# Nanoscale Casimir force softening originated from quantum surface responses

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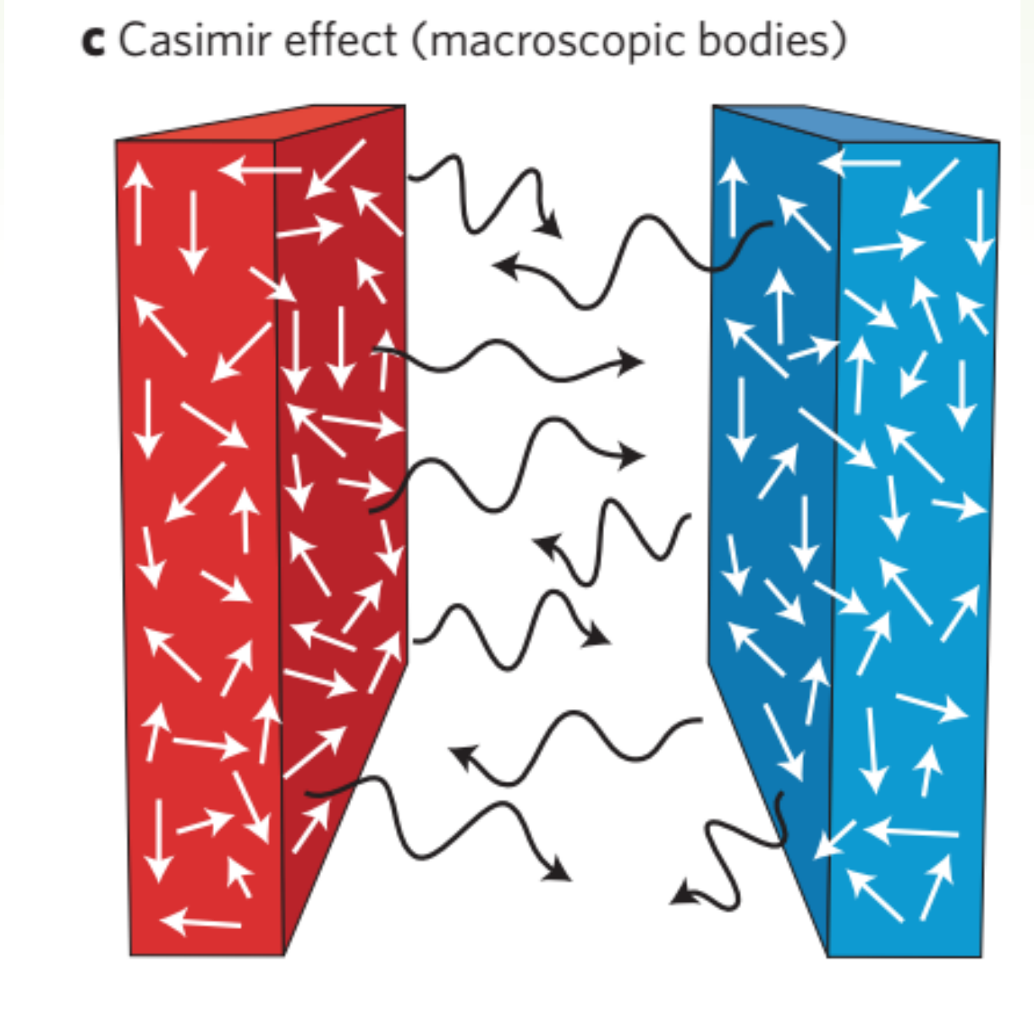
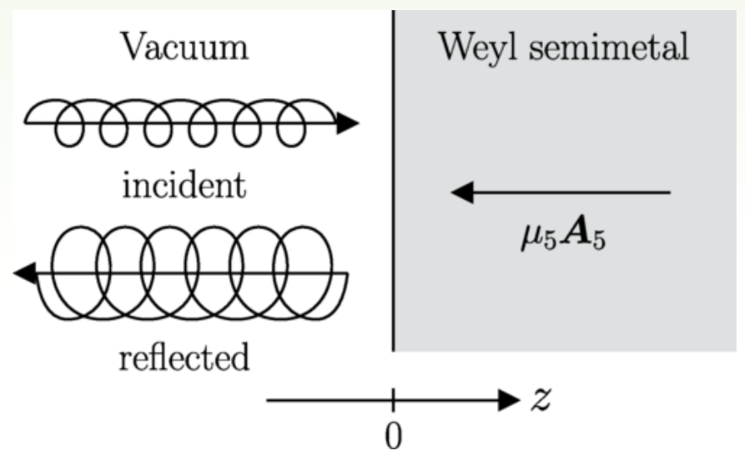
## Abstract

Strong coupling between vacuum fields and quantum matter occurs at the nanoscale and broadens the horizon of light-matter interaction. Nanoscale Casimir force, as an exhibition of vacuum fields, inevitably experiences the influence of surface electrons due to their quantum character, which are ignorable in micron Casimir force. Here, we develop a three-dimensional conformal map method to tackle typical experimental configurations with surface electron contributions to Casimir force purposely and delicately included. Based on this method, we reveal that quantum surface responses (QSRs) can either enhance or suppress the nanoscale Casimir force, depending on materials and crystal facets. The mechanism is demonstrated to be the Casimir force softening, which results from s QSRs effectively altering the distance seen by the Casimir interaction. Our findings not only highlight the interaction between QSRs and vacuum fields but also provide a recipe for theoretical and experimental investigation of nanoscale fluctuation-type problems.

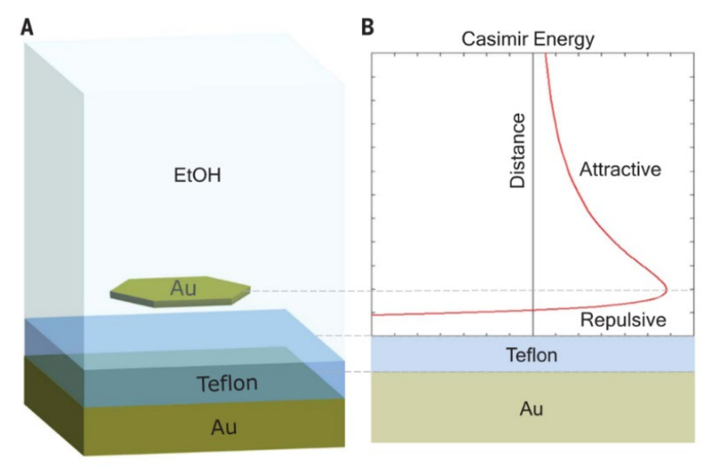
## Background: Casimir effect, strong coupling, surface electrons



A review: Nat. Photon. **5**, 211 (2011)



PRL **130**, 096903 (2023)



Science **364**, 984 (2019)

RESEARCH

QUANTUM OPTICS

**Breakdown of topological protection by cavity vacuum fields in the integer quantum Hall effect**

Felice Appugliese[1*], Josefine Enkner[1], Gian Lorenzo Paravicini-Bagliani[1†], Mattias Beck[1], Christian Reichl[2], Werner Wegscheider[2], Giacomo Scalari[1], Cristiano Ciuti[3], Jérôme Faist[1*]

Science **375**, 1030 (2022)

RESEARCH

REVIEW SUMMARY

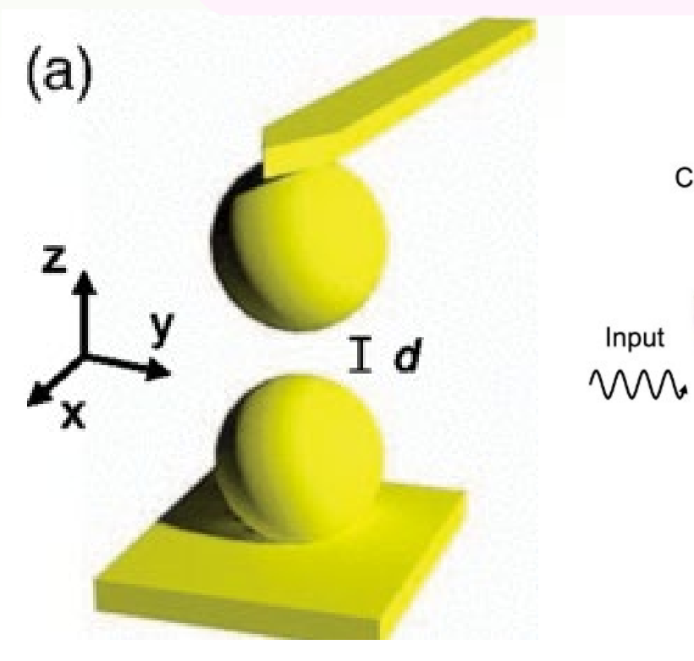
LIGHT-MATTER COUPLING

**Manipulating matter by strong coupling to vacuum fields**

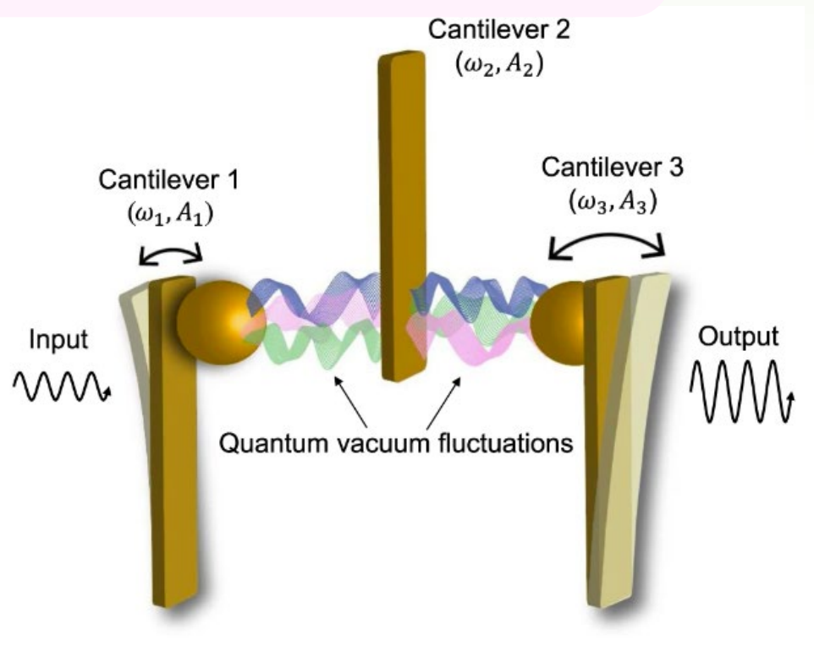
Francisco J. Garcia-Vidal*, Cristiano Ciuti*, Thomas W. Ebbesen*

Science **373**, 178 (2021)

### Typical experimental setups

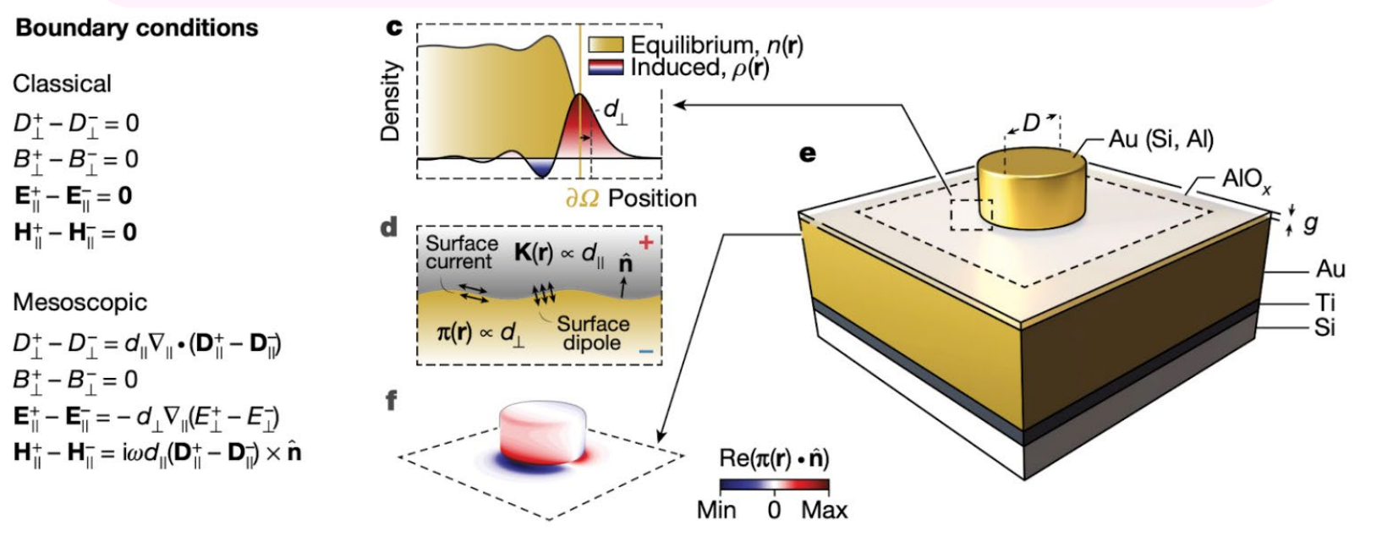


PRL **120**, 040401 (2018)



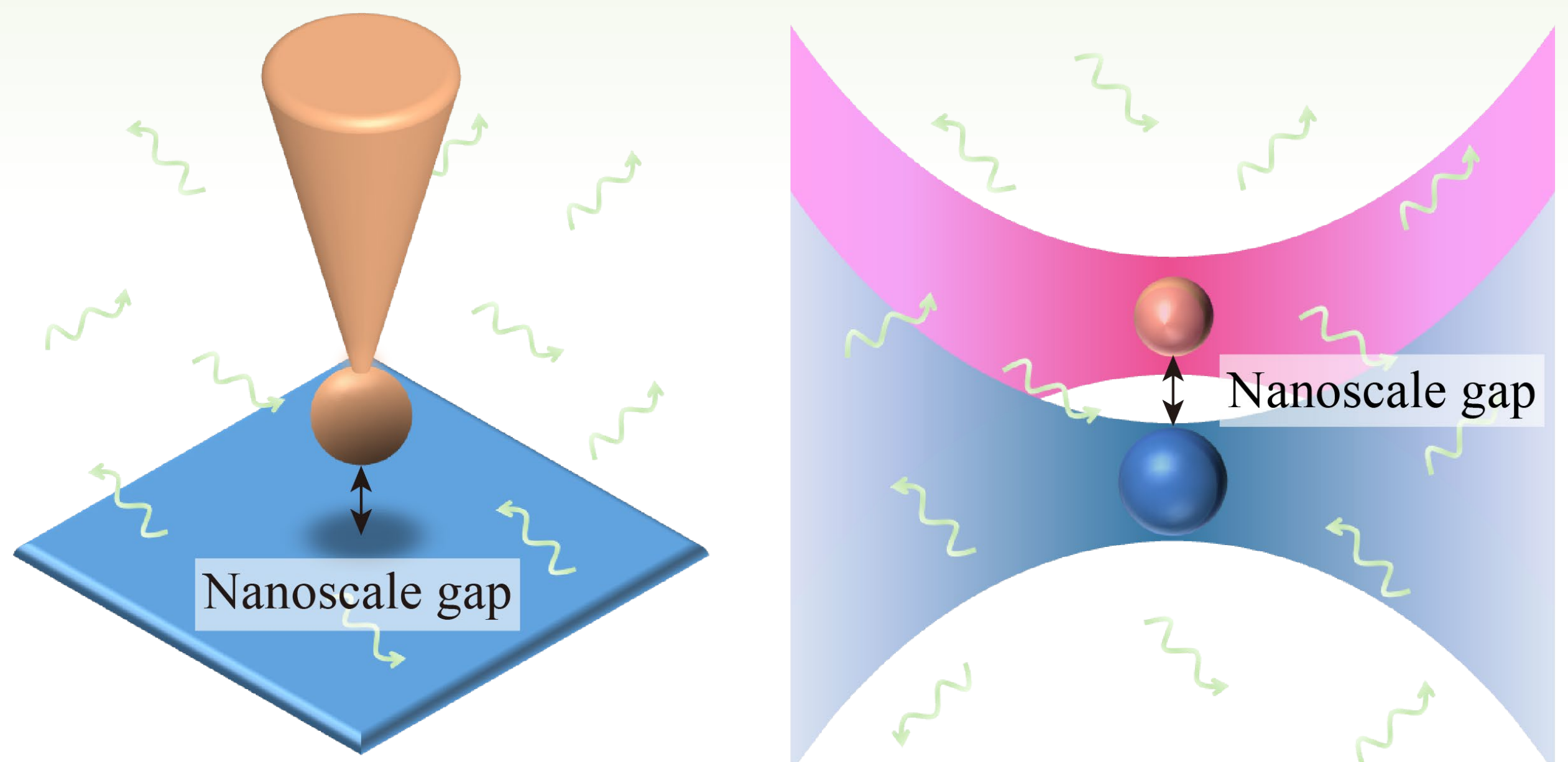
Nat. Commun. **13**, 6148 (2022)

### Feibelman *d*-parameters ($d_\perp$, $d_\parallel$)



Nature **576**, 248 (2019)

## Motivation: *Nanoscale Casimir Force?*

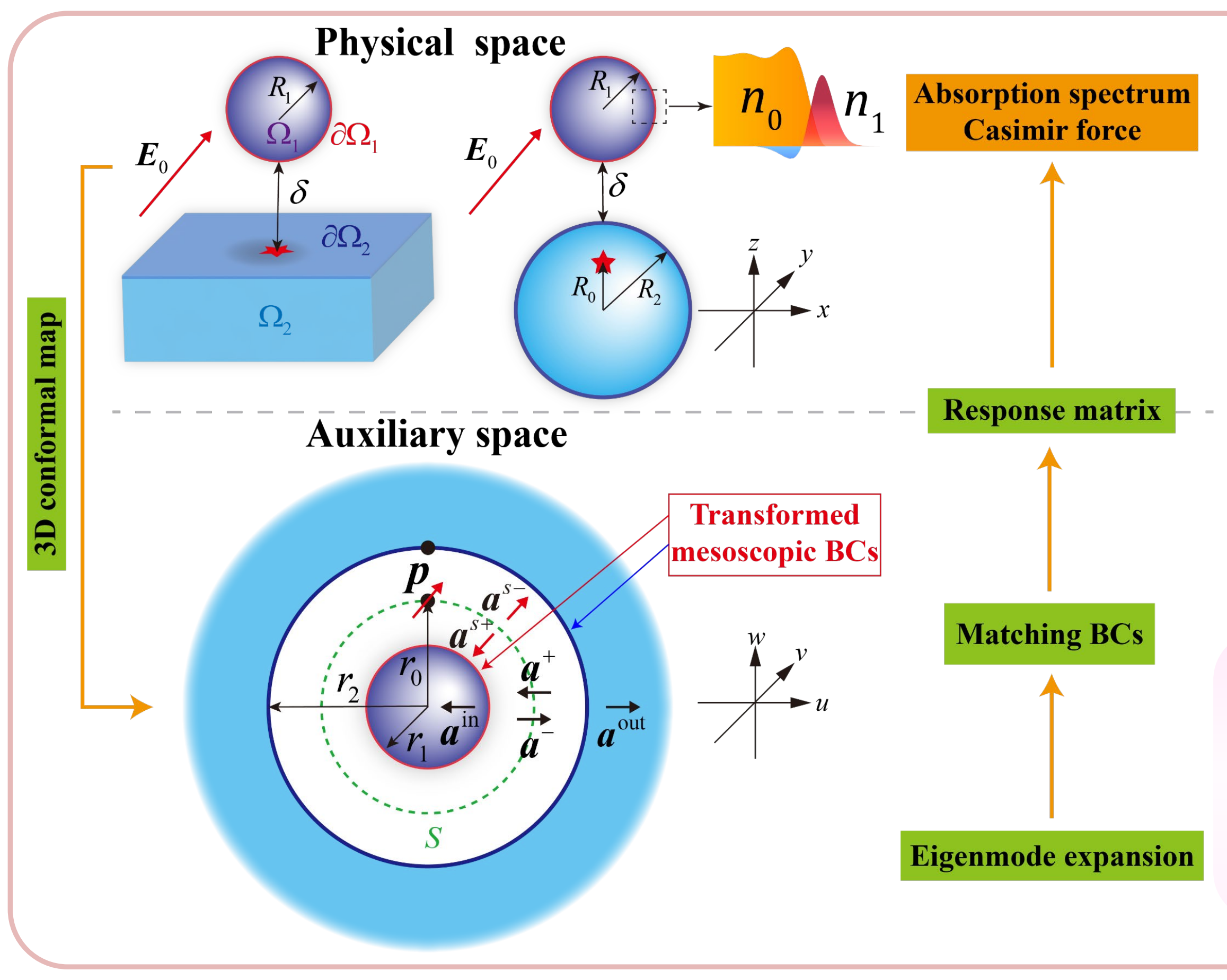


**Casimir formula**: $E = \frac{\hbar}{4\pi}\sum_{m=0}^{+\infty}\int_{-\infty}^{+\infty}\ln f_m(\omega = i\xi)\,\mathrm{d}\xi$

***Challenges:*** **Scattering matrix** of all EM modes (to obtain $\boldsymbol{f_m}$)

- Handle such **experimental structures** (not only planes)
- Include the response of **surface electrons** (beyond ε and μ)
- Require calculations to be **reliable** and **efficient** (all ω and *k*)

## 3D-CM Method



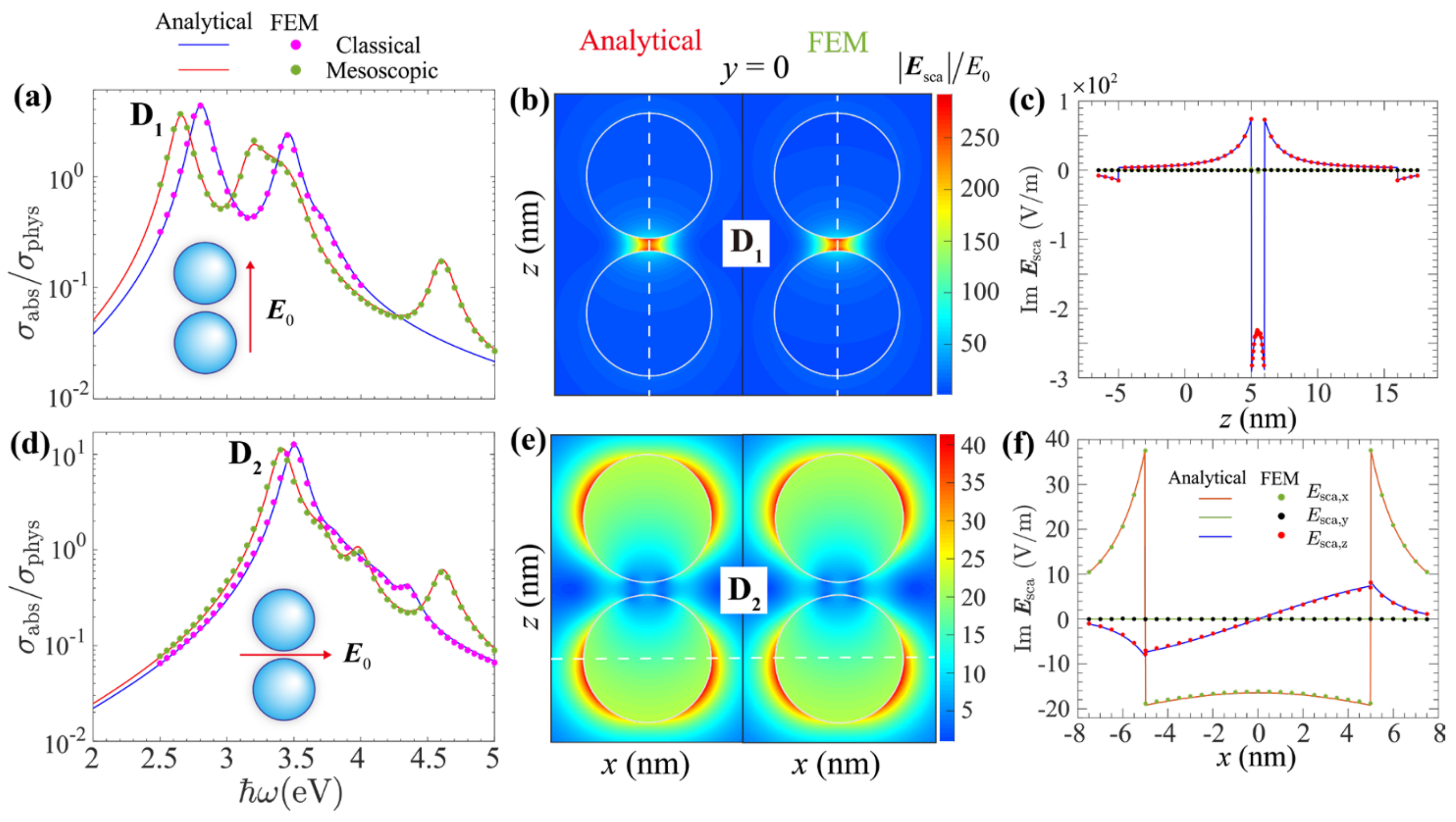
### Response matrix equation

$$\mathbf{S}_{m,\mathrm{wd}}\begin{pmatrix}\boldsymbol{a}_m^{+}\\ \boldsymbol{a}_m^{-}\end{pmatrix} = \begin{pmatrix}\boldsymbol{a}_m^{s+}\\ \boldsymbol{a}_m^{s-}\end{pmatrix},$$

$$\mathbf{S}_{m,\mathrm{wd}} = \begin{pmatrix} -\mathbf{I} & {\mathbf{S}_{m,\mathrm{wd}}^{++}}^{-1}\mathbf{T}_{m,\mathrm{wd}}^{+-} \\ {\mathbf{S}_{m,\mathrm{wd}}^{--}}^{-1}\mathbf{T}_{m,\mathrm{wd}}^{-+} & -\mathbf{I} \end{pmatrix}$$

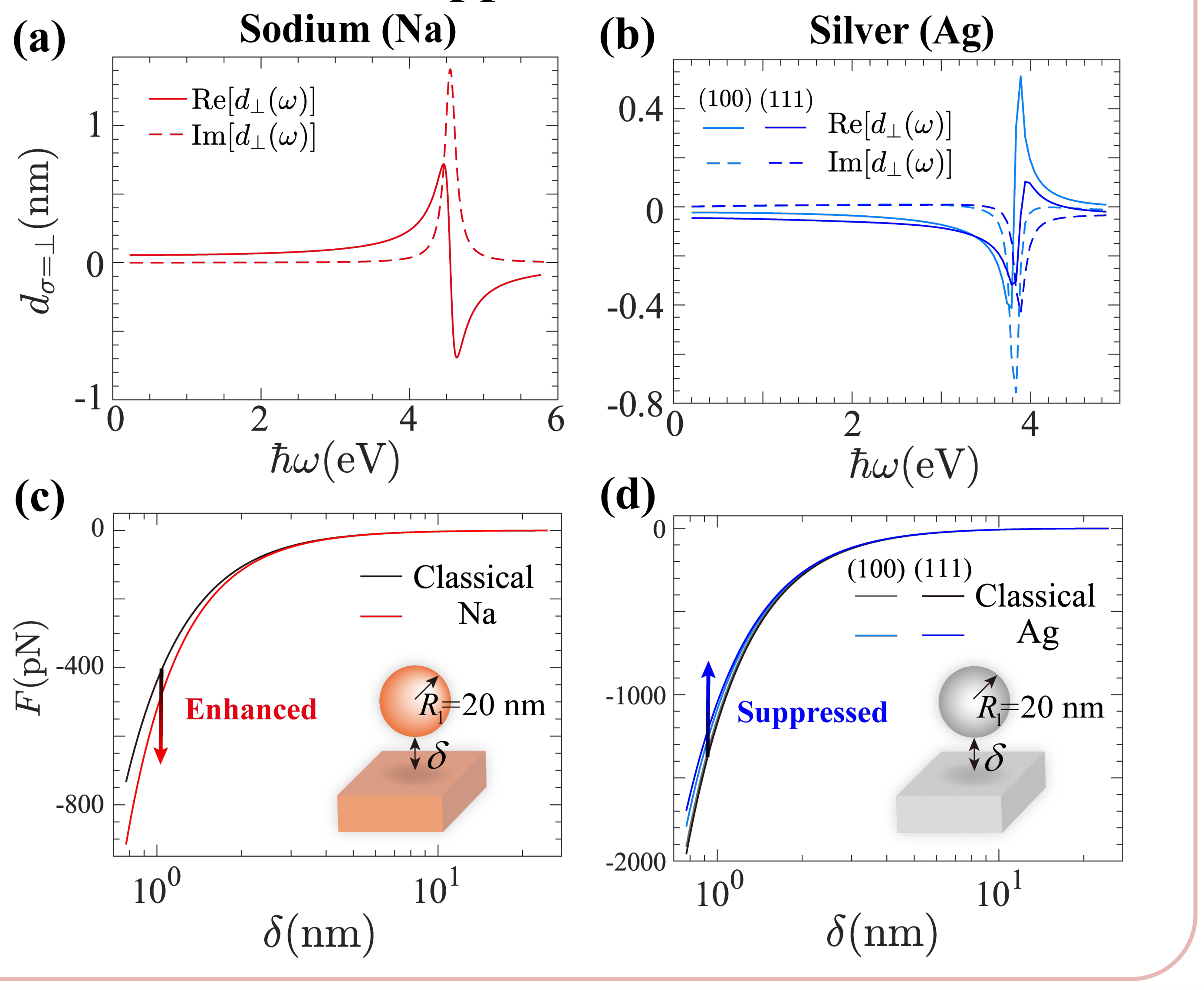
- **Key**: transform mesoscopic BCs to the auxiliary space.
- Obtain scatterings of full EM waves **efficiently** and **accurately**.

## Benchmark: absorption spectrum and field profiles

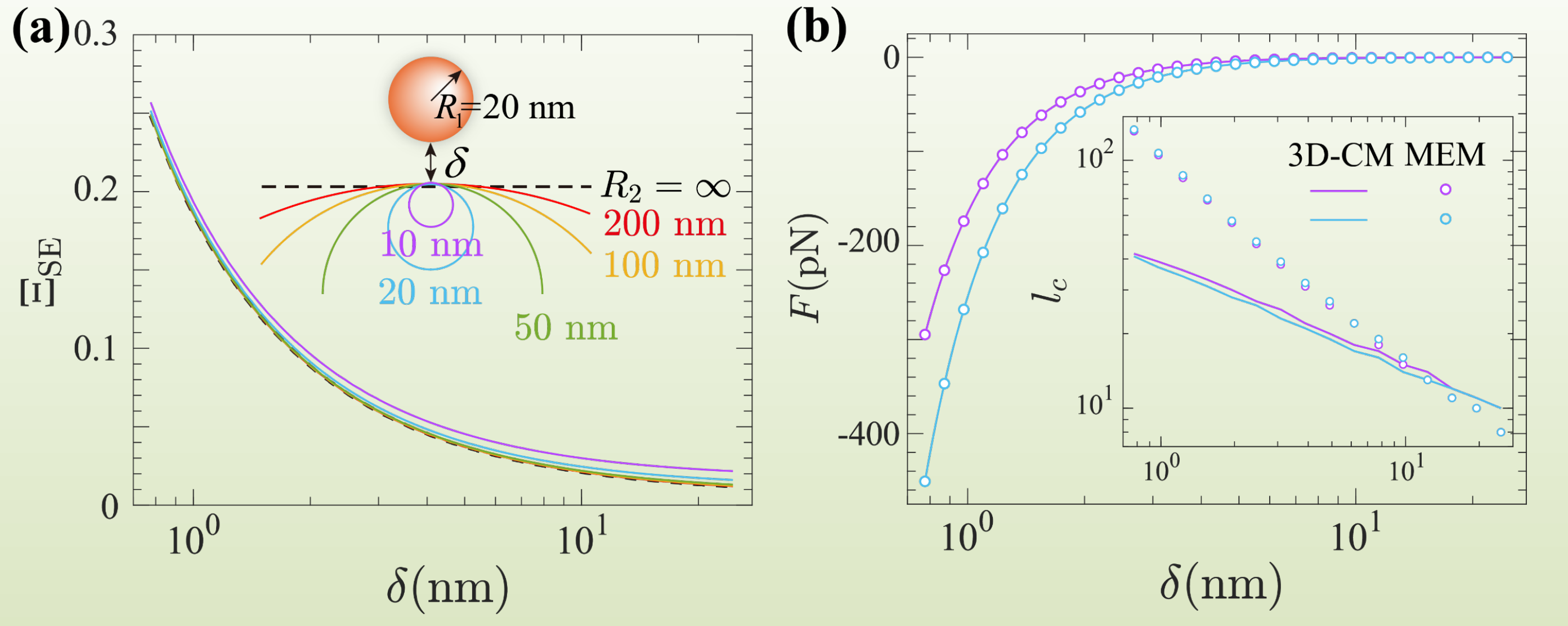


## Nanoscale Casimir force: softening

### Enhancement/suppression of Casimir force

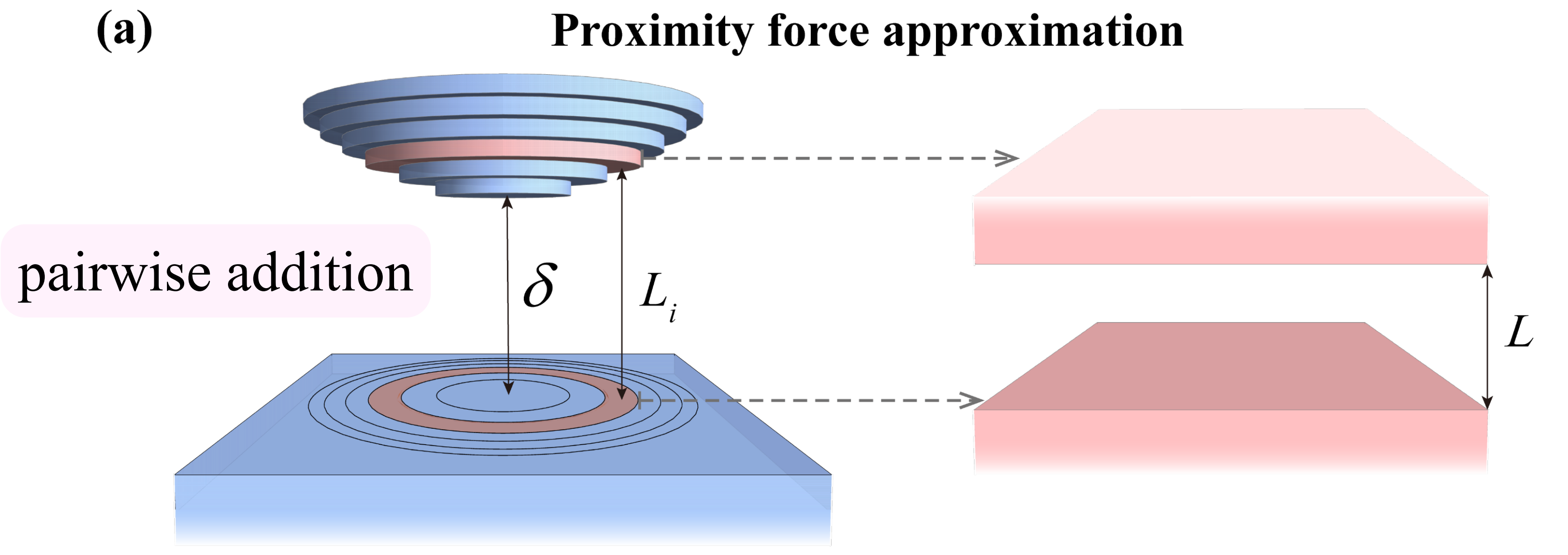


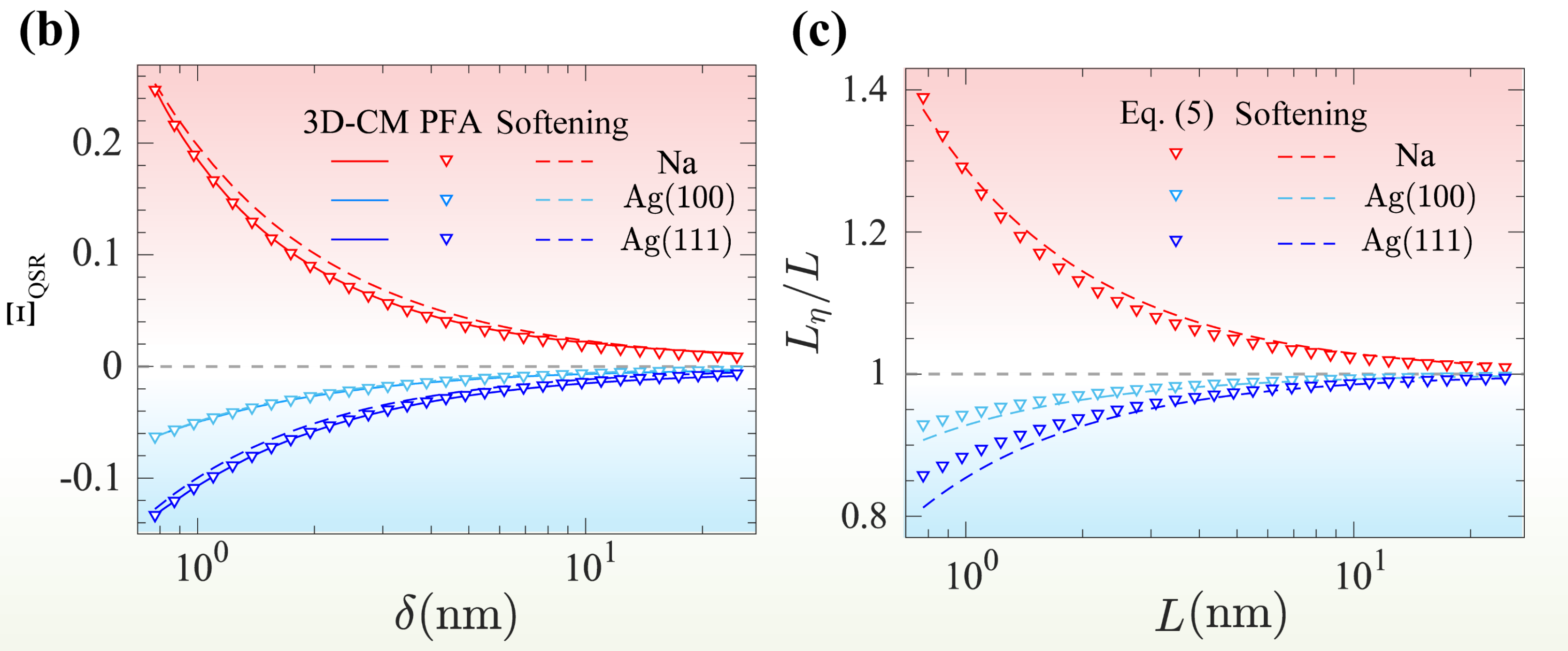
### Extend to bi-sphere



- **MEM:** multipole expansion method

### Casimir softening under PFA





**Surface electron correction factor**: $\Xi_{\mathrm{SE}} = \frac{F_{\mathrm{wd}}}{F_{\mathrm{cl}}} - 1$

Classical **reduction factor**: $\eta_F = \frac{F_{\mathrm{m}}}{F_{\mathrm{PEC}}} = \alpha_0\frac{L}{\lambda_p},\ \alpha_0 = 1.193,\ L \ll \lambda_p$

- In the nanoscale, $\boldsymbol{L} \rightarrow \boldsymbol{L_\eta}$: nanoscale Casimir force softening distance [the *Full* markers]
- Assume that $d_\sigma$ is weakly dispersive and $d_\sigma \ll L$, $\boldsymbol{L_\eta}$ is [*Approx.* lines]
  $\boldsymbol{L_\eta} = L + \Sigma,\ \boldsymbol{\Sigma} = C_\perp d_\perp(i\xi = 0) + C_\parallel d_\parallel(i\xi = 0),\ \boldsymbol{C_\sigma}$**: determined by ε and >0**
- $\boldsymbol{\Sigma}$ only depends on $d_\perp$ and $d_\parallel$
- $\boldsymbol{d_\perp}$: centroid of induced surface electrons; $\boldsymbol{d_\parallel}$: total surface electrons

## Conclusions

- ✓ A three-dimensional conformal map (3D-CM) method can handle **typical experimental geometries**, such as sphere-plate and bi-sphere.
- ✓ Our 3D-CM method is **efficient** and **accurate** whether it is compared with FEM or MEM.
- ✓ **Enhancement**/**suppression** of nanoscale Casimir force.
- ✓ Casimir **softening** distance $L_\eta = L + \Sigma$.

Hewan Zhang and Kun Ding†, Phys. Rev. Lett. **134**, 146901 (2025)