Gravitational Lensing effect by Regular Black Holes without Cauchy Horizons

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Motivation and Goal

- Many regular black-hole models (e.g. Hayward, Bardeen) replace the singularity by a regular core, but often introduce an inner Cauchy horizon.
- Recent work shows that broad classes of static, spherically symmetric geometries can arise as exact vacuum solutions of generally covariant gravity theories.
- Hayward-like and Bardeen-like black holes provide regular geometries without a Cauchy horizon.
- This makes them interesting candidates for testing possible deviations from Schwarzschild through black-hole observations.
- Study how the regulator parameter ℓ and the choice of regular core modify lensing observables.
- Compute weak-field and strong-deflection lensing for both Hayward-like and Bardeen-like black holes.
- Discuss possible constraints from existing EHT results and future lensing observations.

Regular BHs in the ρ

- Metric with generalized areal radius:

$$ds^2 = -A(\rho)\,dt^2 + B(\rho)\,d\rho^2 + R(\rho)^2 d\Omega^2,$$

where

$$A(\rho) = 1 - \frac{2M(\rho)}{\rho}, \qquad B(\rho) = \left(1 - \frac{2M(\rho)}{\rho}\right)^{-1}, \qquad R(\rho) = \frac{m}{M(\rho)}\rho.$$

- **For the Hayward-like case:**

$$M_H(\rho) = \frac{m\rho^3}{\rho^3 + 2m\ell^2}, \quad R_H(\rho) = \rho + \frac{2m\ell^2}{\rho^2}.$$

  - $R_H(\rho)$ has a minimum at $\rho_\ell = (4m\ell^2)^{1/3}$, we restrict to the branch $\rho \geq \rho_\ell$
- **For the Bardeen-like case:**

$$M_B(\rho) = \frac{m\rho^3}{(\rho^2+\ell^2)^{3/2}}, \quad R_B(\rho) = \frac{(\rho^2+\ell^2)^{3/2}}{\rho^2}.$$

  - $R_B(\rho)$ has a minimum at $\rho_0 = \sqrt{2}\ell$, and we restrict to the branch $\rho \geq \rho_0$.
- **Both regular BHs:**
  - **Regulator ℓ has an upper bound,** $\ell < \frac{4m}{3\sqrt{3}}$.
  - Only a single event horizon remains $\Rightarrow$ no inner Cauchy horizon.
- The presence of a minimal areal radius and the branch restriction removes the central singularity and avoids an inner Cauchy horizon.

Lensing Setup

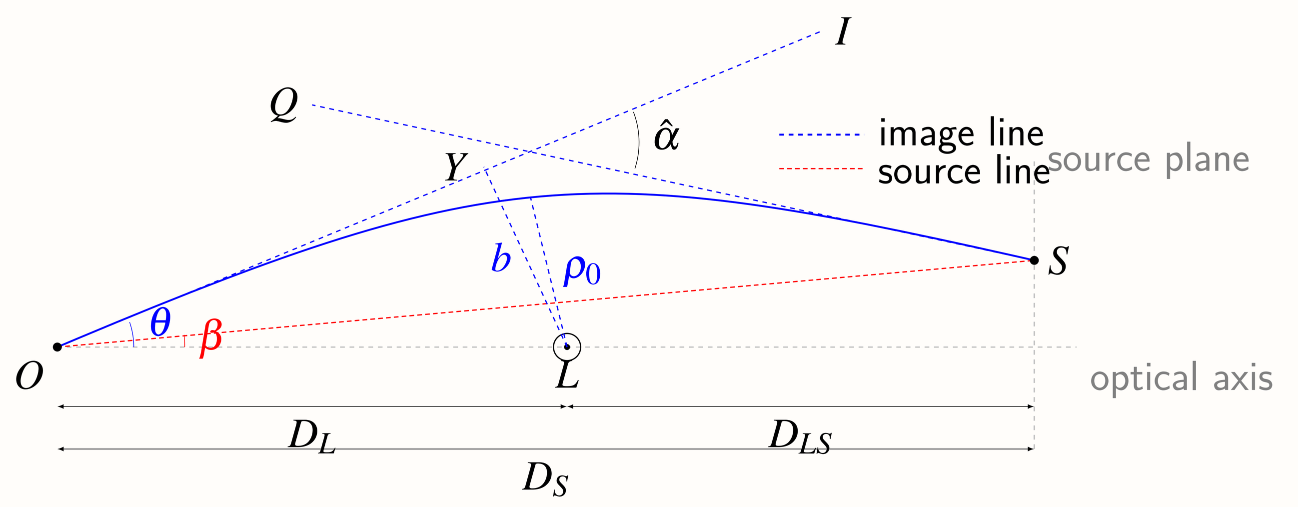


Figure 1: Gravitational lensing geometry, presented in the equatorial $\theta = \pi/2$, (x-y) plane of the lens.

- Static, spherically symmetric line element:

$$ds^2 = -A(\rho)\,dt^2 + B(\rho)\,d\rho^2 + R(\rho)^2(d\theta^2 + \sin^2\theta\, d\varphi^2).$$

- Deflection angle:

$$\alpha(b) = 2\int_{\rho_0}^{\infty} \frac{\left(\frac{B(\rho)}{R(\rho)^2}\right)^{1/2}}{\left(\frac{R(\rho)^2 A(\rho_0)}{R(\rho_0)^2 A(\rho)} - 1\right)^{1/2}} d\rho - \pi$$

Weak-Field Lensing and Deflection angle

- In the weak-field region, the deflection angle is small, we have a thin-lens description and simple lens equation

$$\beta = \theta - \frac{D_{LS}}{D_S}\hat{\alpha} = \theta - \alpha$$

Deflection angle of Hayward-like BH

- Higher-order corrections of α_H, one needs a higher order correction of $\rho_{\min}$, expansion of α_H:

$$\alpha_H \simeq \frac{4m}{b} + \frac{15\pi}{4}\frac{m^2}{b^2} + \frac{16}{3}\frac{m\ell^2}{b^3} + \mathcal{O}(m^3, m^2\ell^2).$$

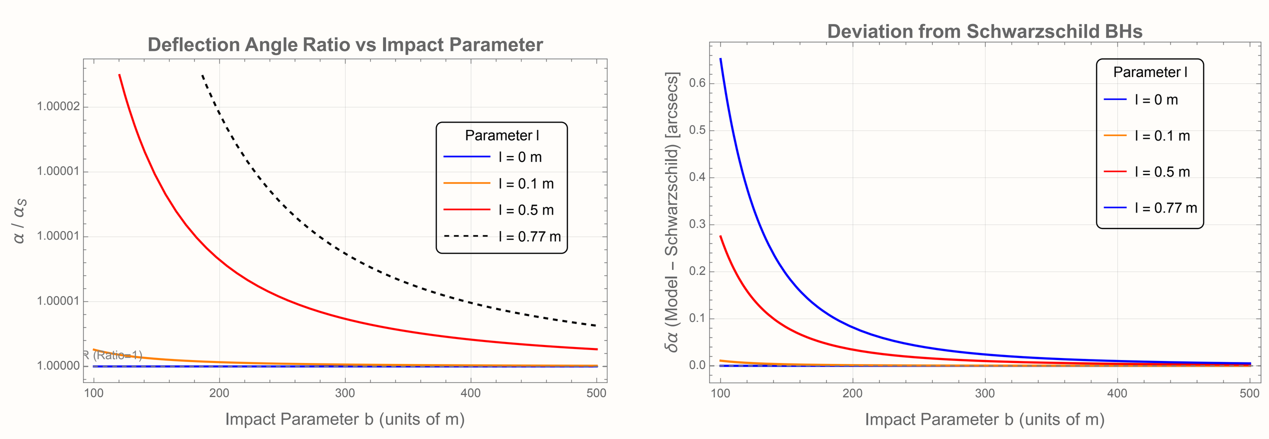
  - First two terms exactly match Schwarzschild: Einstein deflection and standard post-Newtonian correction.
  - Positive sign of the correction terms of order $\sim m\ell^2/b^3$, unlike standard Bardeen BHs giving a negative correctoin of order $\sim m^2\ell^2/b^4$.

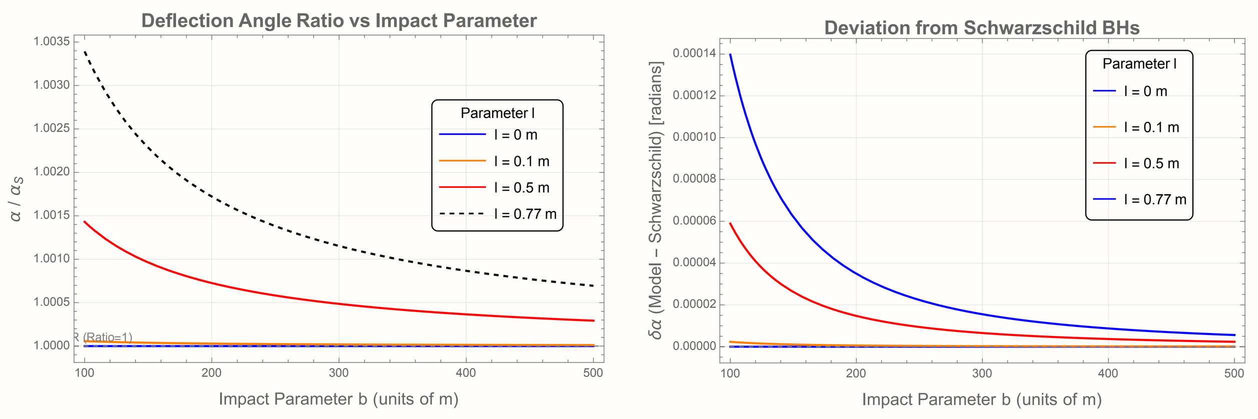
Deflection angle of Bardeen-like BH

- Expansion of α_B:

$$\alpha_B \simeq \frac{4m}{b} + \left(\frac{15\pi}{4}m^2 + \frac{3\pi}{4}\ell^2\right)\frac{1}{b^2} + 8\frac{m\ell^2}{b^3} + \mathcal{O}(m^3, m^2\ell^2).$$

  - Positive sign of the correction terms of order ℓ^2/b^2, unlike standard Bardeen BHs giving a negative correctoin of order $m\ell^2/b^3$.





Strong-Deflection Limit (SDL)

- The SDL formalism developed by Bozza, the deflection angle for both metrics can be written as

$$\hat{\alpha}(b) = -\bar{a}\log\left(\frac{b}{3\sqrt{3}m} - 1\right) + \bar{b} + + \mathcal{O}(b - b_m),$$

where $\bar{a}$ is solved analytically and $\bar{b}$ is computed numerically

- **Hayward-like:**

$$\bar{a}_H = \sqrt{\frac{2A(\rho_m)B(\rho_m)}{R''(\rho_m)A(\rho_m) - A''(\rho_m)R(\rho_m)}} = \frac{\rho_m}{3\rho_m - 6m},$$

where ρ_m is the largest real root of $2m\ell^2 - 3m\rho_m^2 + \rho_m^3 = 0$

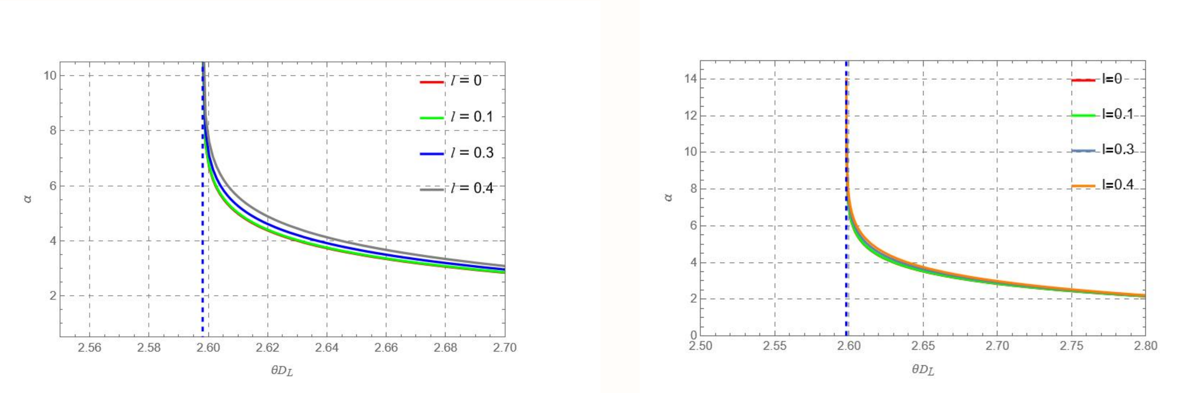
$\ell/2m$	$\bar{a}$	$\bar{b}$	b_m/R_s
0	1.00000	-0.40023	2.5981
0.1	1.00603	-0.410826	2.5981
0.3	1.06296	-0.528106	2.5981
0.4	1.12865	-0.677135	2.5981

- **Bardeen-like:**

$$\bar{a}_B = \sqrt{\frac{2A(\rho_m)B(\rho_m)}{R''(\rho_m)A(\rho_m) - A''(\rho_m)R(\rho_m)}} = \left(\frac{\rho_m}{3\sqrt{\rho_m^2 + l^2} - 6m}\right).$$

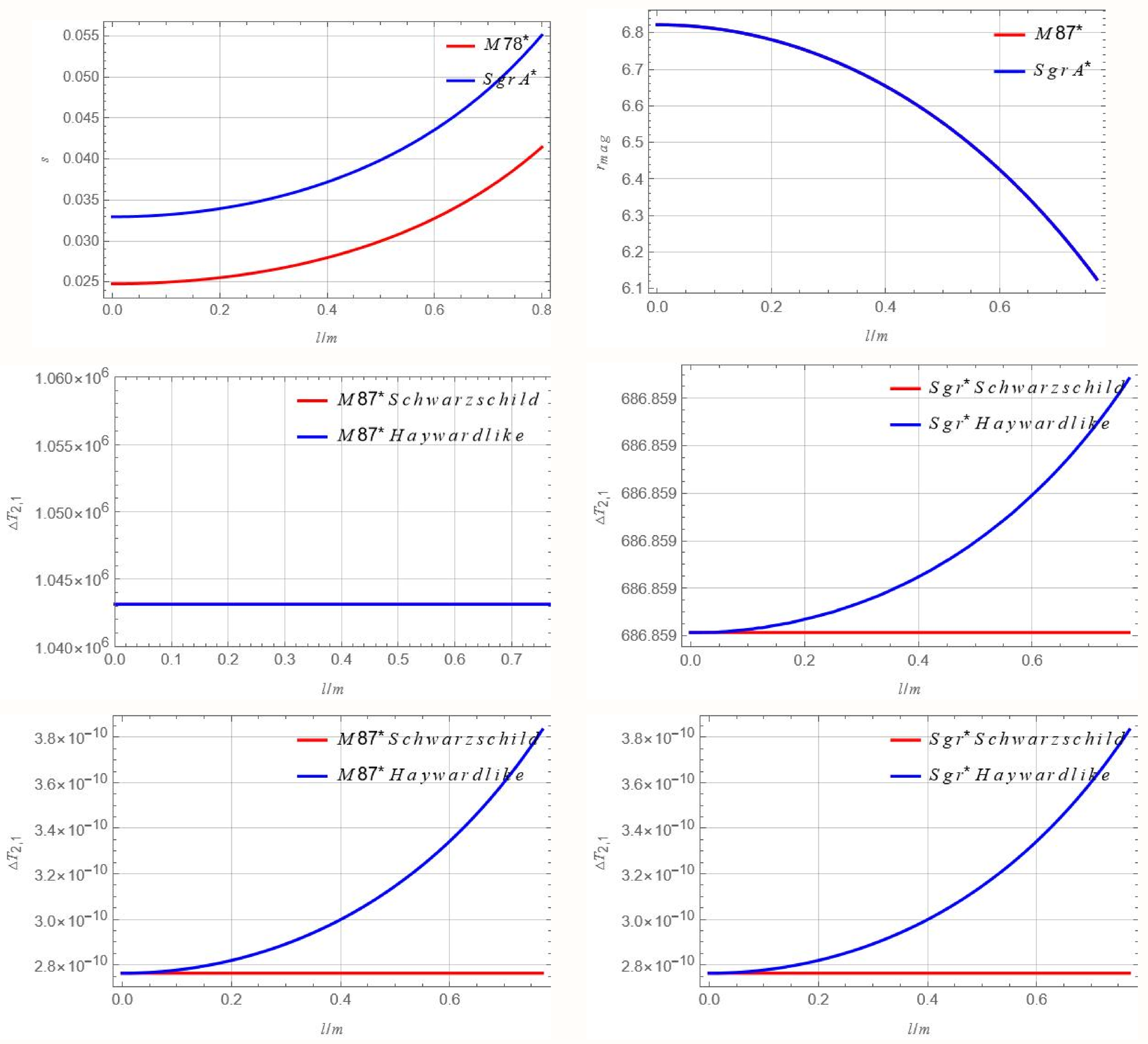
ρ_m is the largest real root of $(\rho_m^2 + \ell^2)^{3/2} - 3m\rho_m^2 = 0$

$\ell/2m$	$\bar{a}$	$\bar{b}$	b_m/R_s
$\ell = 0$	1.00000	-0.40023	2.59808
$\ell = 0.1$	1.00683	-0.407874	2.59808
$\ell = 0.3$	1.07636	-0.532713	2.59808
$\ell = 0.4$	1.17461	-0.72538	2.59808

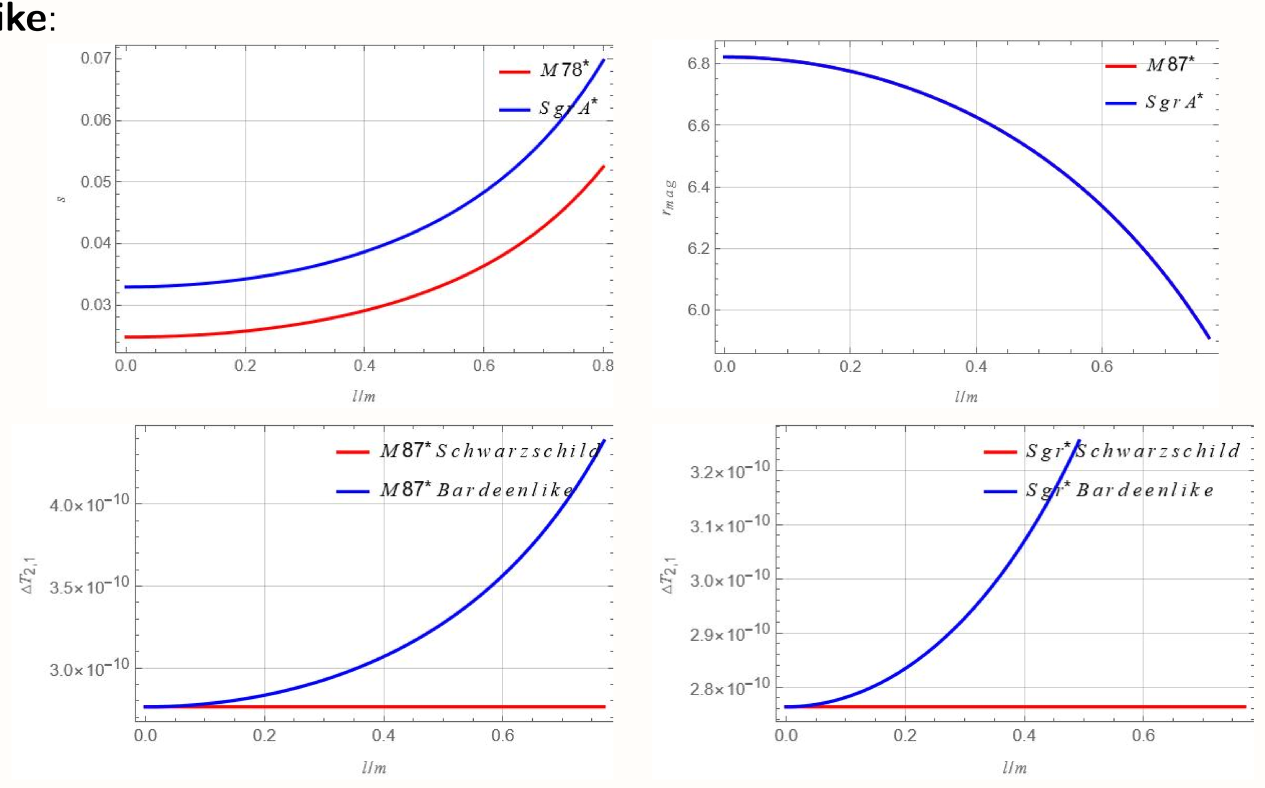


SDL Observables

- **Hayward-like:**



- For M87* $\theta_\infty = 19.80\,\mu$as, s from 25 to 40 nanoarcsecond (nas). For Sgr A* $\theta_\infty = 26.53\,\mu$as, s range from 33 to 53 nanoarcsecond (nas). $r_{\rm mag}$ from 6.82 to 6.13 for both cases. $(\mu_1, \mu_2, \theta_1, \theta_2)$ are calculated.
- **Bardeen-like:**



- For M87* $\theta_\infty = 19.80\,\mu$as, s from 25 to 49 nanoarcsecond (nas). For Sgr A* $\theta_\infty = 26.53\,\mu$as, s range from 33 to 66 nanoarcsecond (nas). $r_{\rm mag}$ from 6.82 to 5.91 for both cases. $(\mu_1, \mu_2, \theta_1, \theta_2)$ are calculated.