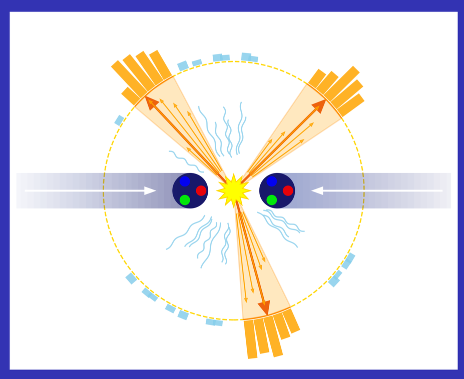

q_T-slicing with multiple jets

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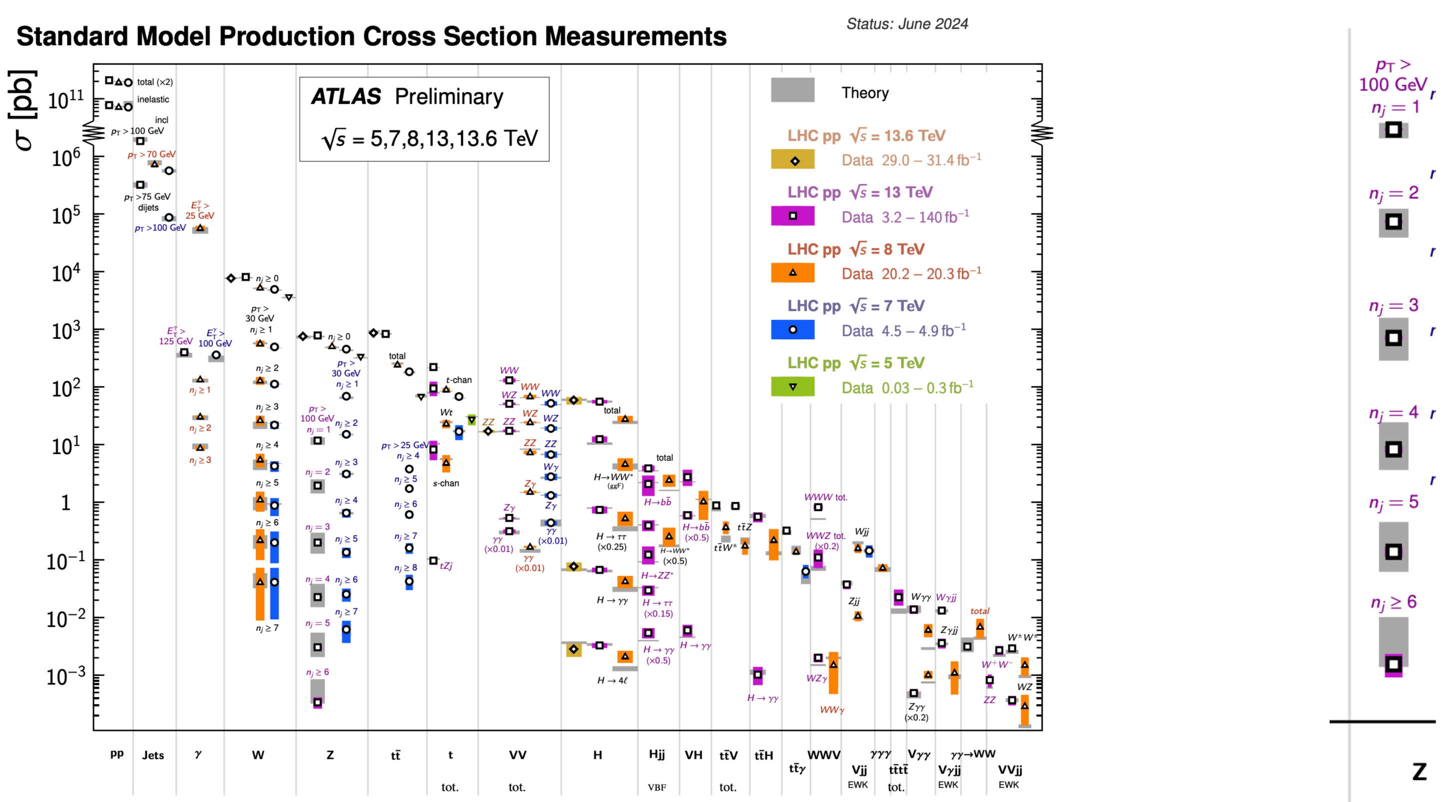
Ref: Phys. Rev. Lett. 135, 171903 (2025)

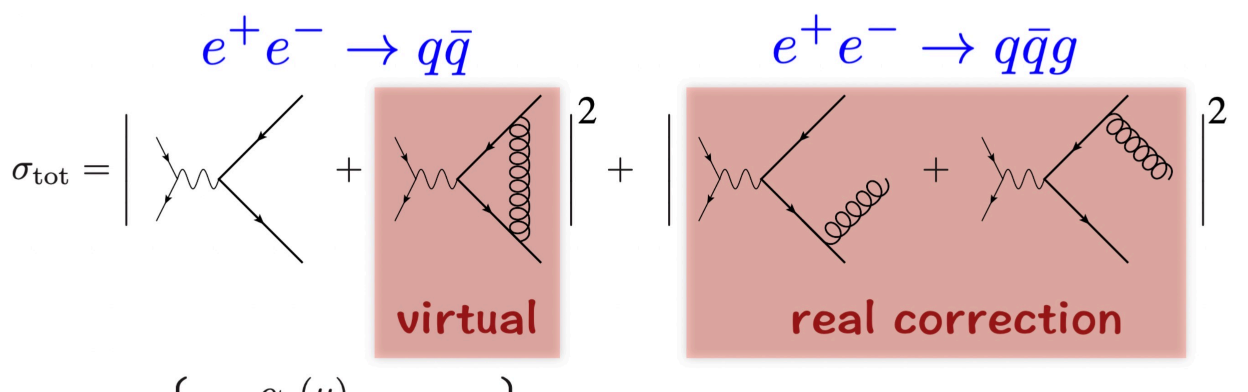
0: Motivation: SM Plots [ATLAS Collaboration '24] & Factorization-violation



Factorization-violation: All proton collisions include forward proton remnants (spectators). Absence of factorization-violation due to Glauber gluons is important for Drell-Yan factorization. [Bodwin '85; Collins, Soper, Sterman '85]

e.g. Transverse momentum dependent (TMD) factorization is violated in dijet production [Collins, Qiu '07 ...] due to exchange of two extra gluons giving non-factorization in unpolarized cross sections.

1. Introduction: Infrared Cancellation



$$= \sigma_0 \left\{1 + \frac{\alpha_s(\mu)}{\pi} + \mathcal{O}(\alpha_s^2)\right\}$$

Real and virtual corrections suffer from soft and collinear infrared divergences, e.g., in $d = 4 - 2\epsilon$:

$$\sigma_{\rm real} = \sigma_0 \frac{\alpha_s(\mu)}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{57}{6} - \frac{7\pi^2}{6}\right)$$

$$\sigma_{\rm virtual} = \sigma_0 \frac{\alpha_s(\mu)}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{7\pi^2}{6}\right)$$

Divergences cancel in the sum! [Kinoshita 1962; Lee & Nauenberg 1964]

2. The Slicing Approach

Slicing capitalizes on large cancellations between singular and regular terms:

$$\frac{d\sigma^{(m)}_{\rm N^kLO}}{dX} = \int_0^\delta dq_T \frac{d\sigma^{(m)}_{\rm N^kLO}}{dX\,dq_T} + \int_\delta^{q_T^{\max}} dq_T \frac{d\sigma^{(m+1)}_{\rm N^{k-1}LO}}{dX\,dq_T}$$

$$\equiv \int_0^\delta dq_T \frac{d\sigma_{\rm SCET}}{dq_T}[1 + \mathcal{O}(\delta^p)] + \int_\delta^{q_T^{\max}} dq_T \frac{d\sigma_{\rm QCD}}{dq_T}$$

Modern slicing variables:

1. Transverse momentum (q_T) of a colorless/massive system. (Unsuitable for jets because $q_T = 0$ for in-cone radiation). [Catani, Grazzini '07]
2. Jettiness (τ_N). [Boughezal, Focke, Liu, Petriello '15 ...]
3. For most color-singlet processes, q_T performs better, motivating an extension of q_T to processes involving jets. [Campbell, Ellis, Seth '22]

3. q_T-Slicing for Jets: Winner-Take-All Scheme

We propose two generalizations of q_T that can be used for jet processes [RJF, Rahn, Shao, Waalewijn, Wu '24]:

The key ingredient is the use of a recoil-free jet axis!

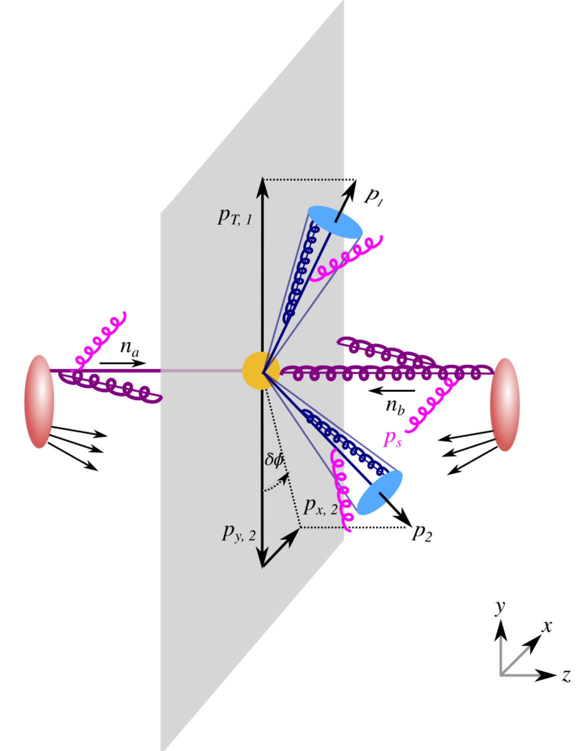
$$p_{T,r} = p_{T,i} + p_{T,j}, \quad \hat{n}_r = \begin{cases} \hat{n}_i, & p_{T,i} \geq p_{T,j} \\ \hat{n}_j, & p_{T,i} < p_{T,j} \end{cases}$$

Soft radiation inside the jet influences the jet axis through momentum conservation, similar to radiation outside the jet, leading to a non-zero q_T.

- **Azimuthal decorrelation** $\delta\phi = q_x/p_{T,1}$: $q_x = p_{x,1}^{\rm WTA} + p_{x,2}^{\rm WTA}$
- **Magnitude of total transverse momentum** $|\vec{q}_T|$: $\vec{q}_T = \sum_i \vec{p}_{T,i}^{\rm WTA}$

4. Azimuthal Decorrelation (q_x)

Suitable for planar Born processes ($pp \to V$+jet, $pp \to 2$jets, $e^+e^- \to 3$jets).



By using the WTA scheme, the transverse momentum component q_x (or $\delta\phi$), which is perpendicular to the scattering plane, is a suitable slicing variable.

Ingredients: hard scattering, collinear initial- and final-state radiation, and soft radiation.

Factorization Formula ($pp \to 2$jets):

$$\frac{d\sigma_{\rm SCET}}{dp_{T,1}\,d\eta_1\,d\eta_2\,dq_x} = \int \frac{db_x}{2\pi} e^{iq_xb_x} \sum_{i,j,k,\ell} B_i(x_a, b_x)\, B_j(x_b, b_x) \mathcal{J}_k(b_x) \mathcal{J}_\ell(b_x)$$
$$\times\, {\rm tr}\left[\hat{\mathcal{H}}_{ij\to k\ell}(p_{T,1}, \eta_1 - \eta_2)\, \hat{S}_{ijk\ell}(b_x, \eta_1, \eta_2)\right]$$

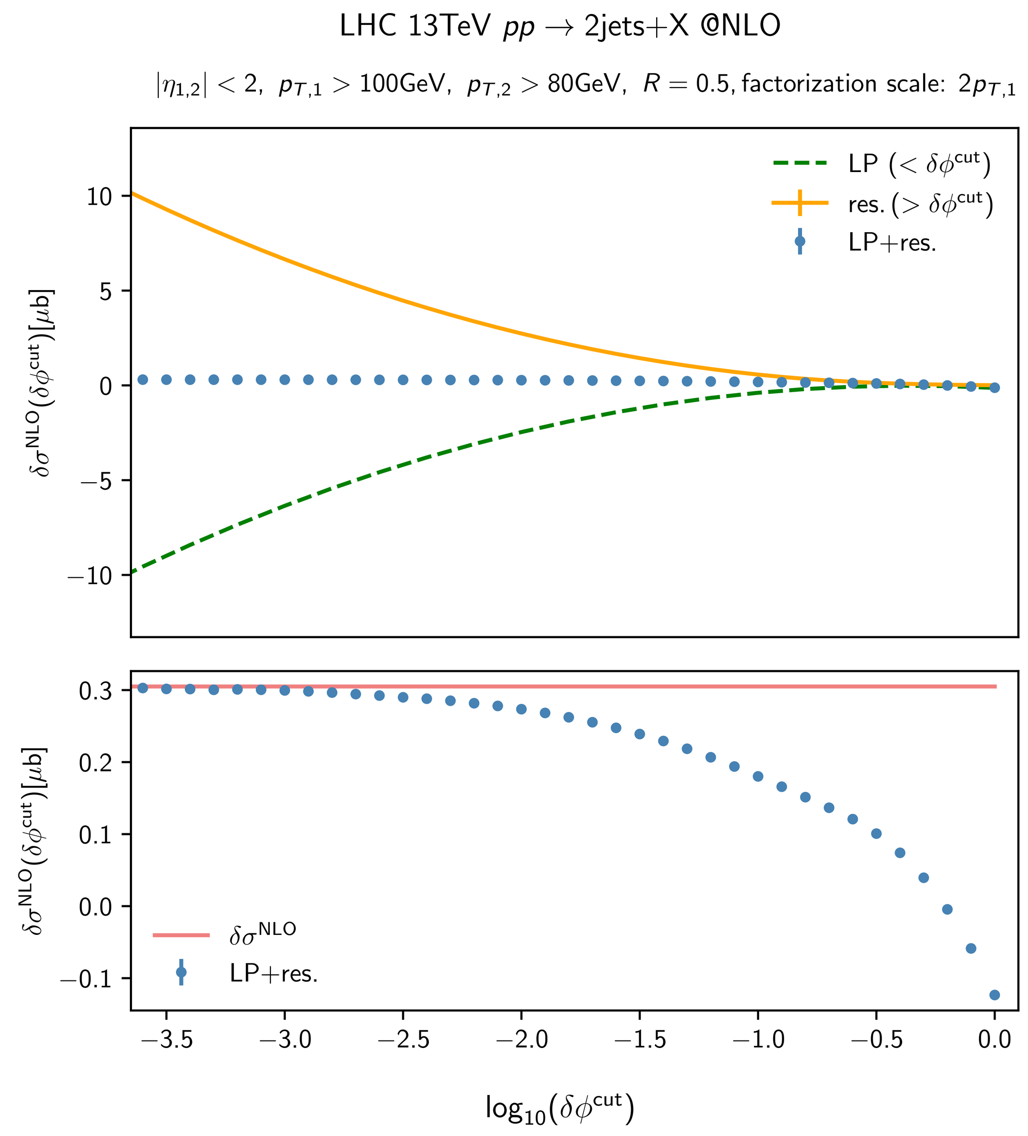


Figure: Slicing capitalizing on large cancellations, achieving convergence at NLO.

5. Transverse Momentum Imbalance ($|\vec{q}_T|$)

q_T-slicing can be applied to non-planar kinematics ($pp \to Z + 2$jets, $pp \to 3$jets). Only the soft function changes, refactoring into global and collinear-soft parts:

$$\hat{S}_{ijk\ell}(\vec{b}_T, \eta_1, \eta_2, R) = \hat{S}^{\rm global}_{ijk\ell}(\vec{b}_T, \eta_1, \eta_2, R) S^{cs}_k(\vec{b}_T, \eta_1, R) S^{cs}_\ell(\vec{b}_T, \eta_2, R) + \mathcal{O}(R^{2n})$$

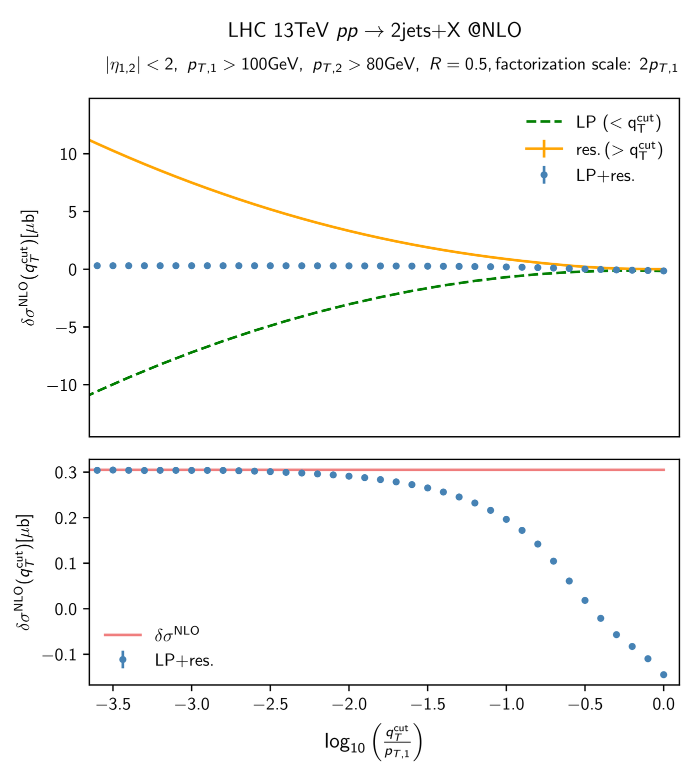


Figure: We include power corrections up to $\mathcal{O}(R^1)$, achieving a 1% precision for the cross section at $R = 0.5$.

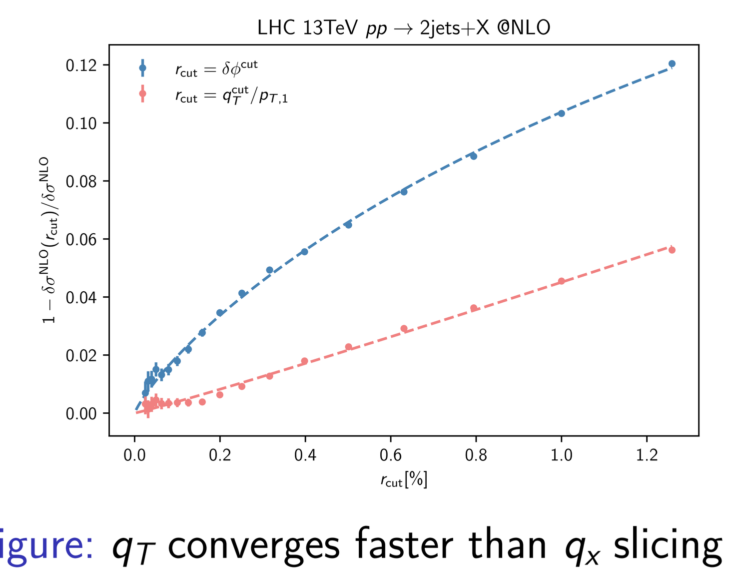


Figure: q_T converges faster than q_x slicing at the expense of a more complicated soft function.

6. Summary

- **Challenge:** IR divergences are complex for processes involving multi-jet final states.
- **Innovation:** Two novel extensions of slicing variables (q_x and q_T) are proposed, tailored for jet processes using the Winner-Take-All scheme.
- **Performance:** q_x offers soft function simplification (planar only), while q_T converges faster and handles non-planar kinematics. Both slicing approaches are successfully demonstrated at NLO.
- **NNLO Progress:** The NNLO collinear-soft function has been calculated, paving the way for multi-jet precision at NNLO.