

α clustering in hot and dense nuclear matter

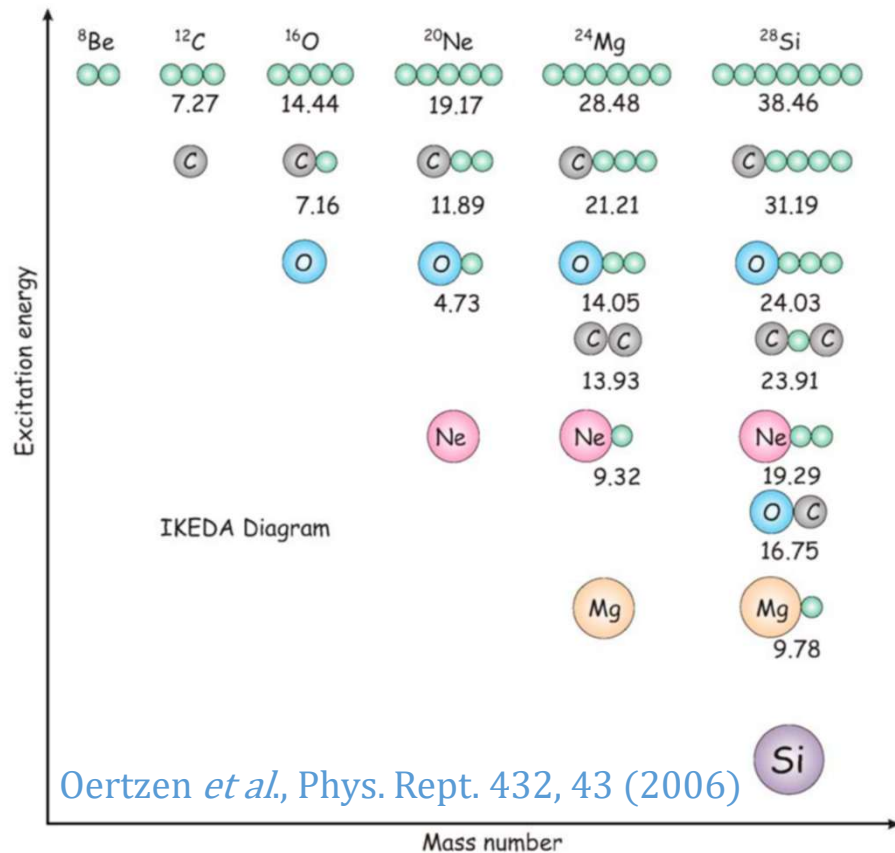
高温高密核物质中的 α 粒子结团

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Nuclear clustering

In light nuclei

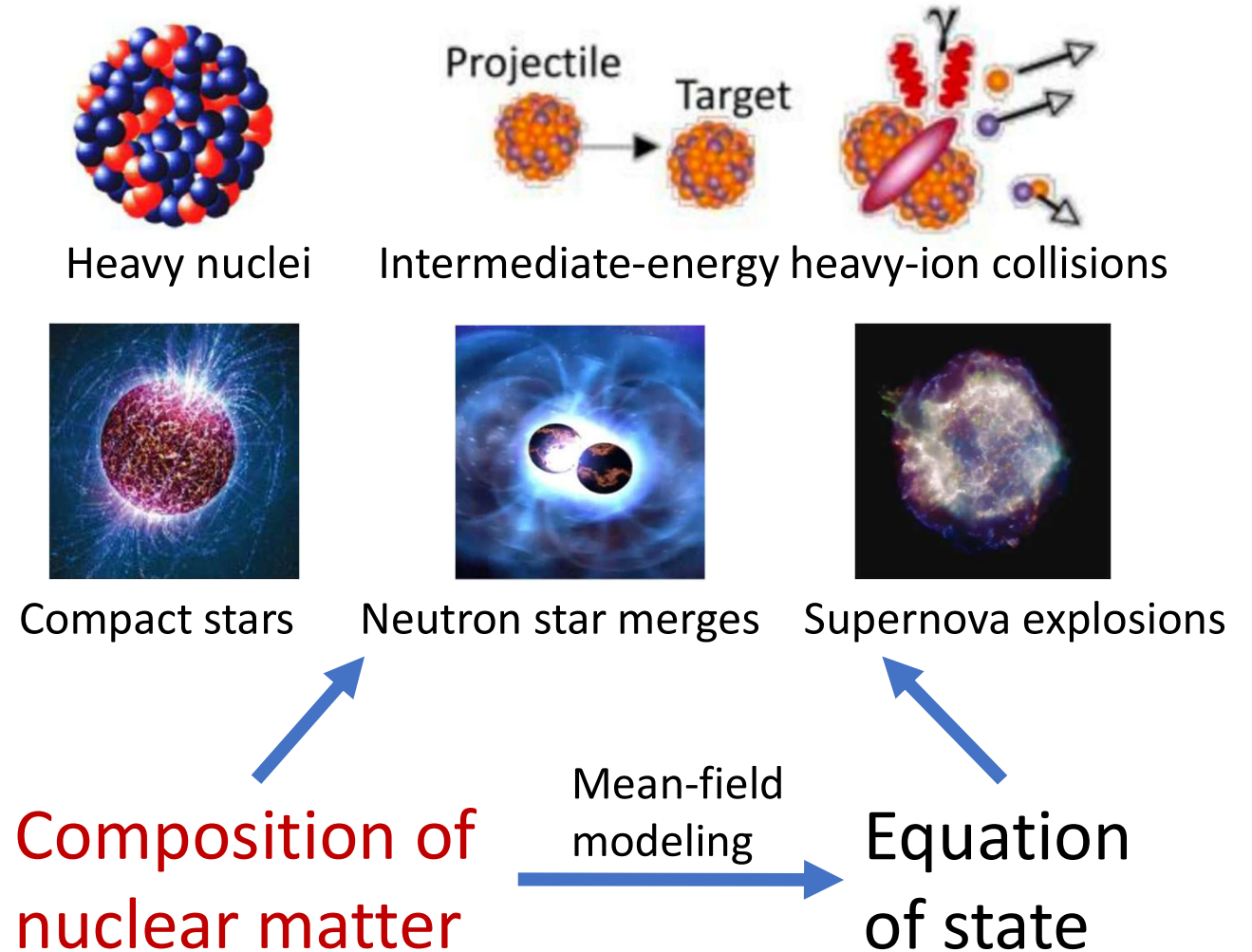


Can be studied by both low-energy reactions and relativistic heavy-ion collisions

verses

in nuclear matter

Nuclear matter exists in various environments:



Composition of Nuclear matter

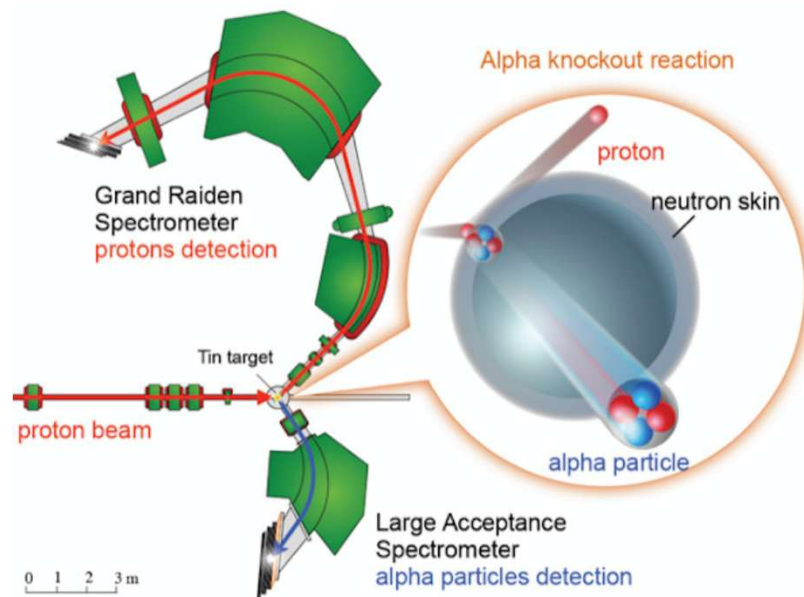
A fundamental question!

The role of nuclear clustering

Boltzmann gas: $\rho_\nu \sim \rho_n^N \rho_p^Z$?

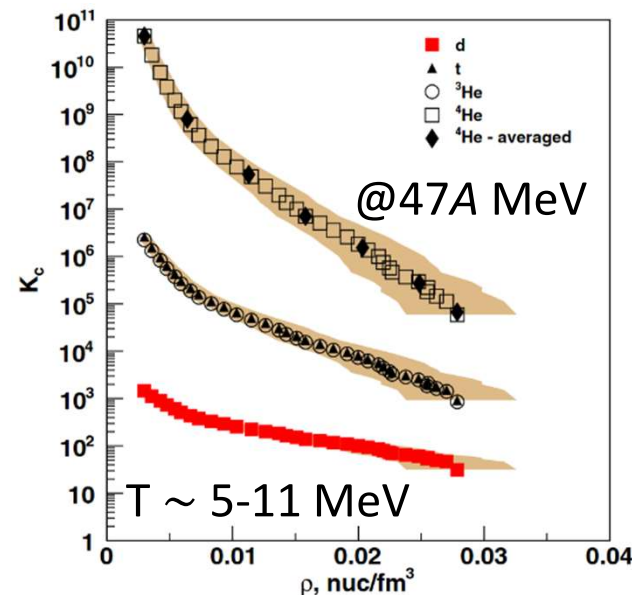
Nuclear clustering is suppressed by the surrounding medium

Knock-out reactions:
 α clusters in dilute matter
(surface of heavy nuclei)

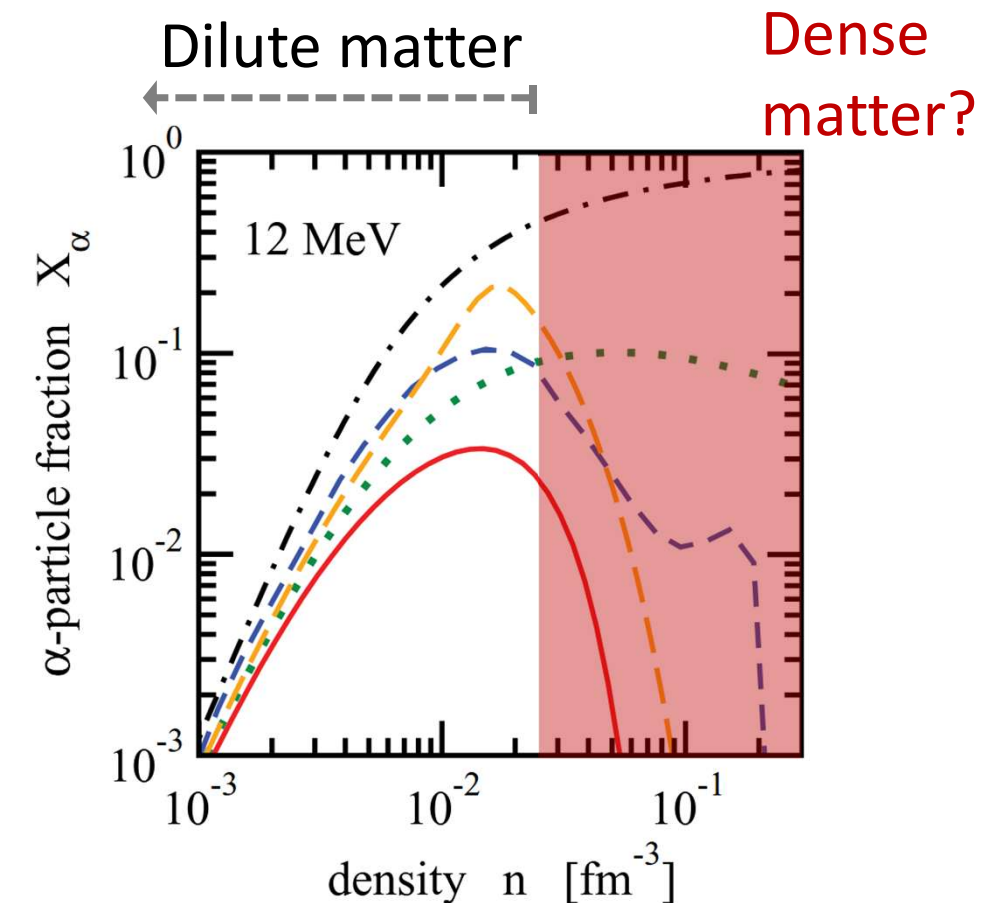


J. Tanaka *et al.*, Science 371, 260–264 (2021)

Fermi energy heavy-ion collisions:
Fraction of light cluster in warm,
dilute nuclear matter



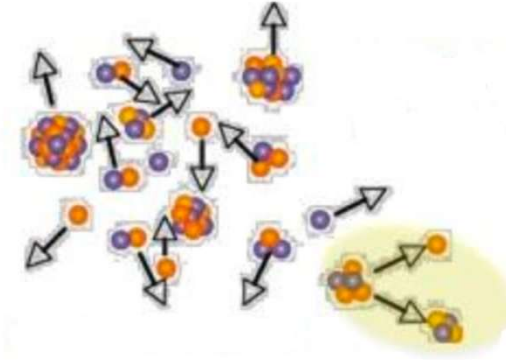
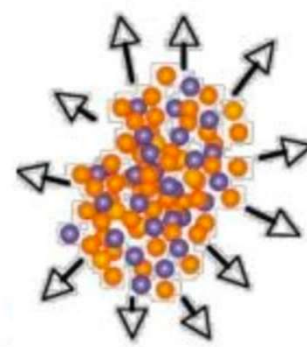
L. Qin, *et al.*, PRL108, 172701 (2012)
See also: PRL108, 062702 (2012); PRL125, 012701 (2020); PRL134, 082304 (2025)



S. Typel, *et al.*, PRC81, 015803 (2010)

Intermediate-energy Heavy-ion collisions

Beyond the
Fermi-energy
regime, up to
a few GeV/A



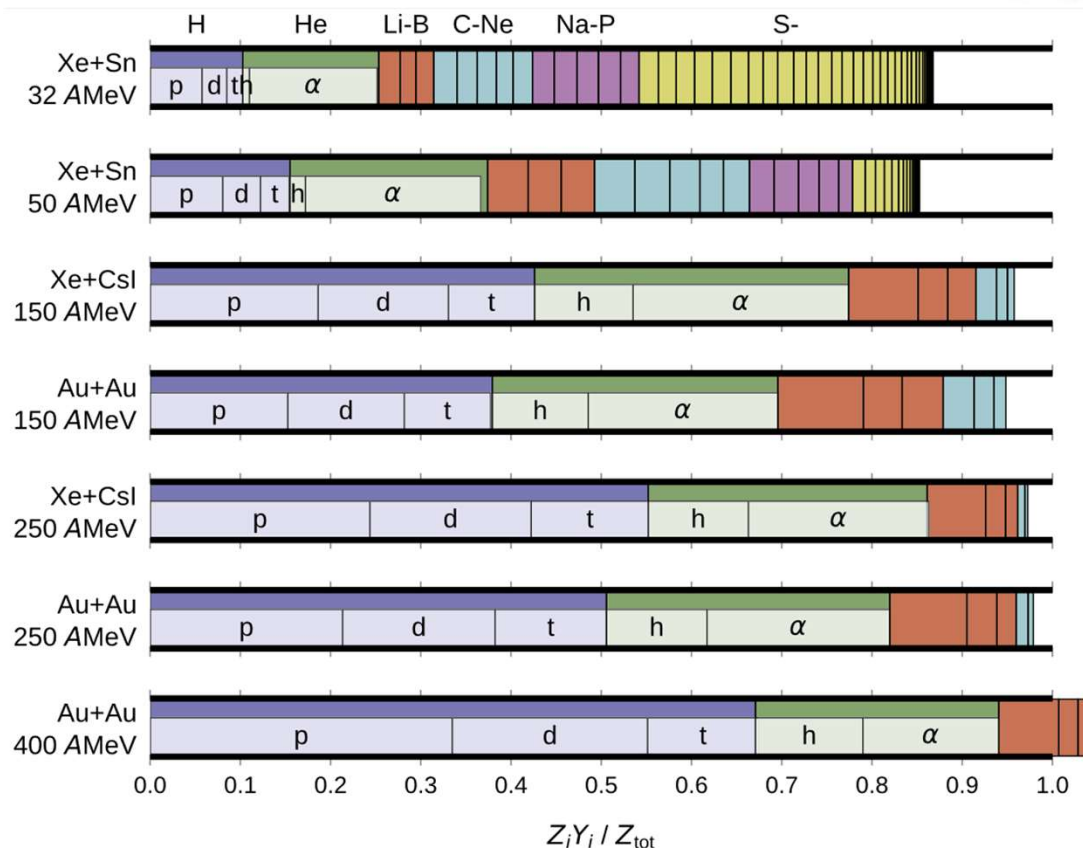
Mesons (π , K, ...)

Nucleons (n , p)

Hyperons (Λ , ...)

Light nuclei (d , ...)

Large fragments



A. Ono, Progress in Particle and Nuclear Physics 105 (2019) 139–179

- Light clusters (mass number ≤ 4) account for a large portion of the measured final-state charged particles
- They could affect the reaction dynamics

- Global thermal equilibrium unlikely
- Coalescence at freeze-out fails
- Realistic modeling of light-clusters dynamics in these collisions is important!

Light nuclei in kinetic approach

Kinetic equations are derived based on the [closed time-path Green's function formulism](#)

For example, in the deuteron case, the two-body Green's function G_2 satisfies an equation

$$G_2 = \mathcal{G}_2 + \frac{1}{4}\mathcal{G}_2 v G_2$$

The [light nuclei are realized as poles](#) of the many-body Green's function.

In the vicinity of the pole, we have

$$i\langle x|G_2^<(P, \Omega, R, T)|x'\rangle \sim \langle x|\phi(P, R, T)\rangle\langle\phi(P, R, T)|x'\rangle f_2(P, R, T)2\pi\delta[\Omega - E(P, R, T)]$$

$$i\langle x|G_2^>(P, \Omega, R, T)|x'\rangle \sim \langle x|\phi(P, R, T)\rangle\langle\phi(P, R, T)|x'\rangle[1 + f_2(P, R, T)]2\pi\delta[\Omega - E(P, R, T)]$$

[P. Danielewicz and G. F. Bertsch, Nuclear Physics A533, 712-748 \(1991\)](#)

Finally leads to equations of the occupation number f_τ of light nuclei

$$(\partial_t + \vec{\nabla}_p \epsilon_\tau \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon_\tau \cdot \vec{\nabla}_p) f_\tau = \mathcal{K}_\tau^<[f_n, f_p, f_d, \dots](1 \pm f_\tau) - \mathcal{K}_\tau^>[f_n, f_p, f_d, \dots] f_\tau,$$

Single particle potential

Related to nuclear
equation of state

$$\tau = n, p, d, t, h, \alpha$$

Kinetic approach and light clusters

Originates from the
Green's function
formalism

Kinetic equation of the occupation number f_τ

$$(\partial_t + \vec{\nabla}_p \epsilon_\tau \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon_\tau \cdot \vec{\nabla}_p) f_\tau = \mathcal{K}_\tau^<[f_n, f_p, f_d, \dots](1 \pm f_\tau) - \mathcal{K}_\tau^>[f_n, f_p, f_d, \dots] f_\tau, \quad \tau = n, p, d, t, h, \alpha$$

For example,
the loss term of
 α particles

$$\begin{aligned} K_\alpha^> f_\alpha = & \frac{\mathcal{S}_{5'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{5'} \frac{d\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{d\vec{p}_N}{(2\pi\hbar)^3 2E_N} |\overline{\mathcal{M}_{N\alpha \rightarrow NNNNN}}|^2 g_N f_N \prod_{i=1'}^{5'} (1 \pm f_i) (2\pi)^4 \delta^4(\sum_{i=1'}^5 p_i - p_N - p_\alpha) \\ & + \frac{\mathcal{S}_{3'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{3'} \frac{d\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{d\vec{p}_N}{(2\pi\hbar)^3 2E_N} |\overline{\mathcal{M}_{N\alpha \rightarrow NNNt}}|^2 g_N f_N \prod_{i=1'}^{3'} (1 \pm f_i) (2\pi)^4 \delta^4(\sum_{i=1'}^3 p_i - p_N - p_\alpha) + t \rightarrow h \\ & + \frac{\mathcal{S}_{2'} f_\alpha}{2E_\alpha} \int \prod_{i=1'}^{2'} \frac{d\vec{p}_i}{(2\pi\hbar)^3 2E_i} \frac{d\vec{p}_N}{(2\pi\hbar)^3 2E_N} |\overline{\mathcal{M}_{N\alpha \rightarrow dt}}|^2 g_N f_N \prod_{i=1'}^{2'} (1 \pm f_i) (2\pi)^4 \delta^4(\sum_{i=1'}^2 p_i - p_N - p_\alpha) + t \rightarrow h \\ & + \text{elastic part.} \end{aligned}$$

Impulse
approximation

Light nuclei can be produced and dissociated through many-body scatterings, **nucleon few-body correlations** are included effectively

- $A = 2$ $\pi NN \leftrightarrow \pi d, NNN \leftrightarrow Nd$
- $A = 3$ $\pi NNN \leftrightarrow \pi t(h), \pi Nd \leftrightarrow \pi t(h), NNNN \leftrightarrow Nt(h), NNd \leftrightarrow Nt(h)$
- $A = 4$ $\pi NNNN \leftrightarrow \pi\alpha, \pi NNd \leftrightarrow \pi\alpha, \pi Nt(h) \leftrightarrow \pi\alpha, NNNN \leftrightarrow N\alpha, NNNd \leftrightarrow N\alpha, NNt(h) \leftrightarrow N\alpha, dt(h) \leftrightarrow N\alpha$

Ideal for central collisions at intermediate energies!

- Many body **transition** amplitudes e.g., $|M_{Npn \leftrightarrow Nd}|^2$
- The in-medium effects on light clusters – **Mott effect**

RW, Y.-G. Ma, L.-W. Chen *et al.*, Physical Review C 108, L031601 (2023)

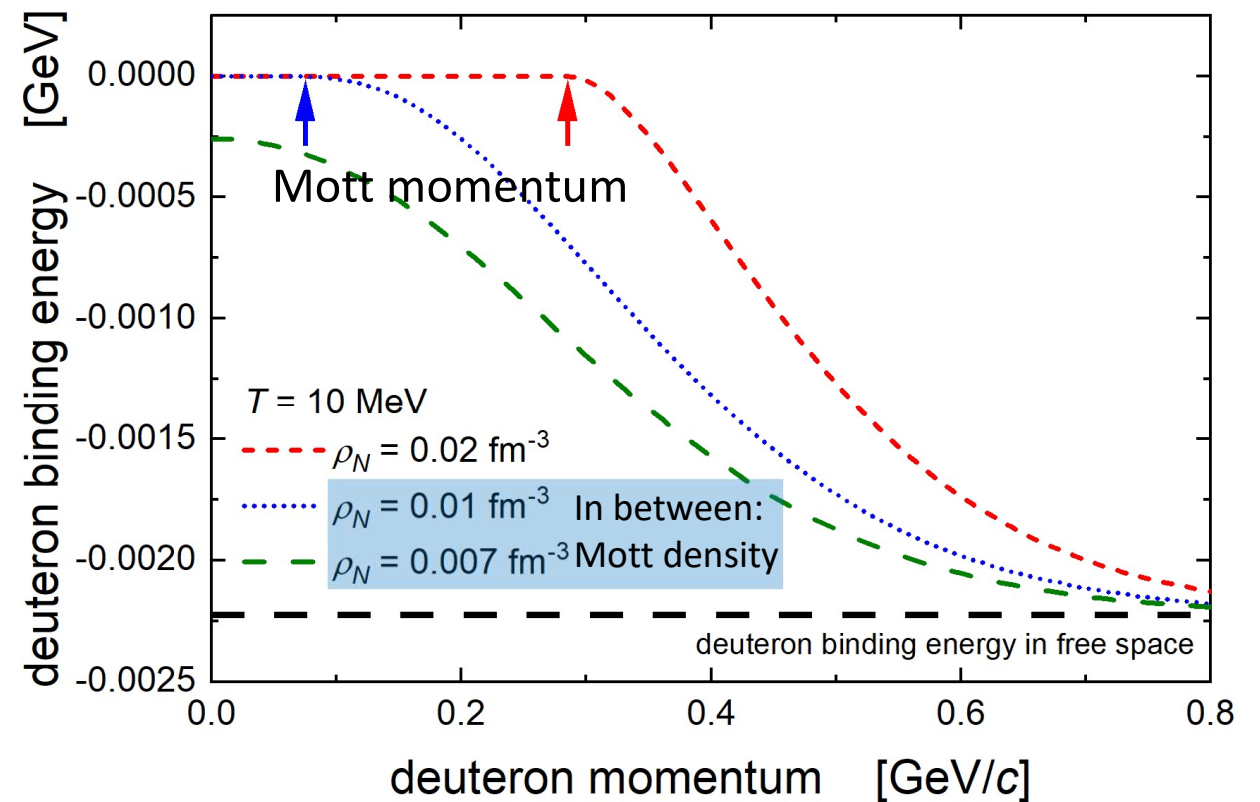
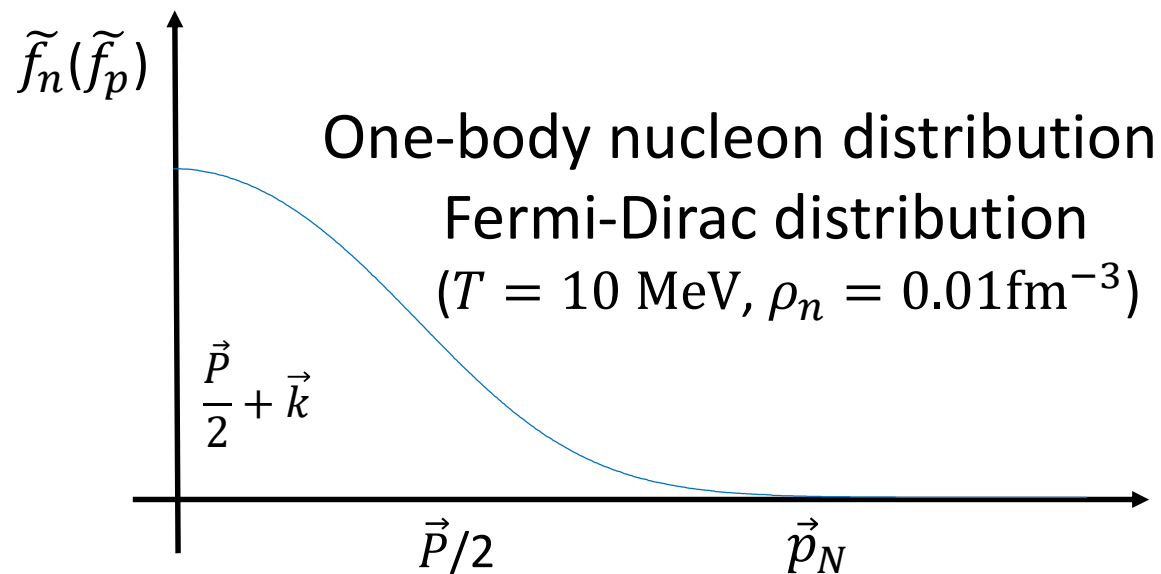
In-medium Schrodinger equation

Originates from the
Green's function
formalism

$$\begin{aligned} & [\epsilon_{\tau_1}(\vec{p}_1) + \epsilon_{\tau_2}(\vec{p}_2) + \cdots + \epsilon_{\tau_A}(\vec{p}_A) - \epsilon_\nu(\vec{P})] \Psi_{\nu, \vec{P}}(1, 2, \cdots, A) \\ & + \sum_{i < j} [1 - \tilde{f}_{\tau_i}(\vec{p}_i) - \tilde{f}_{\tau_j}(\vec{p}_j)] \int \prod_l^{1', 2', \cdots, A'} \frac{d\vec{p}_l}{(2\pi\hbar)^3} V(ij, i'j') \prod_{k \neq i, j} \delta_{kk'} \Psi_{\nu, \vec{P}'}(1', 2', \cdots, A') = 0 \end{aligned}$$

Mott effect: the binding energy of a light cluster is modified by its surrounding nucleons

Right: the deuteron binding energy as a function of its momentum P in nuclear medium, the Mott momentum is recognized as where the binding energy vanishes.



In-medium effects on light clusters

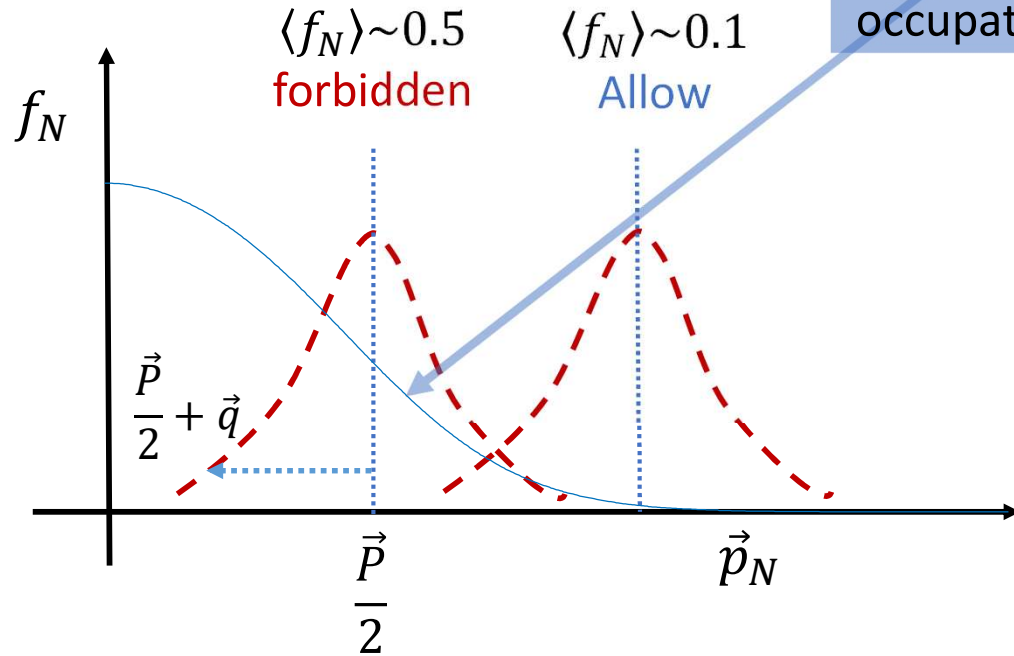
The Mott effect can be **effectively** introduced into the kinetic approach through a **phase-space cutoff parameter**.

$$\langle f_\tau \rangle_\nu(\vec{P}) \equiv \int \tilde{f}_\tau \left(\frac{\vec{P}}{A} + \vec{q} \right) |\phi_\nu(\vec{q})|^2 d\vec{q} \leq F_A^{\text{cut}}$$

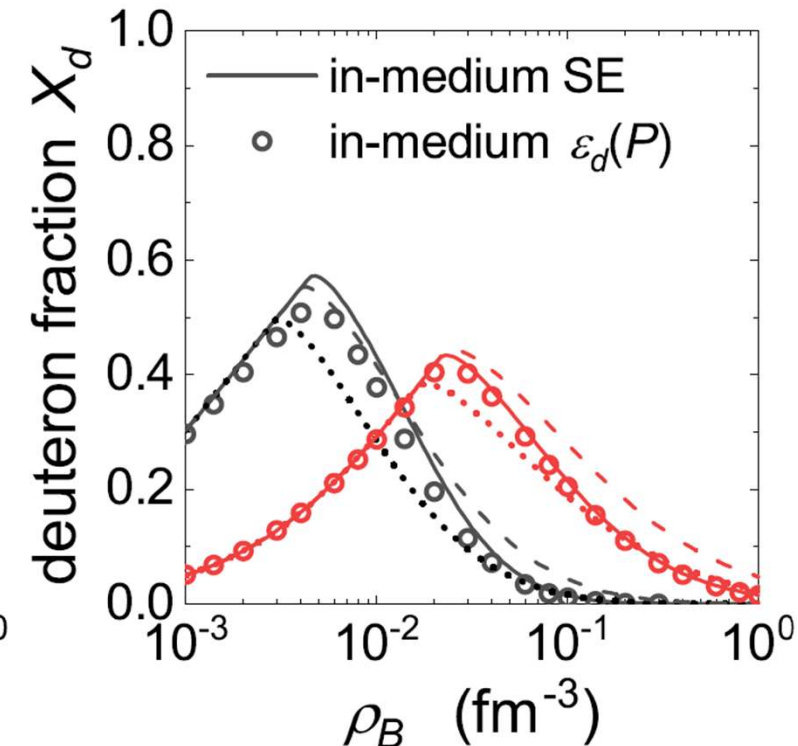
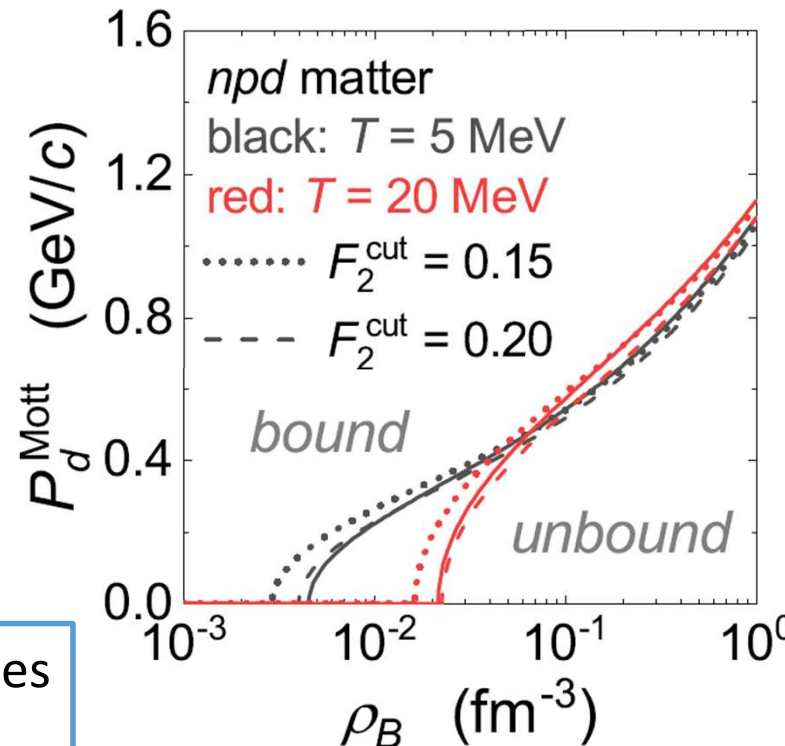
A larger F_A^{cut} corresponds to a weaker Mott effect

One-body nucleon occupation

Momentum distribution of the constituent nucleons, related to the **light-nuclei internal wave function**



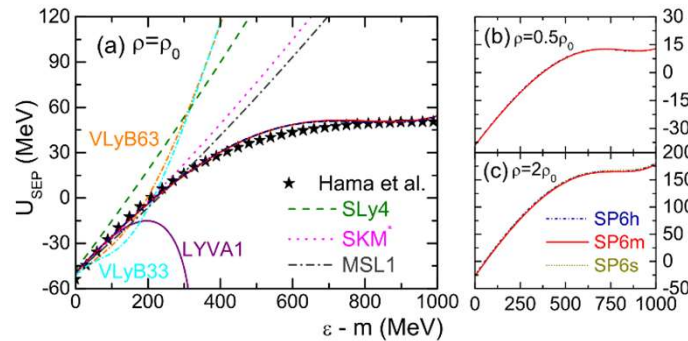
F_2^{cut} , F_3^{cut} , and F_4^{cut} can be treated as surrogates of the strength of light-nuclei Mott effect



RW, Z. Zhang, S. Burrello *et al.*, Physical Review C 113, 034624 (2026)

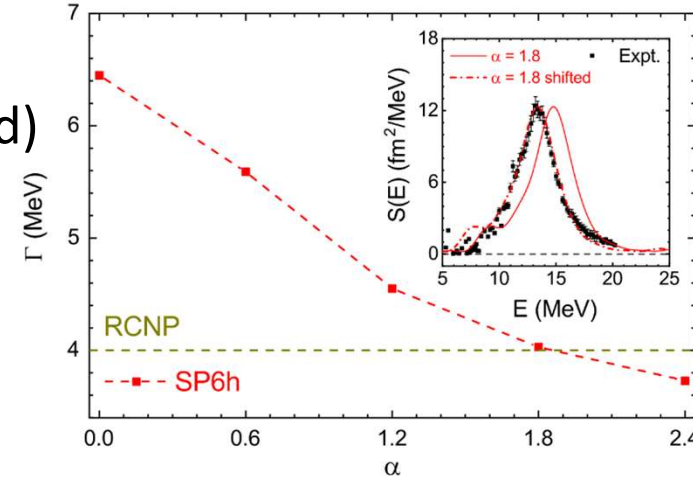
Kinetic approach and light clusters

Appropriate single particle potential



RW, L.-W. Chen, and Y. Zhou, *Physical Review C* **98**, 054618 (2018)

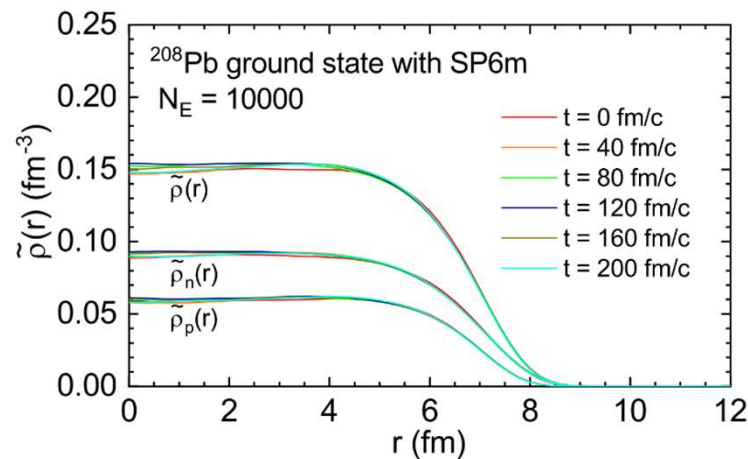
+ Collision integral (Stochastic method)



RW, Z. Zhang, L.-W. Chen, *et al.*, *Physics Letters B* **807** (2020) 135532

+ Kaons, hyperons, ...
for future HIC facilities
(e.g. HIAF, Huizhou)

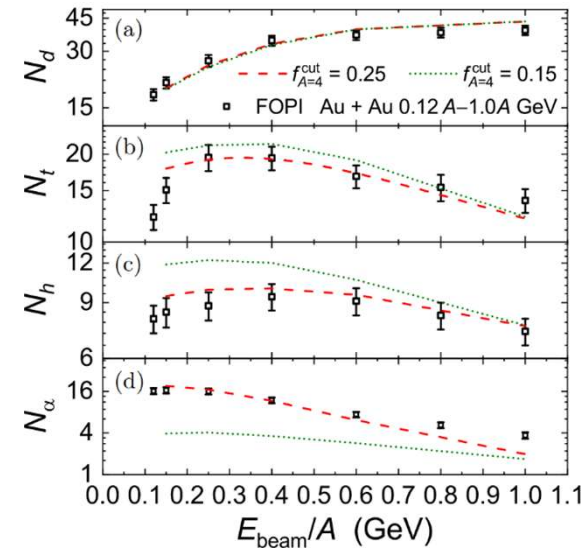
In progress



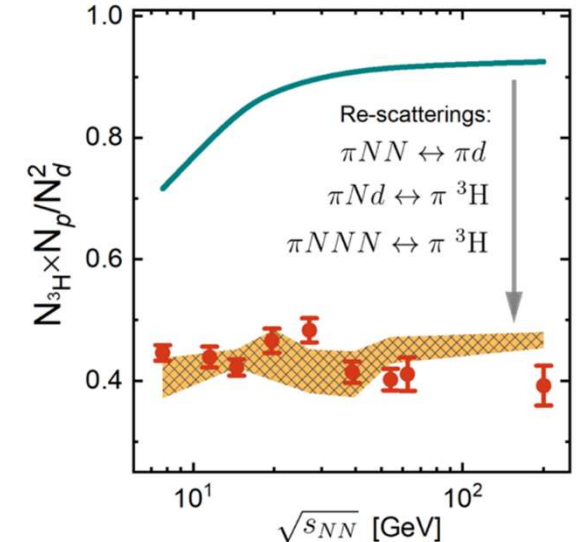
RW, L.-W. Chen, and Z. Zhang,
Physical Review C **98**, 054618 (2018)

Vlasov equation
(Lattice Hamiltonian method)

+ Light clusters
+ Mott effect



RW, Y.-G. Ma, L.-W. Chen, *et al.*,
Physical Review C **108**, L031601 (2023)



K.-J. Sun, RW, C. M. Ko, *et al.*,
Nature Communications (2024) **15**:1074

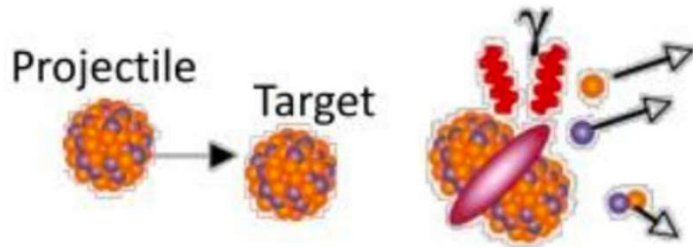
HICs versus nuclear matter

Kinetic equation of the occupation number f_τ

$$(\partial_t + \vec{\nabla}_p \epsilon_\tau \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon_\tau \cdot \vec{\nabla}_p) f_\tau = \mathcal{K}_\tau^<[f_n, f_p, f_d, \dots](1 \pm f_\tau) - \mathcal{K}_\tau^>[f_n, f_p, f_d, \dots] f_\tau, \quad \tau = n, p, d, t, h, \alpha$$

Heavy-ion collisions

By initializing f_τ of each particle species (initially only nucleons) and let them evolve according to the kinetic equation



Parameters characterizing the strength of in-medium effects can be empirically determined by comparison with experimental data, such as light-nuclei yields.

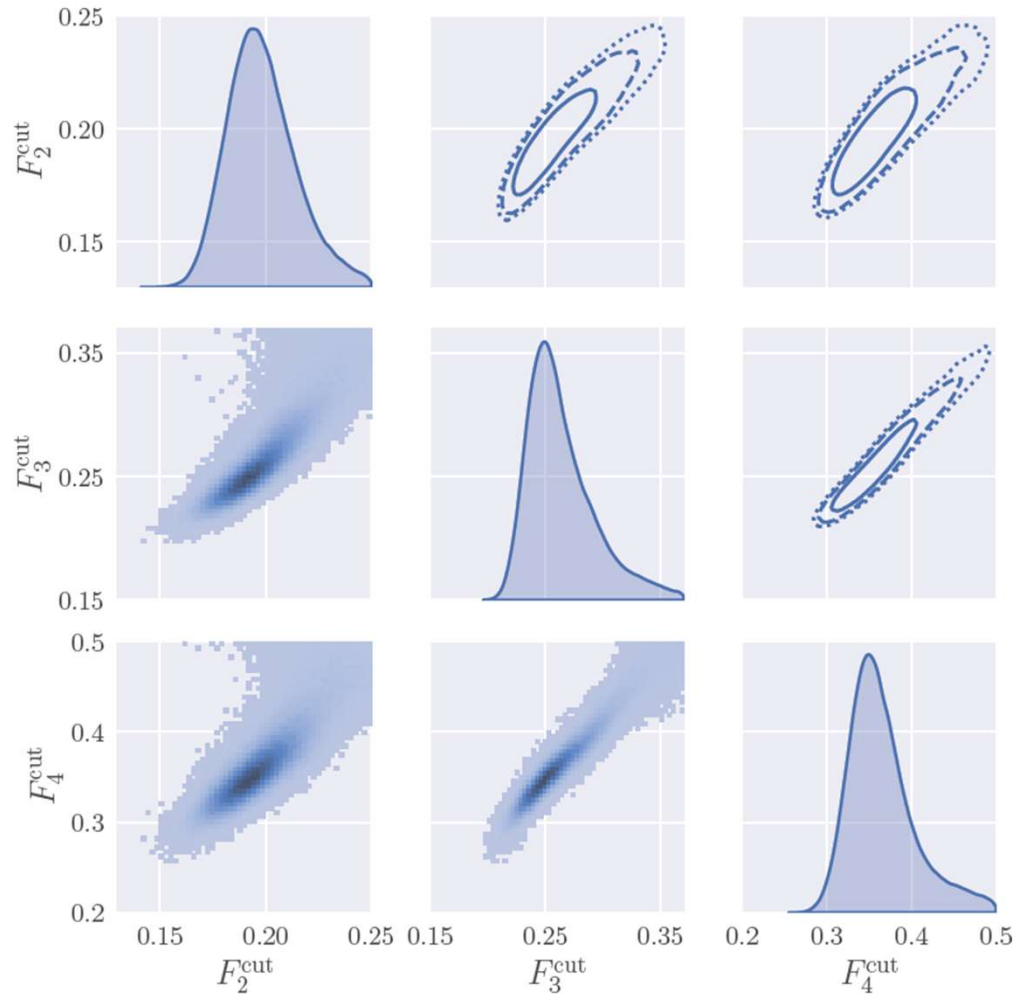
Nuclear matter

By imposing equilibrium conditions on the collision integral $\rightarrow f_\tau^{\text{eq}} \rightarrow$ Particle fractions (Standard thermal model + in-medium effects on light clusters)

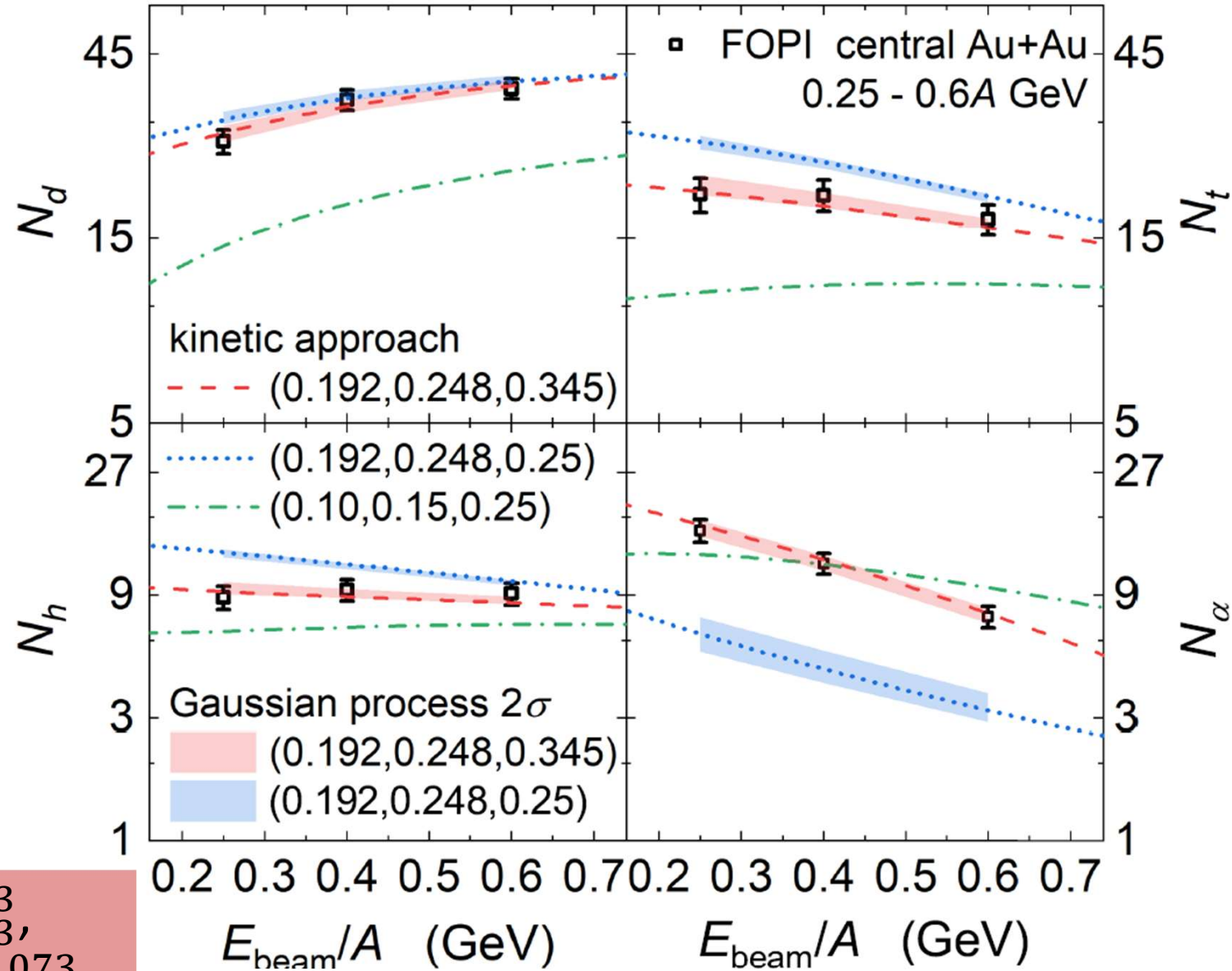


Infer light-cluster compositions of nuclear matter at given conditions (densities and temperatures where light clusters freeze out in HICs)

Light-nuclei yields



90% confidence level: $F_2^{\text{cut}} = 0.192^{+0.023}_{-0.013}$,
 $F_3^{\text{cut}} = 0.248^{+0.046}_{-0.023}$, and $F_4^{\text{cut}} = 0.345^{+0.073}_{-0.030}$.



RW, Z. Zhang, Y.-G. Ma *et al.*, Physical Review Letters 136, 172301 (2026)

α -particle fraction

- According to the present kinetic approach, in intermediate-energy heavy-ion collisions, light nuclei are mainly formatted and freeze-out chemically at high densities ($NNNN \leftrightarrow N\alpha$), especially for α -particles.

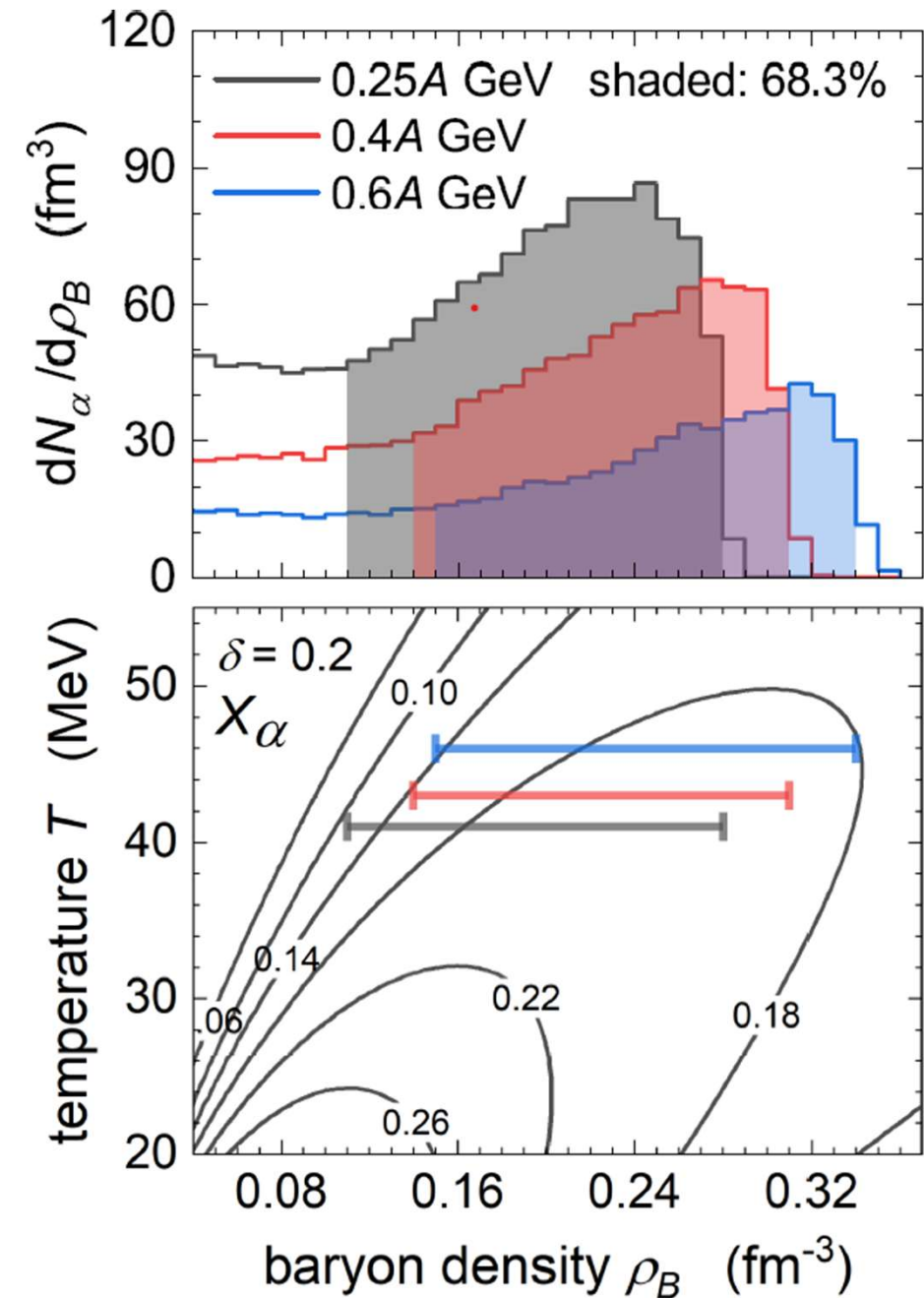
Collision rate $\sim \rho^5$



Contradict!

- It is generally thought that the dense nuclear matter can be regarded almost as a **uniform nucleon liquid**.

α -particle fraction ~ 0.2 in dense nuclear matter at densities of about 1-2 times the saturation density and temperatures of a few tens of MeV.

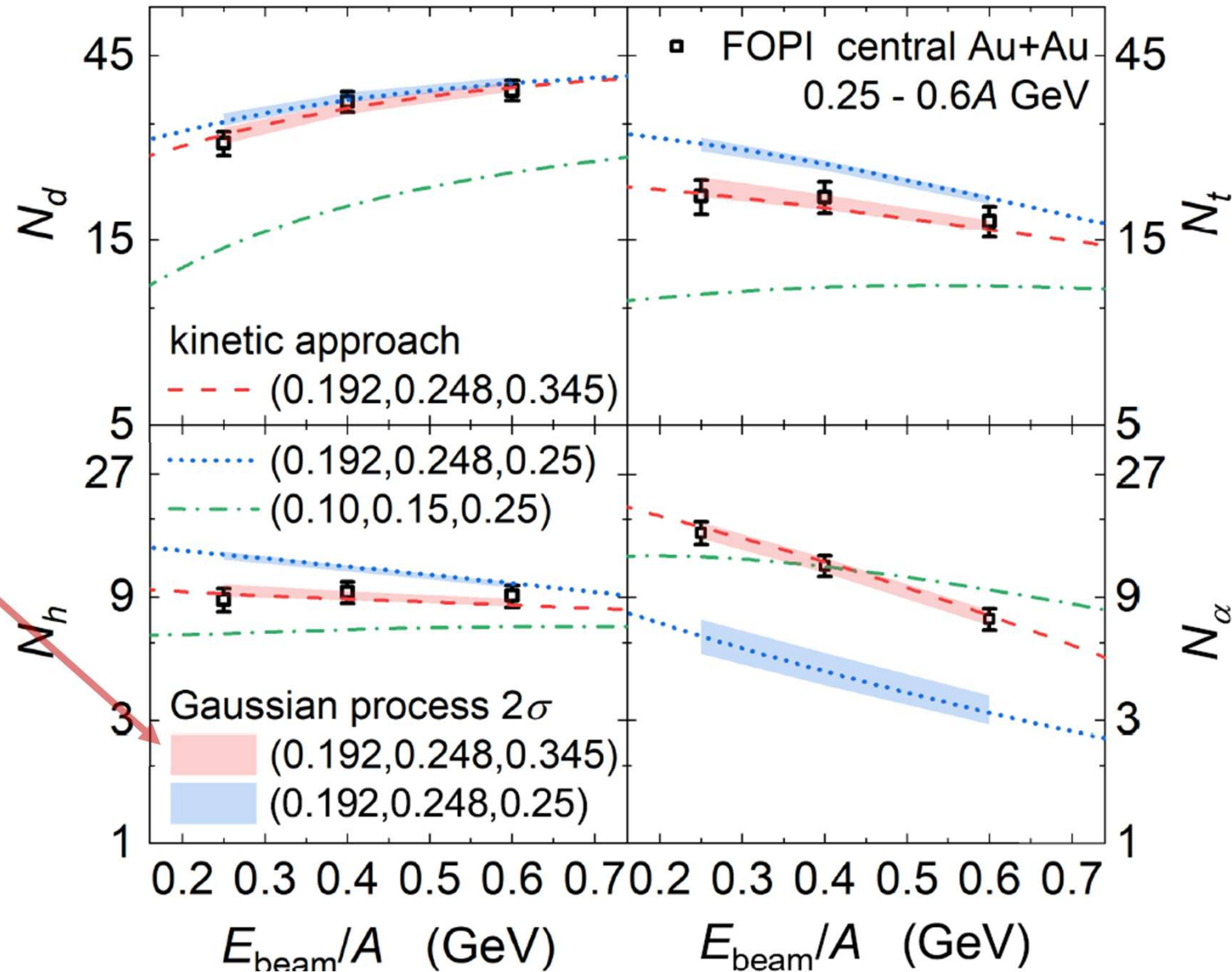
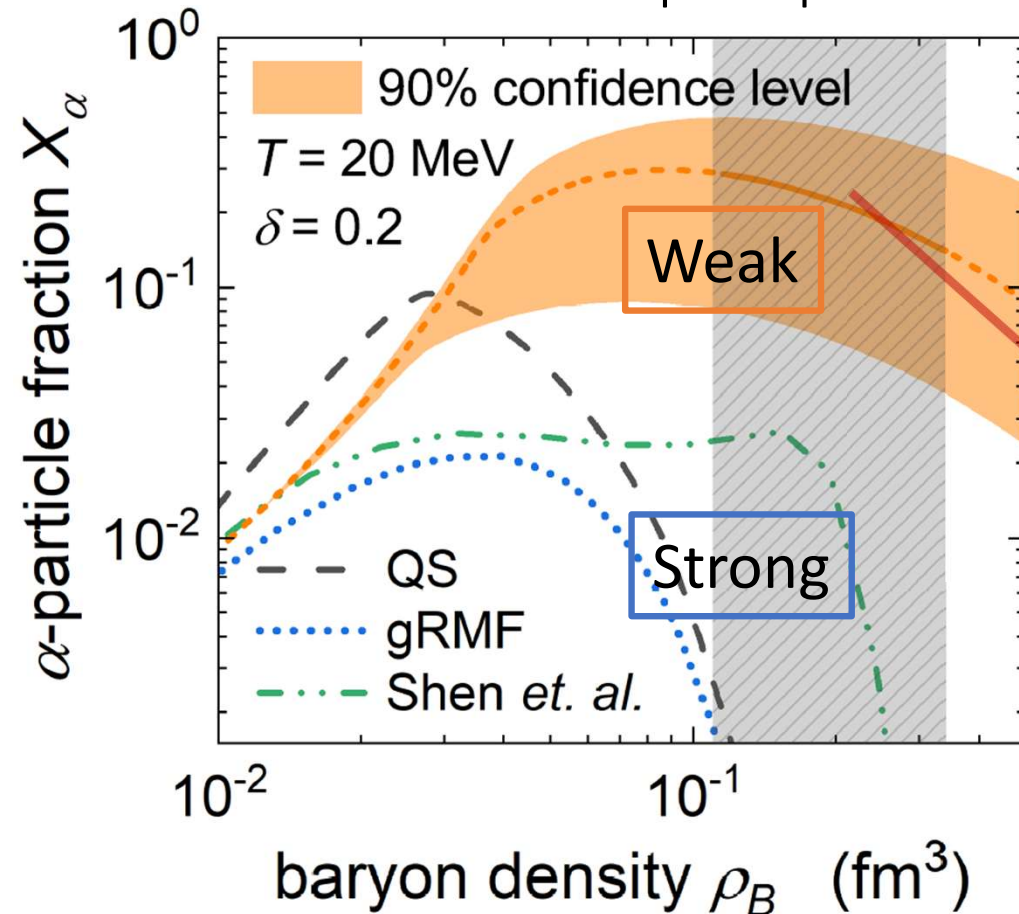


RW, Z. Zhang, Y.-G. Ma *et al.*, Physical Review Letters 136, 172301 (2026)

α -particle fraction

Comparison with other approaches

- QS - Mott momentum from in-medium SE
- gRMF – density dependent (momentum-independent) binding energies
- Shen et. al - excluded volume prescription of α



RW, Z. Zhang, Y.-G. Ma et al., Physical Review Letters 136, 172301 (2026)

Summary and outlook

- We have developed an up-to-date kinetic approach for heavy-ion collisions which includes light clusters up to the α clusters. It enables the probe of in-medium effects on light clusters in hot and dense nuclear matter.
- FOPI measurements on light-nuclei yields in central Au+Au collisions suggest that the in-medium effects on α clusters in these matter is much weaker than previously thought, indicating a clustered nature of such matter.
- The impact of these results on the nuclear equation of state, as well as on neutron star mergers and supernovae, calls for further study.



Thank you!

Fragments in heavy-ion collisions

1. Nucleon correlation

Many-body scatterings like $NNN \leftrightarrow Nd$ In-medium effects

- Quantum/antisymmetrized molecular dynamics effects
A. Ono, *Journal of Physics: Conference Series* 420 (2013) 012103
H.-G. Cheng and Z.-Q. Feng, *Physical Review C* 109, L021602 (2024)
- Kinetic approach/Boltzmann–Uehling–Uhlenbeck equation
Light clusters d , t , ${}^3\text{He}$, and ${}^4\text{He}$ have been included.
P. Danielewicz and G. F. Bertsch, *Nuclear Physics A* 533, (1991) 712-748
R. Wang, Y.-G. Ma, L.-W. Chen, et.al, *Physical Review C* 108, L031601 (2023)

Final-state interactions (coalescence)

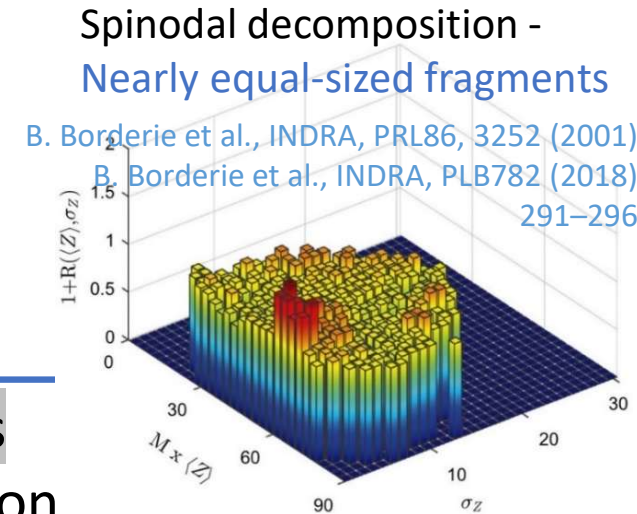
2. Spinodal decomposition

Low incident energies

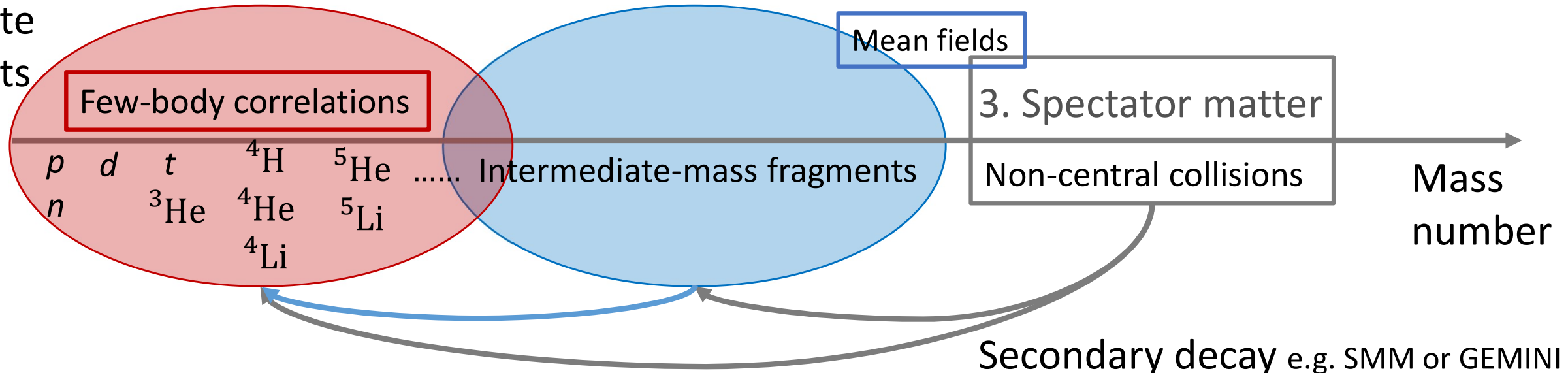
Phase-space fluctuation

- e.g., Statistical mean-field (SMF) or Boltzmann-Langevin equation

F.-S. Zhang and E. Suraud, *Physical Review C* 51, 3201-3210 (1995)
M. Colonna, et. al, *Nuclear Physics A* 642 (1998) 449-460



Final-state fragments



Light clusters in nuclear medium

Thermodynamic Green's function technique

$$G_2^{\text{ladd}}(1, 1', 2, 2') = \delta_{11'}\delta_{22'} - \text{crossed lines} + G_2^{\text{ladd}}(1, 1', 2, 2')$$

$$= \text{series of diagrams} + \dots$$

Bethe-Salpeter equation

$$G_A^{\text{ladd}}(1 \cdots A, 1' \cdots A') = G_A^0(1 \cdots A, 1' \cdots A') + \sum_{(')(*)} G_A^0(1 \cdots A, 1'' \cdots A'') V(1'' \cdots A'', 1^* \cdots A^*) G_A^{\text{ladd}}(1^* \cdots A^*, 1' \cdots A')$$

The free A-(quasi)particle Green function

$$G_A^0(1 \cdots A, 1' \cdots A', z_A) = \frac{[1 - f(\epsilon_1)] \cdots [1 - f(\epsilon_A)] \pm f(\epsilon_1) \cdots f(\epsilon_A)}{z_A - \sum_i \epsilon_i + \mu_A}$$

In-medium modification

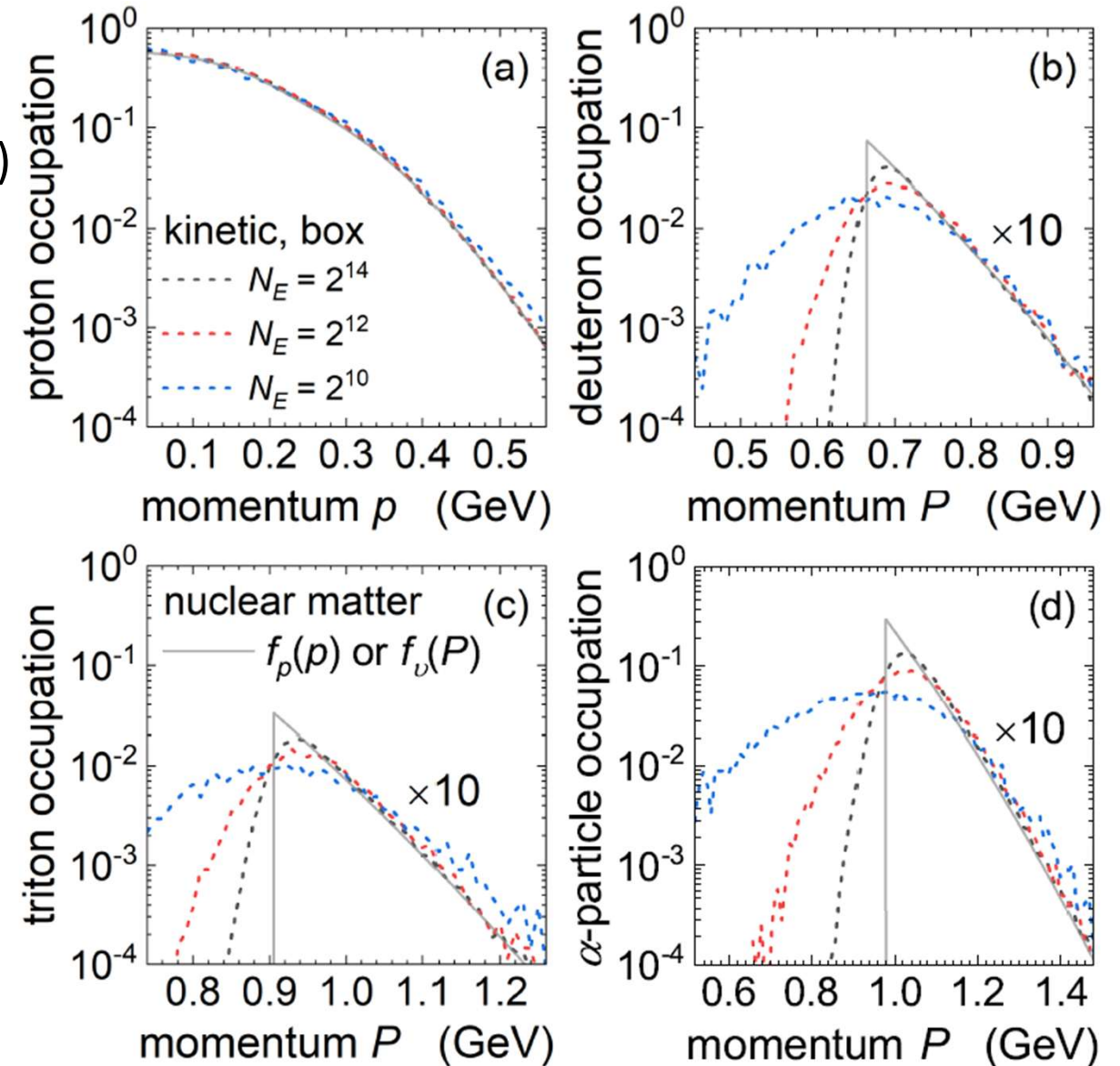
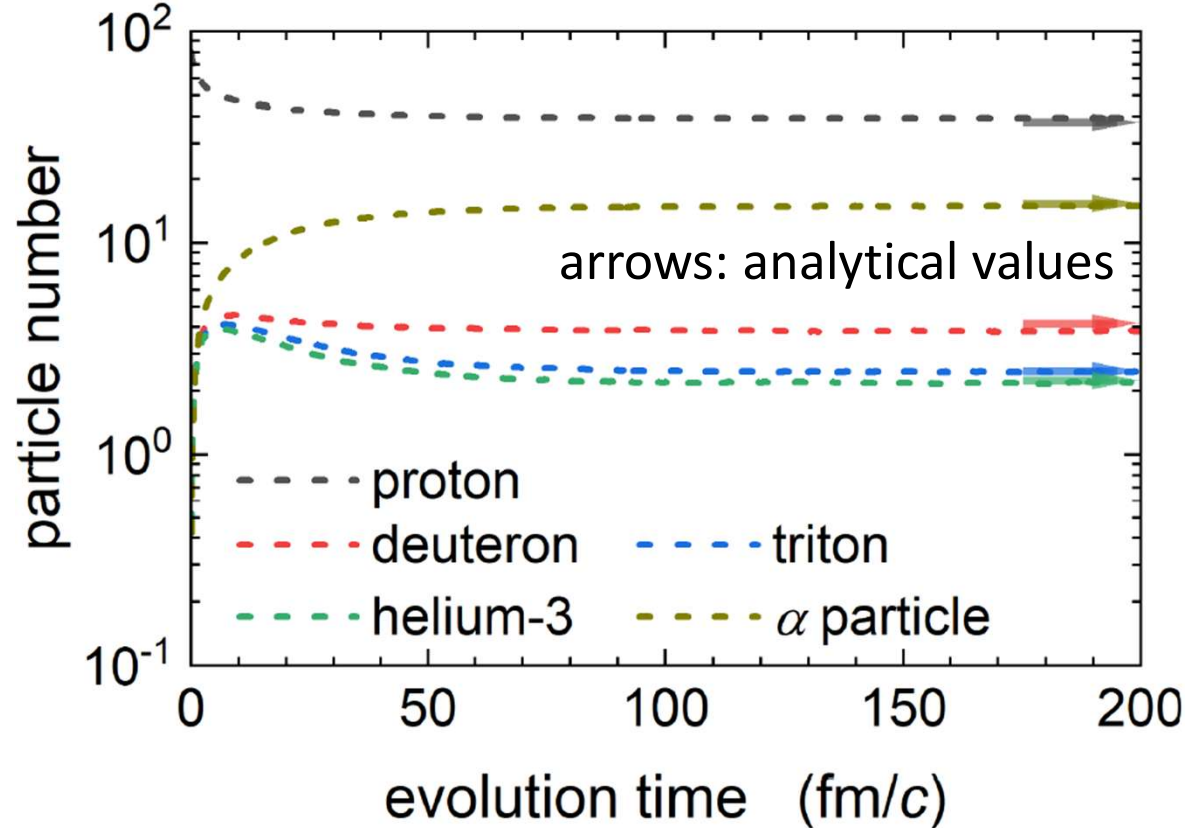
The Bethe-Salpeter equation is satisfied by

$$G_A^{\text{ladd}}(1 \cdots A, 1' \cdots A', z_A) = \sum_{nP} \Psi_{nP}(1 \cdots A) \frac{1}{z_A - \epsilon_{nP} + \mu_A} \Psi_{nP}^*(1' \cdots A')$$

Numerical calculation of nuclear matter

Validation: box calculation

As the number of ensembles N_E (or number of test particles) used in the kinetic approach increases, the resulting light-cluster occupations approach their analytical counterparts



RW, Z. Zhang, Y.-G. Ma, *et al.*, arXiv:2507.16613 [nucl-th]