

# Conformal Operator Flows of the Deconfined Quantum Criticality from SO(5) to O(4)

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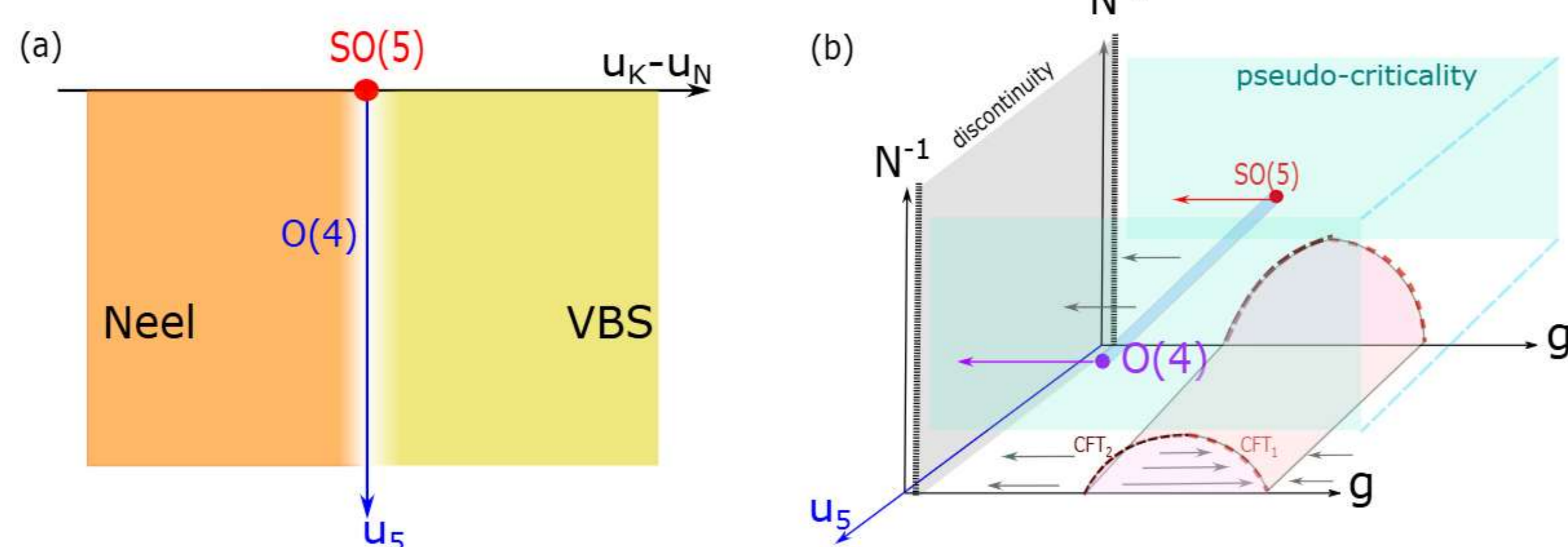
## Abstract

The deconfined quantum critical point (DQCP), which separates two distinct symmetry-broken phases, was conjectured to be an example of (2+1)D criticality beyond the standard Landau-Ginzburg-Wilson paradigm. However, this hypothesis has been met with challenges and remains elusive. Here, we perform a systematic study of a microscopic model realizing the DQCP with a global symmetry tunable from SO(5) to O(4). Through the lens of fuzzy sphere regularization, we uncover the key information on the renormalization group flow of conformal operators. We reveal O(4) primaries decomposed from original SO(5) primaries by tracing conformal operator content and identifying the “avoided level crossing” in the operator flows. In particular, we find that the existence of a scalar operator, in support of the nature of pseudo-criticality, remains relevant, persisting from SO(5) to O(4) DQCP. This work not only uncovers the nature of O(4) DQCP but also demonstrates that the fuzzy sphere scheme offers a unique perspective on the renormalization group flow of operators in the study of critical phenomena.

## 1. Review of the DQCP and RG diagram

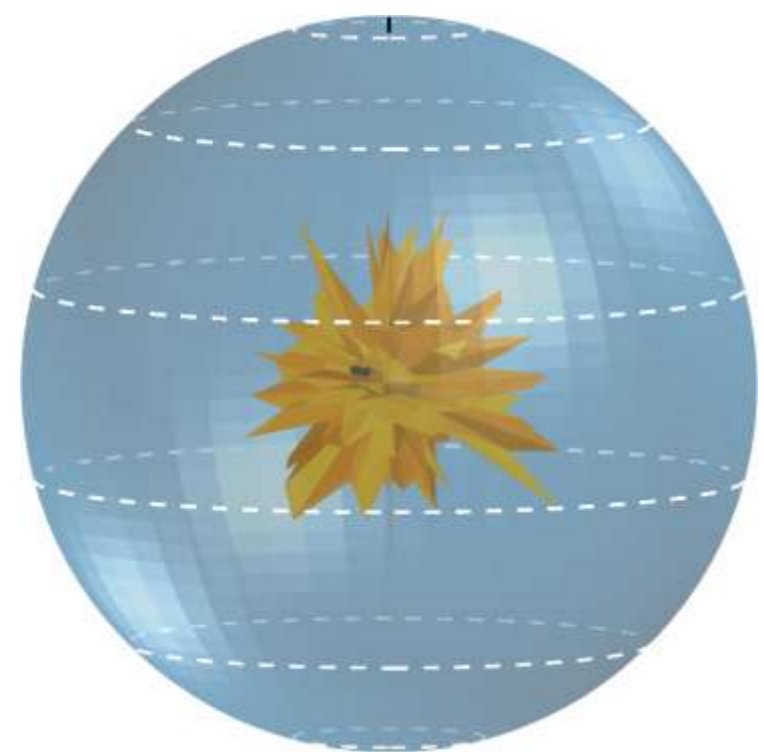
Original proposal: direct continuous transition between two symmetry breaking phase.

Apart from large N case, various works support the DQCP falls into pseudo-critical region



- Is a hypothetical O(4)-symmetric fixed point plausible, which was previously endorsed by the duality?
- Does the O(4) DQCP share the same physics with SO(5) case?

Fuzzy sphere regularization of non-linear sigma model:



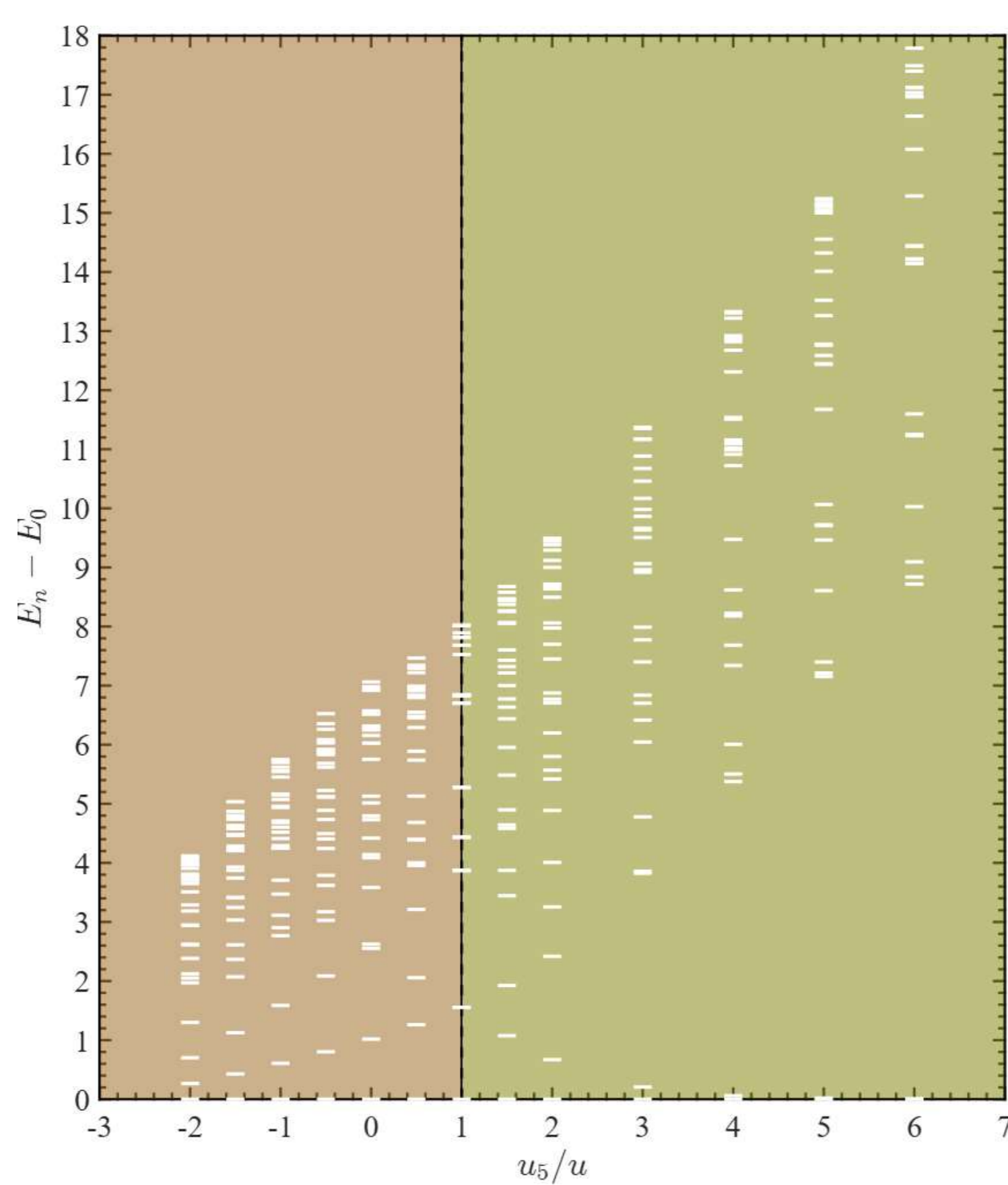
$$H = \int d\mathbf{r}_1 d\mathbf{r}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[ \mathbf{n}_0(\mathbf{r}_1) \mathbf{n}_0(\mathbf{r}_2) - \sum_{i=1}^5 u_i \mathbf{n}_i(\mathbf{r}_1) \mathbf{n}_i(\mathbf{r}_2) \right]$$

$$\mathbf{n}_i(\mathbf{r}) = \psi^\dagger \Gamma^i \psi \quad \psi(\mathbf{r}) = \frac{1}{\sqrt{N_0}} \sum_{m=-s}^s Y_{sm}^{(s)}(\mathbf{r}) \mathbf{c}_m$$

$$\Gamma^{i=1, \dots, 5} = \{ \tau_x \otimes \mathbb{I}, \tau_y \otimes \mathbb{I}, \tau_z \otimes \vec{\sigma} \}$$

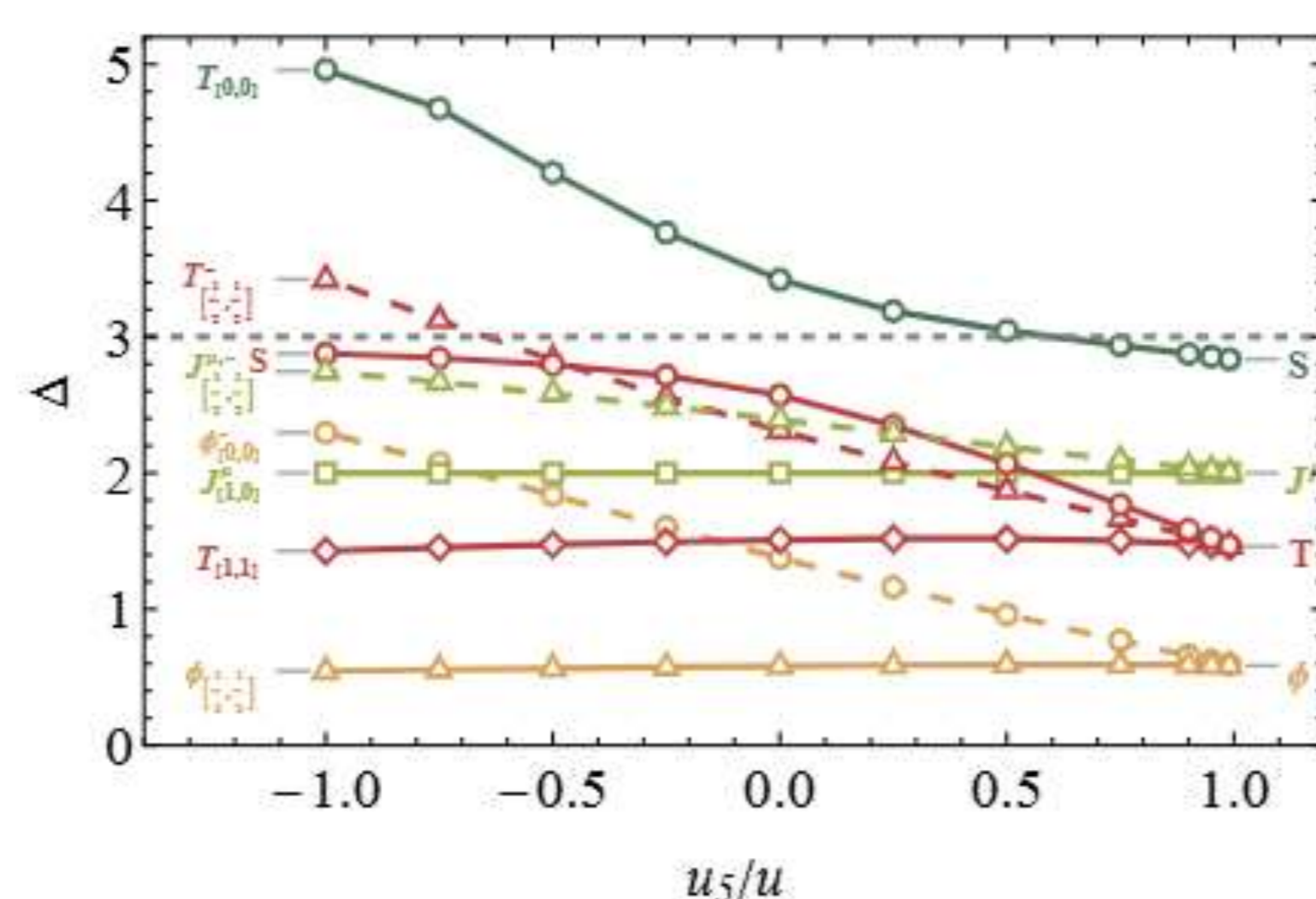
Setting  $u_5 < u_{1,2,3,4}$  Reduce symmetry SO(5)  $\rightarrow$  O(4)

## 2. Tracing operator flow from SO(5) to O(4)



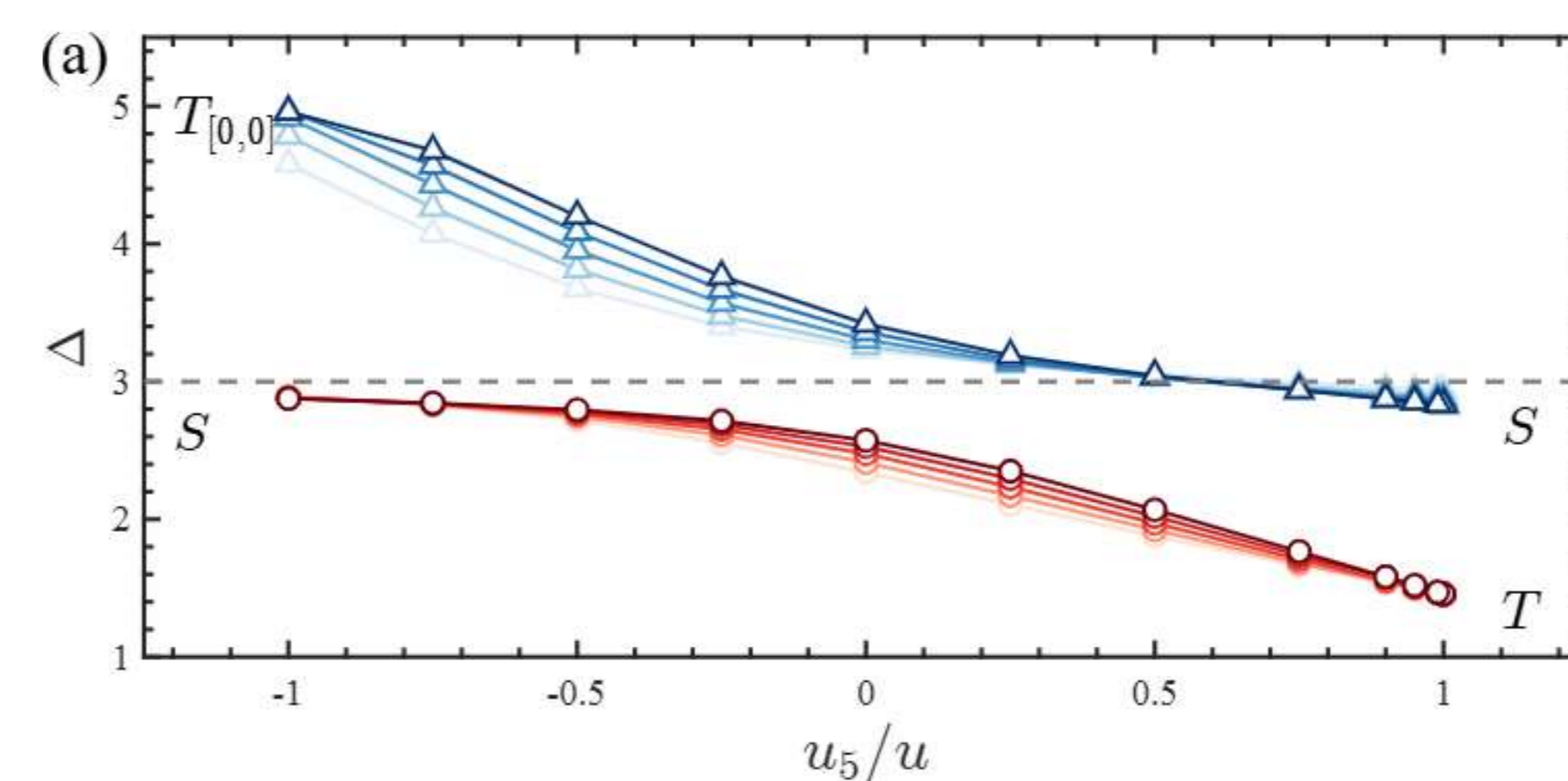
By reducing symmetry, the SO(5) primaries split and decompose into O(4) fields according to branch rule.

- Some splitted states are scaled to be gapped and thus do not belong to the conformal operator spectrum of approximate O(4) fixed point
- while others remain nearly gapless and become new primary operators at the O(4) fixed point



$[p, q]_5$	$\text{Dim}_{SO(5)}$	$\text{Casimir}_{SO(5)}$	Branch rule of $\mathfrak{so}(5) \supset \mathfrak{so}(4)$
$[0, 0]_5$	1	0	$[0, 0]_4$
$[1, 1]_5$	5	2	$[0, 0]_4 \oplus [\frac{1}{2}, \frac{1}{2}]_4$
$[2, 0]_5$	10	3	$[1, 0]_4 \oplus [\frac{1}{2}, \frac{1}{2}]_4 \oplus [0, 1]_4$
$[2, 2]_5$	14	5	$[0, 0]_4 \oplus [\frac{1}{2}, \frac{1}{2}]_4 \oplus [1, 1]_4$
$[3, 1]_5$	35	6	$[0, 1]_4 \oplus [\frac{1}{2}, \frac{1}{2}]_4 \oplus [1, 0]_4 \oplus [\frac{1}{2}, \frac{3}{2}]_4 \oplus [1, 1]_4 \oplus [\frac{3}{2}, \frac{1}{2}]_4$
$[3, 3]_5$	30	9	$[0, 0]_4 \oplus [\frac{1}{2}, \frac{1}{2}]_4 \oplus [1, 1]_4 \oplus [\frac{3}{2}, \frac{3}{2}]_4$
$[4, 0]_5$	35	8	$[0, 2]_4 \oplus [\frac{1}{2}, \frac{3}{2}]_4 \oplus [1, 1]_4 \oplus [\frac{3}{2}, \frac{1}{2}]_4 \oplus [2, 0]_4$

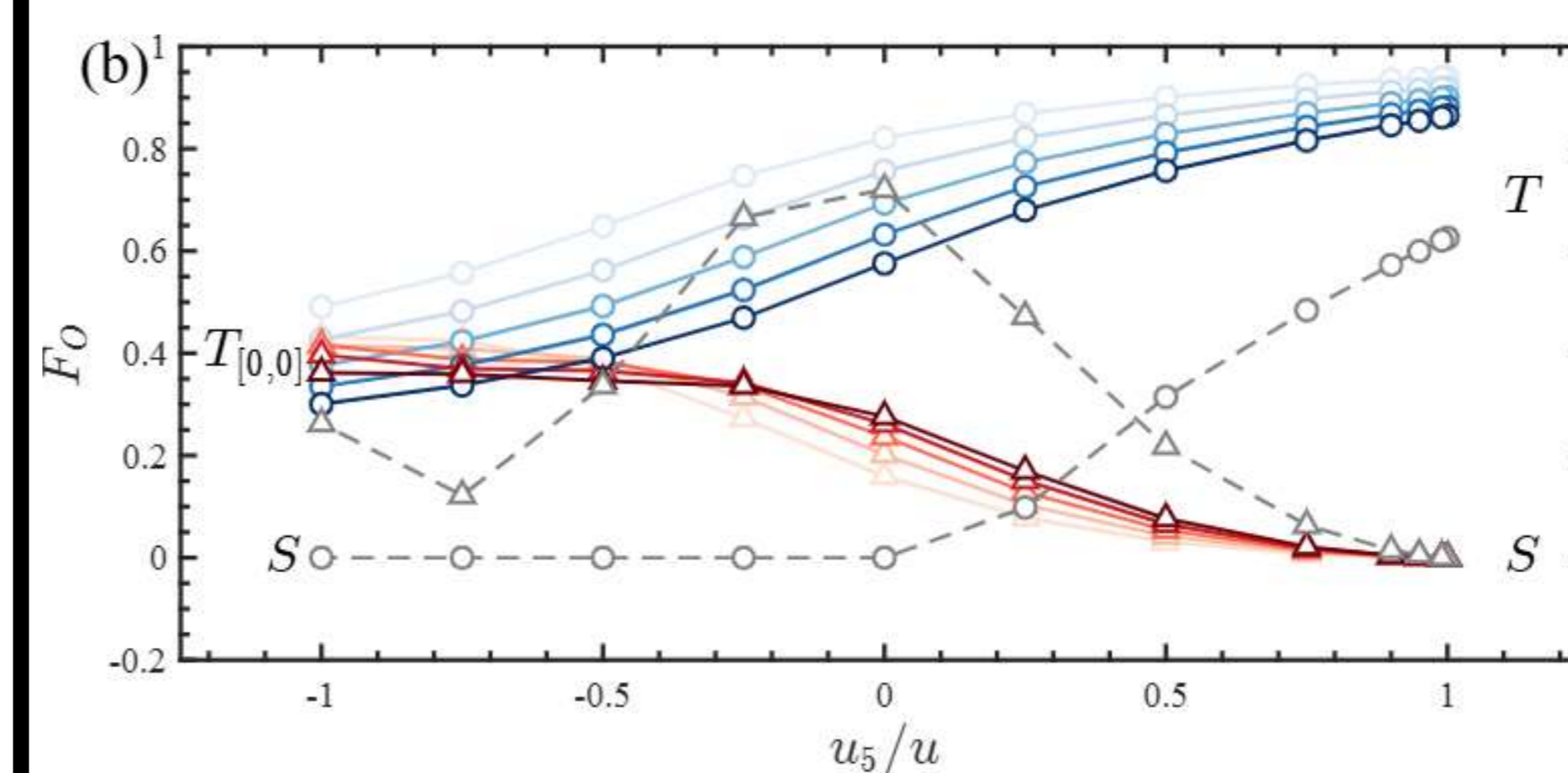
## 3. Level crossing between S and T<sub>55</sub>



The gap first decrease and then increase, suggest a “avoided level crossing behavior”

Probe operator:

$$\hat{T}_{55}(\mathbf{r}) = \int d\mathbf{r}' \delta(\mathbf{r} - \mathbf{r}') \left[ n_5(\mathbf{r}) n_5(\mathbf{r}') - \frac{1}{5} \mathbf{n}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r}') \right]$$



operator content carried by the two lowest singlets exchange by varying  $u_5$

## 4. Approximate conformal symmetry

Rescale the energy spectrum:

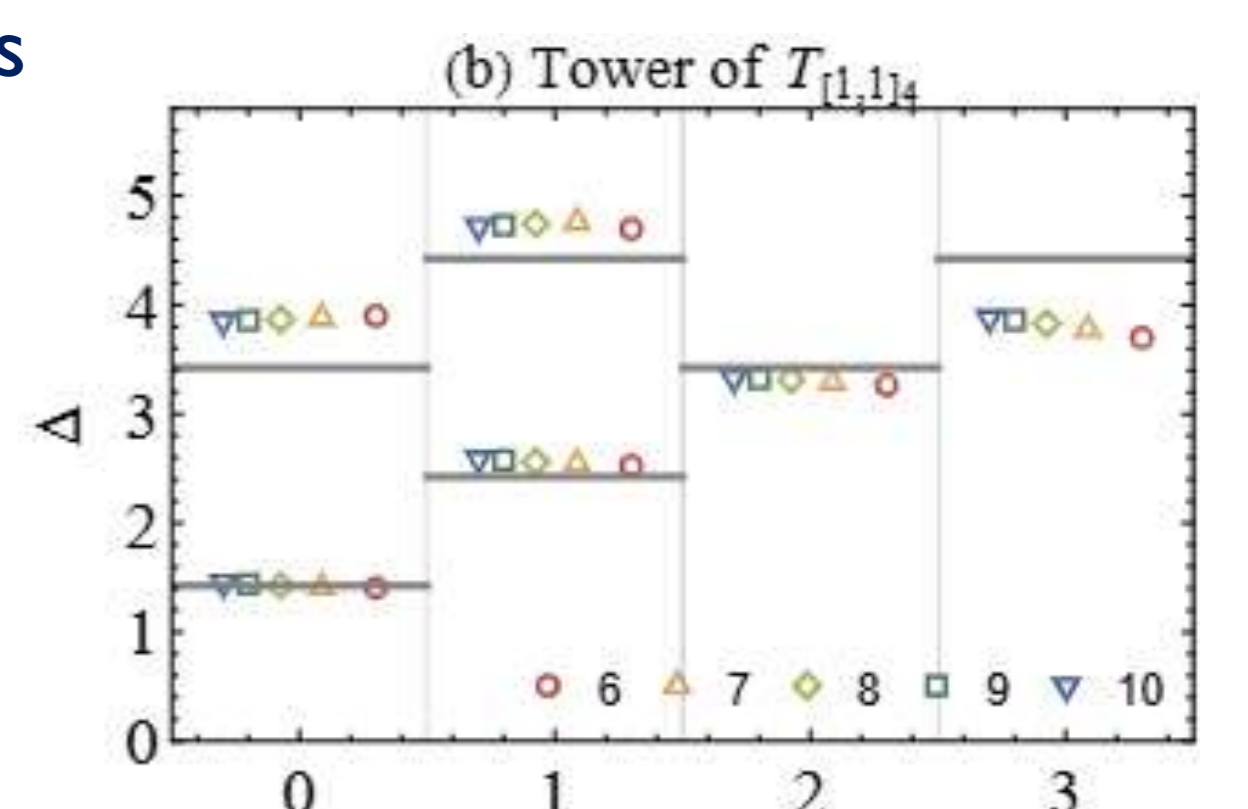
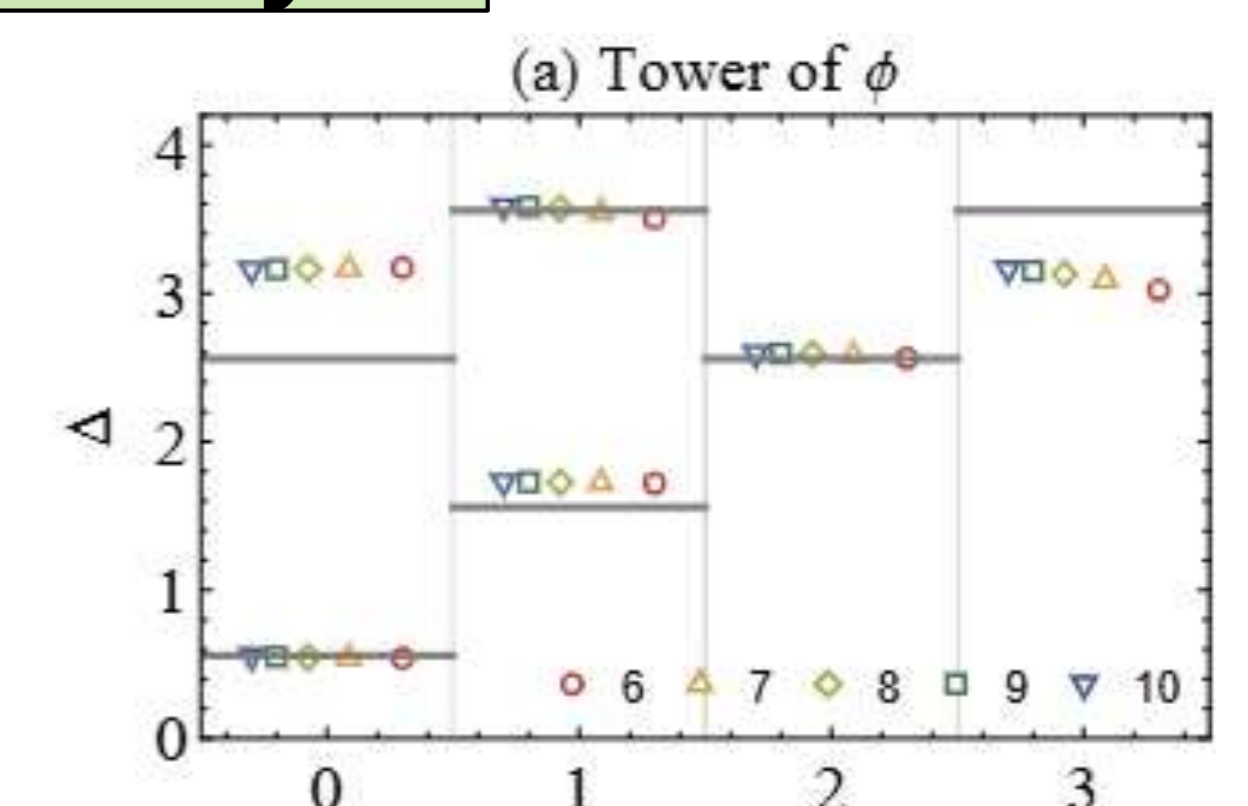
$$3 \frac{E_n - E_0}{E_T - E_0} \approx 3 \frac{\Delta_n}{\Delta_T}, \Delta_T = D$$

Integer level between primaries and descendants

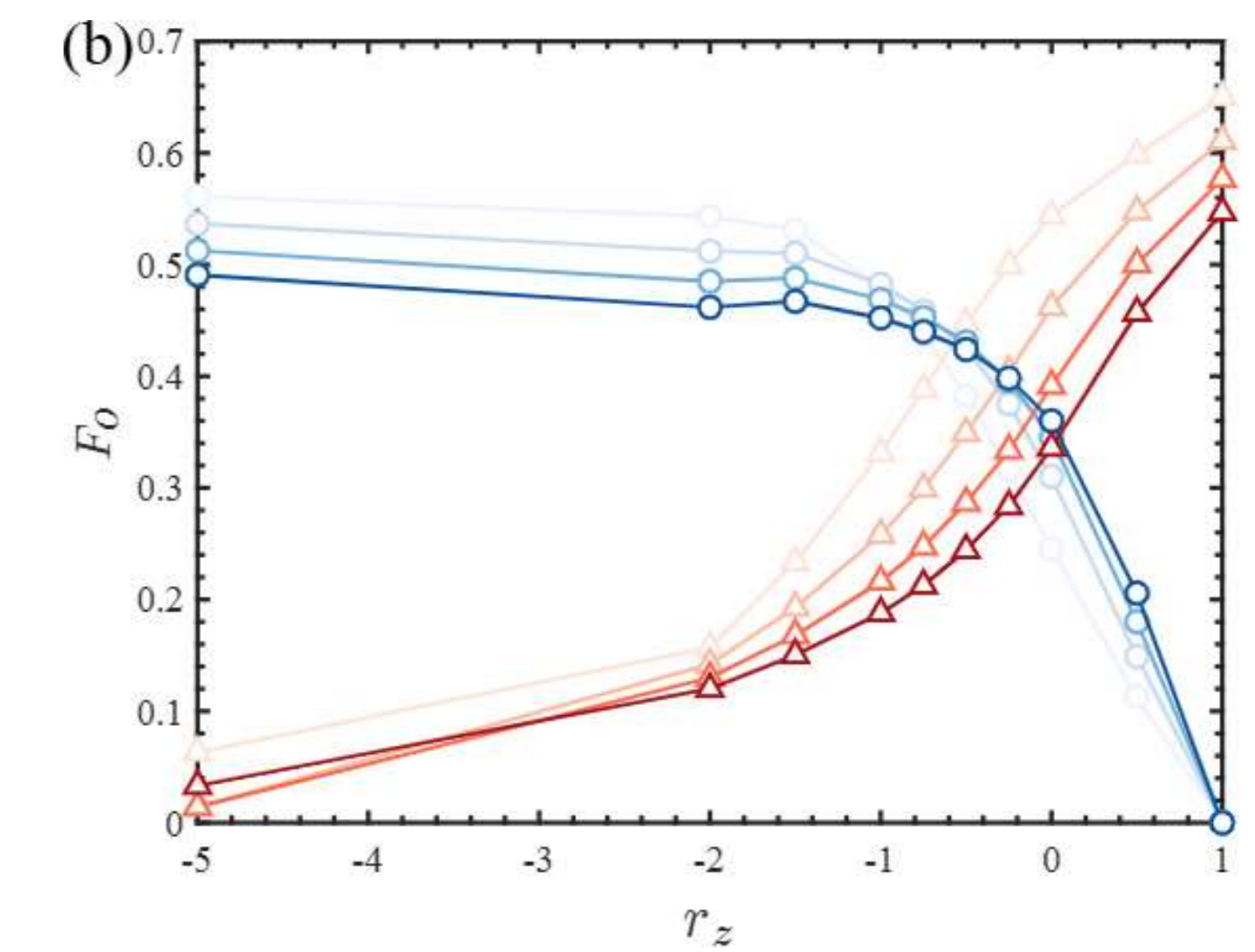
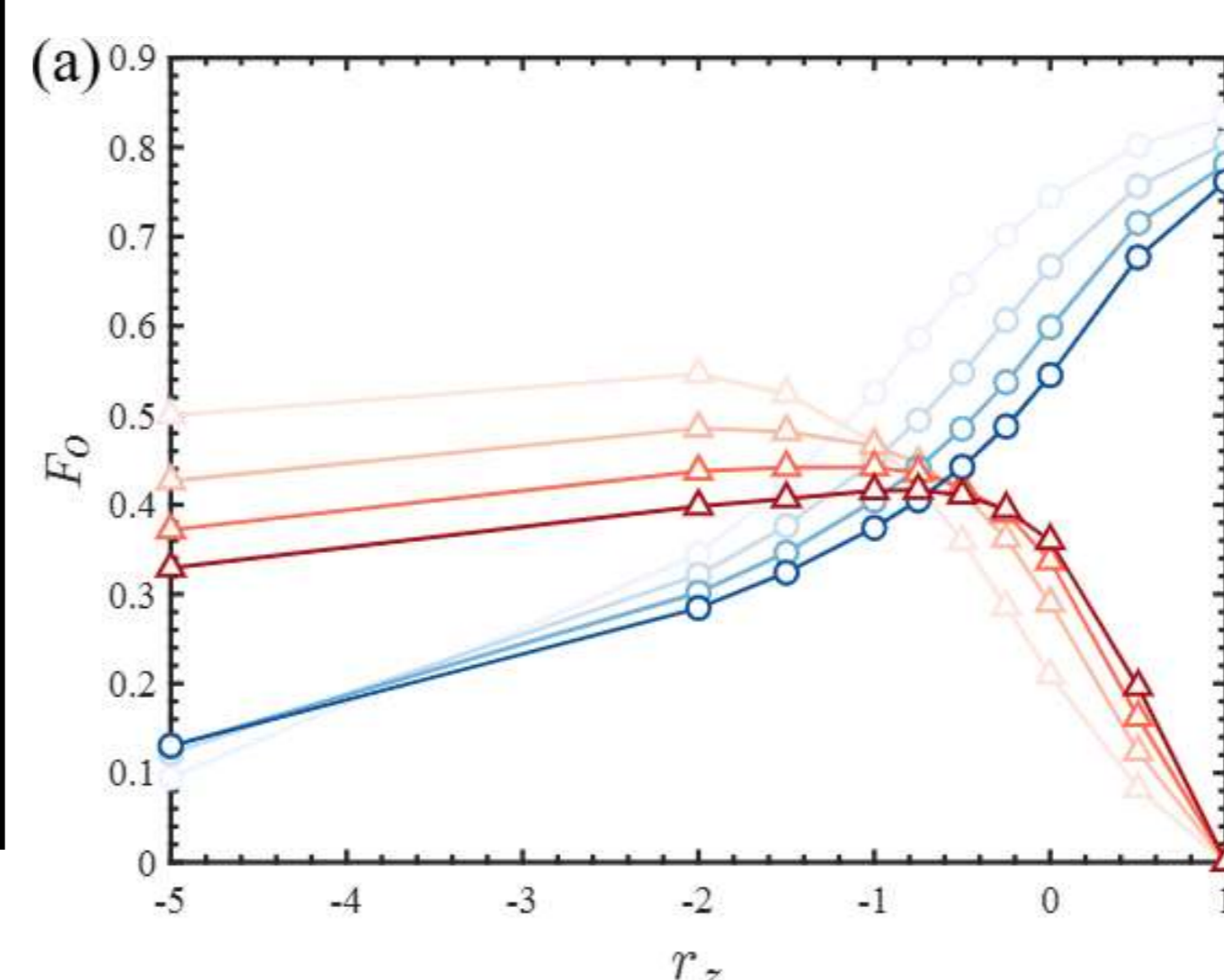
$$\mathcal{O} \rightarrow \Delta_{\mathcal{O}}$$

$$\partial_{\mu_1} \cdots \partial_{\mu_\ell} \square \mathcal{O}$$

$$\rightarrow \Delta = \Delta_{\mathcal{O}} + \ell + 2n$$



## 5. Operator flow from Wilson-Fisher O(3) to O(2) fixed point



$$\bar{S}(\mathbf{r}) = \int d\mathbf{r}' \delta(\mathbf{r} - \mathbf{r}') \mathbf{n}_1(\mathbf{r}) \cdot \mathbf{n}_2(\mathbf{r}')$$

$$T_{zz}(\mathbf{r}) = \int d\mathbf{r}' \delta(\mathbf{r} - \mathbf{r}') \left[ n_1^z(\mathbf{r}) n_2^z(\mathbf{r}') - \frac{1}{3} \mathbf{n}_1(\mathbf{r}) \cdot \mathbf{n}_2(\mathbf{r}') \right]$$

## 6. Conclusion

- We present a detailed characterization of operator flows of the DQCP from SO(5) to O(4).
- We identify a novel avoided level crossing phenomenon of the O(4) operators in energy spectrum.

- The results support that a relevant scalar operator remains relevant throughout the renormalization group flow, suggesting that the O(4) DQCP is weakly first-order transition, akin to its SO(5) counterpart.
- This work outlines a general procedure to analyze the renormalization group flows of operators in the microscopic model, which goes beyond the traditional methods.

References: Shuai Yang, Liang-dong Hu, Chao Han, W. Zhu and Yan Chen, arXiv:2507.01322