

高能与低能核物理交叉学术研讨会

2025.5.15.-5.18. 上海

Nuclear Collective Vibrations and deformation

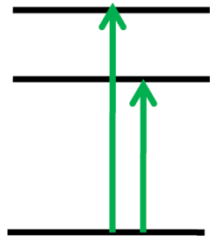
牛一斐

上海交通大学

Nuclear Collective Vibrations

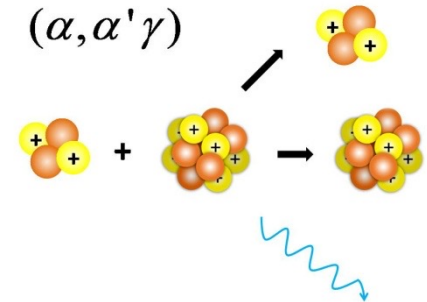
- The vibration excitation of nucleus involving many nucleons

- Non-charge-exchange excitations



Z, N

- ✓ Giant monopole resonances (GMR)
- ✓ Giant dipole resonances (GDR)
- ✓ Giant quadrupole resonances (GQR)
- ...

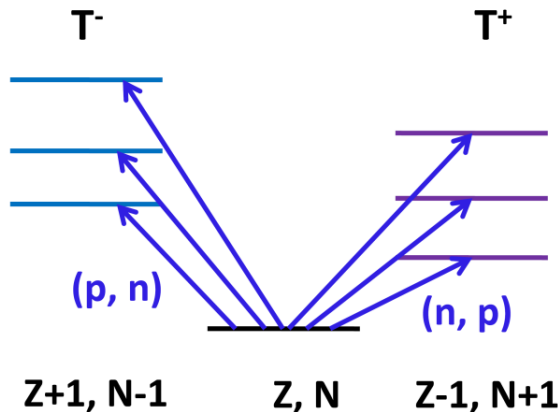


reactions:

$(\alpha, \alpha' \gamma)$,
 (γ, γ') , (γ, n) , (e, e')

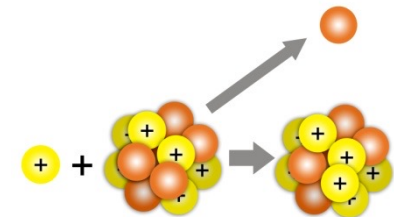
Strong
EM

- Charge-exchange excitations



- ✓ Isobaric Analogue States (IAS)
- ✓ Gamow-Teller Resonances (GTR)
- ✓ Spin-Dipole (SD) excitations
- ...

(p,n) reaction



reactions:

$(p,n) T^-, (n,p) T^+$

Strong

- Nuclear EoS, origin of heavy elements, neutrino properties ...

Nuclear Deformation

□ Spontaneous Symmetry Breaking

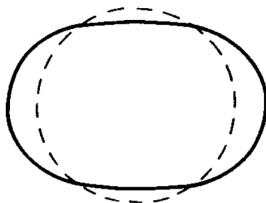
- **Nuclear Hamiltonian:** Rotational symmetry
- **Nuclear wavefunction:** Introduction of deformation breaks rotational symmetry (intrinsic framework)
- ✓ **Purpose:** to include/mimic many-body correlations in the mean-field level

□ Description of deformation

Surface Parametrization

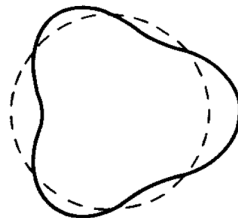
$$R = R(\theta, \phi) = R_0 \left(1 + \alpha_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \phi) \right)$$

Quadrupole



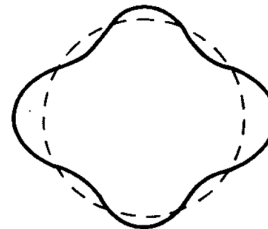
$\lambda=2$

Octupole

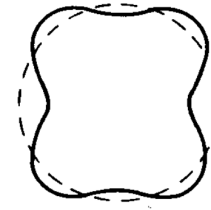


$\lambda=3$

Hexadecupole



$\lambda=4 \ a_{40} > 0$

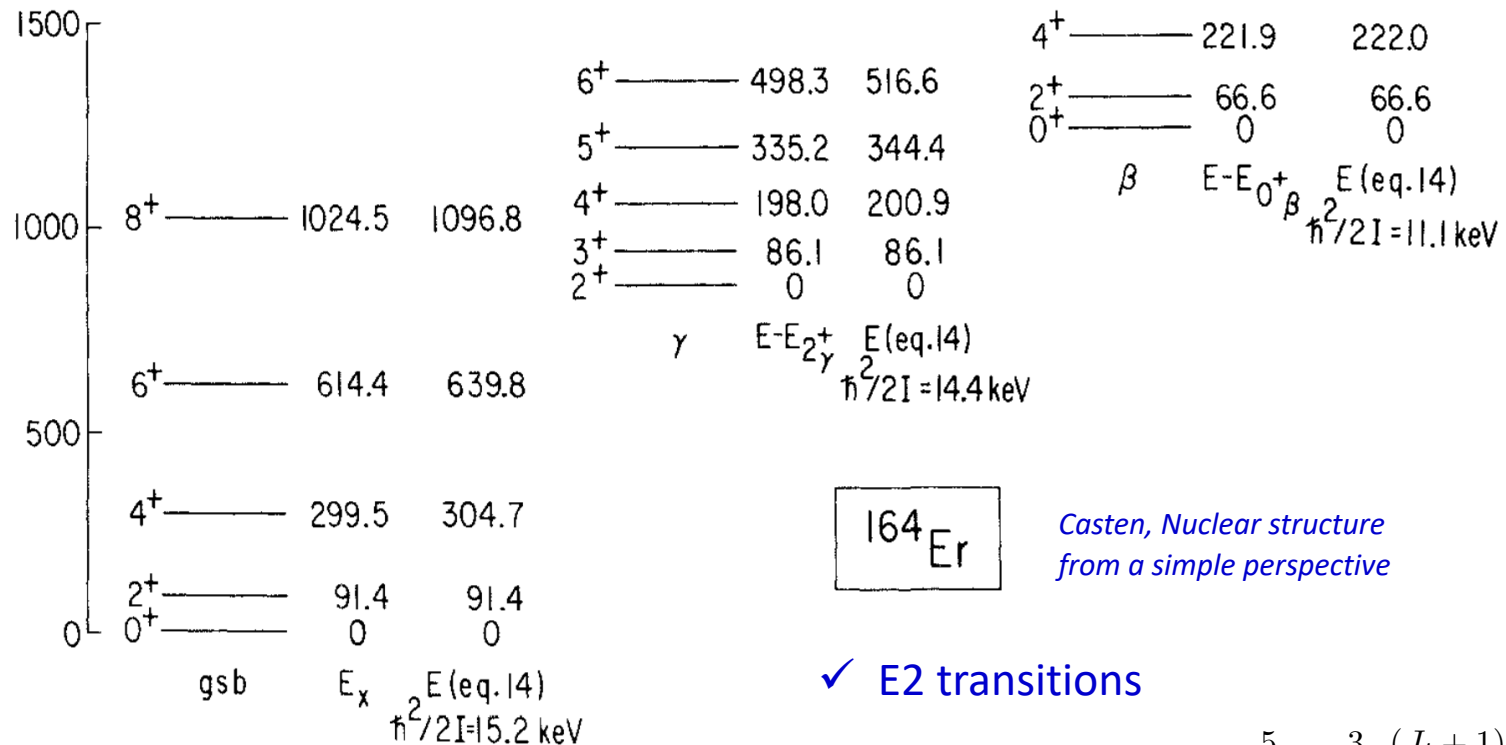


$\lambda=4 \ a_{40} < 0$

Experimental Evidence

□ Low-Energy Nuclear Experiments

- Rotational Band



✓ Rotational energy of a rigid rotor

$$E_{\text{rot}} = \frac{\hbar^2}{2I} [J(J+1) - K(K+1)]$$

✓ E2 transitions

$$B(E2; J_i + 2 \rightarrow J_i) = \frac{5}{16\pi} Q_0^2 \frac{3}{2} \frac{(J_i + 1)(J_i + 2)}{(2J_i + 3)(2J_i + 5)}$$

Intrinsic quadrupole moment

$$Q_0 = \sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} Z e R_0^2 \beta$$

Experimental Evidence

□ High-Energy Nuclear Collisions

- $^{238}\text{U} + ^{238}\text{U}$ $^{197}\text{Au} + ^{197}\text{Au}$

“The nuclear shape influences the geometry of QGP and its collective expansion, imprinting itself on the momentum distribution of the produced particles.”

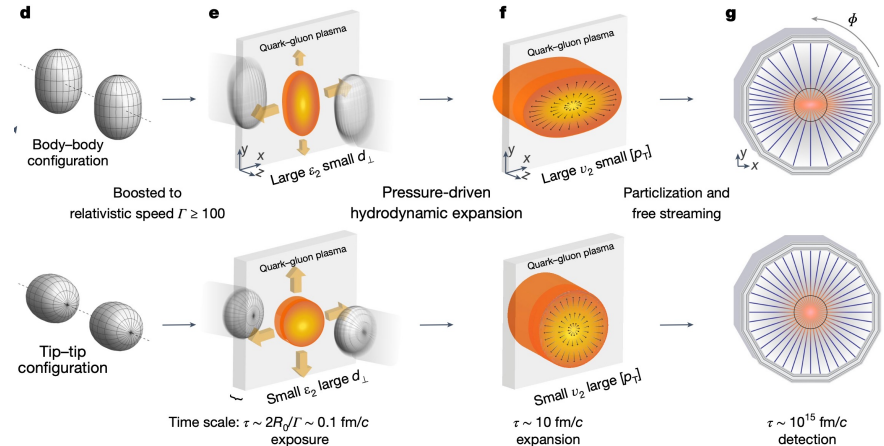
$$^{238}\text{U} \quad \beta_2 = 0.286 \pm 0.025$$

$$\gamma = 8.7^\circ \pm 4.5^\circ$$

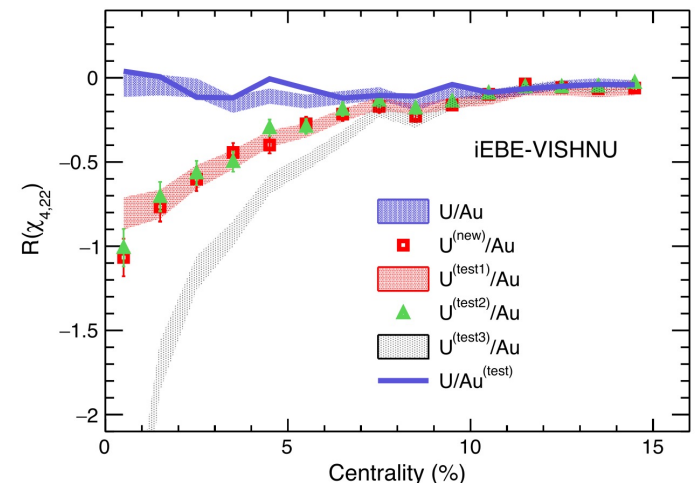
STAR, Nature 635, 67 (2024)

- ✓ The hexadecapole deformation of ^{238}U can be derived from relativistic heavy-ion collisions using a nonlinear response coefficient

H. J. Xu, J. Zhao, and F. Q. Wang, PRL 132, 262301 (2024)



‘figuring out a pocket watch by smashing two together and observing the flying debris’



Nuclear Collective Vibrations

➤ Non-charge-exchange excitations

- ✓ Giant monopole resonances (GMR)
- ✓ Giant dipole resonances (GDR)
- ✓ Giant quadrupole resonances (GQR)

...

γ photon
electron
...

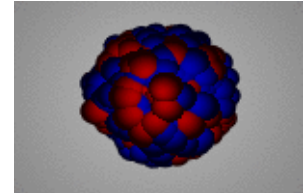
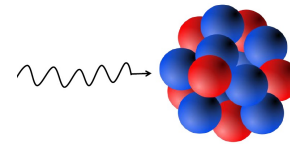
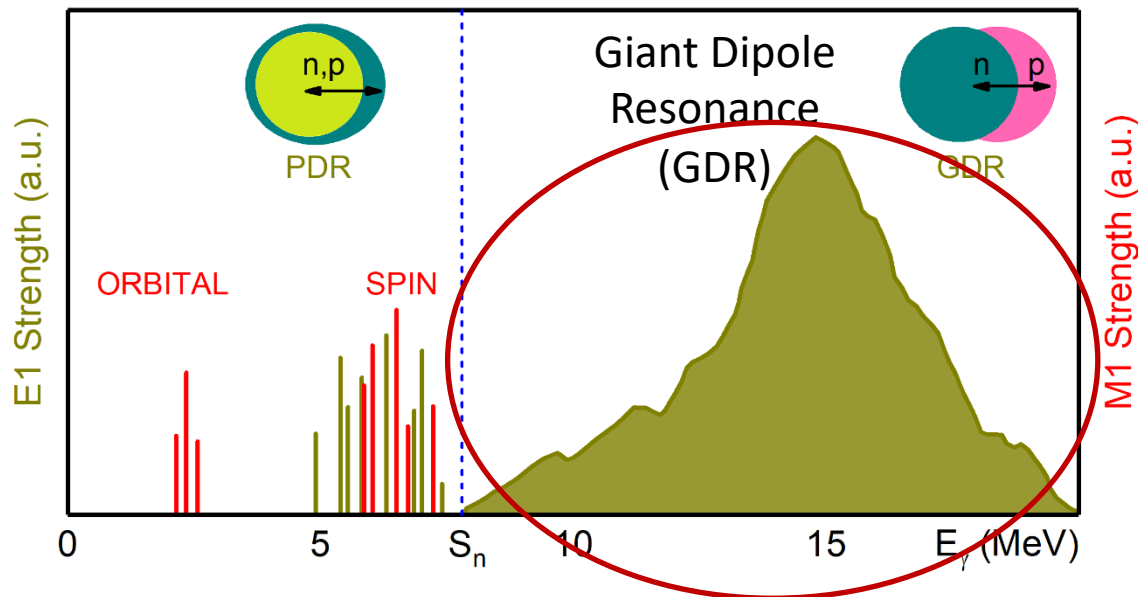


Photo-Nuclear Cross Section

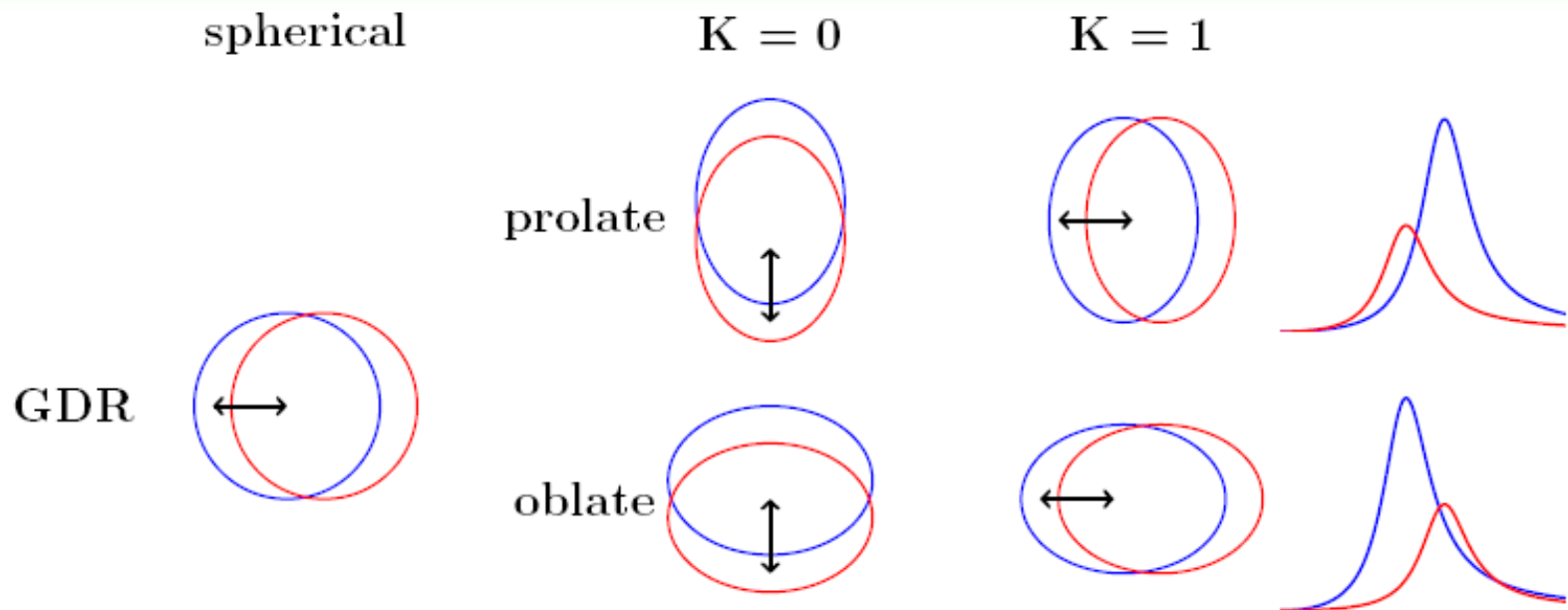
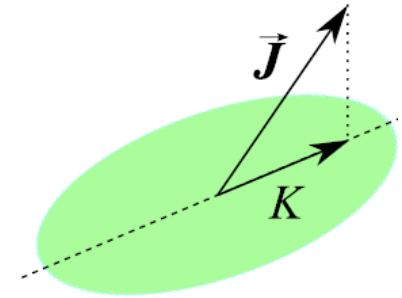


- Electric dipole transition
 - ✓ Giant dipole resonance
 - ✓ Pygmy dipole resonance
- Magnetic dipole transition
 - ✓ Scissors vibration
 - ✓ Spin flip resonance

Deformation degree of freedom

- Deformation effect on GDR

In mean-field approximation: good quantum number K

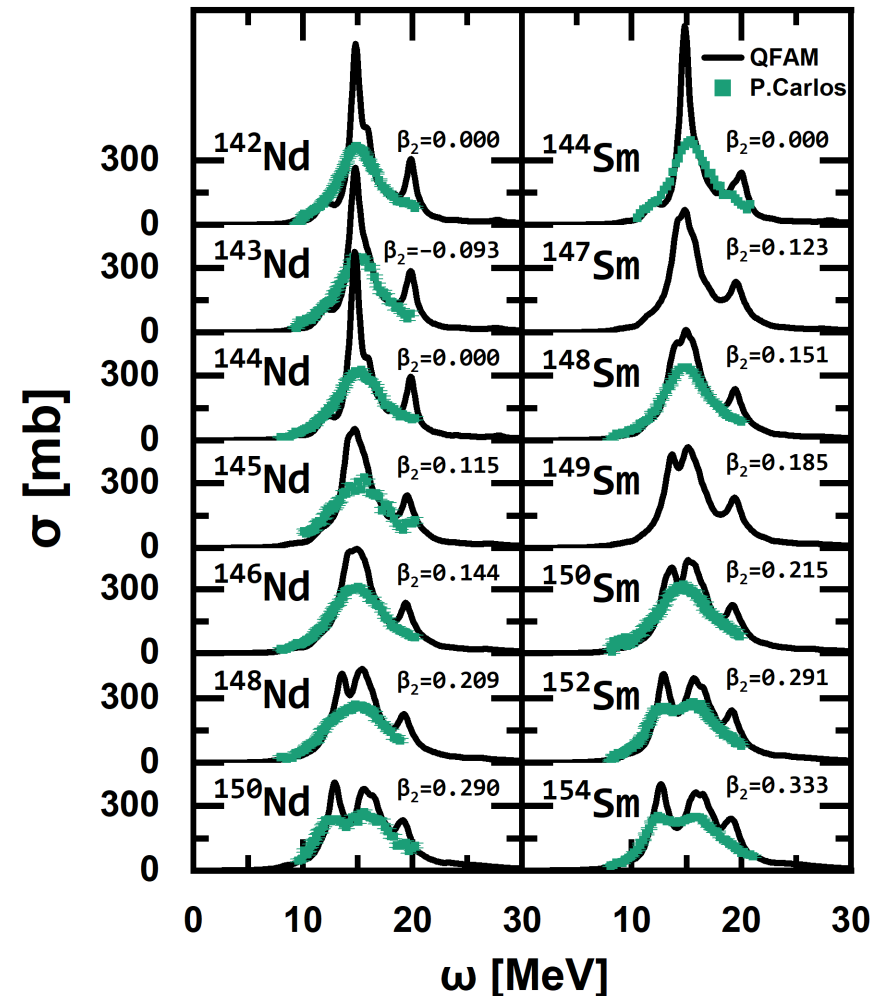


from D. Pena Arteaga

Evolution of cross section with deformation

• Photo-absorption cross sections in Nd & Sm isotopes

Calculated by relativistic quasiparticle finite amplitude method (QFAM) with DD-PC1



- Larger deformations with increasing neutrons from N=82
- Double peaks induced by deformations
- Well-reproduced peak energy
- underestimated resonance width & overestimated peak cross-section; improved in deformed nuclei

$$\text{Exp. } \sigma_{\text{P.Carlos}} = \sigma(\gamma, n) + \sigma(\gamma, pn) + \sigma(\gamma, 2n)$$

Nd: P. Carlos et al., Nucl. Phys. A 172, 437 (1971).

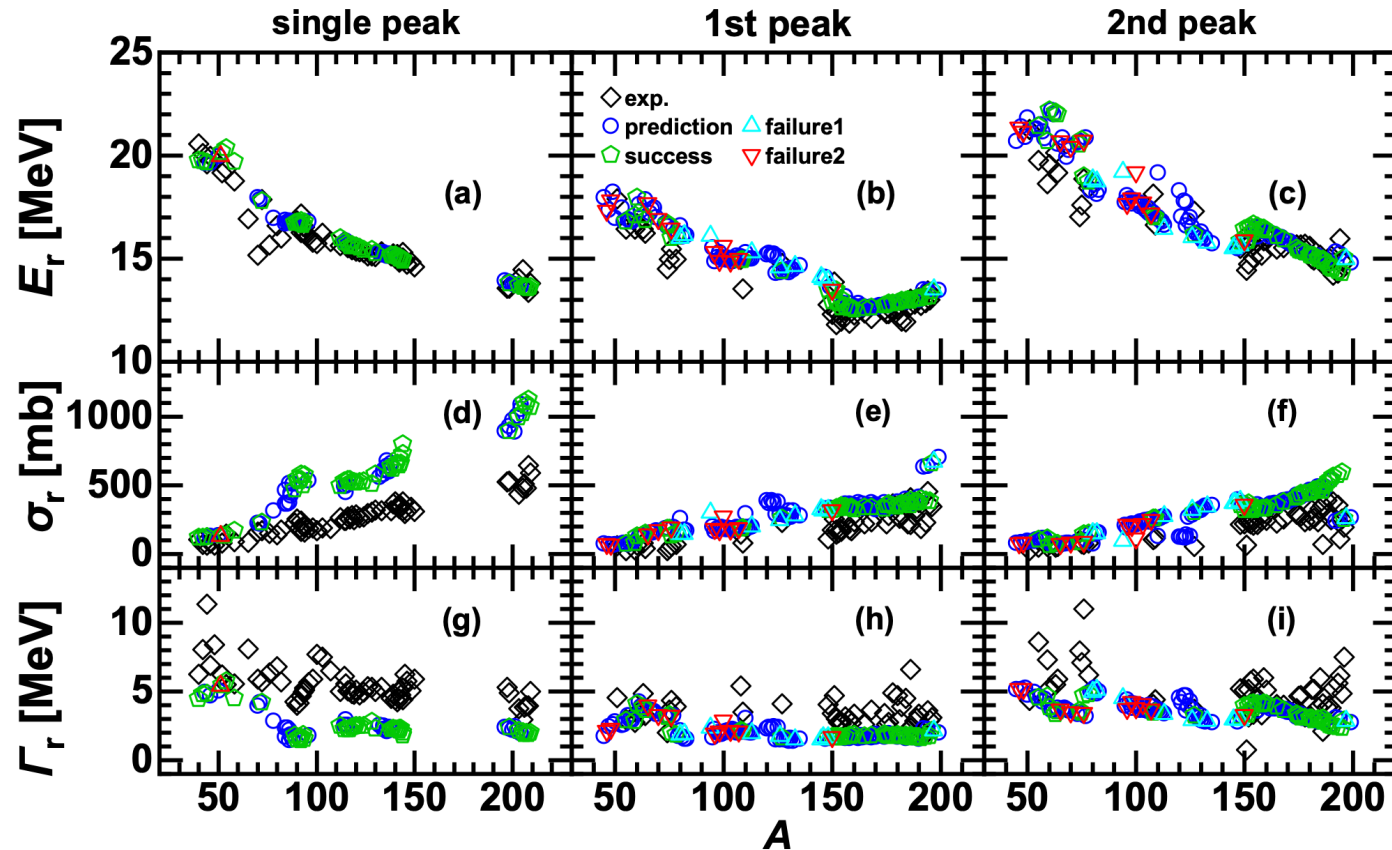
Sm: P. Carlos et al., Nucl. Phys. A 225, 171 (1974).

C. Chen, Y. F. Niu*, R. R. Xu*, and Y. Tian*,

ATNDT 168, 101770 (2026)

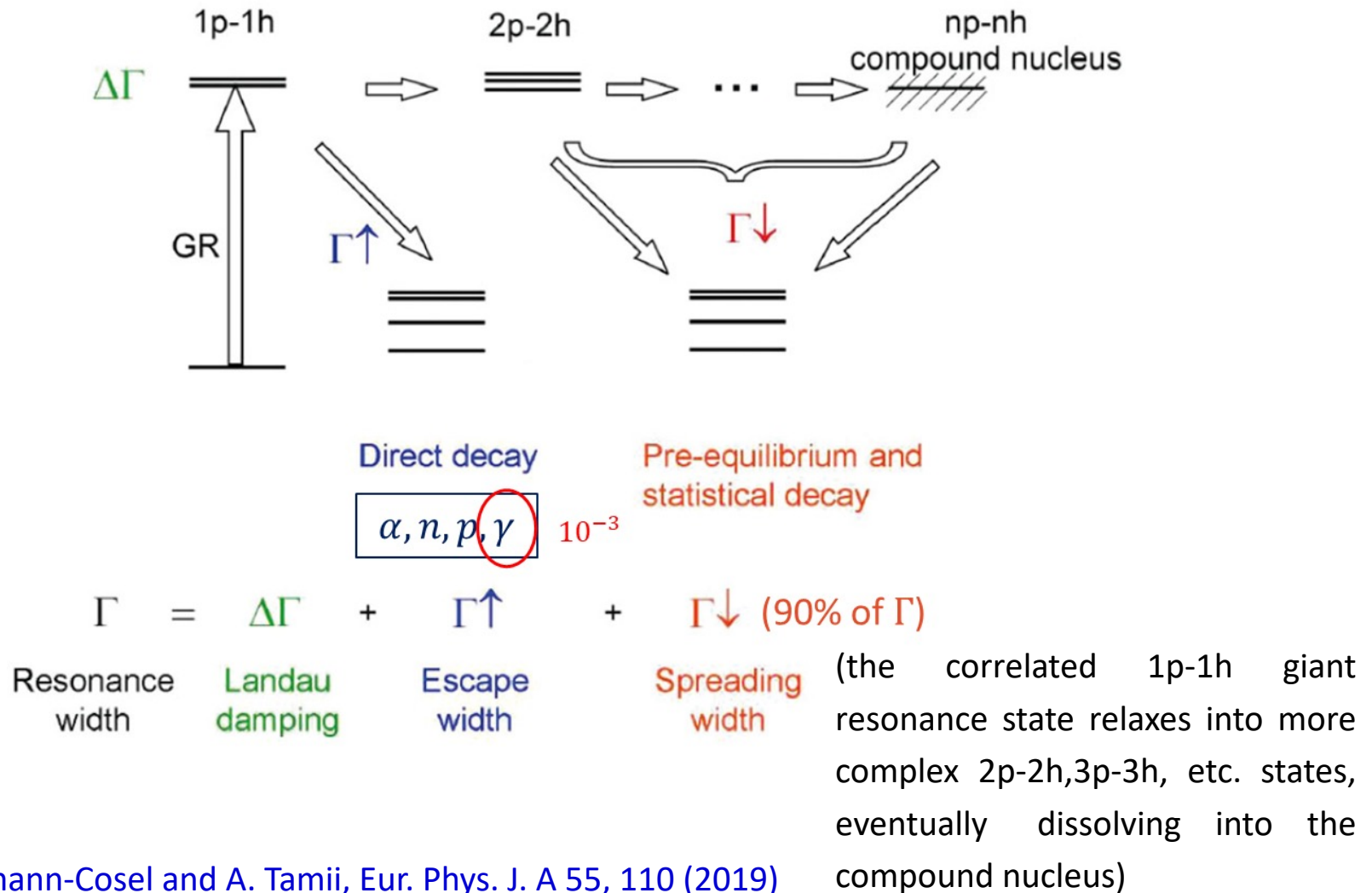
GDR parameters from QFAM calculations

- Systematic calculations of 235 stable nuclei from ^{40}Ca to ^{209}Bi , include 146 even-even nuclei, and 89 odd-A nuclei
- GDR parameters extracted from relativistic QFAM calculations in comparison with those from RIPL-3 library



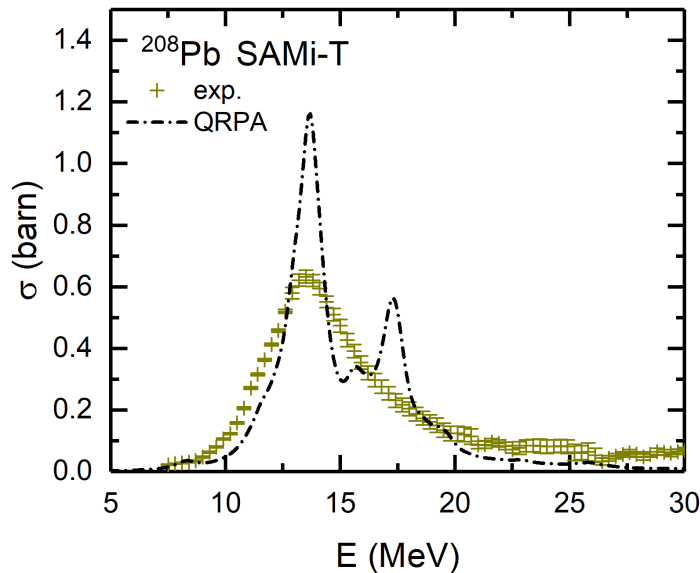
Width of giant resonances

- Typical values of FWHM: **3-5 MeV**



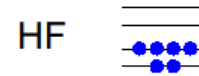
Limits of RPA Description

- (Q)RPA cannot describe the spreading width

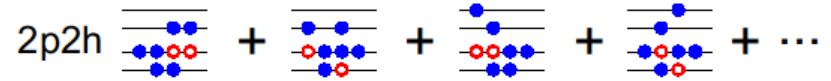
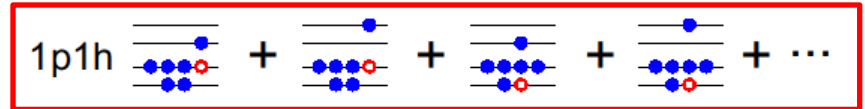


- Spreading Width (Damping Width)
energy and angular momentum of coherent vibrations
→ more complicated states of 2p-2h, 3p-3h, ... character

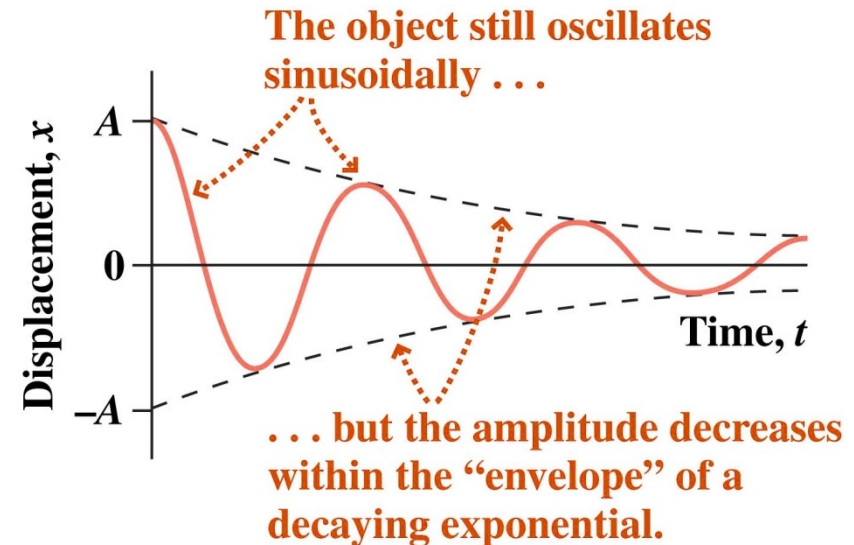
- Correlations beyond RPA



RPA



⋮



Solution: RPA+PVC model

HF 

1p1h  +  +  +  + ...

RPA

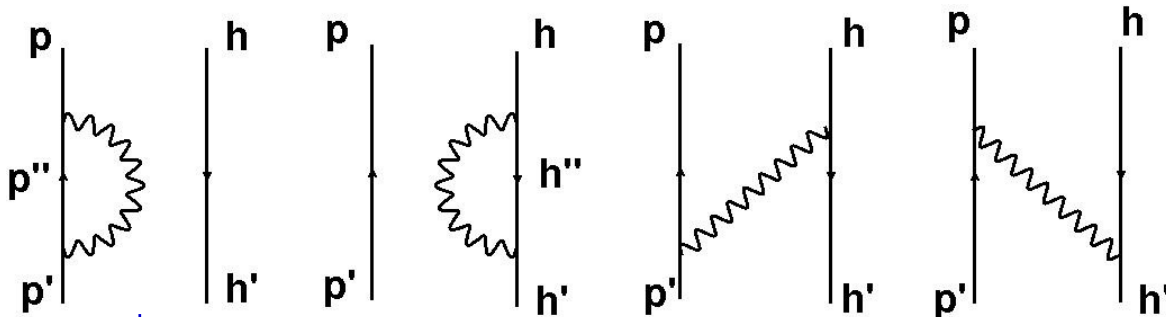
Second RPA

Drozd et al., PR 197, 1 (1990)

Gambacurta et al., PRC 81, 054312 (2010)

Yang et al., PRC 106, 014319 (2022)

- **RPA + PVC (particle vibration coupling)**



Low-lying vibration
phonons $|N\rangle$

G. Colo et al., PRC 50, 1496 (1994)

Y. F. Niu et al., PRC 85, 034314 (2012)

E. Litvinova et al., PRC 75, 064308 (2007)

QRPA+QPVC model

➤ To include pairing correlations for open-shell nuclei

- Quasiparticle RPA + quasiparticle vibration coupling
(**QRPA**) + (**QPVC**)

➤ based on relativistic DFT

Litvinova et al., PRC 78,014312 (2008)

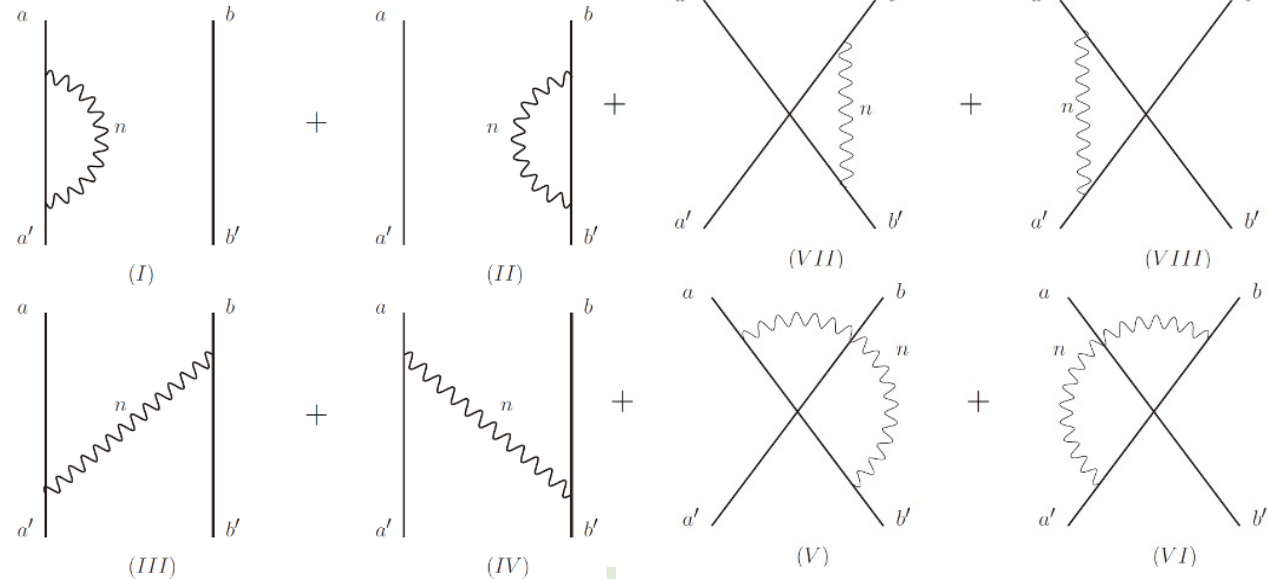
✓ QPVC effect

2 qp $\otimes$ phonons

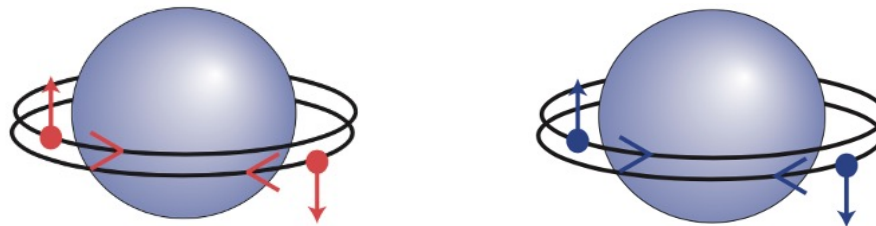
2 quasiparticles(qp)

➤ based on Skyrme DFT

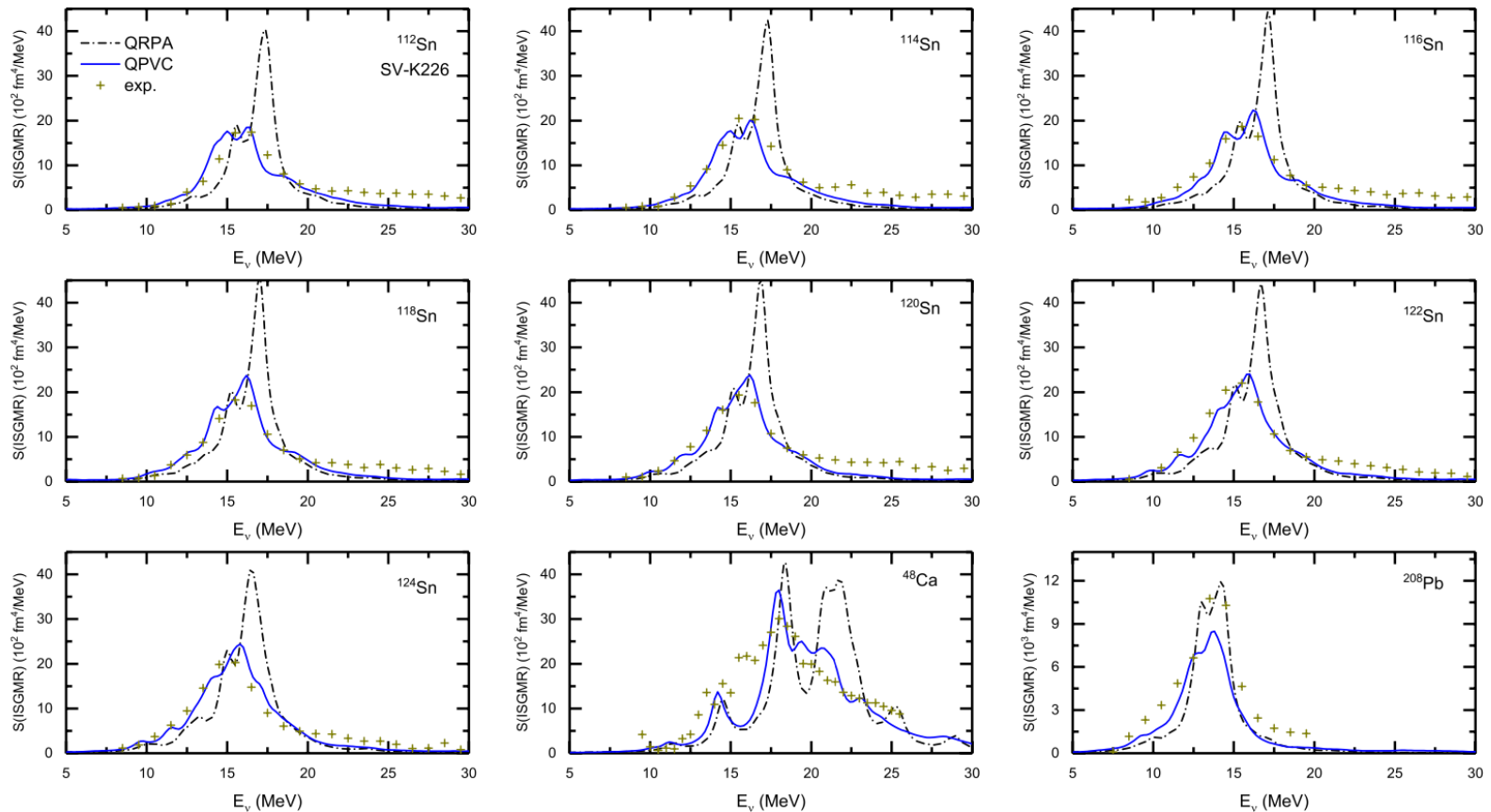
Z. Z. Li, Y. F. Niu, and G. Colo, PRL 131, 082501 (2023)



✓ Pairing effect



Unified description of GMR in Ca, Sn and Pb



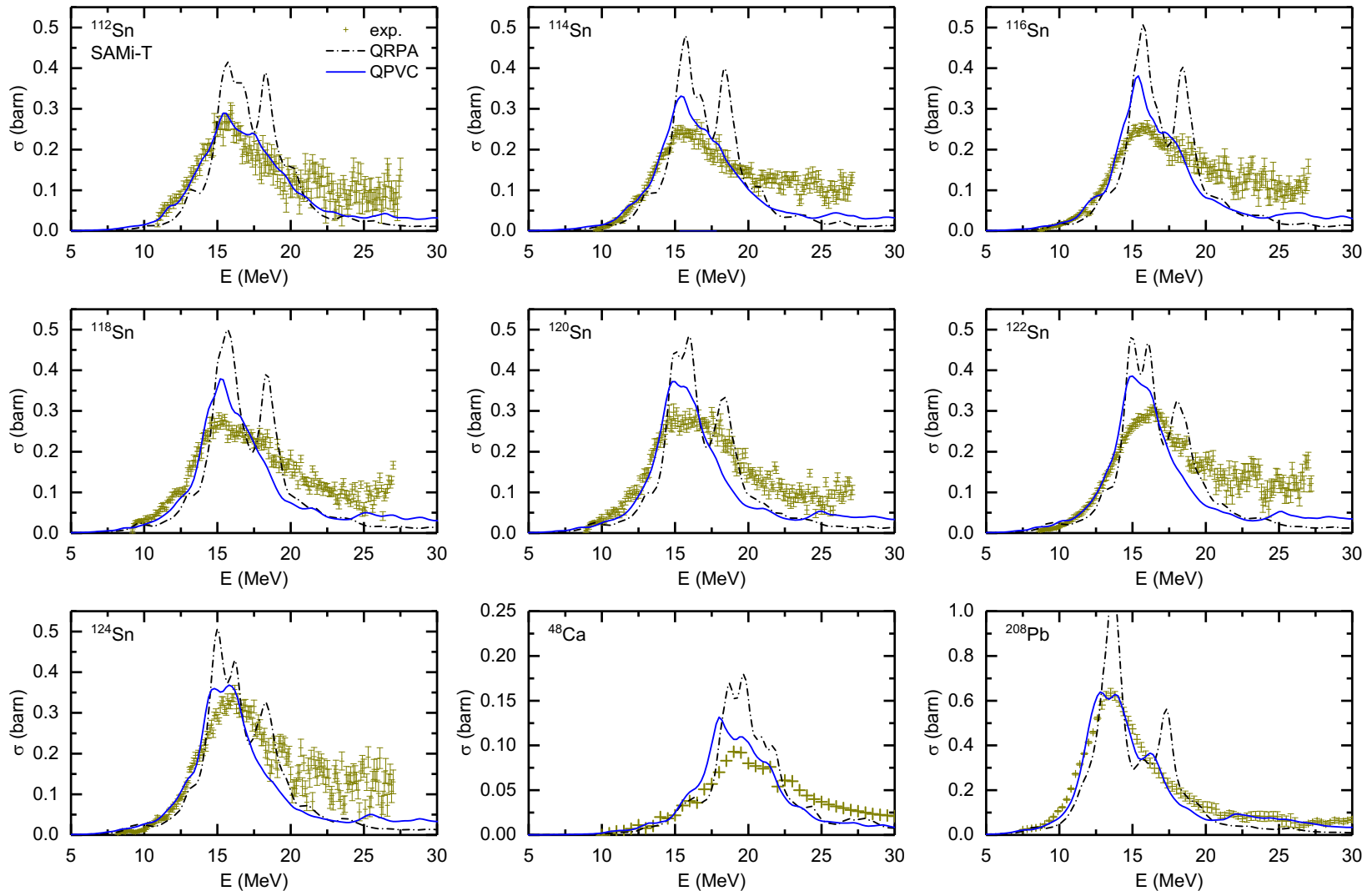
SV-K226: $K_\infty = 226$ MeV

Z. Z. Li, Y. F. Niu, and G. Colo, Phys. Rev. Lett. 131, 082501 (2023)

✓ “Excitation of the isoscalar giant monopole resonance and incompressibility of nuclear matter:
resolution of a long-standing puzzle”

M. N. Harakeh, Science Bulletin 68, 3081 (2023)

Photo-absorption cross sections studied by QPVC



Z. Z. Li, Y. F. Niu, and G. Colo, PRC 110,064317 (2024)

Nuclear Collective Vibrations

➤ Non-charge-exchange excitations

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- ✓ Giant quadrupole resonances (GQR)

...

γ photon
electron
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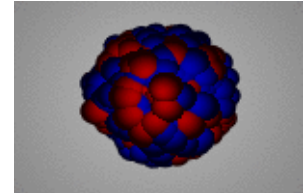
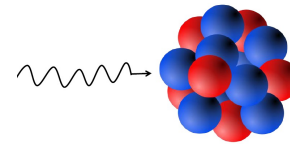
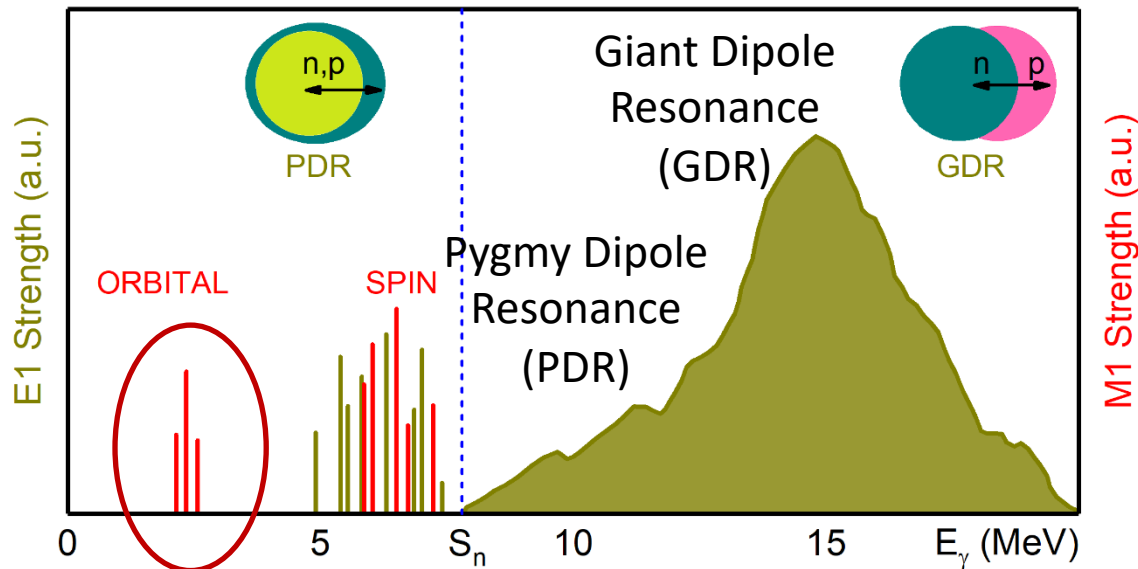


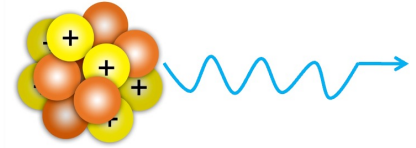
Photo-Nuclear Cross Section



- Electric dipole transition
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 - ✓ Spin flip resonance

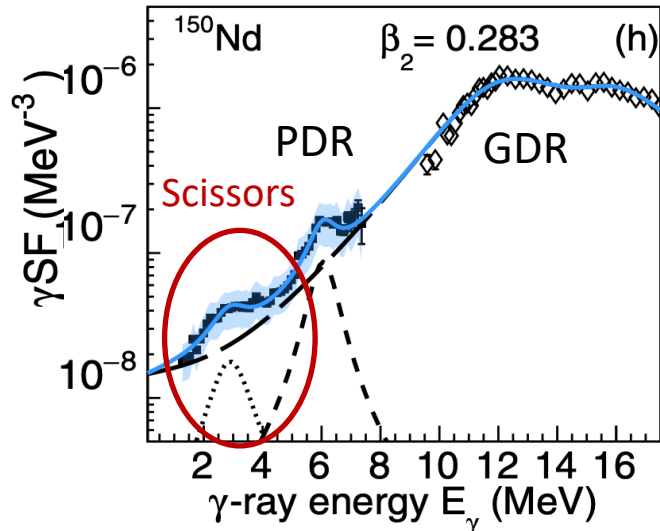
Gamma-ray strength function

➤ **γ -ray strength function:** a statistical measure of γ -decay strength



✓ Exp.: Oslo Method

• Scissors Vibration



M. Guttormsen et al, PRC 106, 034314 (2022)

✓ discovered since 1984

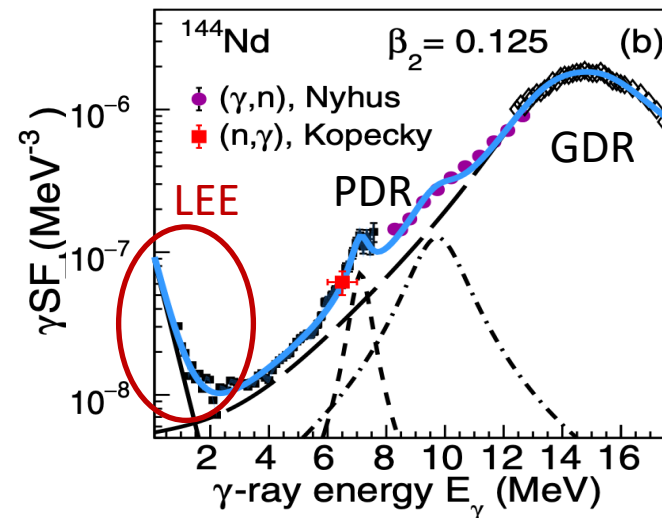
N. Lo Iudice and F. Palumbo, PRL 41, 1532 (1978)

D. Bohle, et al., PBL 137, 27 (1984)

✓ Extensively exist in **well deformed nuclei**

Scissors Vibration

• Low-Energy Enhancement (LEE)



✓ discovered since 2004

A. Voinov et al, PRL 93, 142504 (2004)

✓ extensively exist in **weakly deformed** nuclei

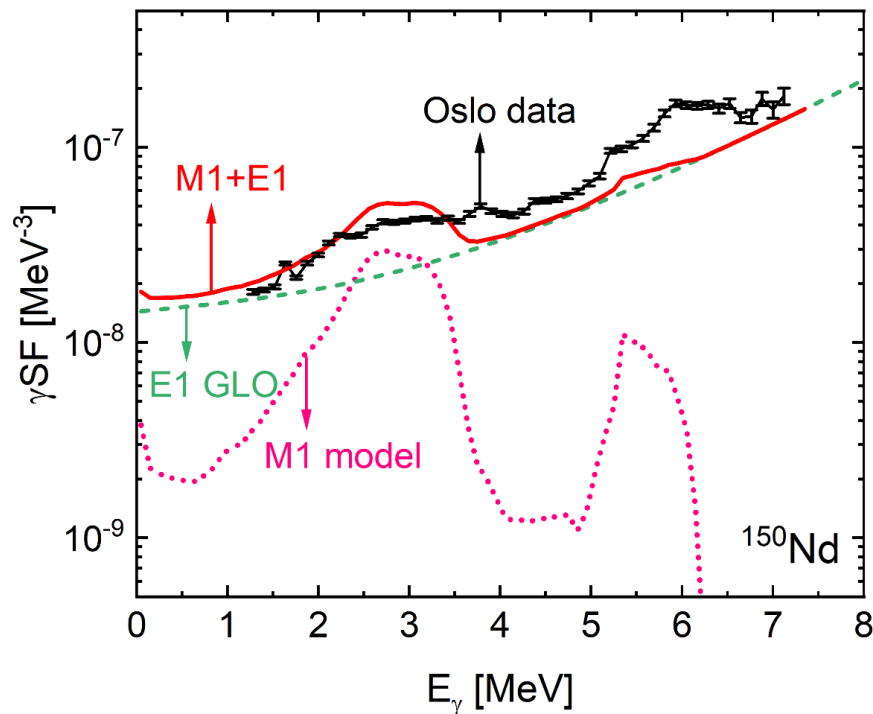
Not understood

Scissors Vibration

✓ Theory: Proton-neutron projected shell model

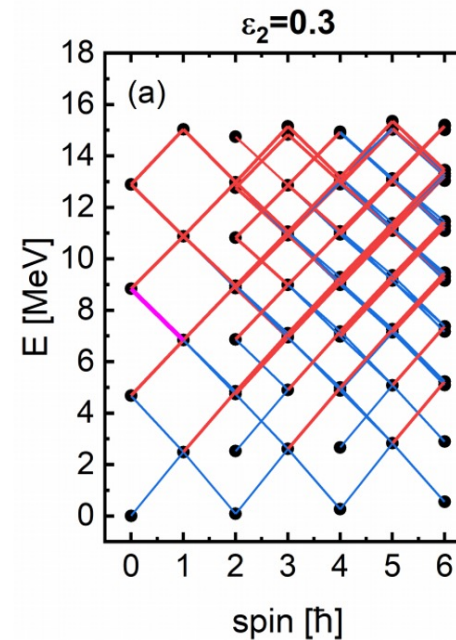
Y. Sun et al, **PRL** 80,672 (1998)

➤ Reproduce the scissors resonance

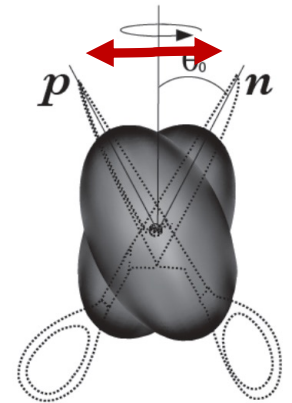


Exp: M. Guttormsen et al, **PRC** 106, 034314 (2022)

➤ Spectrum corresponding to the scissors resonance



Equal distance:
Harmonic Vibration



F. Q. Chen, Y. F. Niu*, Y. Sun* and M. Wiedeking, **PRL** 134, 082502 (2025)

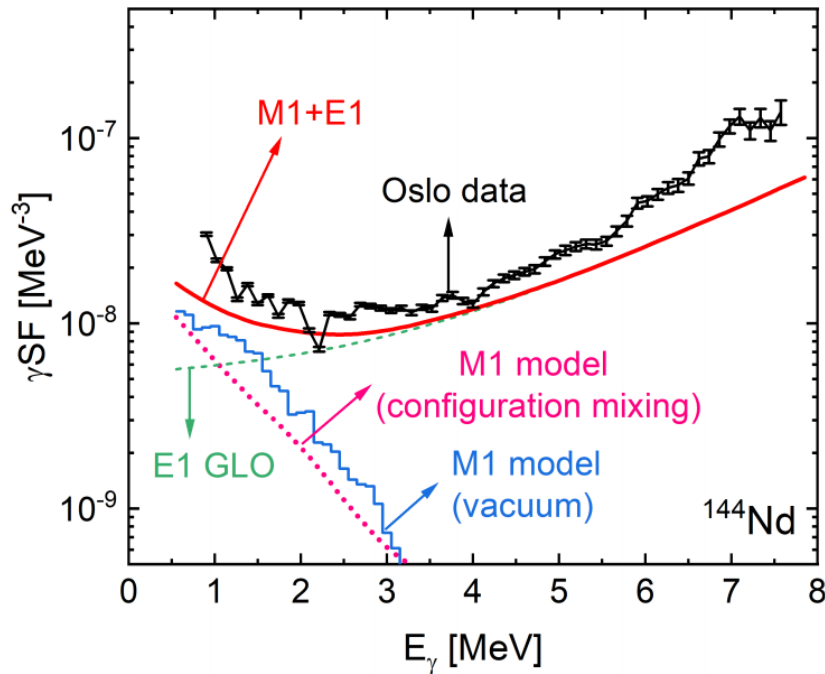
Origin of LEE: Scissors Rotation

✓ Theory: Proton-neutron projected shell model

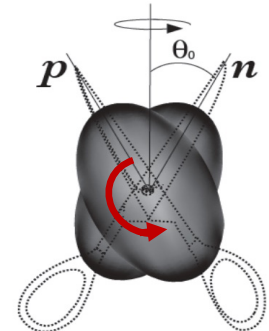
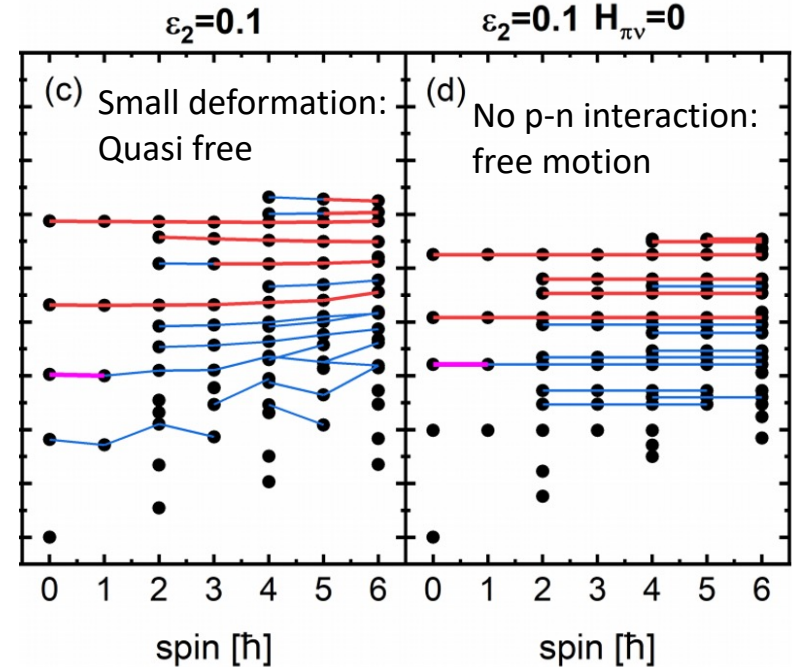
Y. Sun et al, **PRL** 80,672 (1998)

➤ Successfully reproduce LEE

➤ Spectrum corresponding to the LEE



F. Q. Chen, Y. F. Niu*, Y. Sun* and M. Wiedeking, **PRL** 134, 082502 (2025)

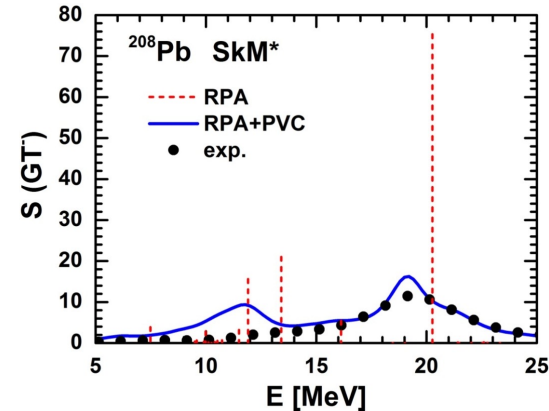
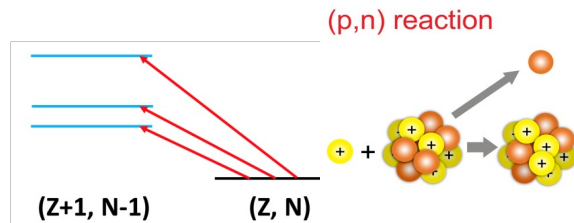


Nuclear Collective Vibrations

➤ Charge-exchange excitations

- ✓ Gamow-Teller Resonances (GTR)

$$\Delta S=1 \quad \Delta L=0 \quad \Delta T=1 \quad \hat{O}_{GT^-} = \sum_{i=1}^A \vec{\sigma}(i) \cdot \vec{\tau}_-(i)$$

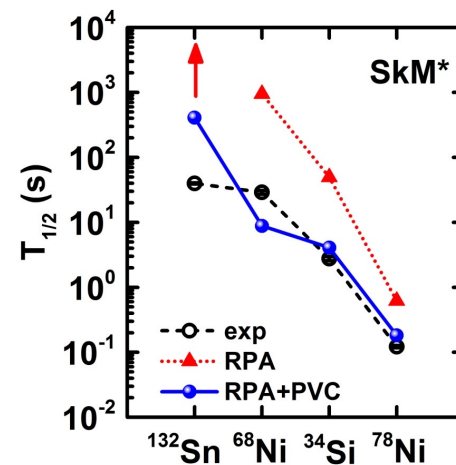
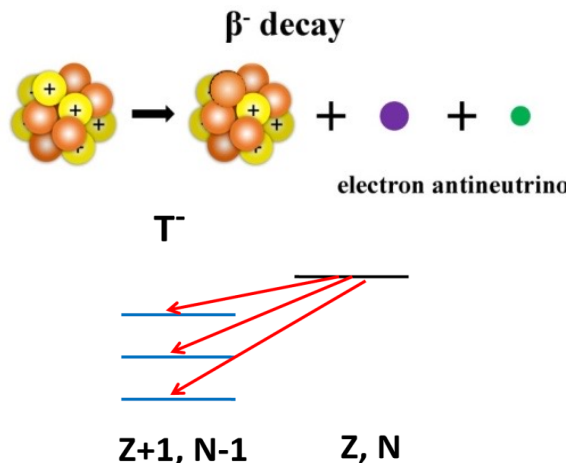


Y. F. Niu, G. Colo, M. Brenna et al., *PRC* 85, 034314 (2012)

Y. F. Niu, G. Colo, and E. Vigezzi, *PRC* 90, 054328 (2014)

Y. F. Niu, G. Colo, and E. Vigezzi, *PRC* 94, 064328 (2016)

- ✓ β^- -decay: dominated by low-energy GT transition



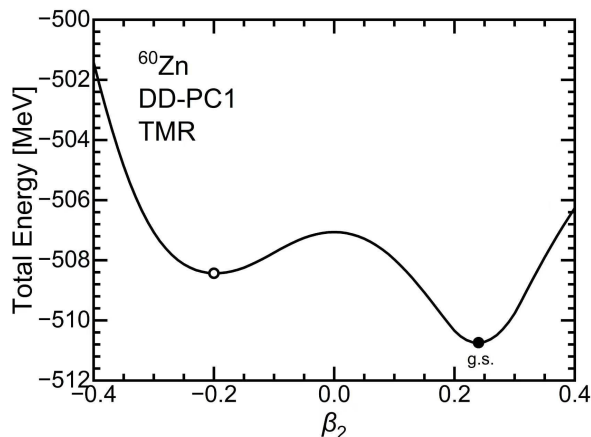
Y.F. Niu, Z. M. Niu, et al., *PRL* 114, 142501 (2015)

Y.F. Niu, Z. M. Niu, et al., *PLB* 780, 325 (2018)

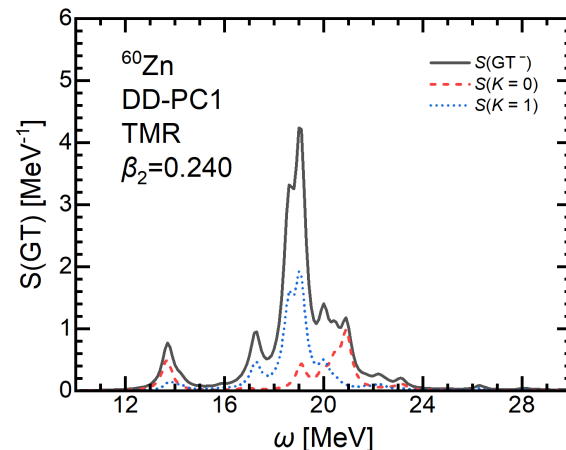
Deformation effect on GTR

- We newly developed relativistic proton-neutron QFAM approach

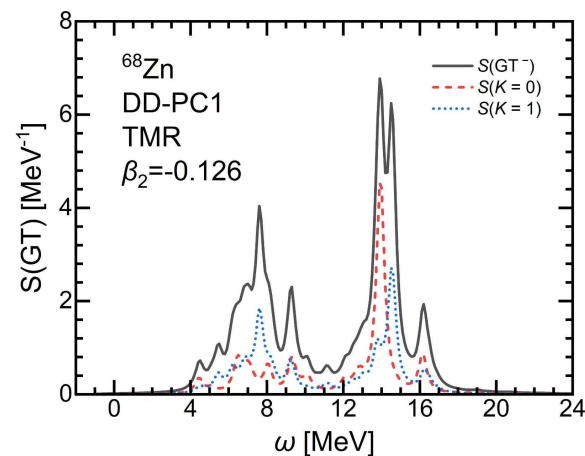
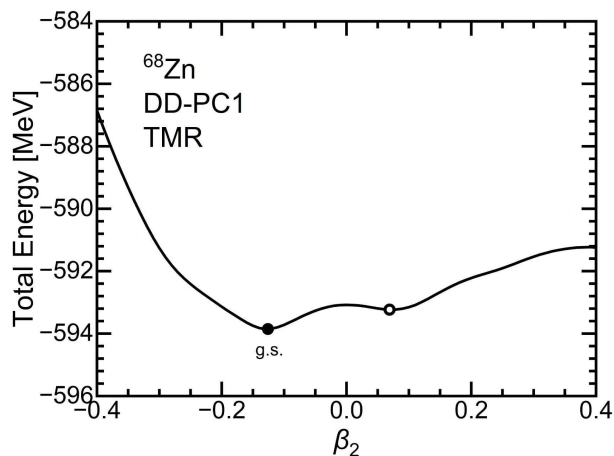
✓ Potential energy surface (PES)



✓ Gamow-Teller transition strength



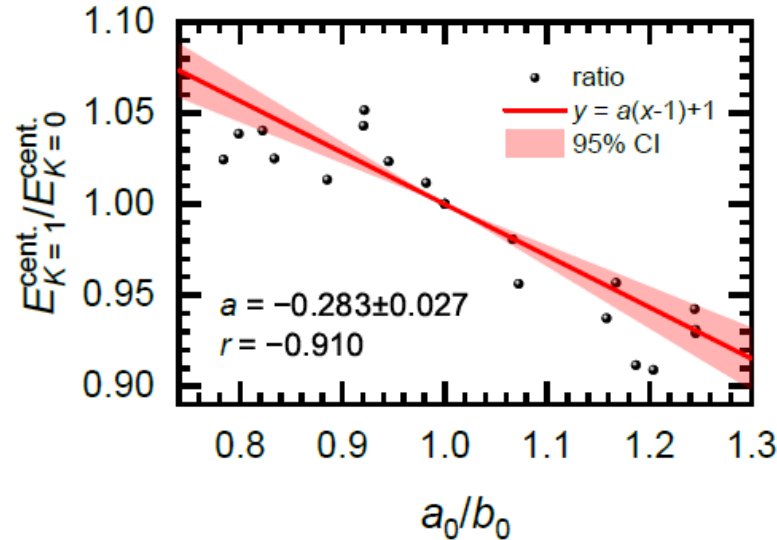
Prolate
GTR
 $E(K=1) < E(K=0)$



Oblate
GTR
 $E(K=1) > E(K=0)$

Deformation effect on GTR

- Zn isotopic chain



$$\frac{a_0}{b_0} = \frac{1 + \sqrt{\frac{5}{4\pi}} \beta_2}{1 - \frac{\sqrt{\frac{5}{4\pi}} \beta_2}{2}}$$

a_0 : symmetry axis
 b_0 : vertical to symmetry axis

C. Chen, Z. Z. Li, Y. N. Zhang, and Y. F. Niu*,
 PRC 112, 064332 (2025)

- Microscopic picture

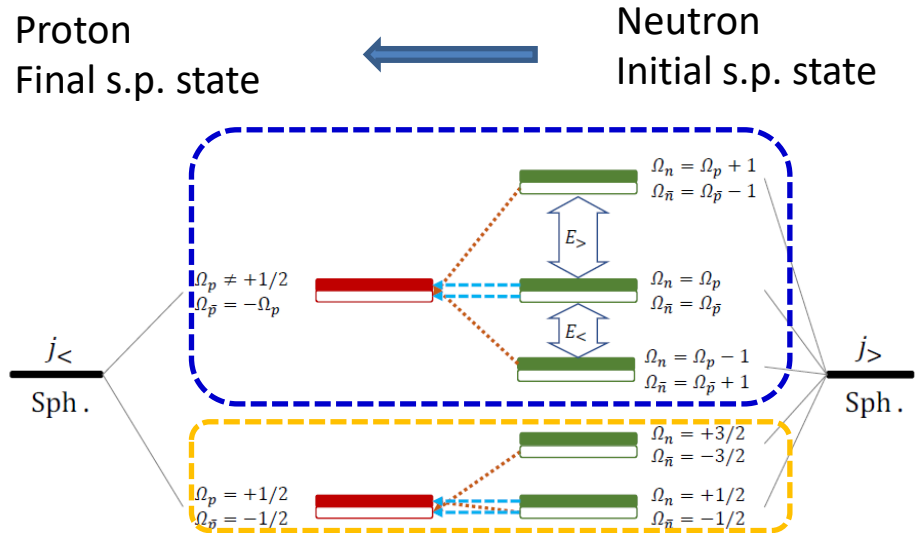
For a given final s.p. state:

K=0 component: assume $E_{K=0} = 0$

K=1 component:

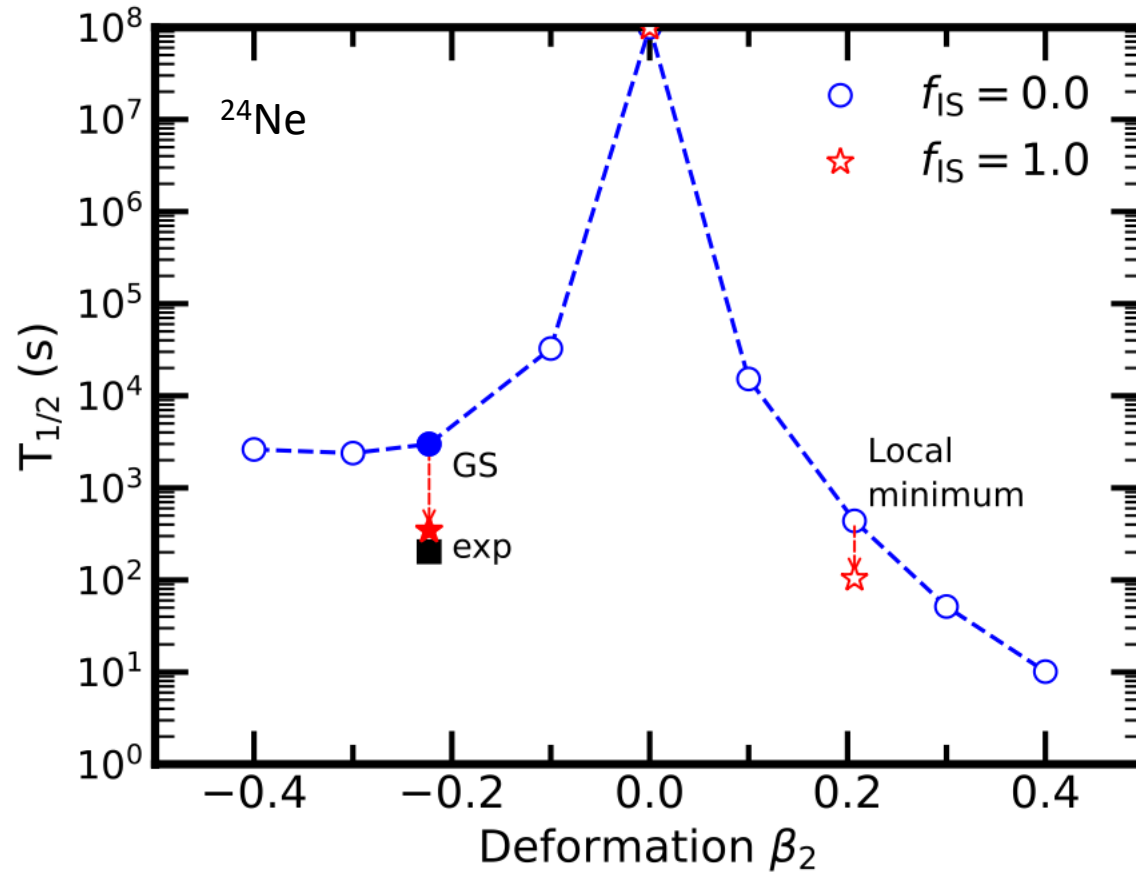
$E_{K=1} = 0;$	$E_{K=1} = E_<$
$E_{K=1} < 0;$	$E_{K=1} = -E_>$

Prolate case



Deformation effect on β -decay

- Relativistic proton-neutron QFAM approach based on deformed relativistic Hartree Bogoliubov in continuum (DRHBc)



Z. Z. Li, X. F. Jiang, Y. F. Niu, X. W. Sun, and J. Meng*, in preparation

Summary and Perspective

Nuclear deformation effect on Nuclear collective Vibration

Non-charge-exchange excitations

- ✓ Deformation manifests itself in the splitting of GDR
- ✓ Deformation is essential in the evolution from scissors vibration to scissors rotation => LEE of gamma-ray strength function

Charge-exchange excitations

- ✓ Deformation manifests itself in the splitting of GTR
- ✓ Deformation is essential to reproduce β -decay half-lives

Perspective

- Deformation + Particle Vibration Coupling

Acknowledgement

Collaborators:

LZU: Chen Chen, Fangqi Chen

PKU: Jie Meng, Zhengzheng Li, Xiaofei Jiang

CIAE: Ruirui Xu, Yuan Tian

Shanghai Jiaotong Uni.: Yang Sun

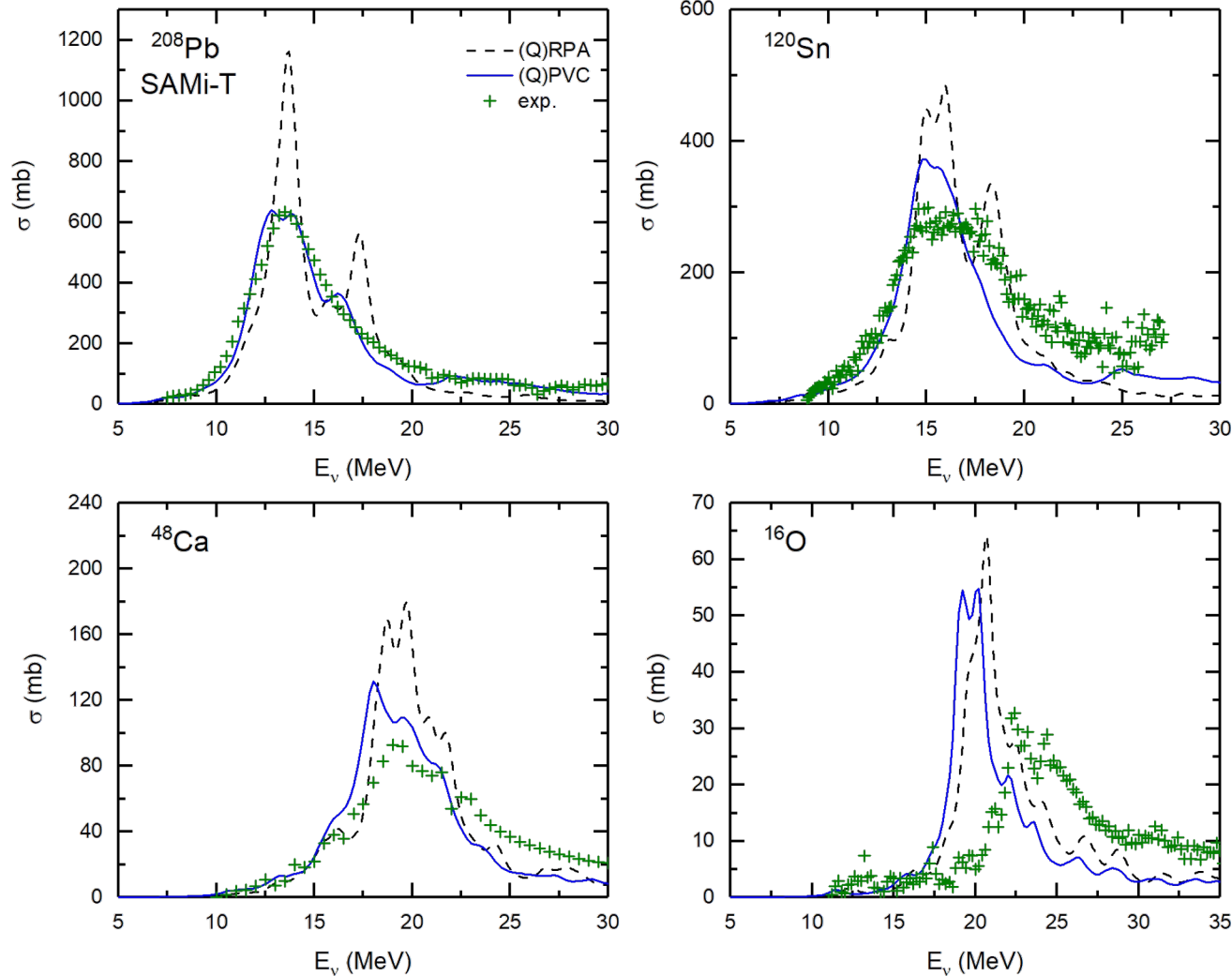
Berkeley: Mathis Wiedeking

Aizu Univ. and RIKEN: H. Sagawa

Milan Univ. : G. Colo, E. Vigizzi

Thank you!

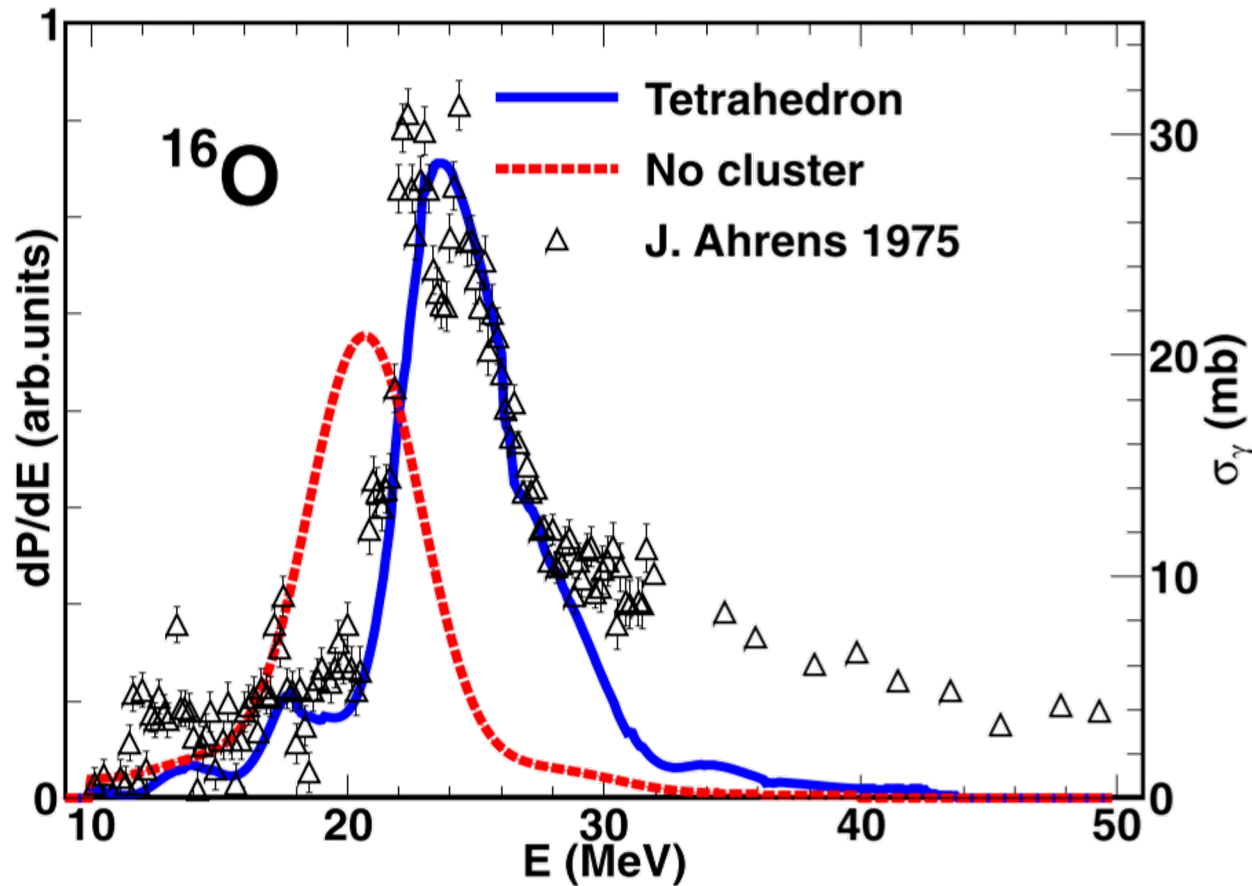
Photo-absorption cross section



Exp: EXFOR

- For ^{208}Pb , ^{120}Sn and ^{48}Ca , QPVC much improves the dipole strength function
- For ^{16}O , the energy is much underestimated

^{16}O : cluster effect?



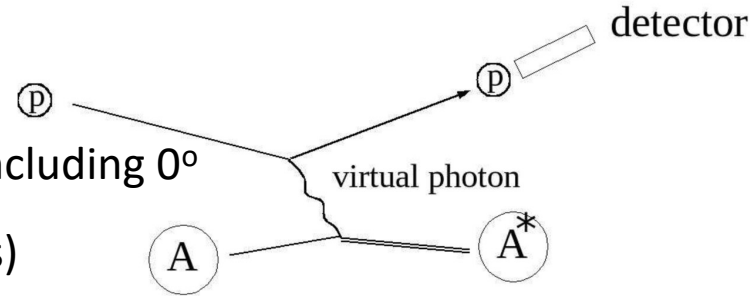
Extended Quantum Molecular Dynamics (EQMD) calculation

W. B. He, et al., PRC 94, 014301 (2016)

Measurements of dipole polarizability

(p,p') reaction

- ✓ polarized proton scattering at angles close to and including 0°
- ✓ excitations via virtual photons (Coulomb excitations)

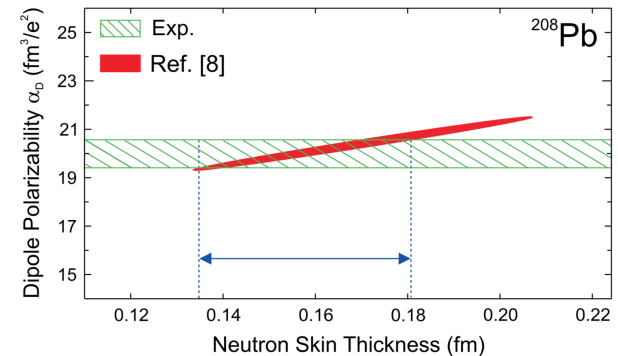


• Dipole Polarizability of ^{208}Pb

This exp. up to 20 MeV + γ absorption data up to 130 MeV

$$\alpha_D = 20.1(6) \text{ fm}^3$$

A. Tamii, et al., Phys. Rev. Lett. 107, 062502 (2011)



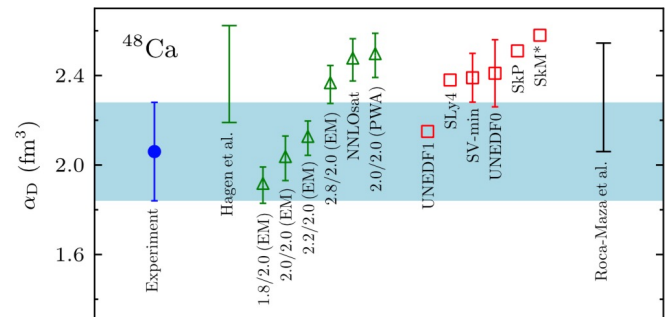
$$r_{\text{skin}} = 0.156^{+0.025}_{-0.021} \text{ fm}$$

• Dipole Polarizability of ^{48}Ca

This exp. between 10 - 25 MeV + (γ, γ') below 10 MeV + photoabsorption data up to 60 MeV

$$\alpha_D = 2.07(22) \text{ fm}^3$$

J. Birkhan, et al., Phys. Rev. Lett. 118, 252501 (2017)



• Dipole Polarizability of ^{120}Sn

This exp. between 5 – 10 MeV + averaged this exp. results and (γ, xn) results between 10-22 MeV+ (γ, γ') below 5 MeV (negligible) + (γ, xn) for high energy up to 135 MeV

$$\alpha_D = 8.93(36) \text{ fm}^3$$

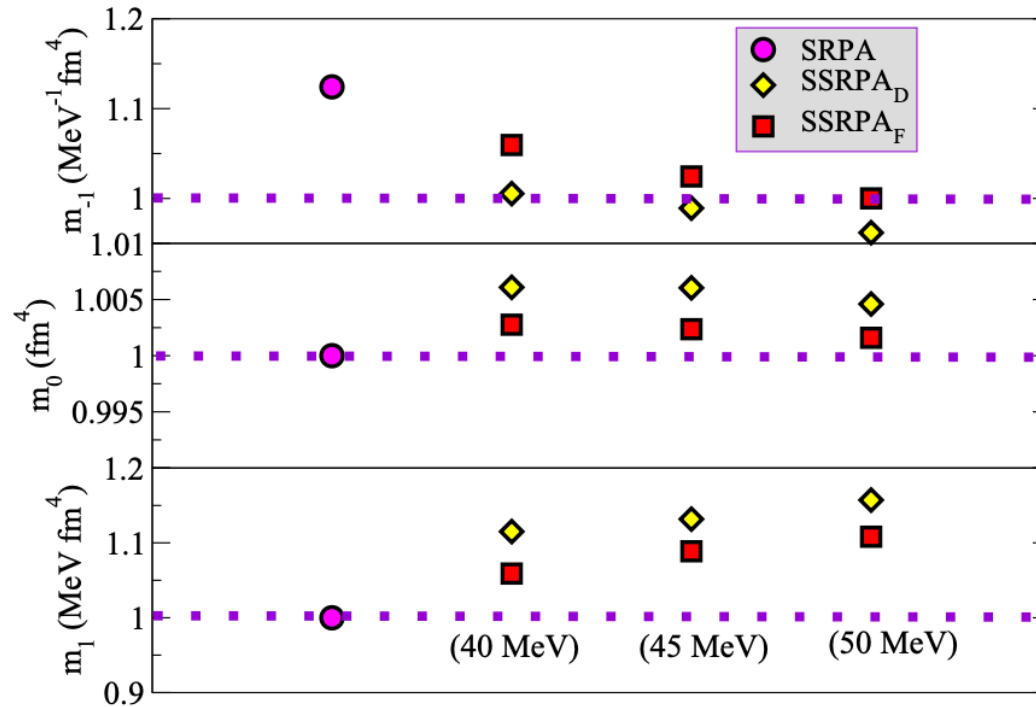
T. Hashimoto, et al., Phys. Rev. C 92, 031305(R) (2015)

$$r_{\text{skin}} = 0.14\text{-}0.20 \text{ fm}$$

Calculation of dipole polarizability: PVC effect

- Second RPA or Particle-vibration coupling (PVC) effect

✓ Second RPA: Quadrupole case



Subtraction method:

$$R(\omega = 0) = R_{\text{KS}}^{\text{RPA}}$$

to avoid double counting

Tselyaev, PRC 88, 054301(2013)

D. Gambacurta, et al., Phys. Rev. C 92, 034303 (2015)

- With subtraction, m_{-1} is kept the same as RPA level
- Without subtraction, m_1 and m_0 are kept the same as RPA level

Quasiparticle Finite Amplitude Method (QFAM)

- Traditional QRPA method: solve the QRPA equation by matrix diagonalization

$$\left[\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} - \omega \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \right] \begin{pmatrix} X(\omega) \\ Y(\omega) \end{pmatrix} = \begin{pmatrix} F^{20}(\omega) \\ F^{02}(\omega) \end{pmatrix}$$

✓ Matrices $A_{\mu\nu\mu'\nu'}$ and $B_{\mu\nu\mu'\nu'}$ are two-body matrix elements

✓ Matrix dimension becomes large for deformed nuclei

[Arteaga et al., PRC 79, 034311 \(2009\)](#)

- QFAM: solve the QRPA equation by iteration

$$\begin{aligned} (E_\mu + E_\nu - \omega) X_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{20}(\omega) &= F_{\mu\nu}^{20}(\omega), \\ (E_\mu + E_\nu + \omega) Y_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{20}(\omega) &= F_{\mu\nu}^{20}(\omega). \end{aligned}$$

✓ Matrices $\delta H_{\mu\nu}^{20}$ and $F_{\mu\nu}^{20}$ are one-body matrix elements

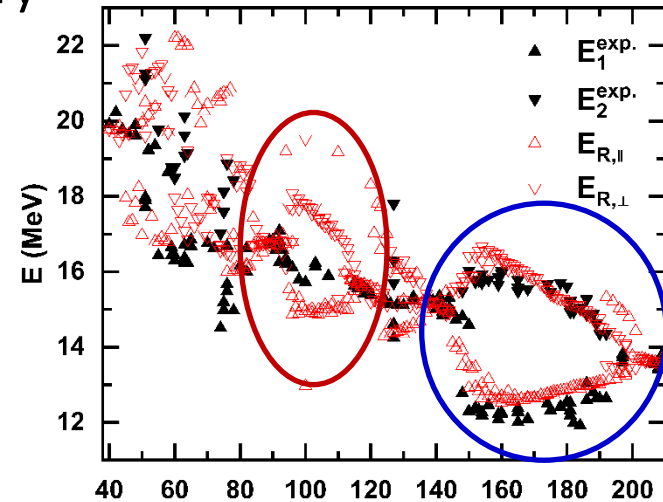
✓ Avoid the matrix diagonalization, feasible for deformed nuclei

[Nakatsukasa et al., PRC 76, 024318 \(2007\)](#)

GDR parameters from QFAM calculations

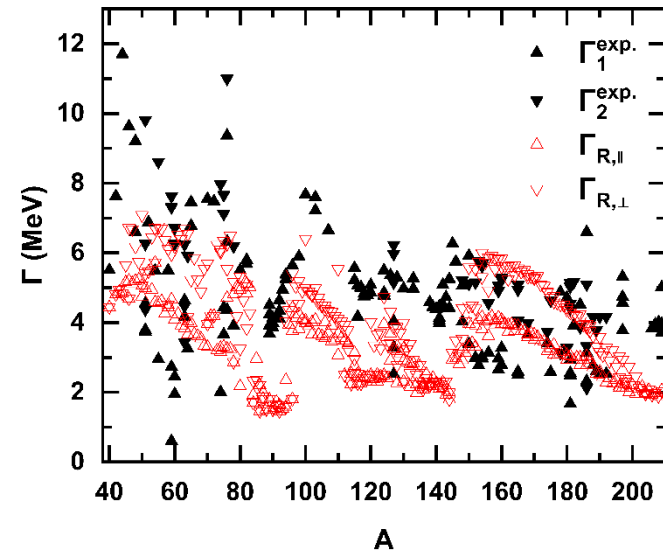
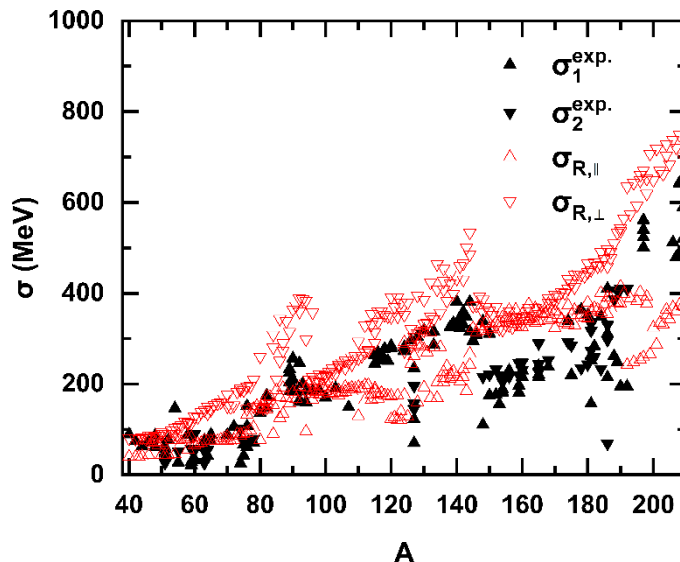
- GDR parameters extracted from relativistic QFAM calculations in comparison with those from RIPL-3 library

- ✓ Resonance **energies** are well reproduced
- ✓ **Widths** are underestimated, correspondingly **peak cross sections** are overestimated, due to **lack of many-body correlations**



wrong classification

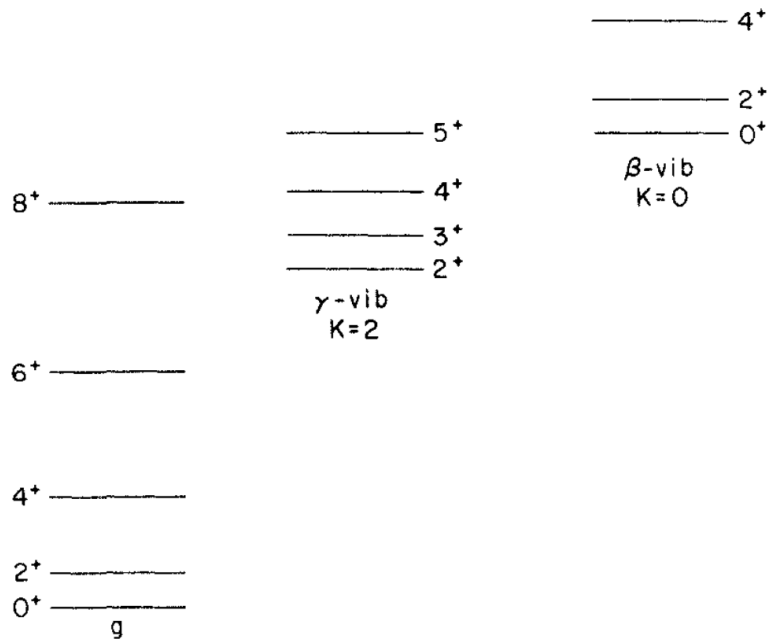
well reproduced



Experimental Evidence

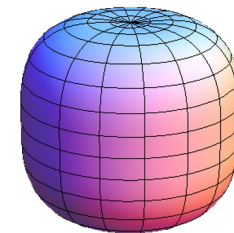
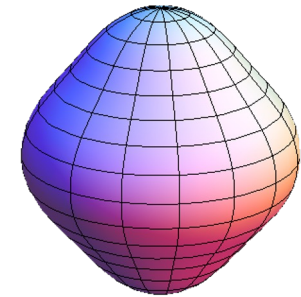
- Low-Energy Nuclear Experiments

Rotational Band

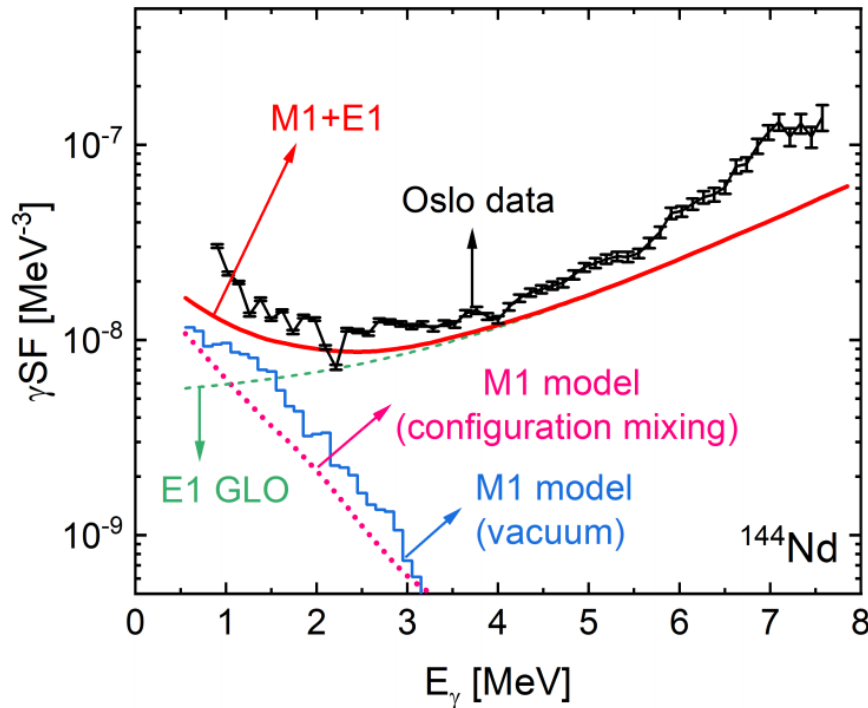


TYPICAL DEFORMED NUCLEUS : $\pi = +$ LEVELS

Positive /Negative β_4



Reproduction of LEE in projected shell model



Exp.: *M. Guttormsen et al, PRC 106, 034314 (2022)*

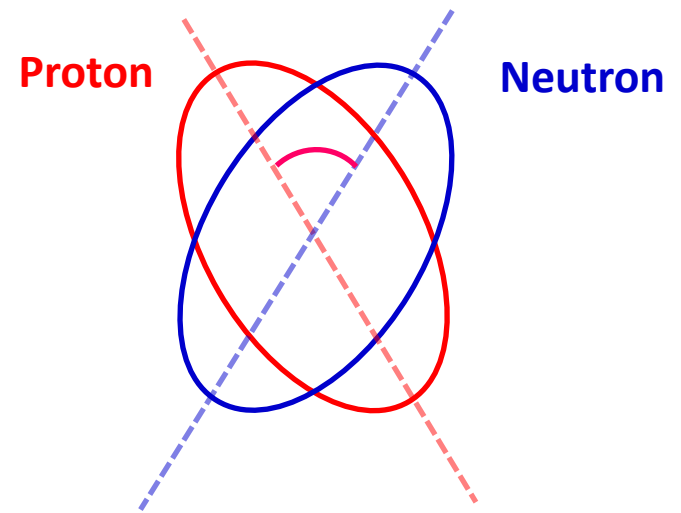
LEE emerges with a fixed **vacuum configuration**



arising from **collective motion** generated by two rotors

F. Q. Chen, Y. F. Niu, Y. Sun and M. Wiedeking, PRL 134, 082502 (2025)

- Projected shell model



Two interacting rotors

$$[\hat{P}_{M_\nu K_\nu}^{I_\nu} |\kappa_\nu\rangle \otimes \hat{P}_{M_\pi K_\pi}^{I_\pi} |\kappa_\pi\rangle]_M^I$$

Y. Sun et al, PRL 80,672 (1998)



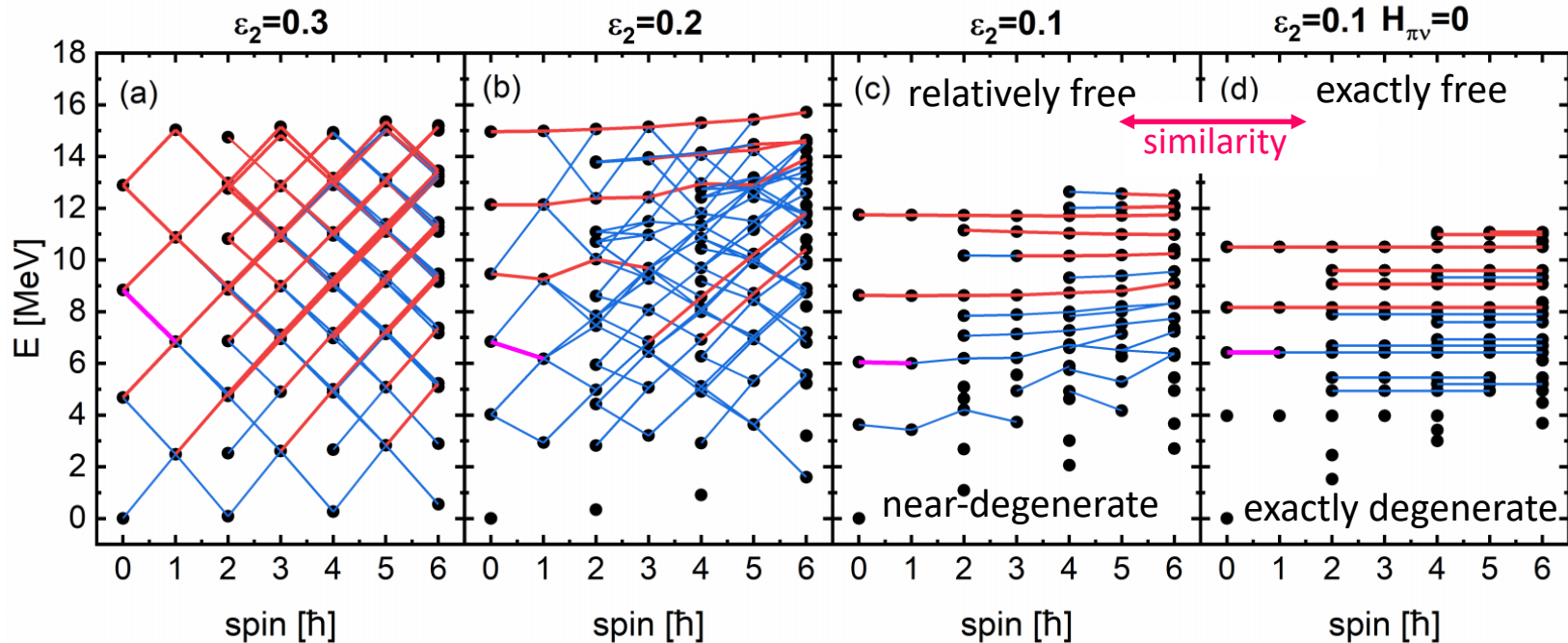
Quasiparticle excitations

C. J. Lv et al, PRL 129, 042502 (2022)

What kind of Collective Motion?

□ From large to small deformation: towards free scissors motion

Energy spectra with vacuum configuration connected by M1 transitions



rhombus-like connection



harmonic oscillation

scissors vibration

erosion of rhombus



start emergence of
LEE bands

near parallel connection



LEE bands

free scissors motion: scissors rotation

parallel

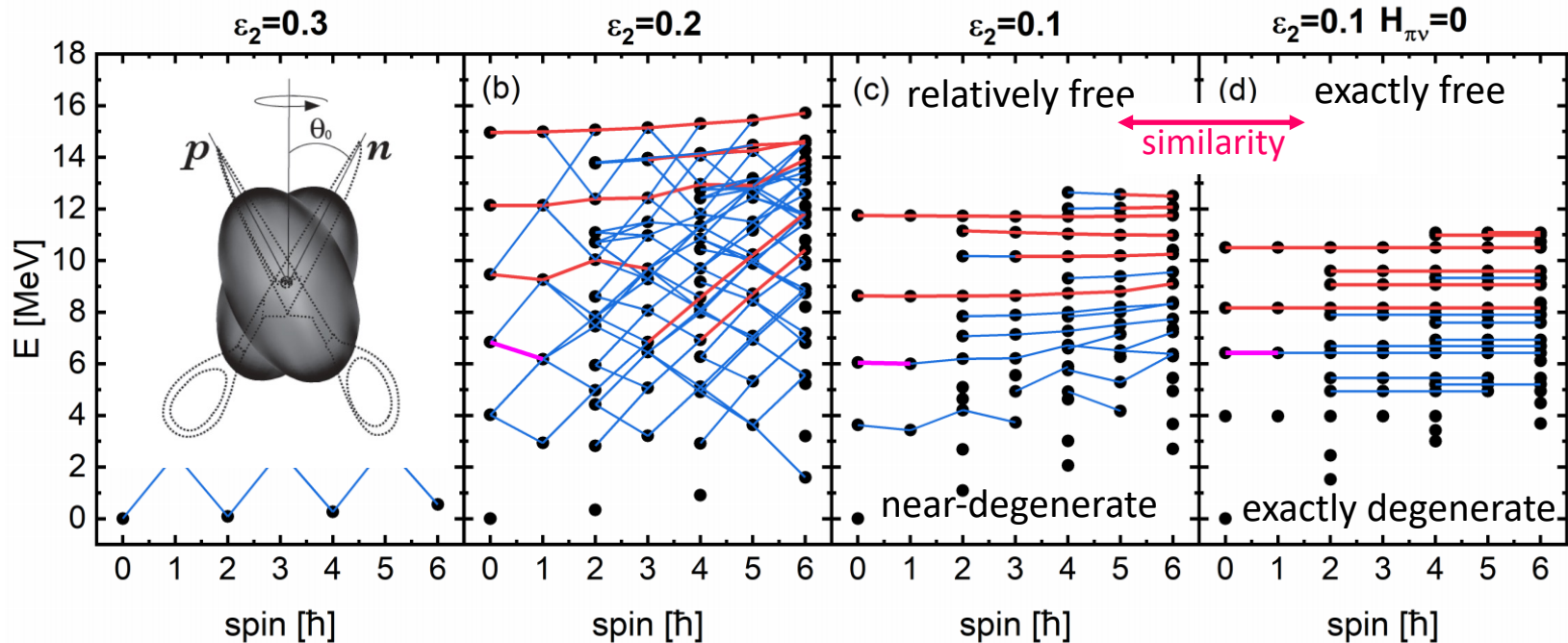


extreme LEE bands

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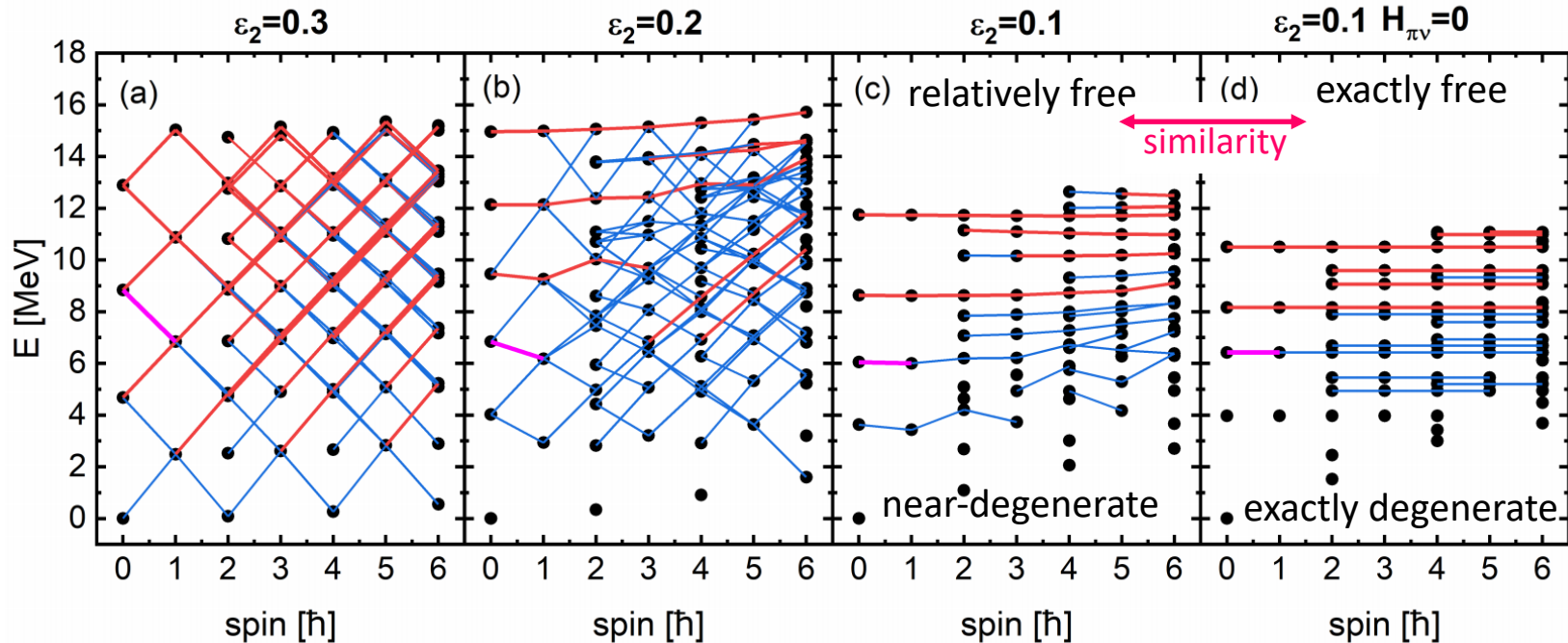


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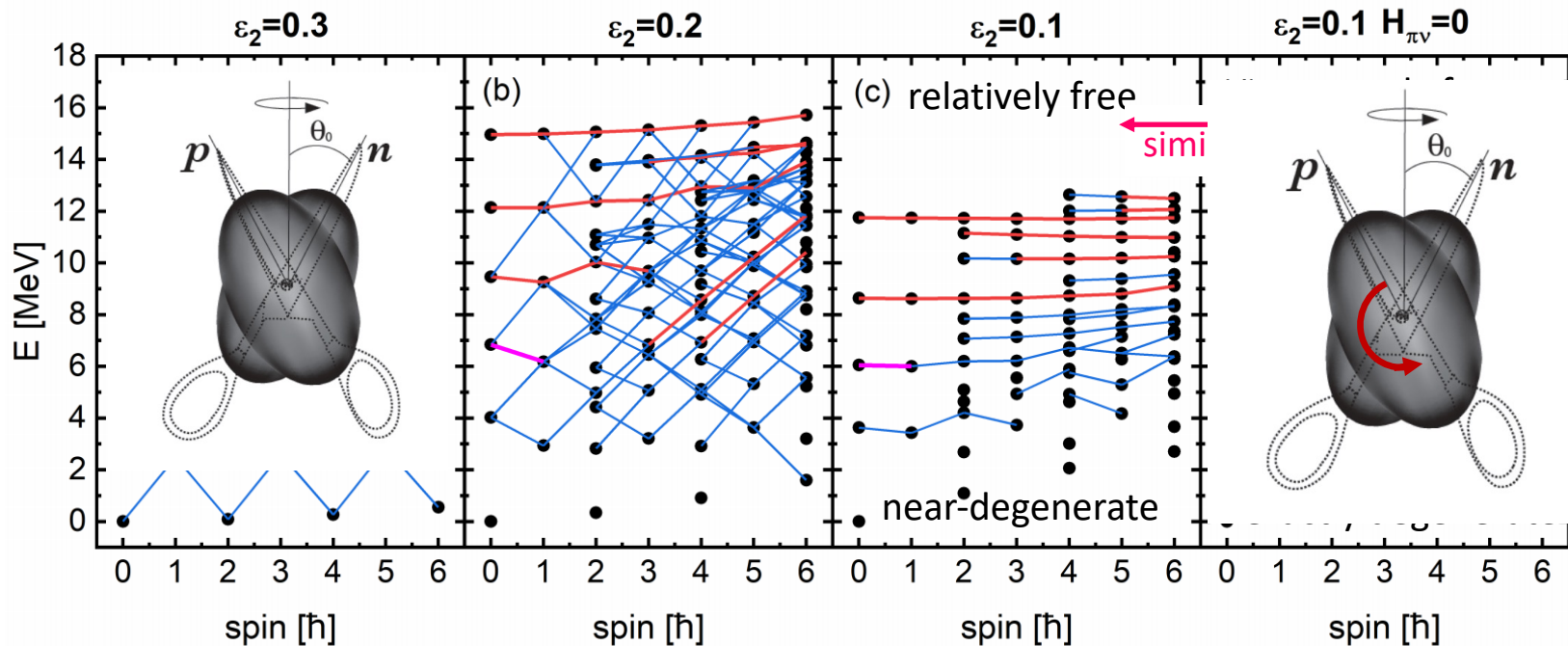


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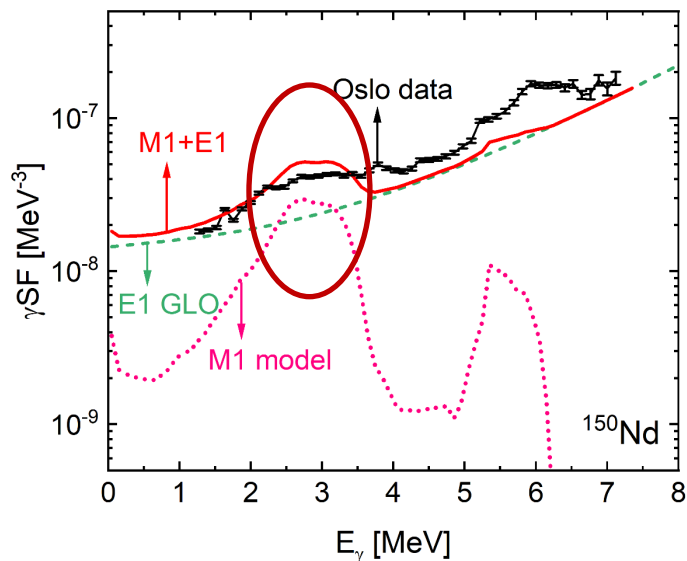
extreme LEE bands

scissors vibration

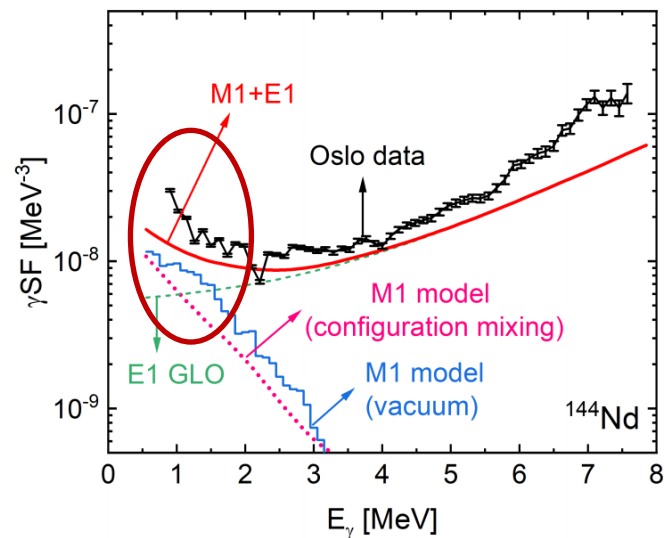
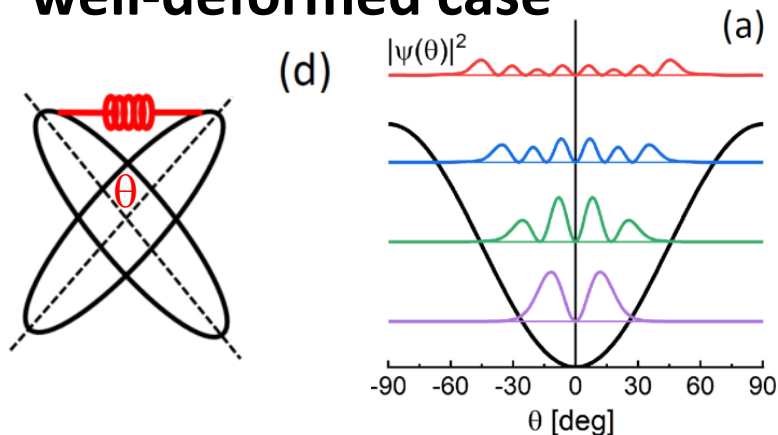
free scissors motion: scissors rotation

Scissors Motion: Vibration v.s. Rotation

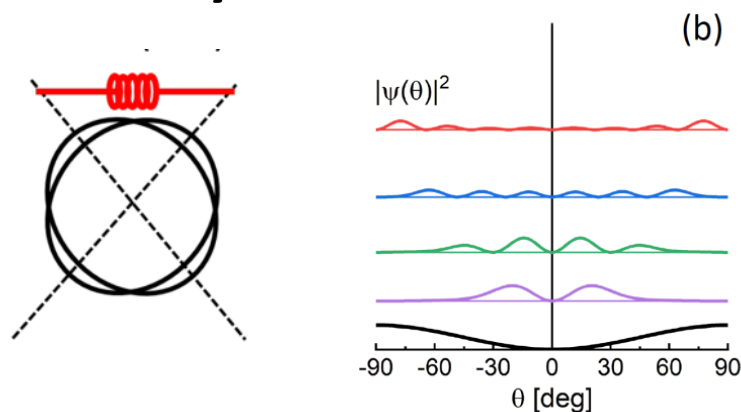
- Scissors Resonance: Scissors Vibration
- LEE: Scissors Rotation



well-deformed case



weakly deformed case



scissors angle $\longrightarrow$ reduction of n-p overlap $\longrightarrow$ energy cost $\longrightarrow$ “scissors potential”

^{238}U studied by density functional theory

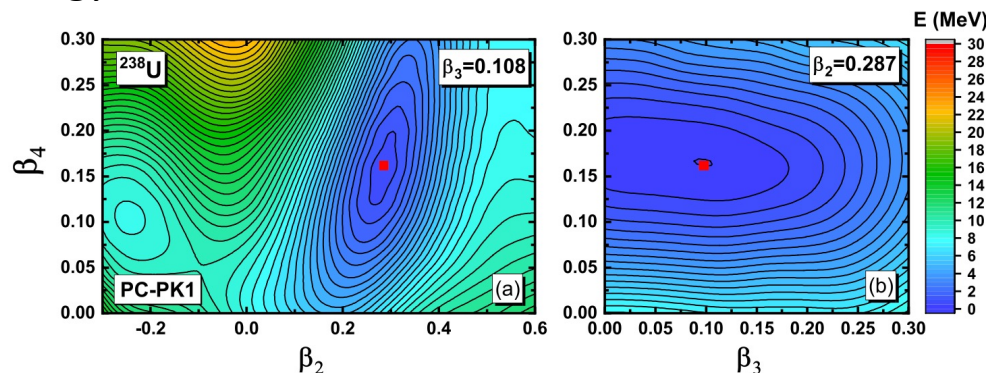
- Density functional theory is an ideal theory for the study of nuclear shapes

Given a nuclear Hamiltonian, the nuclear potential energy surface is obtained self-consistently

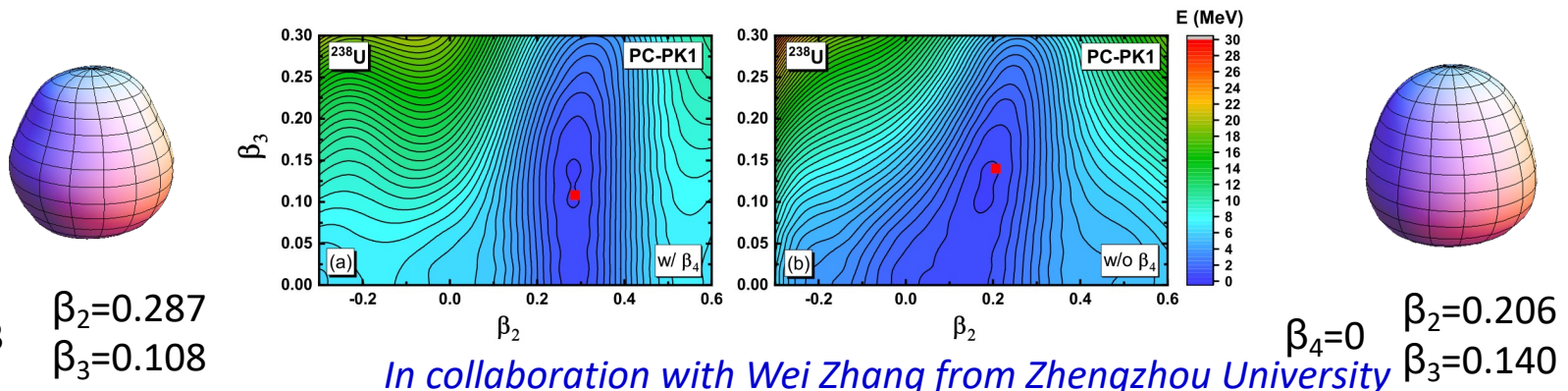
- Relativistic Mean Field + Bardeen Cooper Schrieffer (RMF+BCS) with PC-PK1 interaction

Axial symmetry is assumed, with *quadrupole, octupole and hexadecapole* d.o.f open

- ✓ Potential Energy Surface of ^{238}U



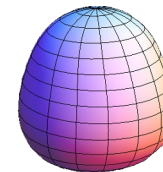
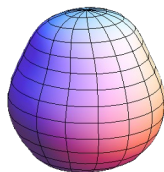
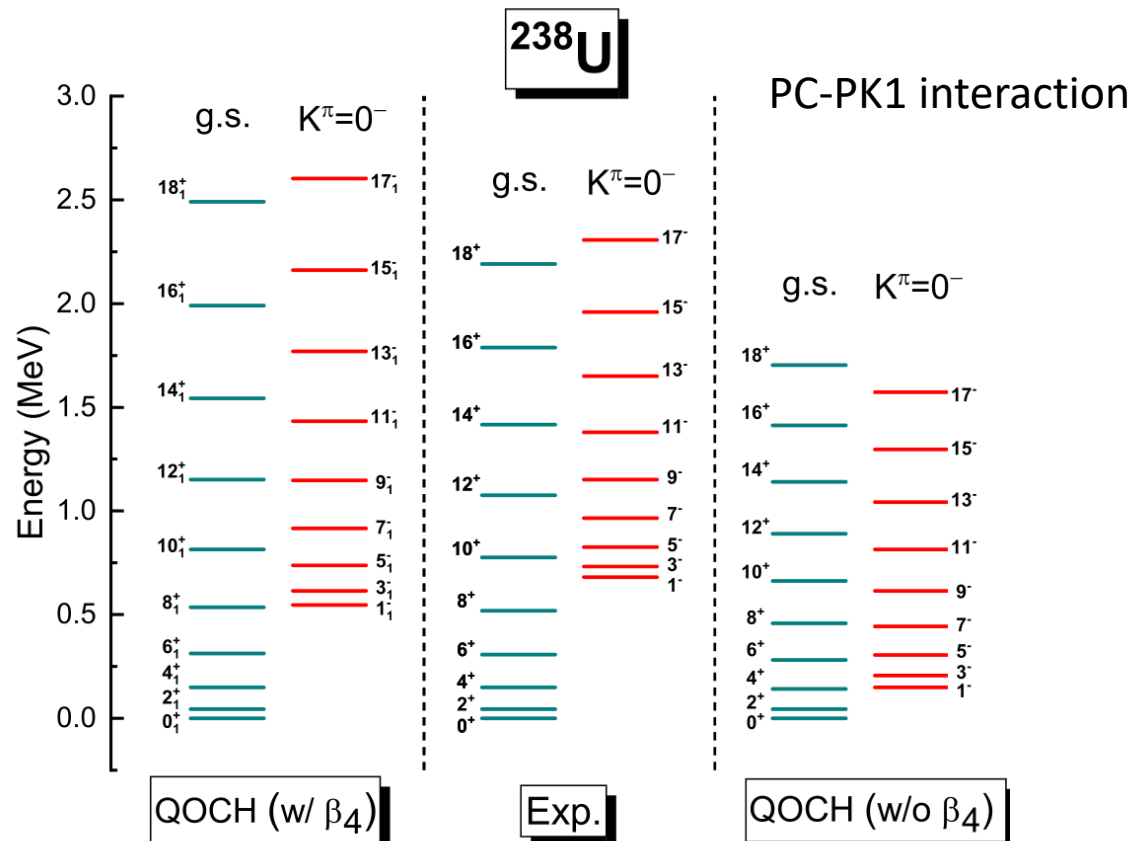
- ✓ Effect of β_4 on potential energy surface



Effects of β_4 on Low-lying spectra of ^{238}U

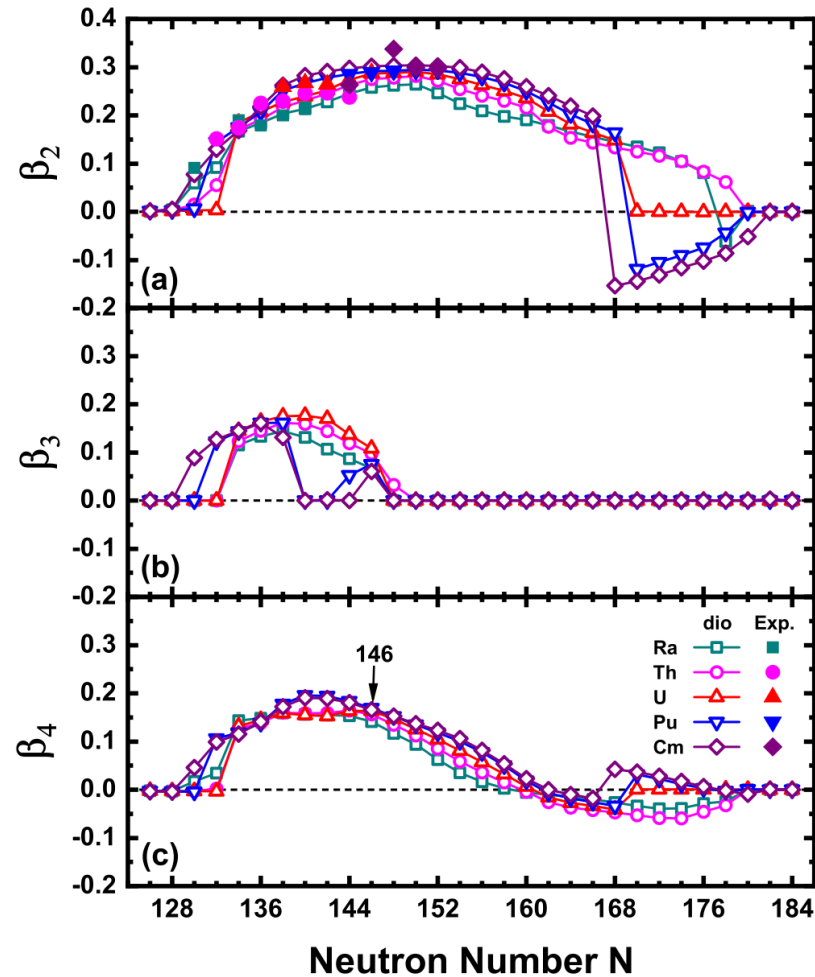
- The quadrupole – octupole collective Hamiltonian (QOCH) model based on covariant DFT

S. Y. Xia, et al., PRC 96, 054303 (2017)



Systematic Trend

- Ground state deformation properties for Z=88-96 even-even nuclei
RMF+BCS theory with PC-PK1 interaction

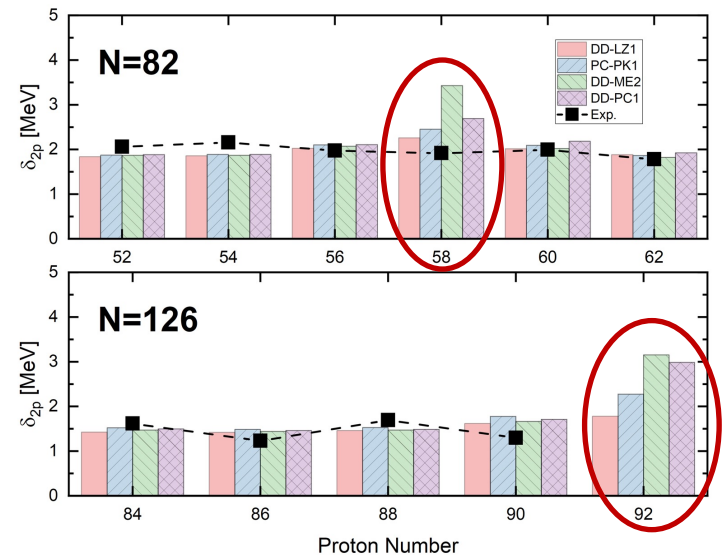


Spurious shells in relativistic density functionals

- Spurious shell closures at $Z(N)=58$ and 92 exist in many relativistic density functionals

- ✓ By changing the density dependence of isoscalar and isovector coupling, the spurious shells can be eliminated in **DD-LZ1** interaction

B. Wei, et al., CPC 44, 074107 (2020)

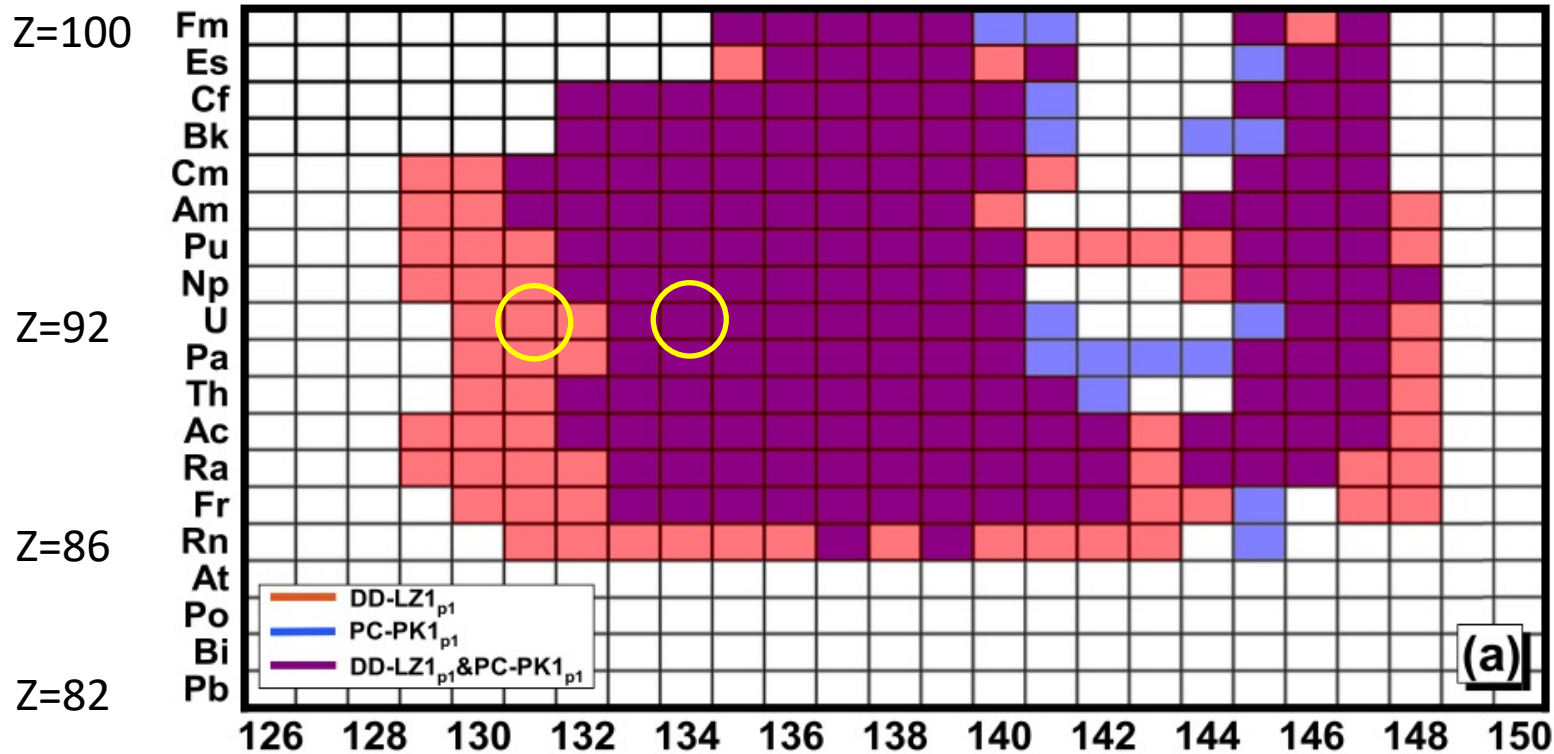


- The regions of nuclei with strong octupole correlations: $N(Z) \sim 34, 56, 88, 134$ associated with the coupling between the intruder subshell (l,j) and the normal-parity subshell $(l-3,j-3)$
P. A. Butler and W. Nazarewicz, RMP 68, 349 (1996)
- ✓ $Z=92$ is in the octupole deformation region

Does the existence of spurious shell gap influence the octupole deformation?

Octupole deformed nuclei: DD-LZ1 vs. PC-PK1

RMF+BCS calculations



Orange: Octupole deformed nuclei ($\beta_3 \geq 0.05$) predicted by DD-LZ1

Blue: Octupole deformed nuclei predicted by PC-PK1

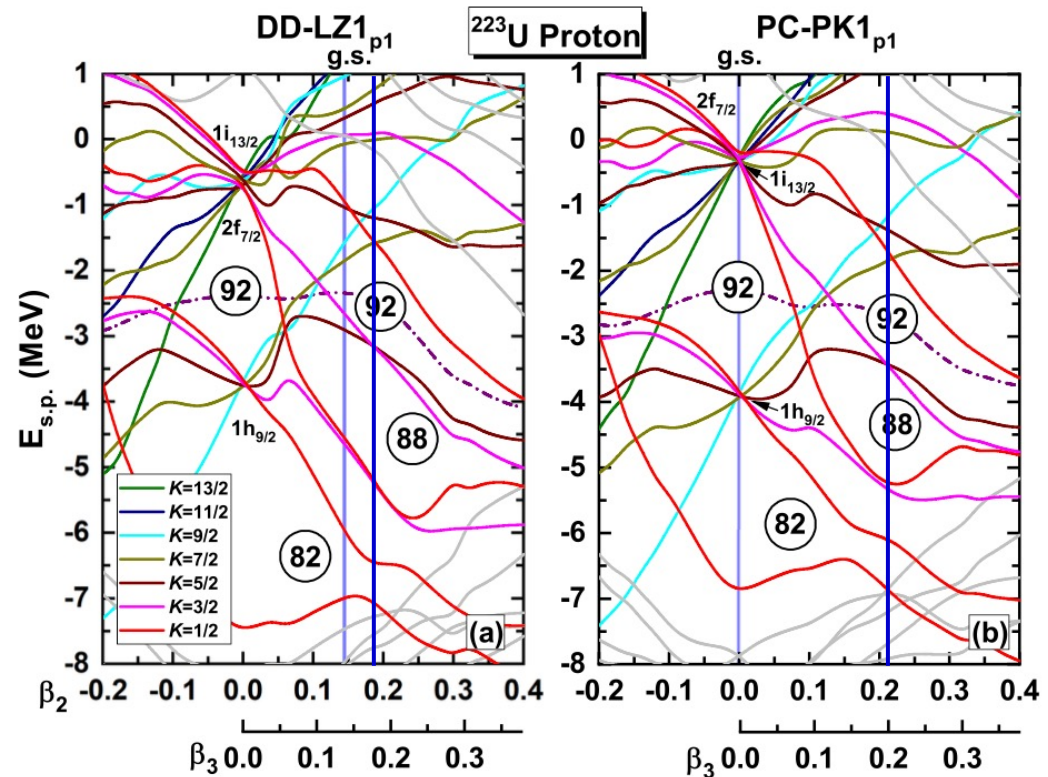
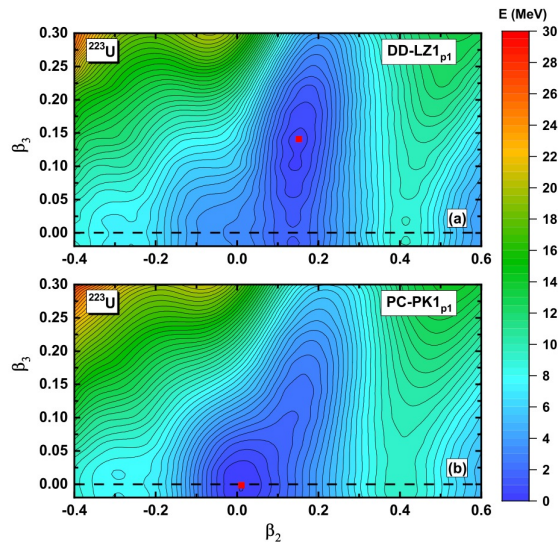
Purple: Octupole deformed nuclei predicted by both DD-LZ1 and PC-PK1

Are the differences between DD-LZ1 and PC-PK1 related to spurious shell gap at Z=92?

In collaboration with Wei Zhang from Zhengzhou University

Spurious shell and octupole deformation

- Take ^{223}U as an example



- ✓ Ground state of ^{223}U

DD-LZ1: $\beta_2=0.14$ $\beta_3=0.14$

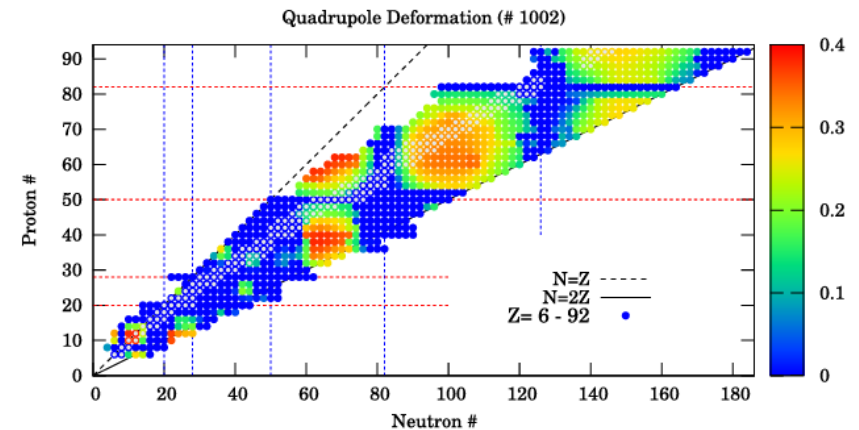
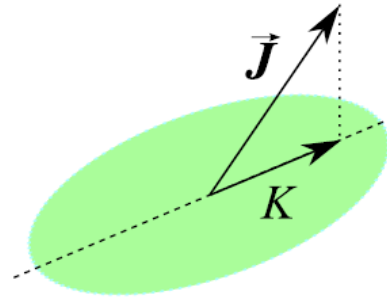
PC-PK1: spherical

- ✓ Smaller shell gap at $Z=92$ (DD-LZ1) makes the appearance of deformed shell gap at a smaller deformation

Deformation degree of freedom

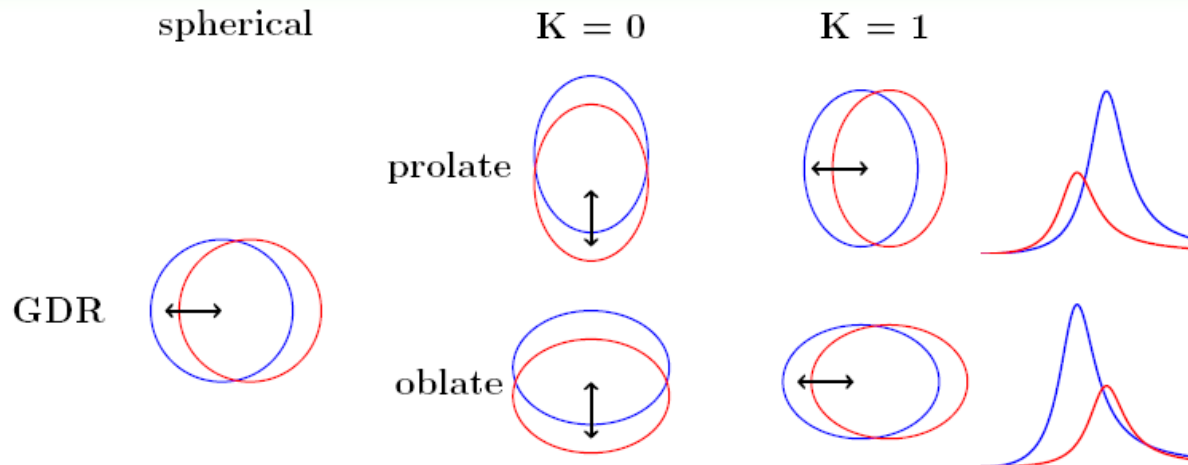
- **Axially deformed nuclei**

In mean-field approximation:
good quantum number K



S. Ebata et al. Phys. Scr. 92, 064005 (2017)

- **Deformation effect on GDR**



from D. Pena Arteaga