

壳模型 配对近似理论

赵玉民

上海交通大学

关键词：壳模型、组态、相互作用、配对

$$\hat{H}\Psi = E\Psi$$

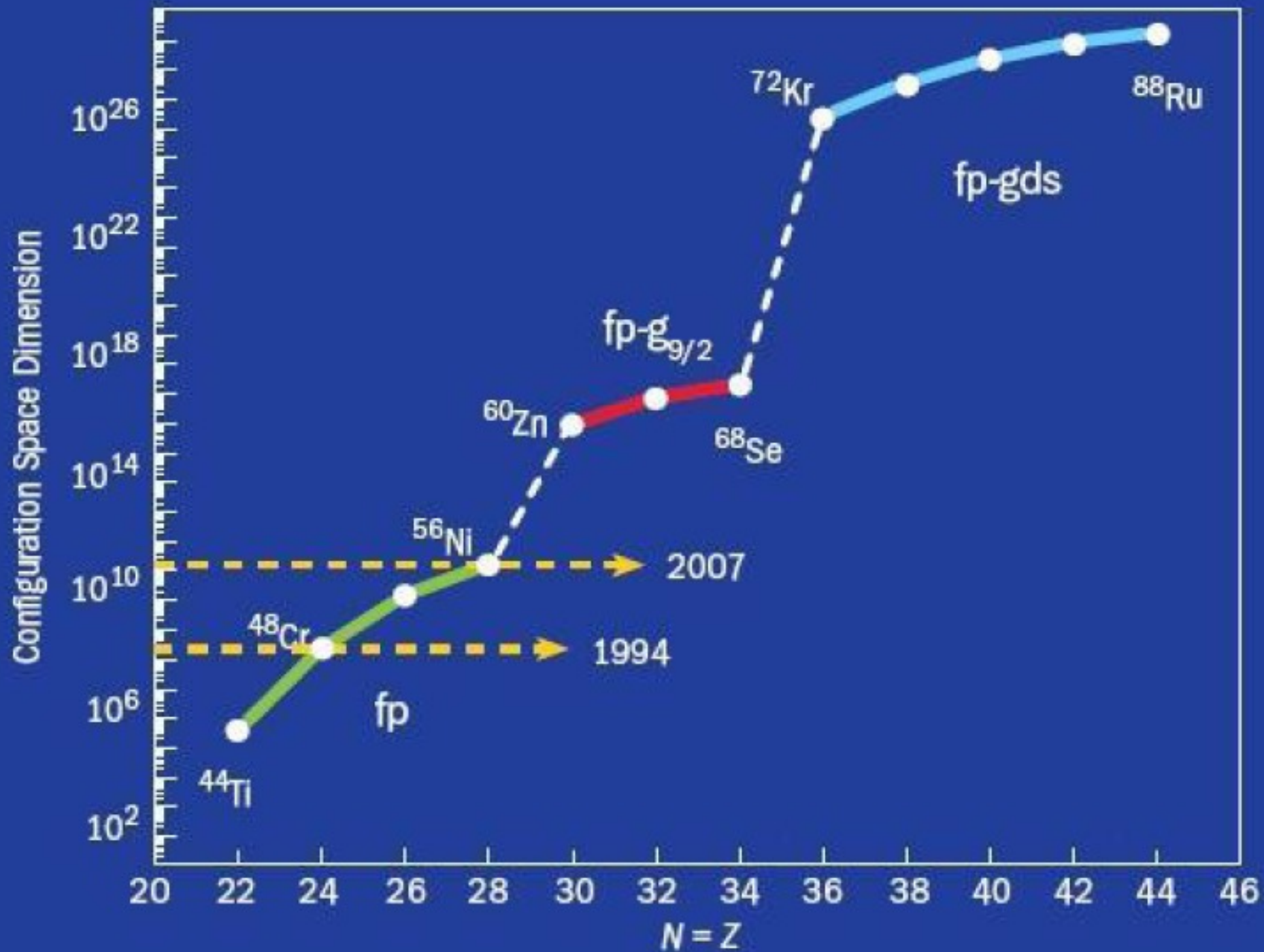
Ψ 是许多基矢量的叠加，基矢空间称为组态空间(重质量核爆炸式增长)

原子核的组态-相互作用壳模型：组态空间大、剩余相互作用复杂、

配对是一种空间截断方案，即留下重要组态、忽略不重要的组态

实际上原子核配对的计算过程也比较复杂... ..

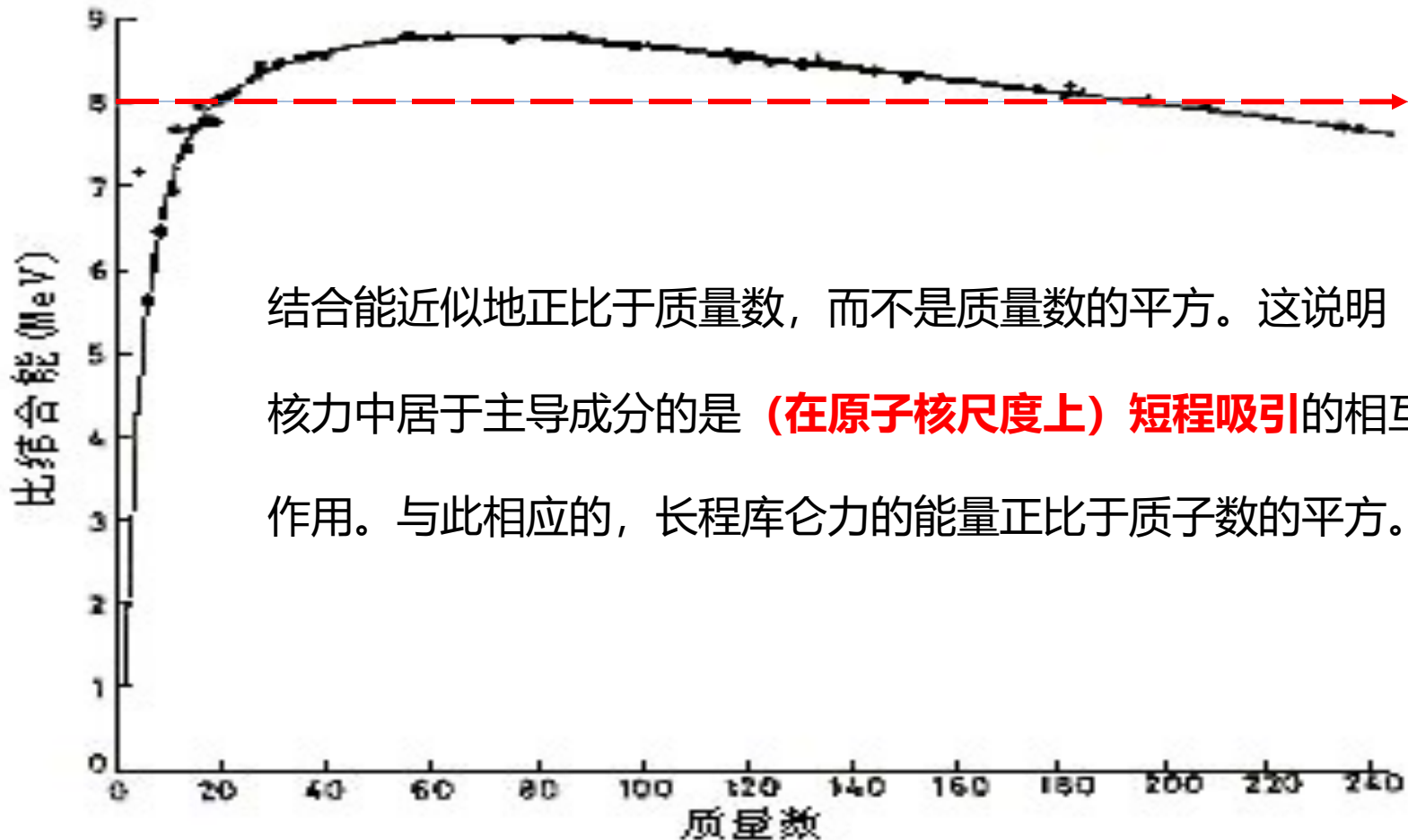
所以人们才做玻色子近似 (the interacting boson model)



Outline

- Part - 1: 配对理论框架
- Part - 2: 配对理论的进展
- Part - 3: 近期具体应用方面的尝试
- Part - 5: 总结

短程吸引的核力



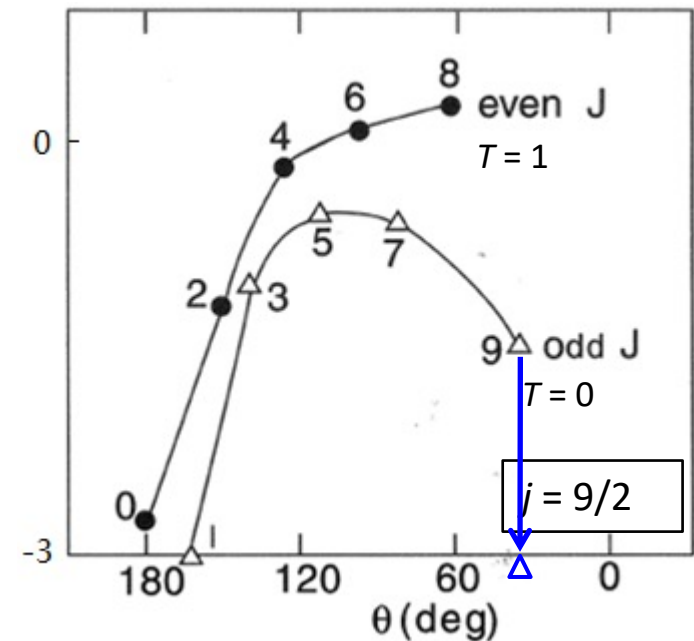
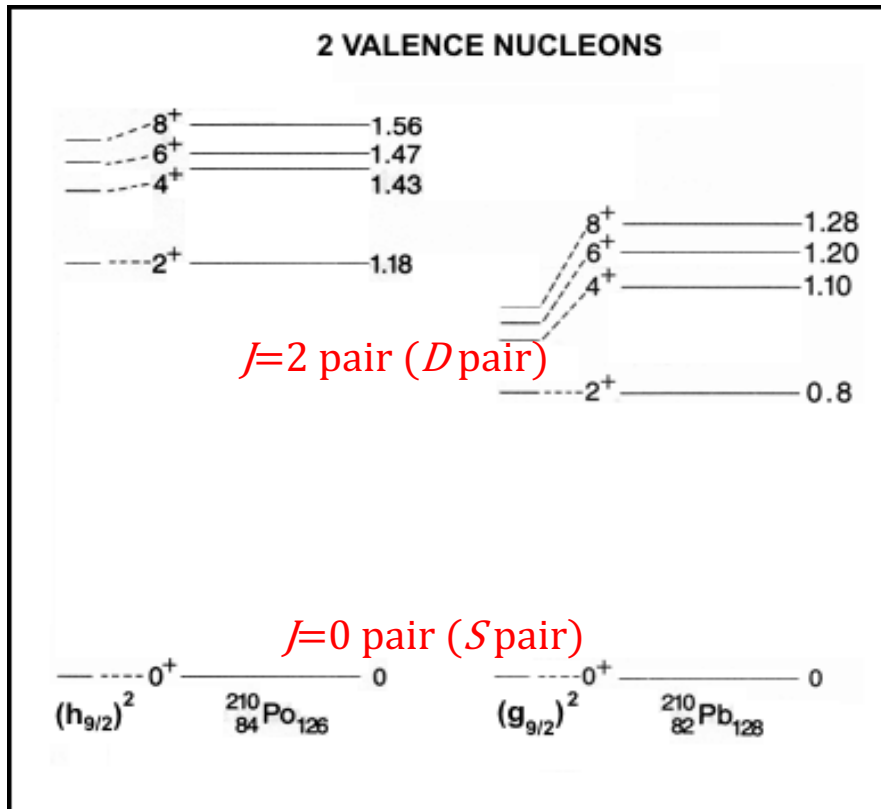
结合能近似地正比于质量数，而不是质量数的平方。这说明核力中居于主导成分的是 **(在原子核尺度上) 短程吸引** 的相互作用。与此相应的，长程库仑力的能量正比于质子数的平方。

简单图像

Nuclear force: attractive and short range
 δ Interaction

$$V_{12}(\delta) = -V_0 \delta(r_1 - r_2)$$

$$= \frac{-V_0}{r_1 - r_2} \delta(r_1 - r_2) \delta(\cos \theta_1 - \cos \theta_2) \delta(\Phi_1 - \Phi_2)$$



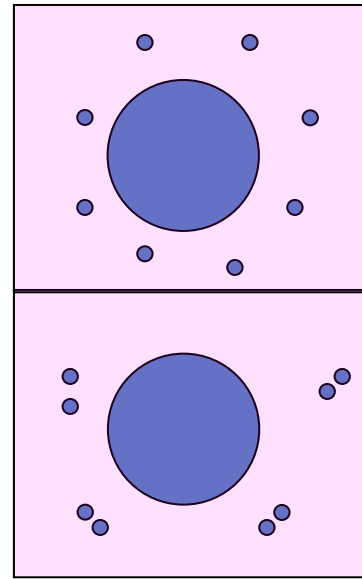
M. G. Mayer, Phys. Rev. 78, 22(1950).

简单图像

Shell-model basis: $|\varphi\rangle = \left(\cdots \left((C_{j_1}^+ \times C_{j_2}^+)^{J_2} \times C_{j_3}^+ \right)^{J_3} \cdots \times C_{j_n}^+ \right)_{M_n}^{J_n} |0\rangle$

nucleon pair: $A^{(r)+} = \sum_{ab} y(abr) A^{(r)+}(ab), \quad A^{(r)+}(ab) = (C_a^+ \times C_b^+)^{(r)}$

pair basis: $|\varphi\rangle = \left(\cdots \left((A^{(r_1)+} \times A^{(r_2)+})^{(J_2)} \times A^{(r_3)+} \right)^{(J_3)} \cdots \times A^{(r_N)+} \right)_{M_N}^{(J_N)} |0\rangle.$



如果考虑所有可能的配对方式, the NPA = the SM

哈密顿量和跃迁算符与壳模型相同。

$$\hat{H}\Psi = E \Psi$$

如何计算配对基矢下的各种矩阵元

$$\langle jr_1 \cdots r_N, J_1 \cdots J_N | V_P^{(s)} | j_0 s_1 \cdots s_N, J'_1 \cdots J'_N \rangle$$

$$A_{M_{J_N}}^{J_N \dagger}(jr_1 r_2 \cdots r_N, J_1 J_2 \cdots J_N) | 0 \rangle \equiv A_{M_{J_N}}^{J_N \dagger} | 0 \rangle = \left(\cdots \left(\left(\left(C_j^\dagger \times A^{r_1 \dagger} \right)^{(J_1)} \times A^{r_2 \dagger} \right)^{(J_2)} \times A^{r_3 \dagger} \right)^{(J_3)} \times \cdots \times A^{r_N \dagger} \right)^{(J_N)} | 0 \rangle$$

配对理论原则上干的就是这个计算，而以上保持角动量的方法、称为 J-scheme

壳模型中 M-scheme versus J-scheme

M-scheme 中的价核子波函数不用角动量耦合，直接在非耦合表象中做对角化。多体波函数对应的角动量则通过计算 J^2 而给出。

J-scheme 中的每个基矢态都有确切的角动量，基矢态构造方式有多种途径，不同用处的构造方式不同。

这两种多体理论方法各有优缺点！各领风骚几十年。总体而言，J-scheme 的需要处理的矩阵维数可以小若干个数量级；因为计算物理方面的发展，M-scheme 矩阵元计算和对角化效率更高。

这些方法的各种讨论见教科书和综述文献。

配对理论一直以来就有 M-scheme 和 J-scheme 途径，不过新的发展是同时具有这两种途径的优点，没有各自的缺点。

M-scheme 配对理论的做法

$$\begin{aligned}
 \langle 0 | \hat{P}_1 \cdots \hat{P}_N | \hat{A}^\dagger \hat{B} | \hat{\mathbb{P}}_1^\dagger \cdots \hat{\mathbb{P}}_N^\dagger | 0 \rangle &= \langle 0 | [\hat{P}_1 \cdots \hat{P}_N, \hat{A}^\dagger \hat{B}] \hat{\mathbb{P}}_1^\dagger \cdots \hat{\mathbb{P}}_N^\dagger | 0 \rangle \\
 &= - \sum_{k=1}^N \frac{1}{2} \text{tr}(p_A p_k) \langle 0 | \hat{P}_1 \cdots \hat{P}_{k-1} \hat{B} \hat{P}_{k+1} \cdots \hat{P}_N | \hat{\mathbb{P}}_1^\dagger \cdots \hat{\mathbb{P}}_N^\dagger | 0 \rangle \\
 &\quad + \sum_{k=2}^N \sum_{i=1}^k \langle 0 | \hat{P}_1 \cdots \hat{P}_{i-1} \hat{\mathcal{P}}_{i,k} \hat{P}_{i+1} \cdots \hat{P}_{k-1} \hat{B} \hat{P}_{k+1} \cdots \hat{P}_N | \hat{\mathbb{P}}_1^\dagger \cdots \hat{\mathbb{P}}_N^\dagger | 0 \rangle.
 \end{aligned}$$

M-scheme 的做法在壳模型里比较成熟，在配对理论上因为涉及到结构系数的变化和传递，**可以做、但是一直没有太好**。例如长期以来，日本同行的做法直接用M-scheme 的壳模型程序，效率上一般，没有听说大的进展；欧洲同行两种方法都有人用，要么M-scheme、要么J-scheme。

在90年代之前

Seniority scheme: 辛弱数(曾谨言书)

原子物理电子态(1943年 Racah)

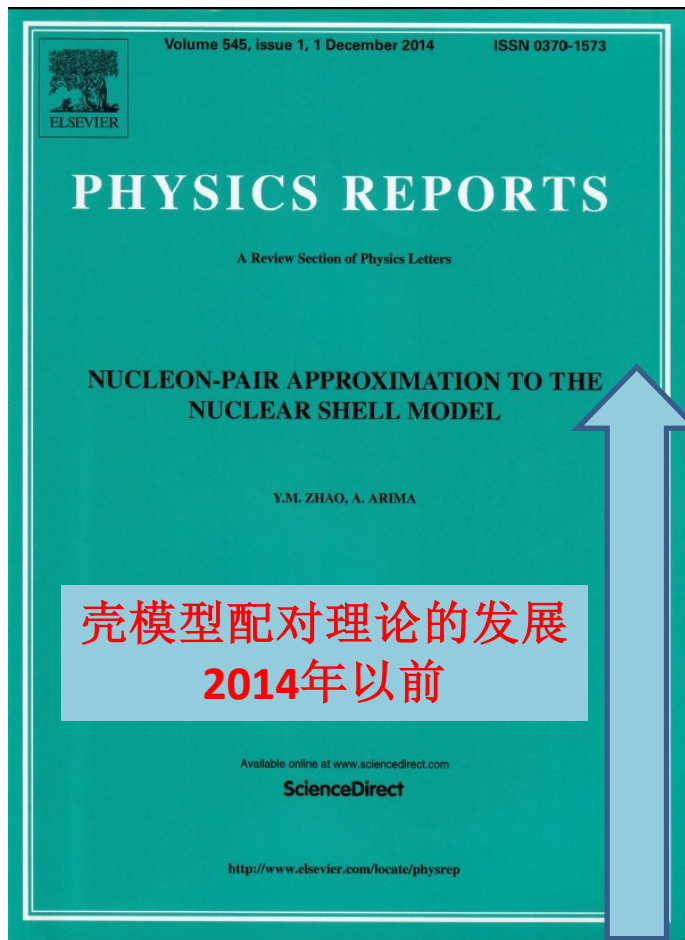
单轨道同类核子配对理论, 准自旋理论 [特别漂亮的形式理论] (1952年)

推广的辛弱数 (generalized seniority) : 七十年代至今

破缺配对近似 (broken pair approximation): 七十和八十年代 欧美流行

费米子动力学对称模型 (Fermion Dynamical Symmetry Model) 八十到九十年代

玻色子模型微观基础的研究



壳模型配对理论的发展 2014年以前

壳模型配对理论的发展 2014年以后

自1949年建立壳模型，人们一直不懈努力，发展壳模型配对近似理论，已经 70 多年

1952年 单轨道 S 配对理论(Racah): 辛弱数

1971年 多轨道 S 配对理论 (Talmi): 推广的辛弱数 (seniority)

1980年代 考虑1个到2个配对破缺贡献的理论 (Allart 等, 破缺配对理论)

1982-87年 带有对称性的配对理论 (费米子动力学对称模型)

(Ginocchio、吴成礼、陈选根、陈金全、冯达旋)

1992-1997年 偶数核子配对方法, 包含 SD 配对 (陈金全)

2000年 同类核子配对统一方法 (赵玉民、陈金全、有马朗人等)

2013年 质子中子配对的方法 (傅冠健、赵玉民、有马朗人等)

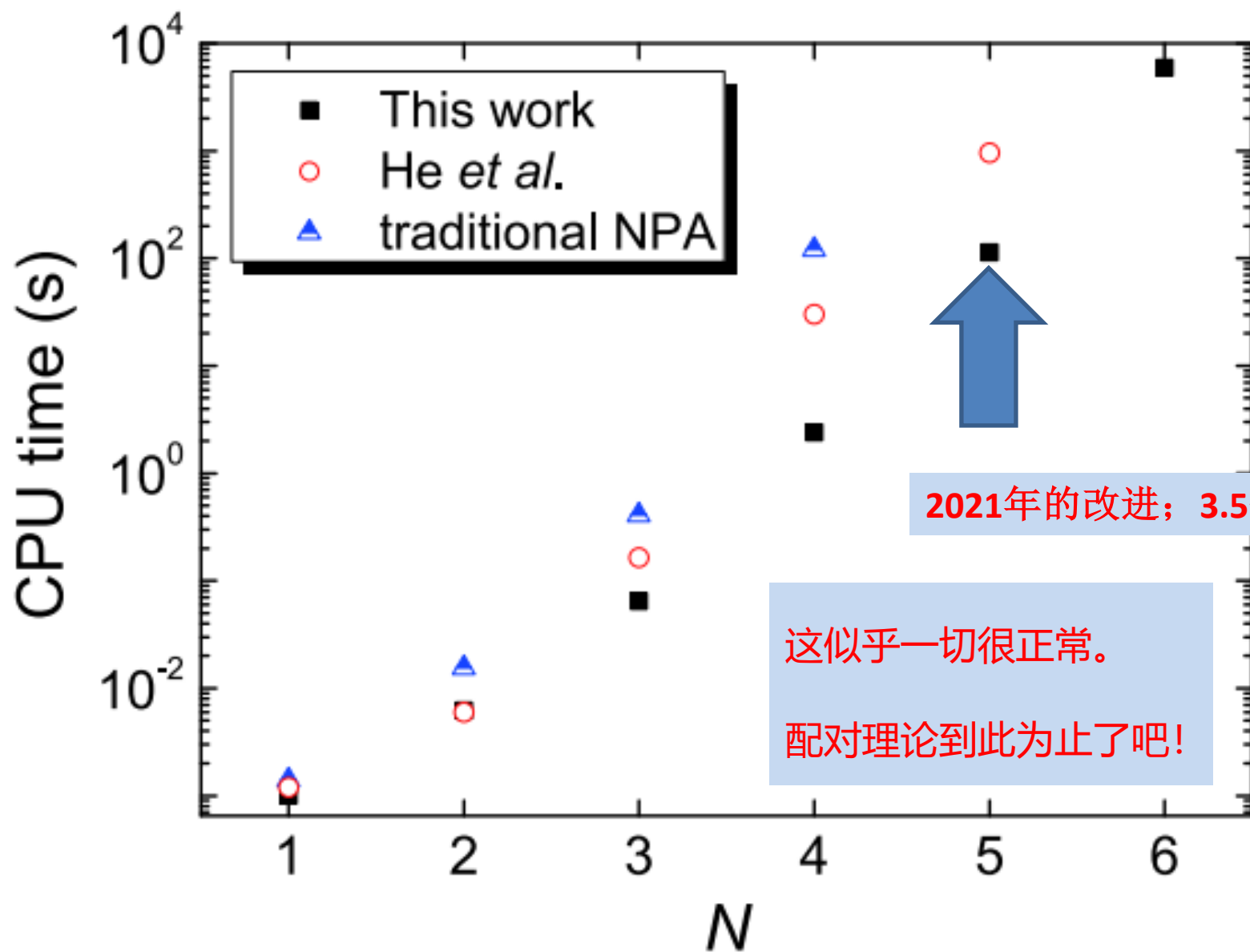
2018年 粒子-空穴的配对理论 (程奕源、赵玉民、有马朗人)

2020年 J -基矢的 M -表象 (何秉承、罗延安、潘峰、Draayer 等)

2021年 配对理论的矩阵表示 (雷杨、路毅、赵玉民)

2023年 配对理论的混合表象 (马畅、尹鑫、赵玉民)

2025-2026年 配对理论的“母分系数”和 芯技术 (潘峰等; 马畅、尹鑫、赵玉民)



2021年的改进；3.5个量级

这似乎一切很正常。

配对理论到此为止了吧！

Development – 7, by 马畅、尹鑫、赵玉民 (混合表象)

State-of-the-art nucleon-pair approximation to the nuclear shell modelC. Ma ¹, X. Yin ¹ and Y. M. Zhao^{1,2,*}¹*Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy,
Shanghai Jiao Tong University, Shanghai 200240, China*²*Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China*

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In this paper a state-of-the-art version of the nucleon-pair approximation to the nuclear shell model is suggested. The configuration space is constructed by stepwise coupling collective nucleon pairs with given spin. The one-body and two-body matrix elements, as well as the overlaps, are calculated in terms of the M -scheme nucleon-basis states, without resorting to any angular momentum couplings. Many redundant commutations are avoided by using the Wigner-Eckart theorem and an algorithm similar to the coefficients of fractional parentage in the nuclear shell model calculations. Numerical experiments demonstrate the overwhelming computational superiority in comparison to all previous approaches of nucleon-pair shell model, particularly for the shell model Hamiltonian in the form of effective interactions. In addition, we exemplify this state-of-the-art version of the nucleon-pair approximation by even-even rare-earth $^{142-152}\text{Nd}$ isotopes, for which the model space is constructed by using $SDGI$ nucleon pairs, and the Hamiltonian is adopted as $V_{\text{low-}k}$ potential. Our calculation of the nucleon-pair approximation is able to reproduce reasonably the low-lying energy states of those Nd isotopes, where the seniority-type, vibrational, transitional, and rotational modes are described on the same footing, without adjusting any parameter.

配对近似的矩阵元与密度矩阵

矩阵元与结构系数定义

配对近似下的哈密顿量矩阵元可由两体相互作用矩阵元和配对基矢（用 ζ 标记）的两体密度矩阵表示：

$$\langle \zeta' | \hat{O} | \zeta \rangle = \sum_{\alpha\beta\gamma\delta} o_{\alpha\beta\gamma\delta} g_{\alpha\beta\gamma\delta}$$

相互作用矩阵元： $o_{\alpha\beta\gamma\delta} = \langle \alpha\beta | \hat{O} | \gamma\delta \rangle$

两体密度矩阵： $g_{\alpha\beta\gamma\delta} = \langle \zeta' | a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\delta} a_{\gamma} | \gamma\delta \rangle$

计算两体密度矩阵，并组合得到哈密顿量矩阵元

C. Ma, et al., Phys. Rev. C 108, 034308 (2023)

密度矩阵的展开结构

$$g_{\alpha\beta\gamma\delta} = 4 \sum_{k=1}^N p_{\alpha\beta}^{(k)} b_{\gamma\delta}^{(k)} + 96 \sum_{k=1}^N \sum_{i=1}^{k-1} \left[p_{\alpha\alpha'}^{(k)} q_{\alpha'\beta'\gamma\delta}^{(i,k)} p_{\beta'\beta}^{(i)} + p_{\alpha\alpha'}^{(i)} q_{\alpha'\beta'\gamma\delta}^{(i,k)} p_{\beta'\beta}^{(k)} \right] + 12 \sum_{k=1}^N \left[s'_{\alpha} p_{\beta\alpha'}^{(k)} - s'_{\beta} p_{\alpha\alpha'}^{(k)} \right] t_{\alpha'\gamma\delta}^{(k)} \Big|_{\text{odd syst.}}$$

$p_{\alpha\beta}^{(k)}$ 配对基矢集体对结构

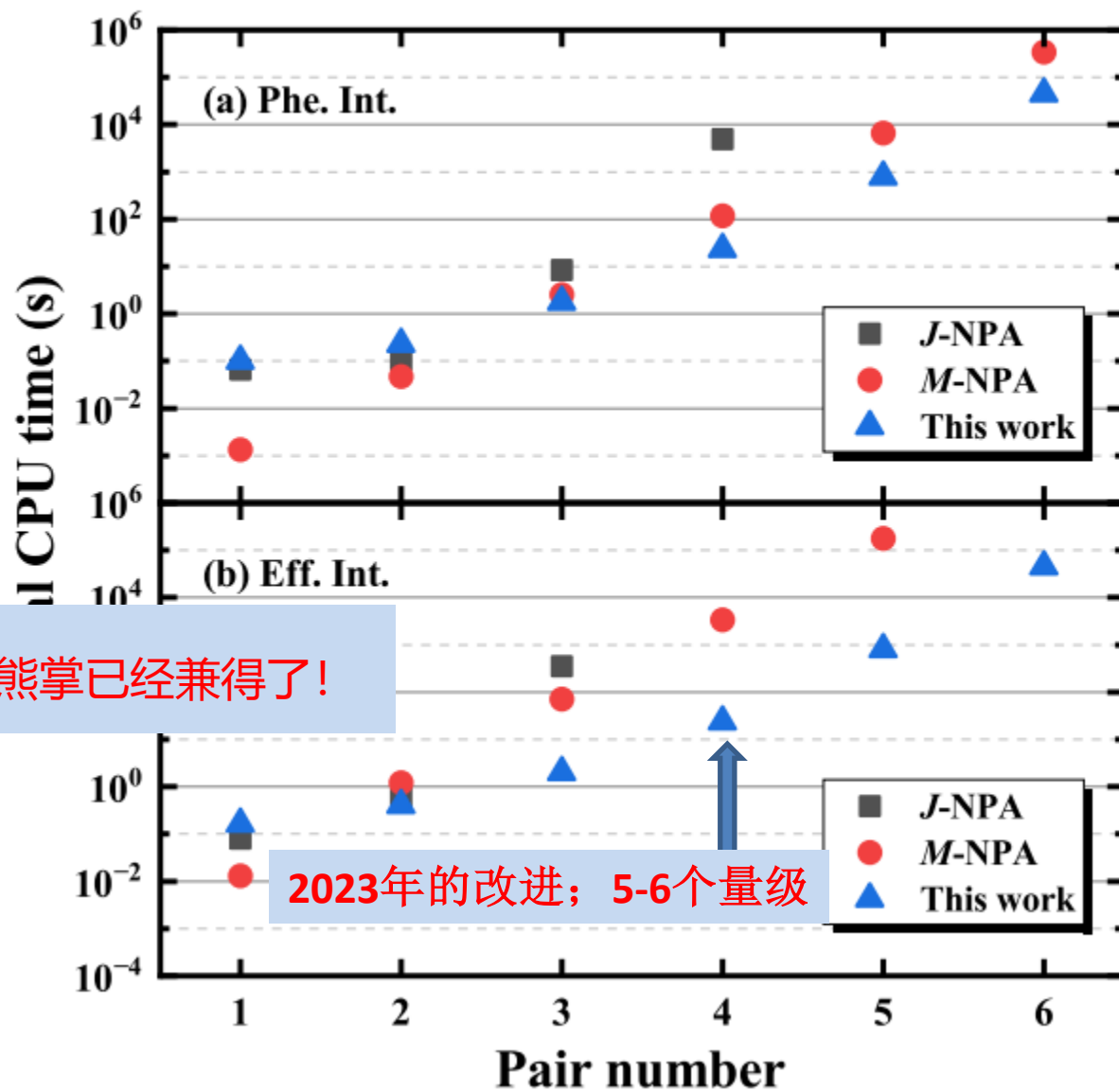
$b_{\alpha\beta}^{(k)} = \langle 0 | a_{\alpha} a_{\beta} \prod_{j \neq k} P_j' | \zeta \rangle$ 两核子集团缩并结构

$t_{\alpha\beta\gamma}^{(k)} = \langle 0 | a_{\alpha} a_{\beta} a_{\gamma} \prod_{j \neq k} P_j' | \zeta \rangle$ 三核子集团缩并结构

$q_{\alpha\beta\gamma\delta}^{(i,k)} = \langle 0 | a_{\alpha} a_{\beta} a_{\gamma} a_{\delta} \prod_{j \neq i,k} P_j' | \zeta \rangle$ 四核子集团缩并结构

q 的计算时间远大于 b 和 t ，通常是主要计算瓶颈。

配对近似计算效率优化的关键在于减少重复的四核子集团计算



差不多了，鱼与熊掌已经兼得了！

2023年的改进：5-6个量级

在大形变转动原子中，“巨大”配对空间能否成功压缩？

PHYSICAL REVIEW C **110**, 014317 (2024)

Toward optimizing the basis states in the nucleon-pair approximation: Variational factorization

C. Ma ¹, X. Yin,¹ and Y. M. Zhao ^{1,2,*}

¹*Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics
and Astronomy, [Shanghai Jiao Tong University](#), Shanghai 200240, China*

²*Collaborative Innovation Center of IFSA (CICIFSA), [Shanghai Jiao Tong University](#), Shanghai 200240, China*



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In this paper we study low-lying states based on the nucleon-pair approximation to the nuclear shell model with our focus on the optimization of basis states constructed by using *SDGI* (nucleon pair with spin zero, two, four, and six) nucleon pairs. We find that the low-lying states in the very large and complex *SDGI* nucleon-pair configuration space can be well represented by much smaller basis states optimized by the variational factorization method of Papenbrock and collaborators [[Phys. Rev. C **67**, 051303\(R\) \(2003\)](#)] (after modifications). This considerably facilitates studies of deformed nuclei in terms of the nucleon-pair approximation.

2026年的新进展： 利用 Hashmap 避免重复计算四核子集团

重复计算如何产生

给定右矢 $|\zeta\rangle$ ，不同左矢 $\langle\zeta'| = \langle 0|P'_1P'_2 \cdots P'_N$



可能选出相同的 $N - 2$ 个集体对子集合

$$\prod_{j \neq i,k} P'_j = \prod_{j \neq \bar{i},\bar{k}} P'_j \rightarrow q_{\alpha\beta\gamma\delta}^{(i,k)} = q_{\alpha\beta\gamma\delta}^{(\bar{i},\bar{k})}$$

注： $q^{(i,k)}$ 和 $q^{(\bar{i},\bar{k})}$ 可能来源于不同的两个左矢，也可能来源于同一个左矢。



出现四核子集团的重复计算，产生算力浪费

利用 Hashmap 避免 Q 重复计算

1. 每个集体对 P'_j 先排序并赋唯一编号： $\text{id}(P'_j)$
2. 从 $P'_1, \cdots, P'_N$ 中取出 $N - 2$ 个集体对子集：
$$I = [\text{id}_{j_1}, \text{id}_{j_2}, \cdots, \text{id}_{j_{N-2}}], \quad j_1 < j_2 < \cdots < j_{N-2}$$
3. 定义 **key**： $\text{key} = \kappa(I)$ ，其中
$$\kappa(I) = 1 + \sum_{r=1}^{N-2} [(i_r - 1) \prod_{s=r+1}^{N-2} \text{maxvals}(s)]$$
4. 定义 **value**： $\text{value} = Q(I)$
5. $Q(I)$ 表示由该 $N - 2$ 集体对子集唯一决定的可复用四维结果（ q 子块），用于后续组装 $g_{\alpha\beta\gamma\delta}$ 。

单线程计算流程



Hashmap 的作用：把“按不同左矢重复计算相同 $Q(I)$ ”改写为“按唯一的 $N - 2$ 集体对子集 I 只计算一次，然后多次复用”。

不同层级的计算会出现相同的对易式

Parallelizable approach for SD -pair shell model matrix elements and overlaps

Feng Pan ^{*}

*Department of Physics, [Liaoning Normal University](#), Dalian 116029, China
and Department of Physics and Astronomy, [Louisiana State University](#), Baton Rouge, Louisiana 70803-4001, USA*

Si-Yao Zhang and Yan-An Luo[†]

School of Physics, [Nankai University](#), Tianjin 300071, China

Bing-Cheng He 

Department of Physics and Astronomy, [University of Tennessee](#), Knoxville, Tennessee 37996, USA

Lianrong Dai [‡]

Department of Physics, School of Science, [Huzhou University](#), Huzhou, Zhejiang 313000, China

Jerry Paul Draayer[§]

Department of Physics and Astronomy, [Louisiana State University](#), Baton Rouge, Louisiana 70803-4001, USA



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We propose a novel procedure for computing overlaps and matrix elements in the M -scheme SD -pair shell model. Our approach demonstrates that calculating the overlap or matrix element of a one- or two-body operator between two N -pair states requires, at most, $N!(N + 1)!/2^N$ products of one-pair overlaps. We also establish an upper bound on the number of independent one-pair overlaps involved. A major advantage of this method is its inherent parallelizability: the one-pair overlaps needed for N -pair overlaps or matrix elements can be computed independently, enabling efficient parallelization to significantly accelerate the overall computation.

3. 利用配对编码加速四核子集团计算效率

四核子集团的计算框架

- 考虑计算给定的四核子集团结构:

$$\begin{aligned}
 q_{\alpha\beta\gamma\delta}^{(N)} &= \langle 0 | a_\alpha a_\beta a_\gamma a_\delta \prod_{j=1}^{N-2} P_j' | \zeta \rangle = -2 \sum_{k=1}^N \text{tr}(p_k p_{N-2}') \langle 0 | a_\alpha a_\beta a_\gamma a_\delta \prod_{j=1}^{N-3} P_j' \prod_{j \neq k}^N P_j^+ | 0 \rangle + \sum_{k=1}^N \sum_{i=1}^{k-1} \langle 0 | a_\alpha a_\beta a_\gamma a_\delta \prod_{j=1}^{N-3} P_j' \prod_{j \neq i, k}^N P_j^+ \mathfrak{P}_{i, k}^+ | 0 \rangle \\
 &= -2 \sum_{k=1}^N \text{tr}(p_k p_{N-2}') q_{\alpha\beta\gamma\delta}^{(N-1, k)} + \sum_{k=1}^N \sum_{i=1}^{k-1} q_{\alpha\beta\gamma\delta}^{(N-1, i, k)}
 \end{aligned}$$

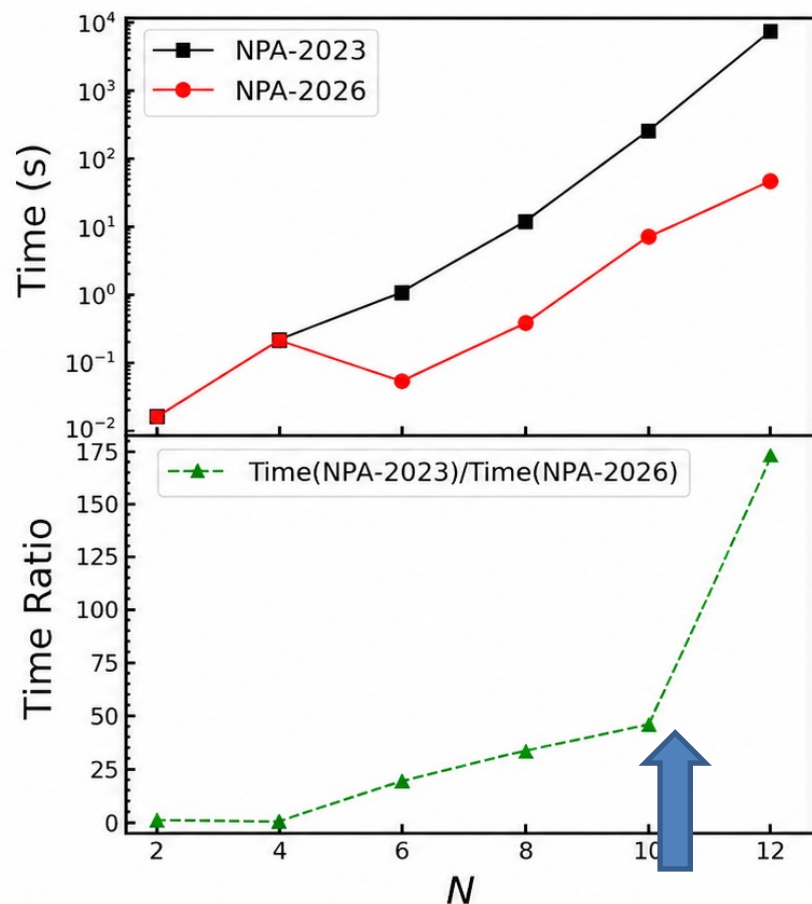
- 将 N 对体系诱导的四核子集团结构转化为 $N-1$ 对体系诱导的四核子集团结构，通过递归计算得到最终结果。
- 重复计算如何产生：重复的耦合集体对 $\mathfrak{P}_{i, k}^+ = [[P_{N-2}', P_i^+], P_k^+]$ & 相同的四核子集团递归分支 $q_{\alpha\beta\gamma\delta}^{(N-1, i, k)}, q_{\alpha\beta\gamma\delta}^{(N-1, k)}$ 。

静态有序的配对编码技术

- 对所有配对近似计算衍生出的耦合集体对 ($\mathfrak{P}_{i, k}^+$) 赋予有序且连续的整数编码。
- 在处理哈密顿量矩阵元之前预先计算耦合集体对并利用整数编码存储在连续内存中，以供后续计算调用。
- 在四核子集团结构递归计算中迭代集体对编码而非集体对本身，并根据编码所以集体对结构，避免耦合集体对的重复计算。
- 根据集体对编码迅速匹配相同递归分支，大幅缩减递归计算的分支数量。

相同的对易关系式进行编码

4. Benchmark



- **N**: 价核子数
- **NPA-2023**: NPA in Ref. [C. Ma, et al., Phys. Rev. C 108, 034308 (2023)]
- **NPA-2026**: **NPA in this work (in preparation)**
- **Time**: 整个NPA计算时间
- **模型空间**:

$1p_{1/2}$	$1p_{3/2}$	$0f_{5/2}$	$0g_{9/2}$	$1d_{5/2}$
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- **配对截断**: S、D pairs.

再次提升了1-2个量级！(现在可计算价质子12个、价中子12个情况)
对于5-6个同类核子配对，比早期的J-scheme 已经7-8个量级

配对理论最近的应用

特殊的配对近似

Seniority (辛弱数) 理论、(推广的)多轨道辛弱数、破对近似、费米子动力学对称模型(FDSM)、玻色子模型微观基础。

普遍性的配对近似（迄今为止的结果）

关于谱学的数值计算、集体性的演化、

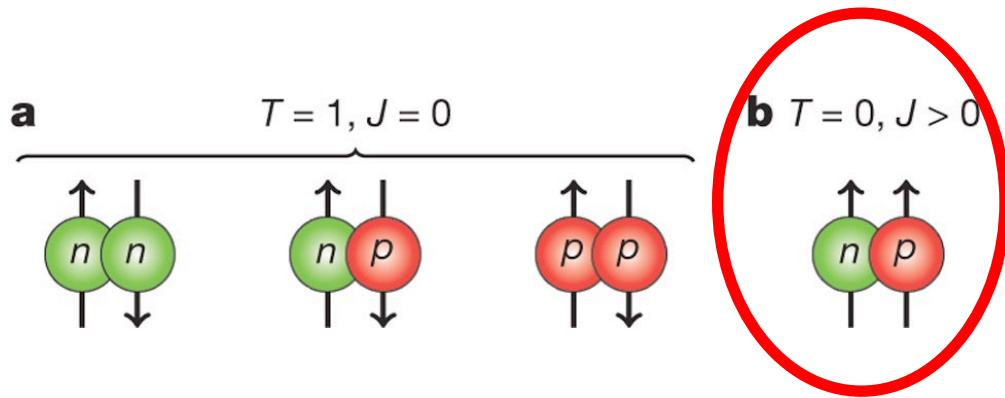
同位旋标量配对现象的寻找

转动模式下**SD** 配对主导性问题

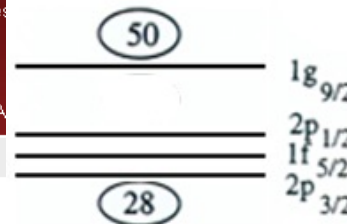
贝塔衰变、形状共存、

八极形变态、转移反应截面、电荷交换反应

Isoscalar spin-aligned p-n pair



当价质子和价中子处于同一个壳层时，尤其在 $N = Z$ 原子核中，质子-中子配对关联可能非常重要 (i.e., $J = J_{\max}$)



NATURE | LETTER

◀ previous article next article ▶

Evidence for a spin-aligned neutron–proton paired phase from the level structure of ^{92}Pd

B. Cederwall, F. Ghazi Moradi, T. Bäck, A. Johnson, J. Blomqvist, E. Clément, G. de France, R. Wadsworth, K. Andgren, K. Lagergren, A. Dijon, G. Jaworski, R. Liotta, C. Qi, B. M. Nyakó, J. Nyberg, M. Palacz, H. Al-Azri, A. Algara, G. de Angelis, A. Ataç, S. Bhattacharyya, T. Brock, J. F. Brown, P. Davies, A. Di Nitto, Zs. Dombrádi, A. Gadea, J. Gál, B. Hadinia, F. Johnston-Theasby, Joshi, K. Juhász, R. Julin, A. Jungclaus, G. Kalinka, S. O. Kara, A. Khaplanov, J. Kownacki, G. Rana, S. M. Lenzi, J. Molnár, R. Moro, D. R. Napoli, B. S. Nara Singh, A. Persson, F. Recchia, M. Sandzelius, J.-N. Scheurer, G. Sletten, D. Sohler, P.-A. Söderström, M. J. Taylor, J. Timár, J. J. Valiente-Dobón, E. Vardaci & S. Williams [Show fewer authors](#)

Affiliations | Contributions | Corresponding author

Nature 469, 68–71 (06 January 2011) | doi:10.1038/nature09644

Physical Review Letters

moving physics forward

PRL 107, 172502 (2011)

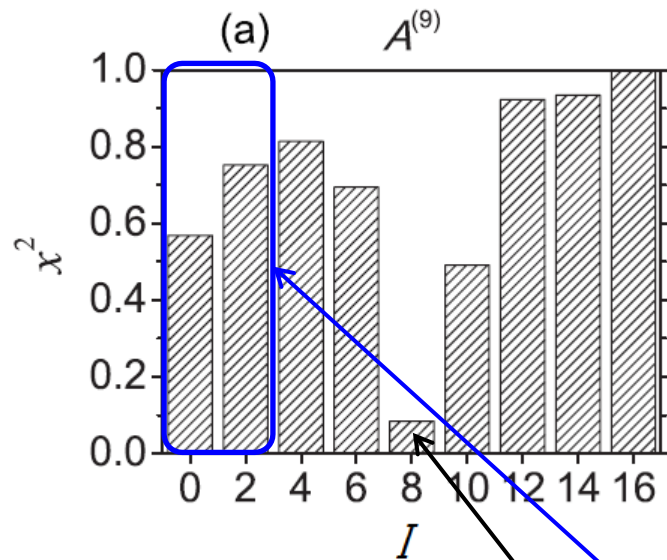
PHYSICAL REVIEW LETTERS

week ending
21 OCTOBER 2011

16^+ Spin-Gap Isomer in ^{96}Cd

B. S. Nara Singh,¹ Z. Liu,² R. Wadsworth,¹ H. Grawe,³ T. S. Brock,¹ P. Boutachkov,³ N. Braun,⁴ A. Blazhev,⁴ M. Górska,³ S. Pietri,³ D. Rudolph,⁵ C. Domingo-Pardo,³ S. J. Steer,⁶ A. Ataç,⁷ L. Bettermann,⁴ L. Cáceres,³ K. Eppinger,⁸ T. Engert,³ T. Faestermann,⁸ F. Farinon,³ F. Finke,⁴ K. Geibel,⁴ J. Gerl,³ R. Gemhäuser,⁸ N. Goel,³ A. Gottardo,² J. Gregosz,⁹ C. Hinke,⁸ R. Hoischen,^{3,5} G. Ilie,⁴ H. Iwasaki,⁴ J. Jolie,⁴ A. Kaşkaş,⁷ I. Kojouharov,³ R. Krücken,⁸ N. Kurz,³ E. Merchán,³ C. Nociforo,³ J. Nyberg,¹⁰ M. Pfützner,¹¹ A. Prochazka,³ Zs. Podolyák,⁶ P. H. Regan,⁶ P. Reiter,⁴ S. Rinta-Antila,¹² C. Scholl,⁴ H. Schaffner,³ P.-A. Söderström,¹⁰ N. Warr,⁴ H. Weick,³ H.-J. Wollersheim,³ P. J. Woods,² F. Nowacki,¹³ and K. Sieja¹³

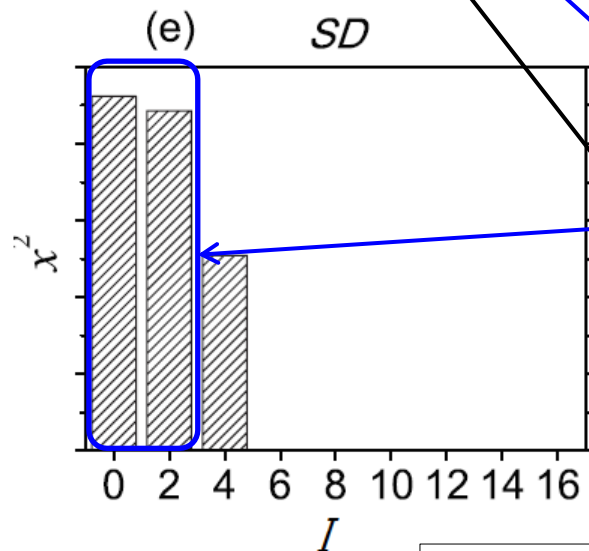
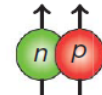
^{96}Cd : dual description



S pair: $J=0, T=1$

D pair : $J=2, T=1$

$A^{(9)}$ pair : $J=9, T=0$

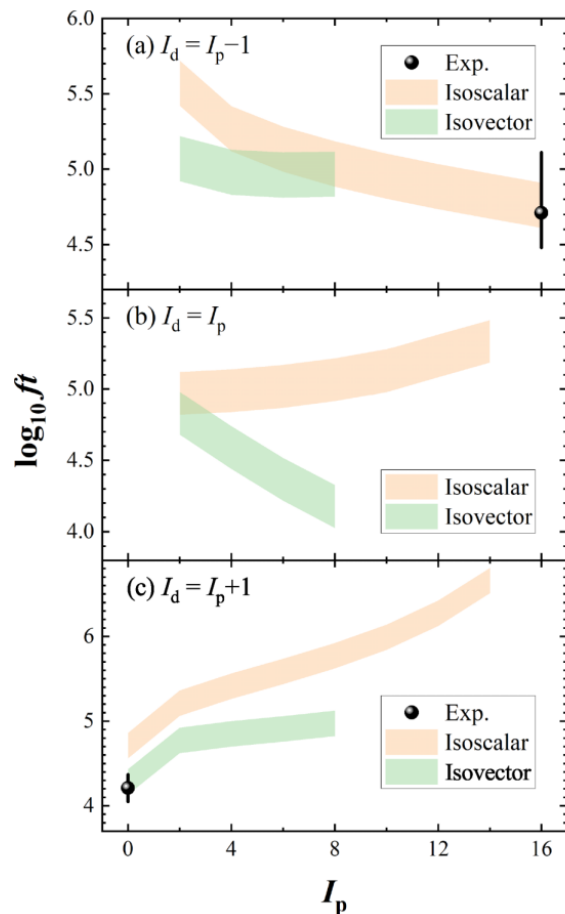


Dual description:
Non-orthogonality feature of
nucleon-pair basis!

Seniority two state

Identification of isoscalar spin-aligned pair condensation versus conventional isovector pairing correlation in ^{96}Cd with Gamow-Teller transitions

C. Ma ^{1,2}, X. Yin,² and Y. M. Zhao ^{2,3,*}



本文是解析得到的... ..

并用壳模型验算过... ..

Log ft 与配对种类的敏感度！！

基态: **isovector**

同核异能态16+: **isoscalar**

Discrimination between isoscalar and isovector pairing correlations using transition sum rules

C. Ma ^{1,*} Y. M. Zhao ^{2,3,†} and G. J. Fu ⁴

¹*School of Science, Yanshan University, Qinhuangdao 066004, China*

²*Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy,
Shanghai Jiao Tong University, Shanghai 200240, China*

³*Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China*

⁴*School of Physics Science and Engineering, Tongji University, Shanghai 200092, China*



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In this paper, we make use of transition sum rules to study isoscalar and isovector pairing correlations in $T = 0$ states of seven self-conjugate nuclei, ^{96}Cd , ^{94}Ag , ^{92}Pd , ^{90}Rh , ^{48}Cr , ^{46}V , and ^{44}Ti . Four typical pairing correlation schemes are investigated, including (i) isoscalar spin-aligned (SA), (ii) isoscalar spin-one (SO), (iii) isovector lowest-seniority (LS), and (iv) isovector SDG pair approximations. Our systematic calculations demonstrate that the SA pairing strongly suppresses Gamow-Teller transitions, while the SO pairing reduces electric quadrupole transition strengths. Comparisons with experimental data show that the SA pairing picture accurately describes high-spin isomers with relatively long half-lives, whereas the isovector pairing (LS/ SDG) modes is more favored by low-spin ground states. In addition, we investigate the robustness of these conclusions by extending our calculations from single- j to multi- j shells with realistic shell-model interactions.

β -decay half-lives of the r -process waiting-point isotones of $N = 81$ and 82 nuclei

C. Ma,¹ Y. Lu,² Y. Lei³, and Y. M. Zhao^{1,4,*}

¹*Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*

²*College of Physics and Engineering, Qufu Normal University, Qufu 273165, China*

³*School of National Defense Science and Technology, Southwest University of Science and Technology, Mianyang 621010, China*

⁴*Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China*

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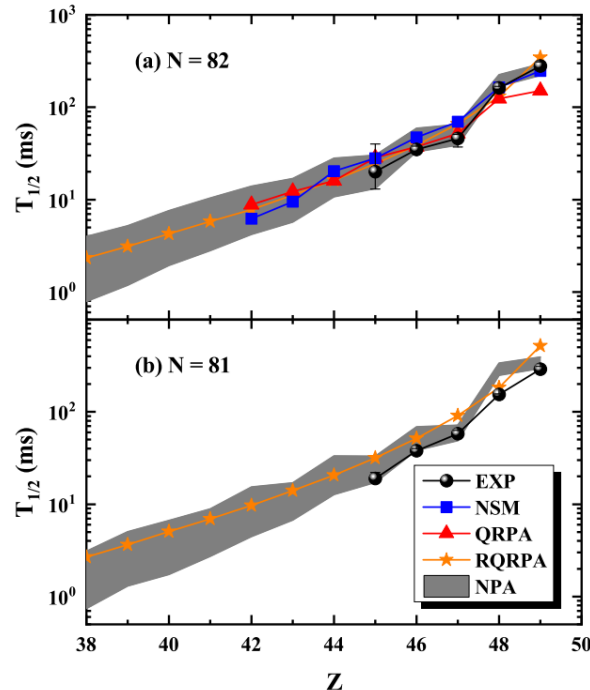


FIG. 5. β -decay half-lives for $N = 82$ and 81 isotones versus proton number Z . (a) corresponds to $N = 82$, and (b) corresponds to $N = 81$. Experimental data, and theoretical results from the NSM [26], the QRPA + HFB21 [21], and the RQRPA + RHB [23] methods, are also presented in black, blue, red, and orange, respectively. The range in dark grey represents our predicted half-lives with uncertainties originated from the nuclear masses.

Alternating-parity doublets of even-even Ba isotopes

X. Yin,¹ C. Ma,¹ and Y. M. Zhao ^{1,2,*}

¹*Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy,
Shanghai Jiao Tong University, Shanghai 200240, China*

²*Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China*



(Received 22 October 2023; revised 25 December 2023; accepted 29 January 2024; published 23 February 2024)

In this paper we study the structure of even-even Ba isotopes with neutron number $N = 84\text{--}90$ within the framework of nucleon-pair approximation of the shell model. We use a phenomenological Hamiltonian, which includes single-particle energies, monopole, quadrupole, and octupole pairing energies, as well as quadrupole-quadrupole and octupole-octupole interactions. Our calculated energy levels and $B(E2)$ values are in good agreement with experimental data. The alternating parity doublets are well reproduced by considering F^- nucleon pairs with spin-three and negative parity, which demonstrates the importance of F^- pair in octupole deformation. The octupole-octupole interaction is found to favor stronger $B(E3)$ transitions and suppress $B(E2)$ transitions in the low-lying states of nuclei in these even-even Ba isotopes.

Pandax 实验 本底中 Pb-214 的噪声贡献特别大

Beta 能谱的计算，如果能做得细致一些，对压低本底好处特别大

白春林、尧江明 + 王小保、马畅(用配对做的)

Coexistence of octupole and quadrupole shapes in ^{144}Ba in the nucleon-pair approximation of the shell model

X. Yin,¹ M. Q. Lin,¹ C. Ma,¹ and Y. M. Zhao ^{1,2,*}

¹*Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy,
Shanghai Jiao Tong University, Shanghai 200240, China*

²*Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China*



(Received 26 December 2024; revised 3 March 2025; accepted 27 March 2025; published 10 April 2025)

The nucleon-pair approximation (NPA) of the shell model, with collective pairs projected from unconstrained Hartree-Fock states as well as a number of so-called noncollective pairs, is applied to study the low-lying states, both the yrast and a few non-yrast, of the ^{144}Ba nucleus. There are the most sophisticated NPA calculations ever performed. The calculated electric quadrupole transition rates are very consistent with experimental data. Our calculations demonstrate robustness of intrinsic quadrupole moment for the yrast states. In addition, our calculations also reproduce an octupole band and a prolate band in low-lying states of ^{144}Ba . With consideration of two major shells both for valence protons and neutrons, our calculations yield a sizable $B(E1, J^- \rightarrow (J - 1)^+)$.

我们当时还对于配对框架下的跨壳激发质心伪态办法非常少，所以跨壳空间取得不大，最近克服了这个矛盾，可以做得更好！

早期关于sd 玻色子模型的著名争论

Features of Nuclear Deformations Produced by the Alignment of Individual Particles or Pairs
A. Bohr and B R Mottelson, 1980 Phys. Scr. 22 468

On the Spectrum of ^{168}Er
A Bohr and B R Mottelson, Phys. Scr. 25 28(1982).

On the Ability of the Interacting Boson Model to Describe Nuclear Deformation Effects
A Bohr and B R Mottelson, Phys. Scr. 25 915 (1982)

Dominance of Monopole and Quadrupole

Pairs in the Nilsson Model

T. Otsuka, A. Arima and N. Yoshinaga

Phys. Rev. Lett. **48**, 387 (1982).

The structure of the S and D pairs of the interacting Boson Model from the Hartree-Fock-Bogolyubov approximation

S. Pittel and J. Dukelsky
Physics Letters B **128** 9 (1983)

Role of high multipole pairs in the description of deformed nuclei

P. Catara, A. Insolia, E. Maglione, and A. Vitturi
Physics Letters B **123** 375 (1983)

Self-Consistent Treatment of the Pairing Plus Quadrupole Force in the Nilsson Plus BCS Model and in the Interacting Boson Model

D R Bes, R A Broglia, E Maglione, and A Vitturi
Phys. Scr. **28** 527 (1983)

Test of the validity of the SD truncation for deformed systems

E. Maglione, A. Vitturi, C.H. Dasso, and R.A. Broglia
Nuclear Physics A **404** 333 (1983)

Microscopic structure of monopole and quadrupole bosons

E. Maglione, F. Catara, A. Insolia, and A. Vitturi
Nuclear Physics A **397** 102 (1983)

当时非常多的定性讨论。最近我们开展了
对于真实原子核的定量检验... ..

在大形变转动原子中，SD 配对到底有多重要？

我们有了新的算法，我们下面还要问同样的问题！！！！

PHYSICAL REVIEW C **109**, 054316 (2024)

Dominant nucleon-pair configurations in low-lying states of deformed nuclei

C. Ma^{},¹ X. Yin,¹ and Y. M. Zhao^{1,2,*}

¹*Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy,
Shanghai Jiao Tong University, Shanghai 200240, China*

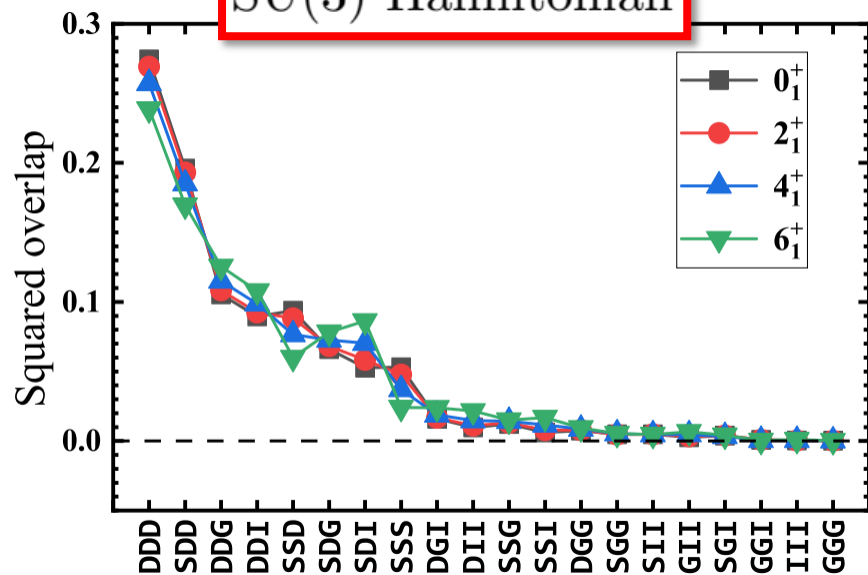
²*Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China*



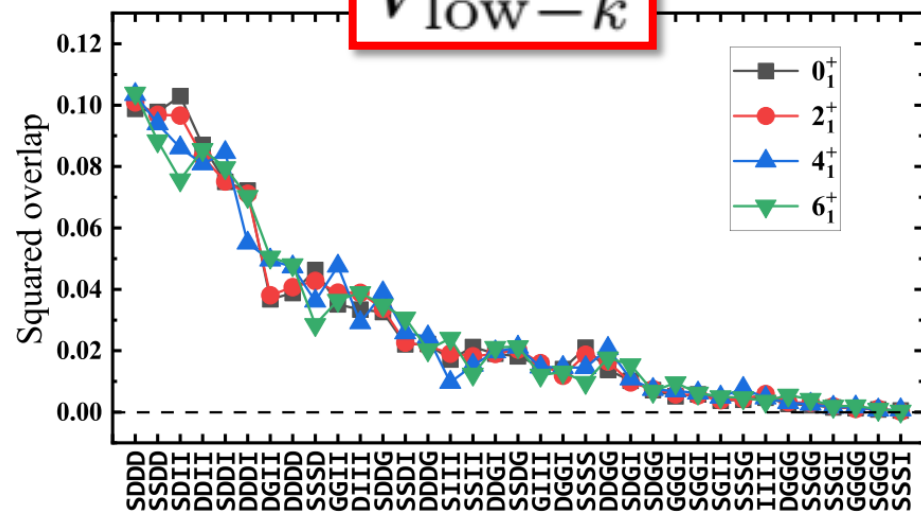
(Received 8 February 2024; revised 15 March 2024; accepted 11 April 2024; published 16 May 2024)

In this paper we explicitly study dominant nucleon-pair configurations in low-lying states of deformed nuclei in two cases; the first is the schematic Elliott's SU(3) model with valence nucleons in the pf shell, and the second is the effective $V_{\text{low-}k}$ interaction with valence protons in the 50-82 shell and valence neutrons in the 82-126 shell, exemplified by ^{146}Ba and ^{148}Ce . We find that the SD nucleon-pair configurations are more predominant in low-lying states of these deformed nuclei than expected. Mixings of nucleon-pair configurations beyond the conventional SD -pair subspace, in particular, one- G and/or one- I nucleon-pair configurations, albeit small, are found to be necessary to reproduce the moment of inertia.

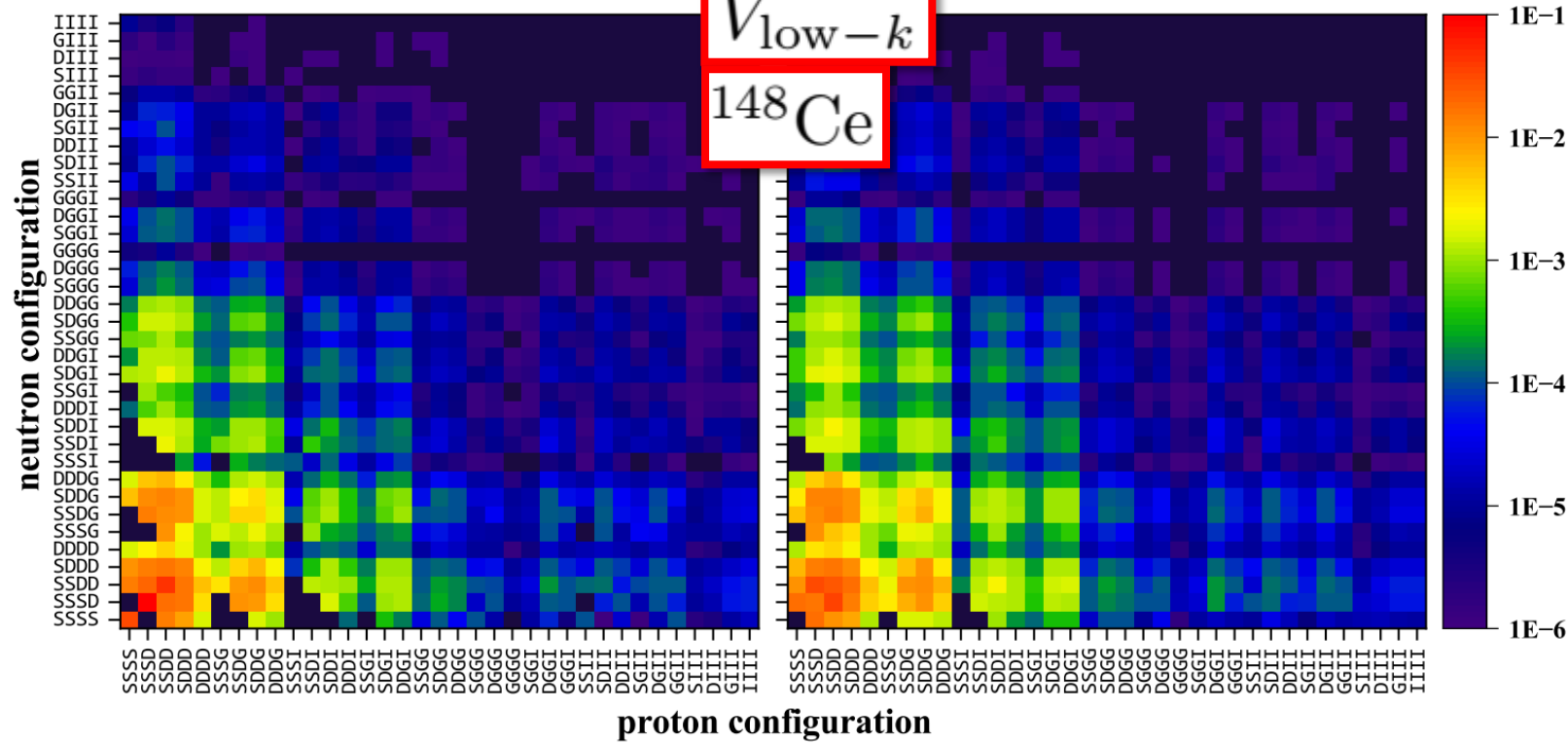
SU(3) Hamiltonian



$V_{\text{low}-k}$



$V_{\text{low}-k}$
 ^{148}Ce



Shell evolution and nucleon-pair structure of yrast states in $N = 80$ isotones

C. Ma^{1,2}, Y. X. Yu³, X. Yin², G. J. Fu³ and Y. M. Zhao^{2,4,*}

¹School of Science, *Yanshan University*, Qinhuangdao 066004, China

²Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy,
Shanghai Jiao Tong University, Shanghai 200240, China

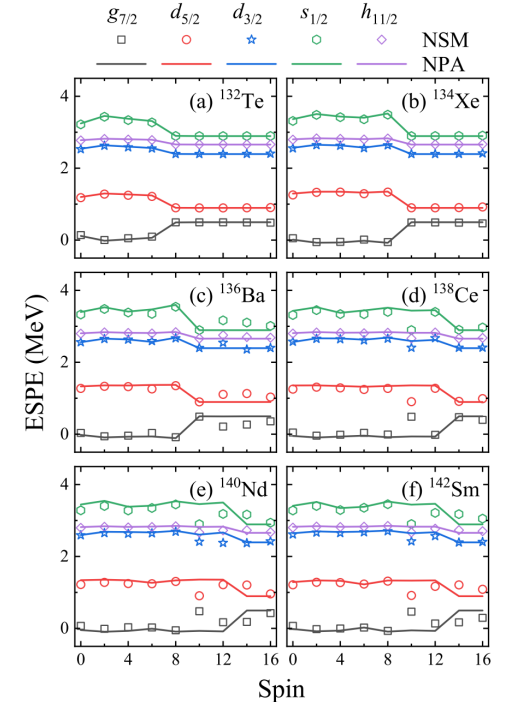
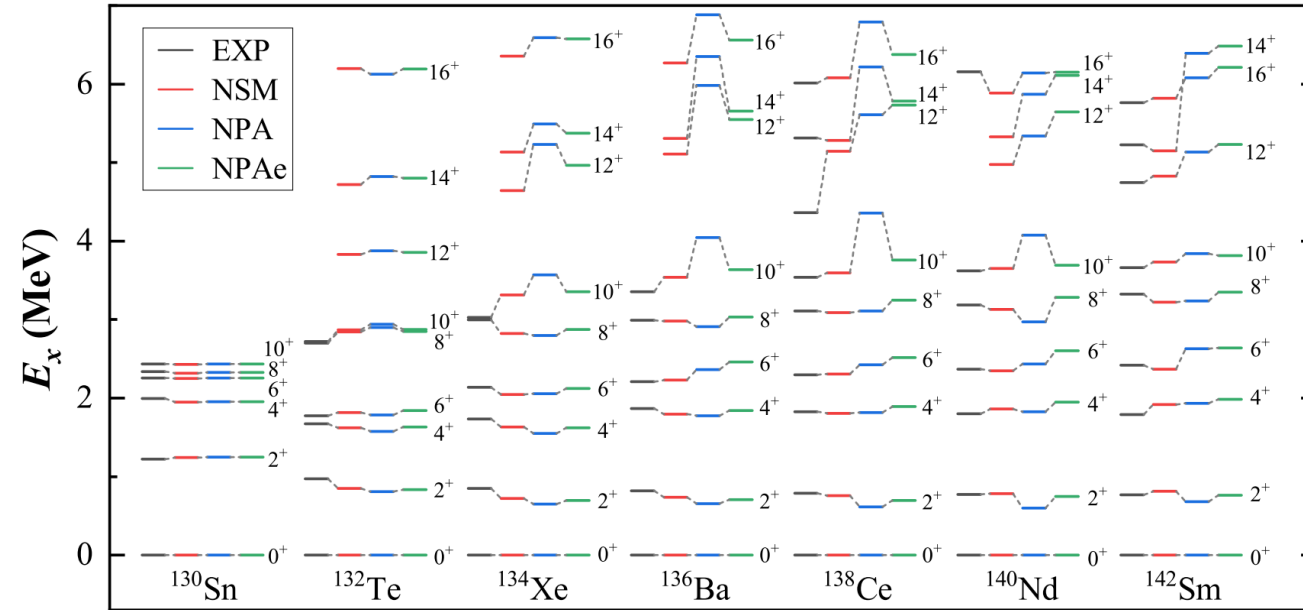
³School of Physics Science and Engineering, *Tongji University*, Shanghai 200092, China

⁴Collaborative Innovation Center of IFSA (CICIFSA), *Shanghai Jiao Tong University*, Shanghai 200240, China



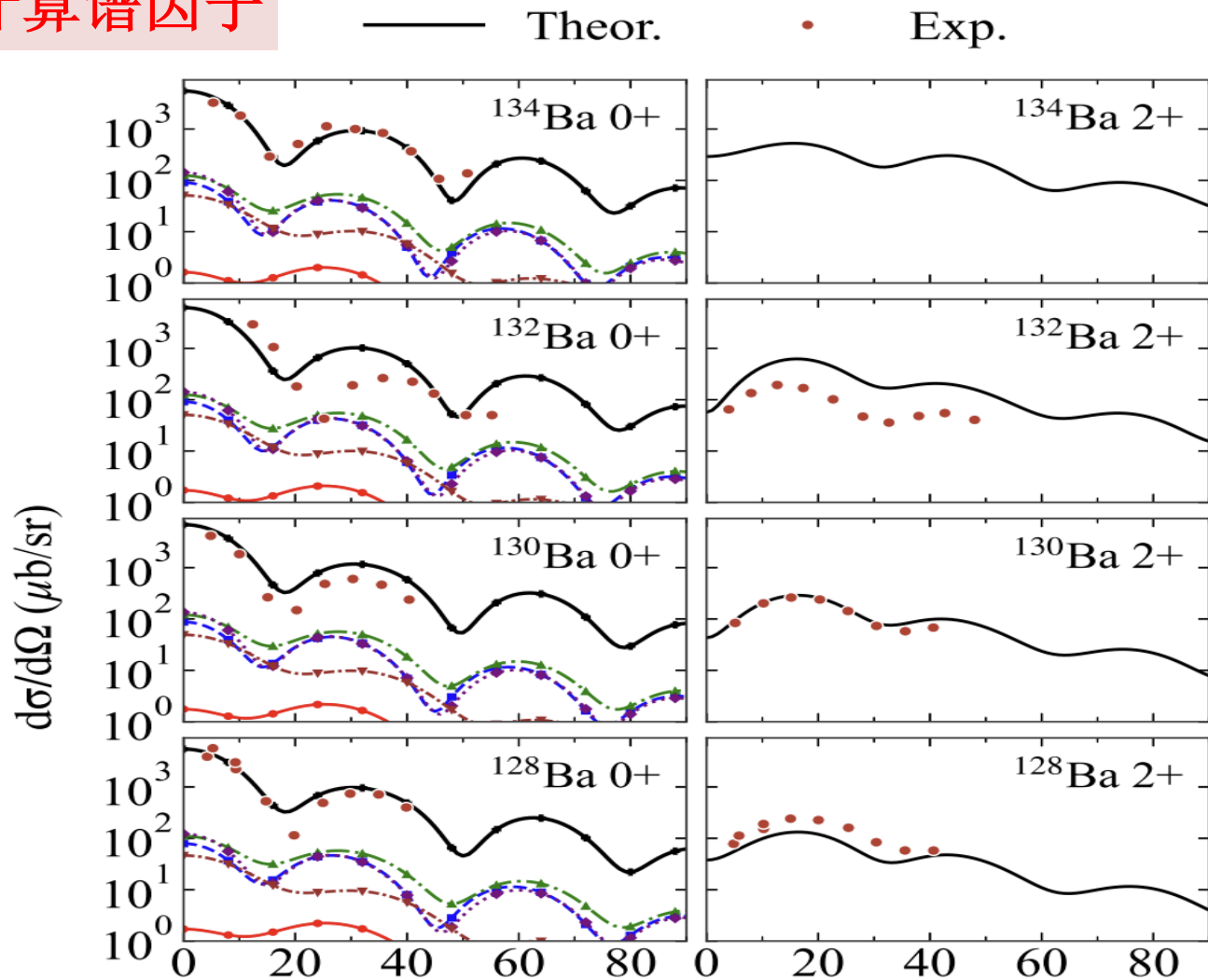
(Received 5 December 2024; revised 25 May 2025; accepted 9 June 2025; published 22 July 2025)

The yrast states in even-even $N = 80$ nuclei are studied within the frameworks of both the large-scale shell model and the nucleon-pair approximation by using optimized effective interaction. Both calculations reproduce most of the experimental data for yrast $2^+ - 8^+$ states. Abrupt inconsistency between these two sets of calculations for the $10^+ - 16^+$ states is found to be a reflection of the type-II shell evolution, which is also seen in the effective single-particle energies of proton levels, at the 10^+ states. This suggests a strong correlation between the type-II shell evolution and the evolution of nucleon-pair structure, which leads to fragility of optimized nucleon pairs of the ground states for states with spins higher than ten. By further consideration of nucleon pairs specifically optimized for the 10^+ state with a new variational procedure in our configuration space, we are able to improve substantially the consistency between calculated results in the nucleon-pair approximation and those of the exact shell-model calculation.

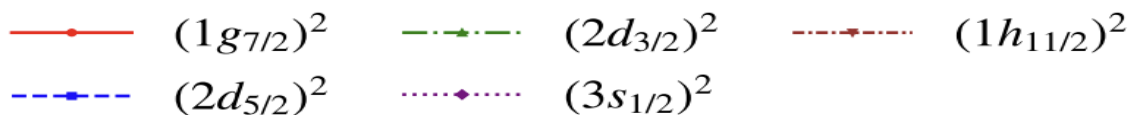


配对理论计算谱因子

转移反应微分截面



这是初步结果... ..



配对理论的未来

“全空间”壳模型计算方法中的M-scheme 和 J-scheme 是分开的,

传统配对理论也是分开的, 各走各的路。

在配对理论中我们把二种方式结合起来构造空间和计算矩阵元, 取得很好效果。

是否到了最佳(**虽然已8个量级!!!**)? 取决于对这两种途径继续**更深入的理解**

或许依然还有很多余地... ... **组态-有效相互作用的做法 走向重质量区**

简化模型基矢正在进行中

Part 4: 配对理论的未来

重质量区域

- 1、更高效的壳模型配对理论方法、可靠性验证等模型问题
- 2、重质量原子核的低激发态结构系统计算
- 3、重质量区跨壳激发现象、壳模型有效相互作用**
- 4、尝试：八极形变、形状共存、形状相变、壳演化、转移反应/电荷交换**

Short summary

Short introduction to the Nucleon-pair approximation of the shell model

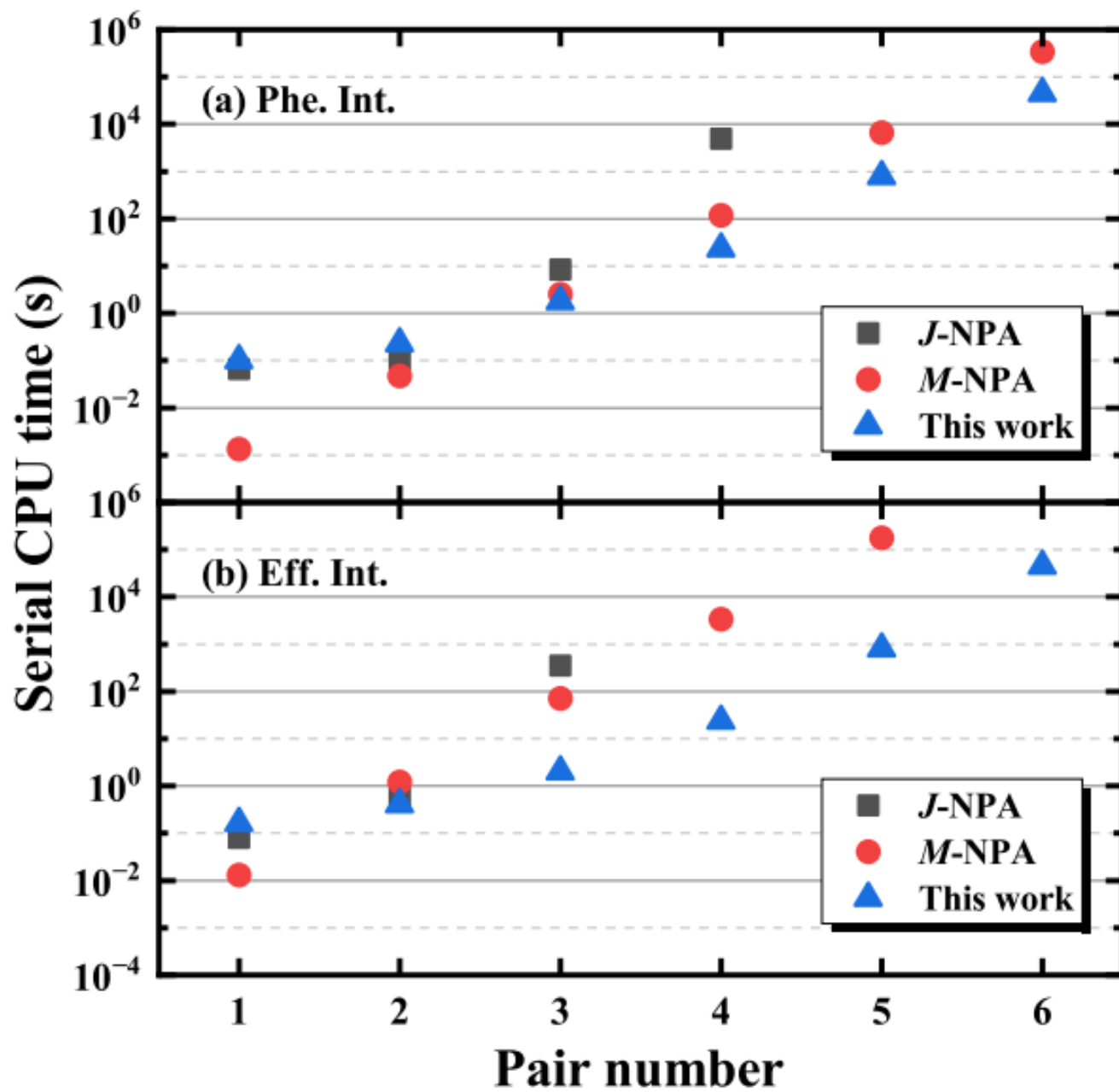
New developments (近十年):

- Guan-Jian Fu (SJTU/Tongji University)
- Li-Yuan Jia (SJTU/University of Shanghai for Science and Technology)
- Yi-Yuan Cheng (SJTU/Eastern China Normal University)
- Bing Cheng He and Yan An Luo (Nankai University)
- Feng Pan (Liaoning Normal University)
- Yi Lu (SJTU / Qufu Normal University) and Yang Lei (SJTU / Mianyang)
- Chang Ma (SJTU / Yanshan University)
- Xin Yin (SJTU / Zhejiang Science-Technology University)

Applications: Hui Jiang, Yang Lei, Yi Lu, Guan-Jian Fu, Yi-Yuan Cheng, Man Bao, Chang Ma, Xin Yin, Heng-Yi Wang, Yu-Min Zhao; Bing-Cheng He and Yan-An Luo.

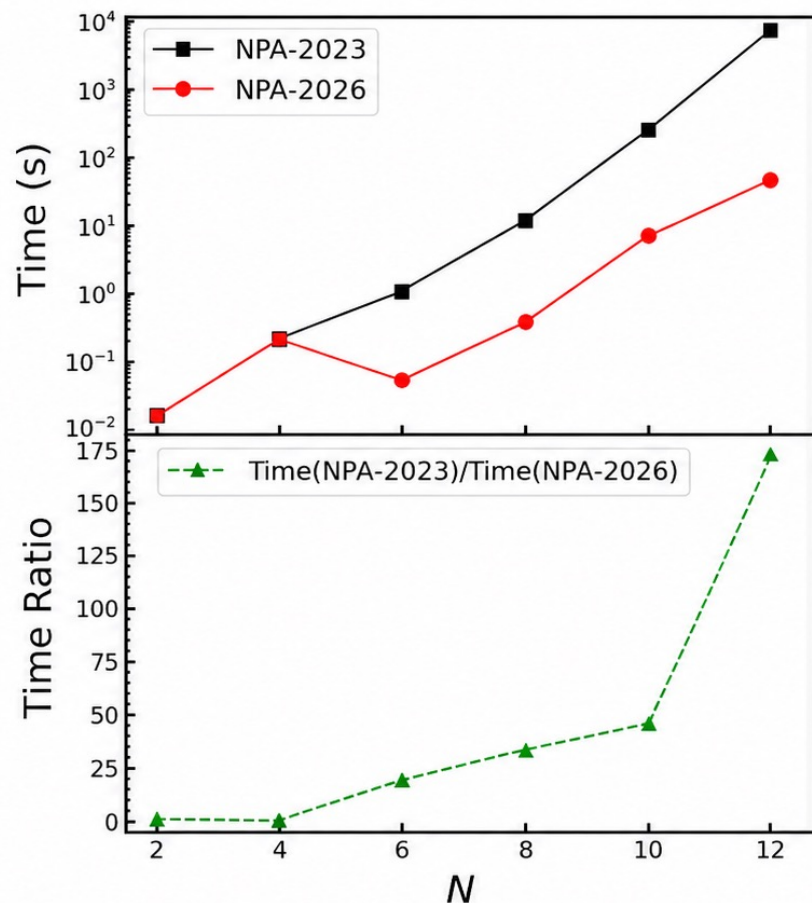
未来: 重质量区核结构各种性质, 目前比较振奋。

谢谢大家！



2023 版本，已经提高了6个量级的效率//主要还是基于对称性

4. Benchmark



- **N**: 价核子数

- **NPA-2023**: NPA in Ref. [C. Ma, et al., Phys. Rev. C 108, 034308 (2023)]

- **NPA-2026**: NPA in this work (in preparation)

- **Time**: 整个NPA计算时间

- **模型空间**:

$1p_{1/2}$	$1p_{3/2}$	$0f_{5/2}$	$0g_{9/2}$	$1d_{5/2}$
------------	------------	------------	------------	------------

- **配对截断**: S、D pairs.

- 因为同类价核子数从8个增加12个，原子核集体运动模式具有很大的变化，从过渡区到稳定形变区！！

主要还是分层编码、寻找重复