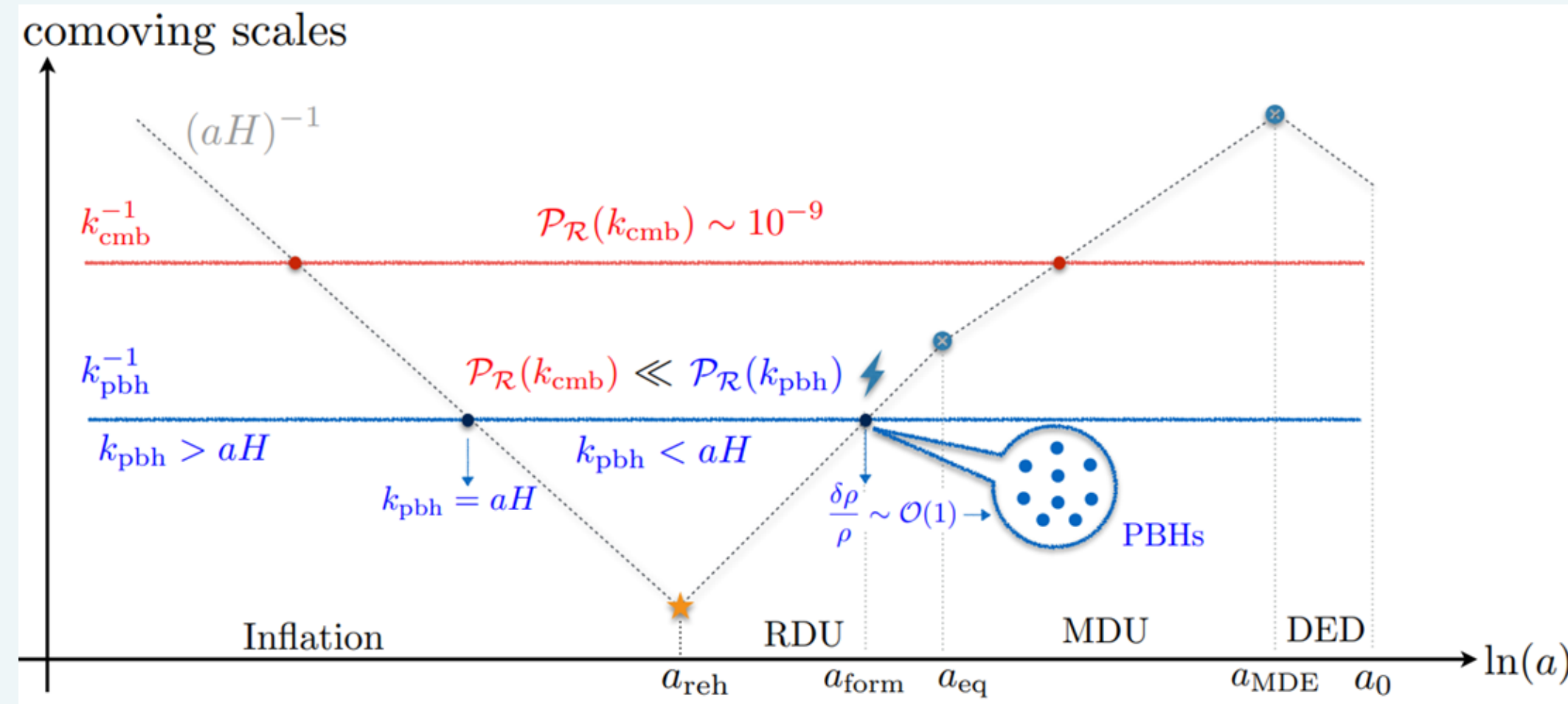


Towards Stochastic Inflation in Higher-Curvature Gravity

Yermek Aldabergenov, Ding Ding, Wei Lin, Yidun Wan
Fudan University; Shanghai Research Center for Quantum Sciences
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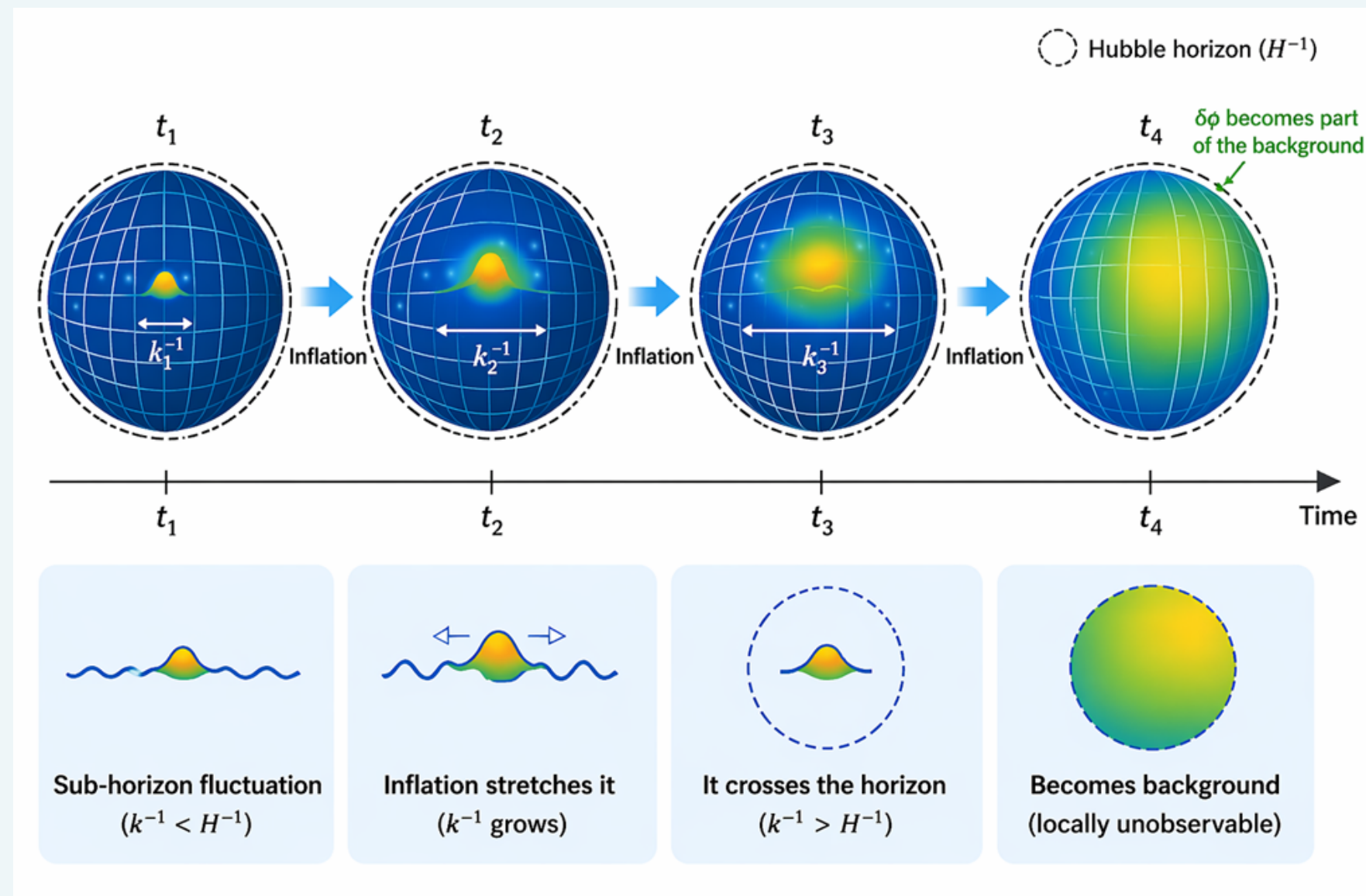
1. Inflation

Inflation is an early accelerated expansion of the universe. During inflation, the comoving Hubble radius $(aH)^{-1}$ shrinks, leading to quantum fluctuations exit the horizon and become super-Hubble perturbations. After inflation, these modes re-enter the horizon. If the perturbation on a small scale is large enough, re-entry can trigger gravitational collapse and form PBHs.



2. Stochastic inflation: coarse-graining horizon-crossing modes

The figure illustrates the basic stochastic-inflation idea: inflation stretches short-wavelength quantum fluctuation modes and eventually, they cross the Hubble scale, joining the locally coarse-grained background. Since newly crossing quantum modes carry random quantum phases and amplitudes, their continual inflow makes the coarse-grained background stochastic rather than purely deterministic.



Mathematically, this is implemented by splitting the scalar field into long- and short-wavelength sectors in a particular Hubble patch:

$$\phi(t, \mathbf{x}) = \phi_c(t) + \phi_q(t, \mathbf{x}), \quad k_\Sigma = \frac{aH}{C_\Sigma}, \quad \Sigma \ll 1.$$

Physical meaning: ϕ_c is the coarse-grained background seen by a local Hubble patch, while ϕ_q contains short modes. The cutoff k_Σ defines the moving boundary between the UV and IR sectors; the small parameter $\Sigma \ll 1$ selects modes that are already well outside the sound horizon.

3. Why Gauss-Bonnet?

Generic curvature-squared corrections besides the Einstein-Hilbert action are natural in early-universe effective gravity, but they usually bring higher-derivative ghost degrees of freedom. Nevertheless, the Gauss-Bonnet combination

$$R_{\text{GB}}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

is special. In four dimensions, R_{GB}^2 alone is topological. With a field-dependent coupling $\xi(\phi)$, it becomes dynamically relevant and modifies the inflaton equations with preserving the ghost-free structure. Moreover, GB terms are well-motivated by Lovelock gravity, effective field theory, and string effective actions. Therefore, we start from the scalar-GB coupling model

$$\sqrt{-g}^{-1} \mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{8}\xi(\phi)R_{\text{GB}}^2 - V(\phi).$$

This also makes a useful setting for testing how stochastic inflation changes beyond standard Einstein gravity.

4. Classical background and stochastic Langevin equation

The homogeneous background field obeys

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi + 3\xi_\phi H^2 (H^2 + \dot{H}) = 0.$$

Physical meaning: The red term is the explicit GB effect. It can partially cancel or enhance the bare-potential slope, changing where slow roll and ultra-slow roll occur.

The stochastic equations for the coarse-grained variables (ϕ_c and its conjugate momentum Π_c) are

$$\begin{aligned} \dot{\phi}_{c,N} &= \Pi_c + \Xi_\phi, \\ \dot{\Pi}_{c,N} &= -(3 - \epsilon_\Pi)\Pi_c - H^{-2}V_\phi - 3\xi_\phi H^2(1 - \epsilon_\Pi) \\ &\quad + X_1\Xi_\Pi - \frac{X_1\mathcal{E}_{,N}\omega}{4\mathcal{E}(1-\omega)}\Xi_\phi. \end{aligned}$$

Color code: black = deterministic classical dynamics, red = explicit GB correction, blue = stochastic noise. The variables Ξ_ϕ and Ξ_Π are random variables rather than deterministic ones:

$$\langle \Xi_I(N)\Xi_J(N') \rangle = A_{IJ}\delta(N - N'), \quad I, J \in \{\phi, \Pi\}.$$

Physical meaning: This Langevin system summarizes the local inflationary dynamics of a Hubble patch. The black terms describe classical dynamics in GR, the red term shows how GB coupling deforms that dynamics, and the blue terms encode the random kicks generated by horizon-crossing quantum fluctuations.

5. Effective Potential and the USR condition

In the slow-roll/quasi-de Sitter limit, the GB effect can be absorbed in an effective potential, $V_\phi^{\text{eff}} \equiv V_\phi + 3\xi_\phi H^4$. USR is a stage with $\mathcal{E}_{,N}/\mathcal{E} \simeq -6$, which can amplify small-scale curvature perturbations relevant for PBH production. In GR, USR usually requires $V_\phi \simeq 0$. In contrast, the condition becomes $V_\phi^{\text{eff}} \simeq 0$ with GB coupling, which leads to V_ϕ itself need not vanish: $V_\phi \simeq -3\xi_\phi H^4$.

6. Slow-roll power spectrum

For the standard SR regime, the power spectrum can be written as a classical part multiplied by a stochastic correction,

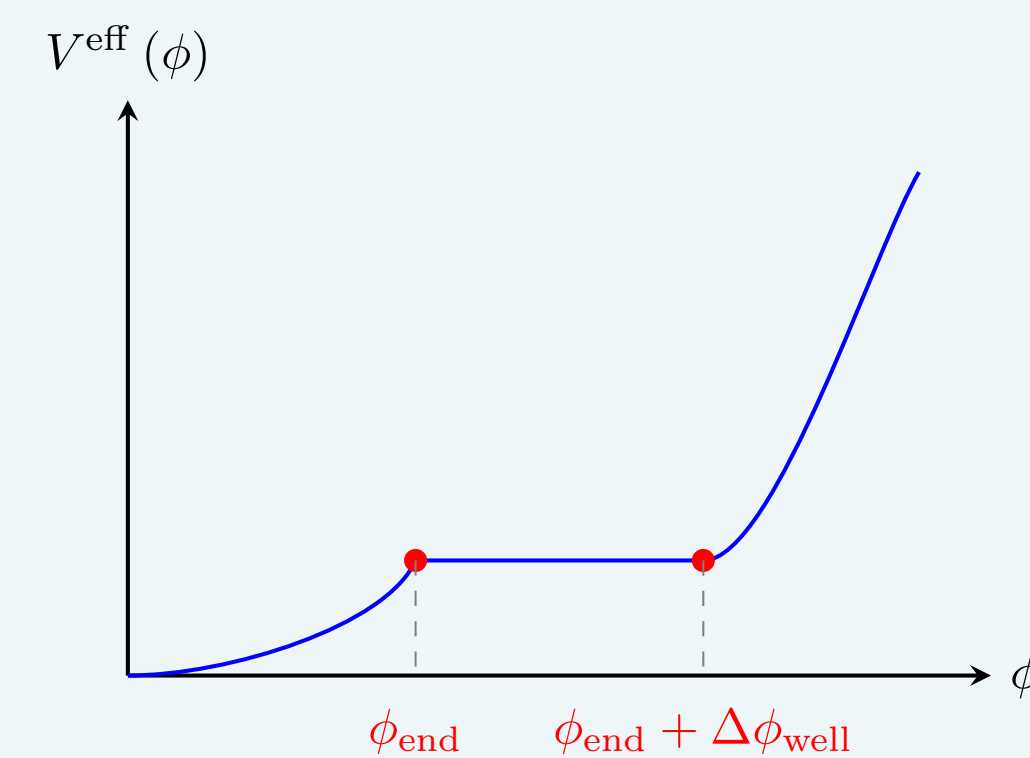
$$\begin{aligned} \mathcal{P}_\zeta(\phi_*) &= \underbrace{\frac{18v_*^3}{(3v_*' + \xi_*'v_*'^2)} \left[1 + \delta\mathcal{P}_\zeta^{\text{noise}}(\phi_*) \right]}_{\text{classical SR part}} \\ \delta\mathcal{P}_\zeta^{\text{noise}} &= -\frac{3v \left(-15v^2 + 3v^2v'\xi' + 12vv'' + 4v^3\xi'' \right)}{(3v' + v^2\xi')^2}, \end{aligned}$$

in the classical limit. Here, $v = V/(24\pi^2)$, $\xi = 24\pi^2\xi$, and starred quantities are evaluated at horizon exit ϕ_* .

Physical meaning: The first factor is the SR power spectrum set by the deterministic dynamics, already modified by GB coupling. The blue piece is the leading correction induced by stochastic noise. It quantifies how random kicks perturb the standard SR result.

7. USR PBH mass fraction

Local USR setup: For the PBH calculation, we approximate the relevant USR stage by a local $V_\phi^{\text{eff}} \simeq 0$ well with width $\Delta\phi_{\text{well}}$. We assume PBH-relevant modes exit during this USR stage, while stochastic noise outside this stage is negligible. We use the normalized variables $x = (\phi - \phi_{\text{end}})/\Delta\phi_{\text{well}} \in [0, 1]$ and $y = -\Pi_c/(3\Delta\phi_{\text{well}})$. It can be identified that x labels the field position inside the USR well and y is the normalized momentum. The results below focus on the stochastic-noise-dominated, small-velocity regime $y_{\text{in}} \ll 1$.



Variables and horizon-exit initial data. The endpoint $x = 0$ is the end of USR, while $x = 1$ is its beginning. For a given perturbation mode, x_{in} and y_{in} are the field and momentum values when that mode is coarse-grained, equivalently when it exits the relevant coarse-graining/horizon scale. The local potential is encoded by ν_0 . The small slope ν'_0 parameterizes the residual bare-potential slope; Under $V_\phi^{\text{eff}} \simeq 0$, the slope is tied to the GB contribution $3\xi_\phi H^4$ and therefore carries information about $\xi(\phi)$.

PBH mass fraction: For a mode corresponding to PBH mass M , $\beta_f(M)$ is the fraction of the total energy density collapsing into PBHs at formation, $\beta_f(M) \equiv \rho_{\text{PBH}}(M, t_f)/\rho_{\text{tot}}(t_f)$. In the stochastic calculation, this fraction is estimated from the probability that the coarse-grained curvature perturbation exceeds the collapse threshold:

$$\beta_f(M) = \int_{\zeta_c}^{\infty} P(\zeta_{\text{CG}}) d\zeta_{\text{CG}}.$$

Thus, the relevant information is the far tail of $P(\zeta_{\text{CG}})$, not only its variance or peak value.

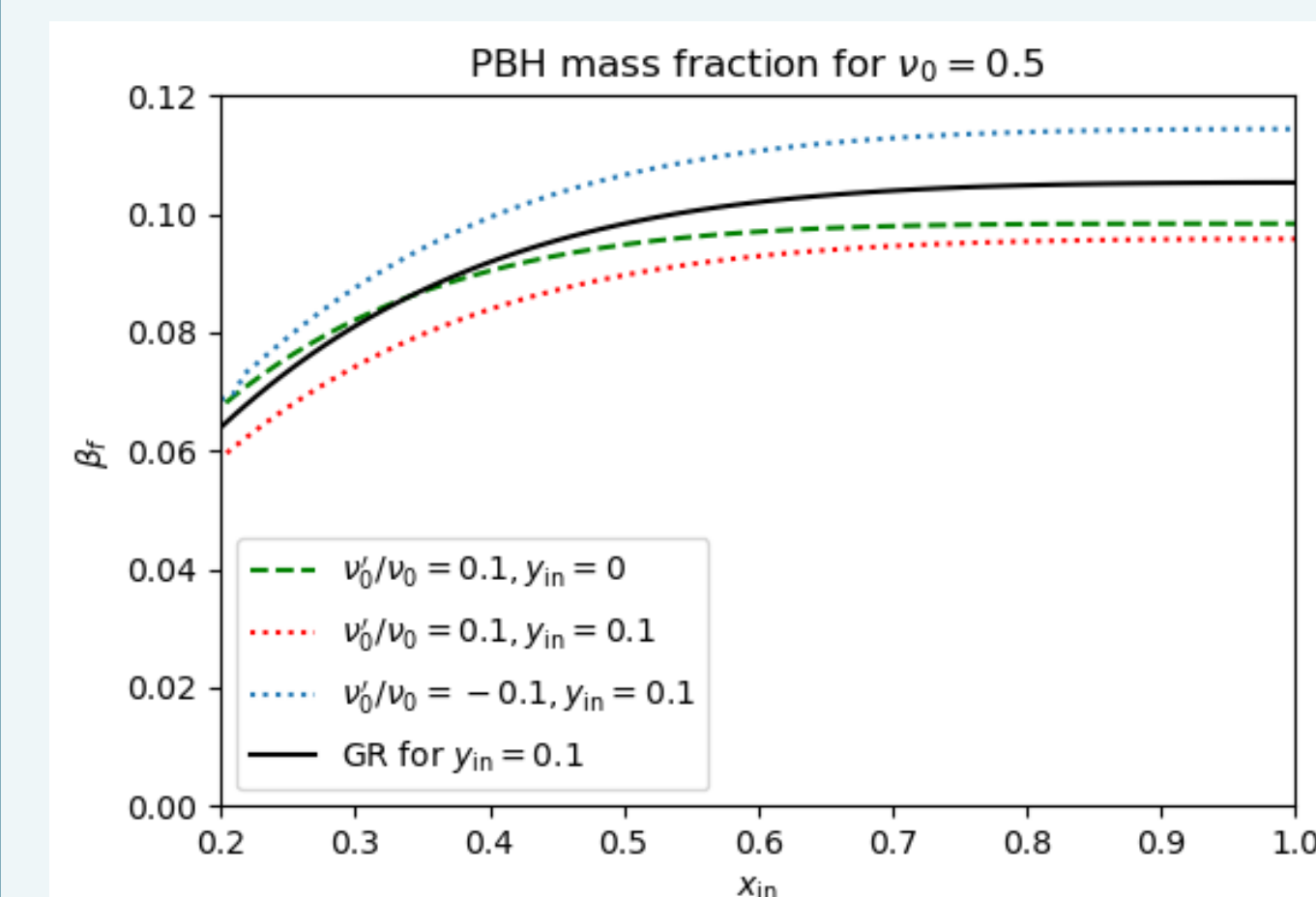


Fig. 6: effect of y_{in} and ν'_0 . A nonzero initial velocity changes β_f , and the sign of ν'_0 shifts the curve relative to GR. The change is visible but not a dramatic order-of-magnitude effect in this parameter range.

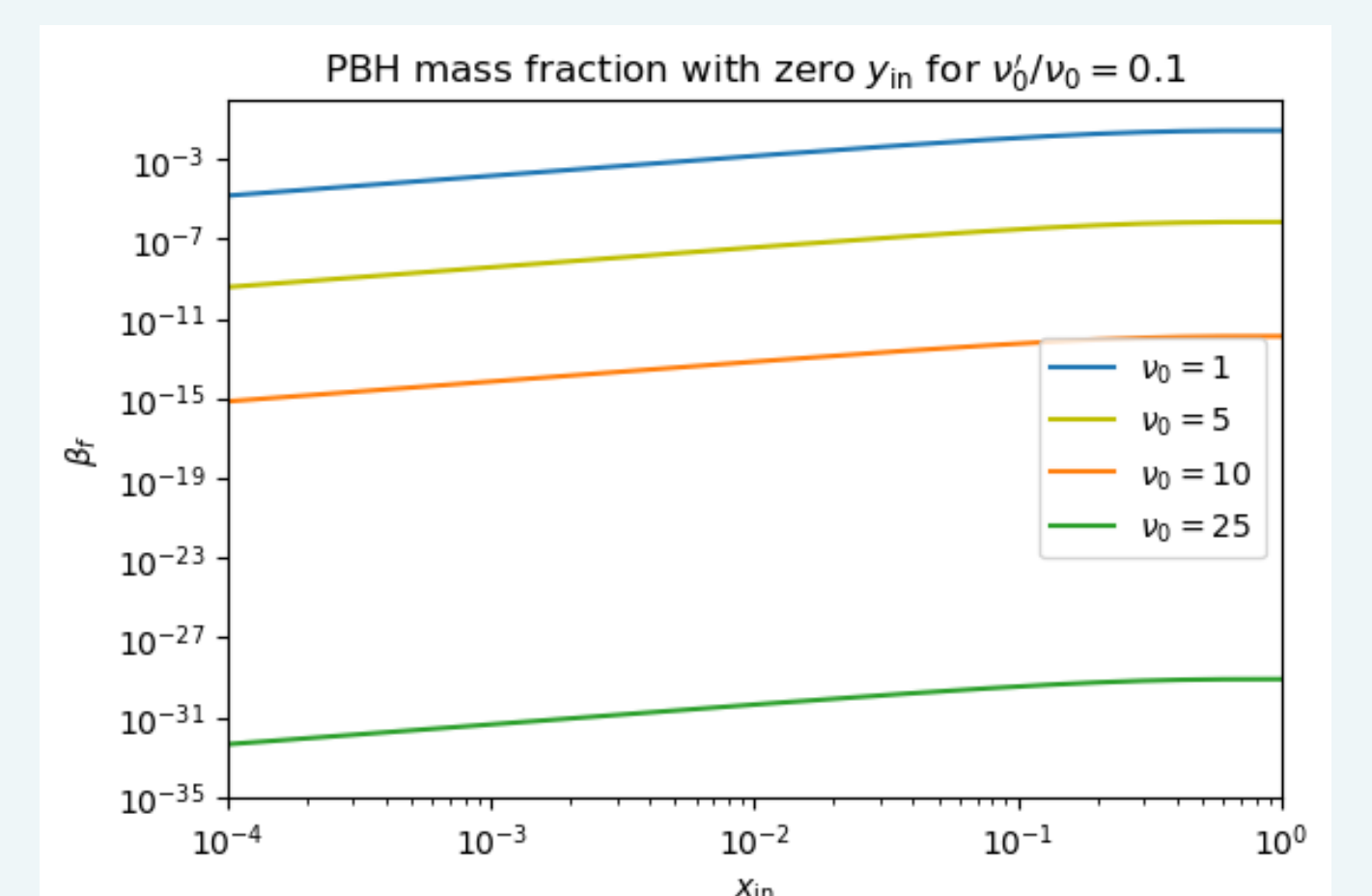


Fig. 7: weak dependence on x_{in} . Changing the initial field position x_{in} affects the curve shape, but the order of magnitude of β_f is mainly controlled by ν_0 . Larger ν_0 gives a smaller PBH fraction.

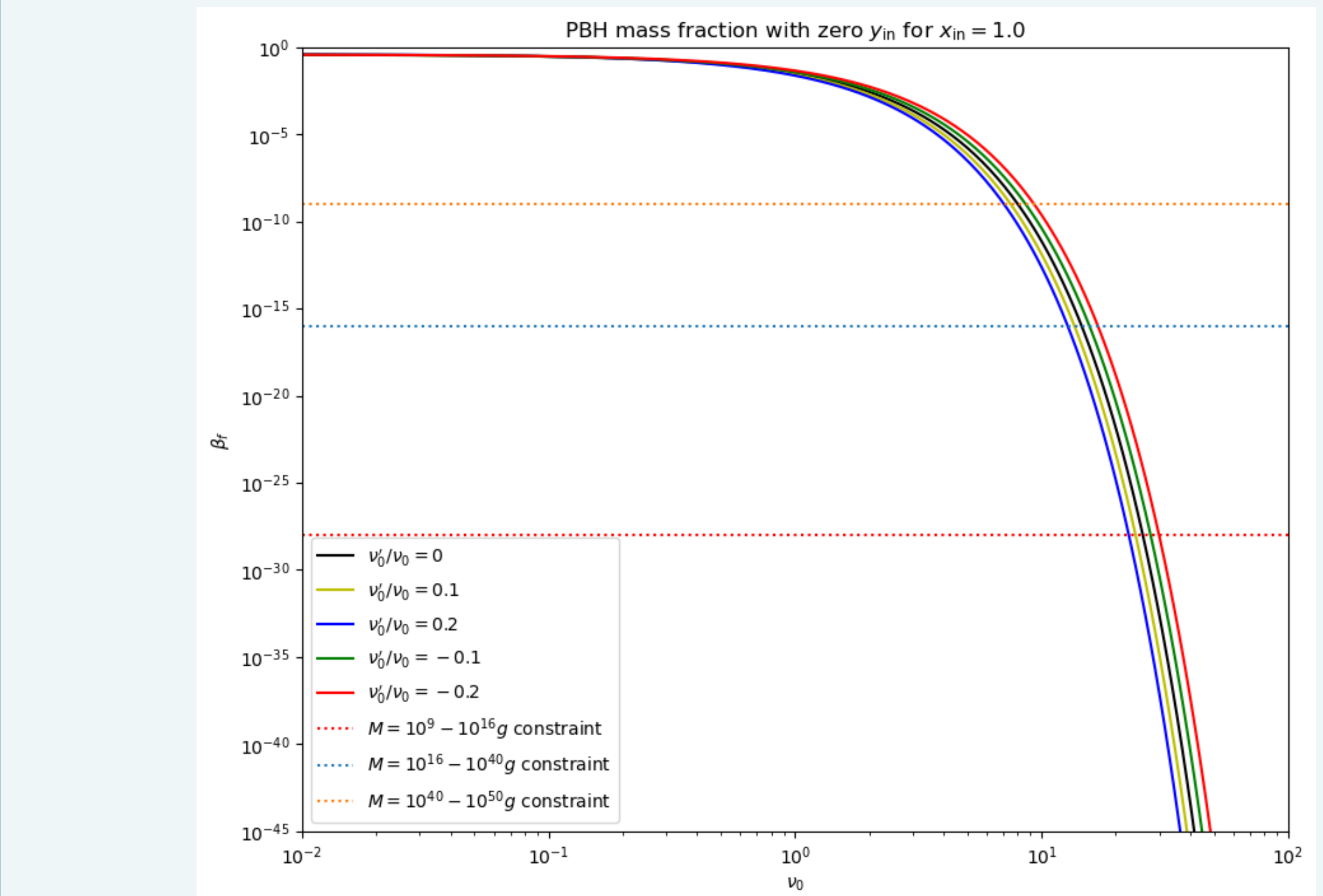


Fig. 8: parameter space and observational constraints. The horizontal dotted lines are observational upper bounds on β_f for different PBH mass windows. Since larger ν_0 suppresses β_f , these bounds translate into lower bounds on ν_0 . A positive ν'_0 suppresses PBH formation and allows smaller viable ν_0 than in GR, while a negative ν'_0 enhances PBH formation and pushes the viable region toward larger ν_0 .

Main message from the PBH analysis. The initial field position x_{in} and small initial velocity y_{in} affect β_f , but the dominant order-of-magnitude control comes from ν_0 and the GB-related slope ν'_0 . In the stochastic-noise-dominated USR regime, satisfying PBH constraints typically requires $\nu_0 \sim O(10)$ or larger, with the precise allowed range shifted by the sign of ν'_0 .

Take-home message. This work develops a stochastic-inflation framework for scalar-Gauss-Bonnet coupling higher-curvature gravity. The stochasticity comes from random quantum modes crossing the coarse-graining scale; The Gauss-Bonnet coupling changes how the coarse-grained background responds to these random kicks.

- Formalism:** Working in spatially flat gauge, the UV/IR split leads to stochastic Klein-Gordon and Langevin equations for the coarse-grained inflaton (ϕ_c, Π_c) . The noise variables Ξ_ϕ, Ξ_Π are random kicks, while the GB coupling modifies the drift and the noise coefficients.
- Effective potential:** In the slow-roll/quasi-de Sitter limit, the effect of GB coupling correction is encoded in $V_\phi^{\text{eff}} = V_\phi + 3\xi_\phi H^4$. Therefore, an USR stage requires $V_\phi^{\text{eff}} \simeq 0$ instead of necessarily $V_\phi \simeq 0$. A nonzero bare slope in the USR region can carry information about $\xi(\phi)$.
- Power spectrum:** In the standard SR regime in classical limit, the stochastic induced scalar power spectrum is the GB-modified classical slow-roll result multiplied by a leading stochastic correction. This shows explicitly how quantum noise corrects the usual deterministic prediction.
- PBH application:** For PBH production, we use a local, stochastic-noise-dominated USR setup rather than a full model scan. The horizon-exit initial data $x_{\text{in}}, y_{\text{in}}$ affect the PBH fraction, but the dominant order-of-magnitude control comes from the local height ν_0 and the GB-related residual slope ν'_0 .
- Observational Constraint shift:** Larger ν_0 suppresses β_f . Positive ν'_0 suppresses PBH production and relaxes the required ν_0 , while negative ν'_0 enhances PBH production and pushes the viable region toward larger ν_0 .