Distinguishing Coherent and Incoherent Errors in Multi-Round Time-Reversed Dynamics via scramblons

Zeyu Liu[1], Pengfei Zhang[1,2]

[1]State Key Laboratory of Surface Physics & Department of Physics, Fudan University, Shanghai, 200438, China

[2]Hefei National Laboratory, Hefei 230088, China

Setup

Question: How to distinguish two different types of errors?

Incoherent: coupling to the environment

$$\mathcal{L}_H[\rho] = -i[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right)$$

Coherent: Imperfection of Hamiltonian

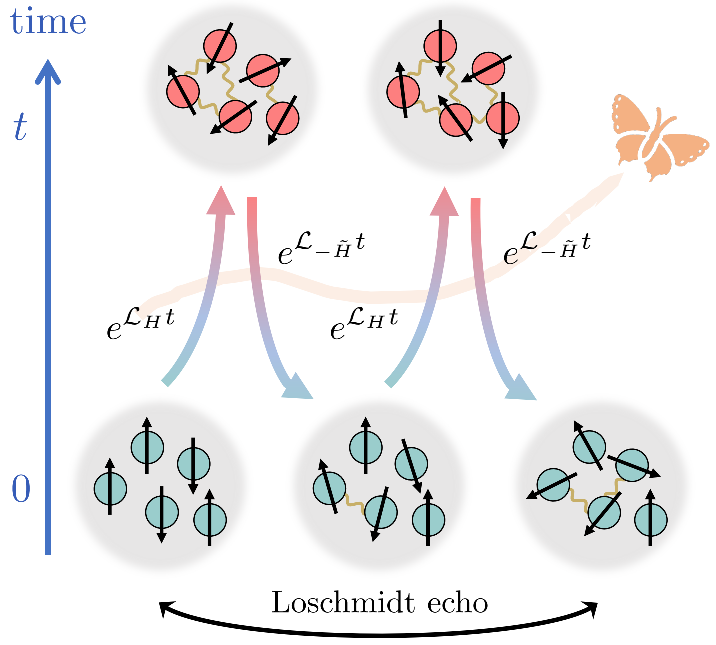
$$\tilde{H} = H + \delta H$$

Setup: To explore the distinct signatures of coherent and incoherent errors, we consider a multi-round time-reversed protocol.

$$\rho_0 \propto (I + \epsilon O)$$

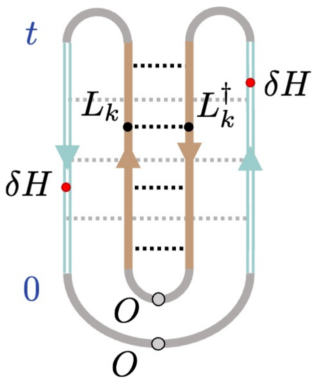
$$\rho_n = \left(e^{\mathcal{L}_{-\tilde{H}} t} e^{\mathcal{L}_H t}\right)^n [\rho_0]$$

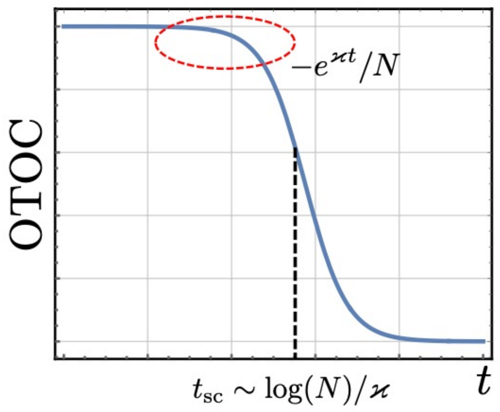
$$\mathrm{Tr}\,(\rho_n O) \equiv \mathrm{Tr}(O^2) F_n(t)$$



Perturbation theory

Both L_k and δH form OTOC with O





Butterfly effect

Perturbation analysis: To second order, the Loschmidt echo $F_n(t)$ receives the contribution from

$$F_n(t) \approx 1 - 2n \int_0^t dt' \sum_k \langle O L_k^\dagger(t')[L_k(t'), O]\rangle \quad \text{OTOC}$$

$$- n^2 \int_0^t dt' dt'' \, \frac{1}{2} \langle [O, \delta H(t')][\delta H(t''), O]\rangle.$$

Coherent errors form n^2 OTOC

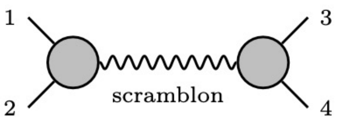
Incoherent errors form $2n$ OTOC

This analysis is true at early time regime

SYK model and scramblon theory

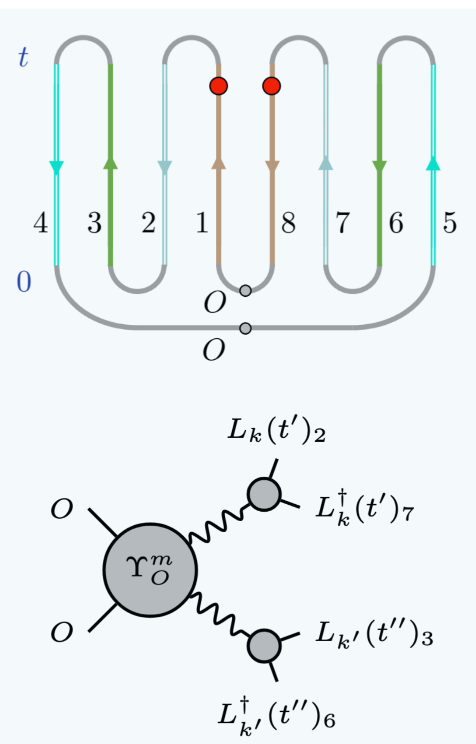
In scramblon theory, we have

OTOC =

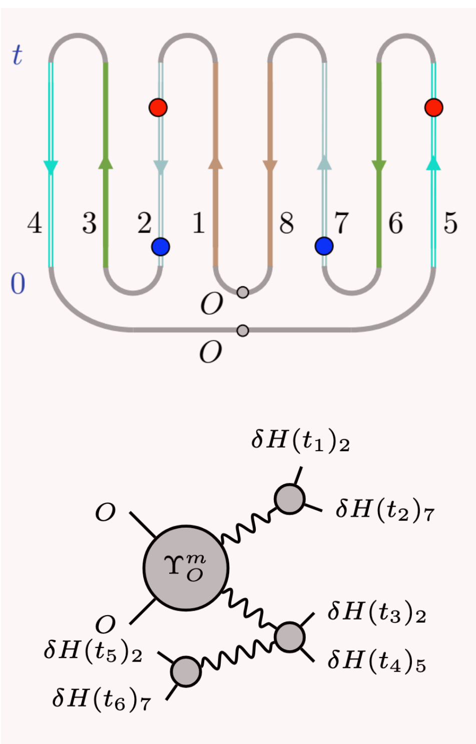


$$= \left(-\frac{e^{\varkappa t}}{Nc}\right) \Upsilon^{\mathrm{R},1}(t_{12}) \Upsilon^{\mathrm{A},1}(t_{34}).$$

Coherent



Incoherent



For incoherent error, scramblon calculation gives

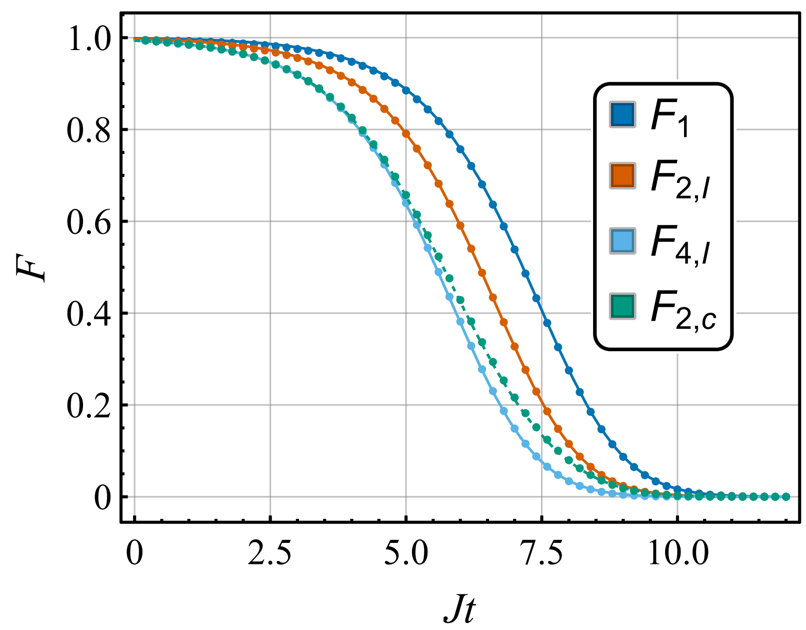
$$F_n(t)_I = \frac{1}{(1 + n\gamma_I e^{\varkappa t})^{2\Delta_O}}$$

For coherent error, scramblon calculation gives

$$F_n(t)_c = \frac{1}{\left(1 + n\gamma_c e^{\varkappa t} + \frac{1}{b\Delta_\delta}\sum_{a=1}^{n-1}\frac{n-a}{a^2}\left[1 - \frac{1}{\left(1+a^2 b\gamma_c e^{\varkappa t}\right)^{2\Delta_\delta}}\right]\right)^{2\Delta_O}}$$

$F_n(t)_c = F_{n^2}(t)_I$ at early time, $F_n(t)_c = F_n(t)_I$ at late time,

Example: SYK numerics



Reference

arXiv:2601.04856