T\mu\nu}^{(0)}\mathcal{A}^{(0)\nu}}) + \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma_{T\mu\nu}^{(0)}\mathcal{A}_\sigma^{(0)}}) + \frac{1}{2m}p_\mu\nabla^\sigma(\widehat{\Sigma_{T\sigma\nu}^{(0)}\mathcal{A}^{(0)\nu}}) - \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma_{T\sigma\nu}^{(0)}\mathcal{A}_\mu^{(0)}}).
\end{aligned}$$

- Nonlocal collision term
- Related to spatial derivatives
- Correlated transport of V&A
- Polarization can be generated in a initially unpolarized syste

the interaction needs to be specified to calculate the off-diagonal self-energy $\Sigma^>$ & $\Sigma^<$

Near equilibrium state

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

- *Collision is the driving force for the system to go from non-equilibrium to equilibrium.*

- *Local equilibrium state is determined by detailed balance principle (loss term and gain term cancel to each other).*

- *Global equilibrium state is determined by detailed balance + Killing condition*

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0, \qquad \beta_\mu = u_\mu / T$$

- *Near-equilibrium state can be described by relaxation time approximation (RTA).*

Relaxation Time Approximation (RTA)

Classical RTA: *[J.Anderson and H.Witting, Physica 74, 466(1974)]*

$$p^\mu \partial_\mu f = -p^\mu u_\mu \frac{\delta f}{\tau}, \qquad \delta f = f - f_{eq}$$

Quantum RTA:

$$\left(\gamma^\mu p_\mu - m\right) W + \frac{i\hbar}{2}\gamma^\mu \partial_\mu W = -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta W}{\tau}, \qquad \delta W = W - W_{eq}$$

A single time scale to control the relaxation process for all the components.

Motivations:

1) A multi-component kinetic theory needs more time scales to control different degrees of freedom.

2) Calculating the relaxation times in Kadanoff-Baym formalism.

RTA from Kadanoff-Baym equations

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

Calculating damping and correlation times for different degrees of freedom with Kadanoff-Baym equations.

Method:

1) Quantum kinetic equations in near equilibrium state:

$$W = W_{eq} + \delta W$$

2) Expanding collision terms around the equilibrium state:

$$C(\Sigma, W) = C'(\Sigma_{eq}, W_{eq})\delta W$$

Note: $C\left(\Sigma_{eq}, W_{eq}\right) = 0 \; in \; equilibrium \; state$

Kadanoff-Baym RTA:

Chiral fermions: $f_\pm = \frac{1}{2}(f_V \pm f_A)$

$$\begin{cases} p^\mu \partial_\mu f_+ = -\frac{\delta f_+}{\tau_+} - \frac{\delta f_+ - \delta f_-}{\tau_{+-}} \\ p^\mu \partial_\mu f_- = -\frac{\delta f_-}{\tau_-} - \frac{\delta f_- - \delta f_+}{\tau_{+-}} \end{cases}$$

$\tau_\pm$: $damping \; time \; for \; chiral \; fermions$

$\tau_{\pm-}$: $correlation \; time \; between \; two \; kinds \; of \; fermions$