

Introduction

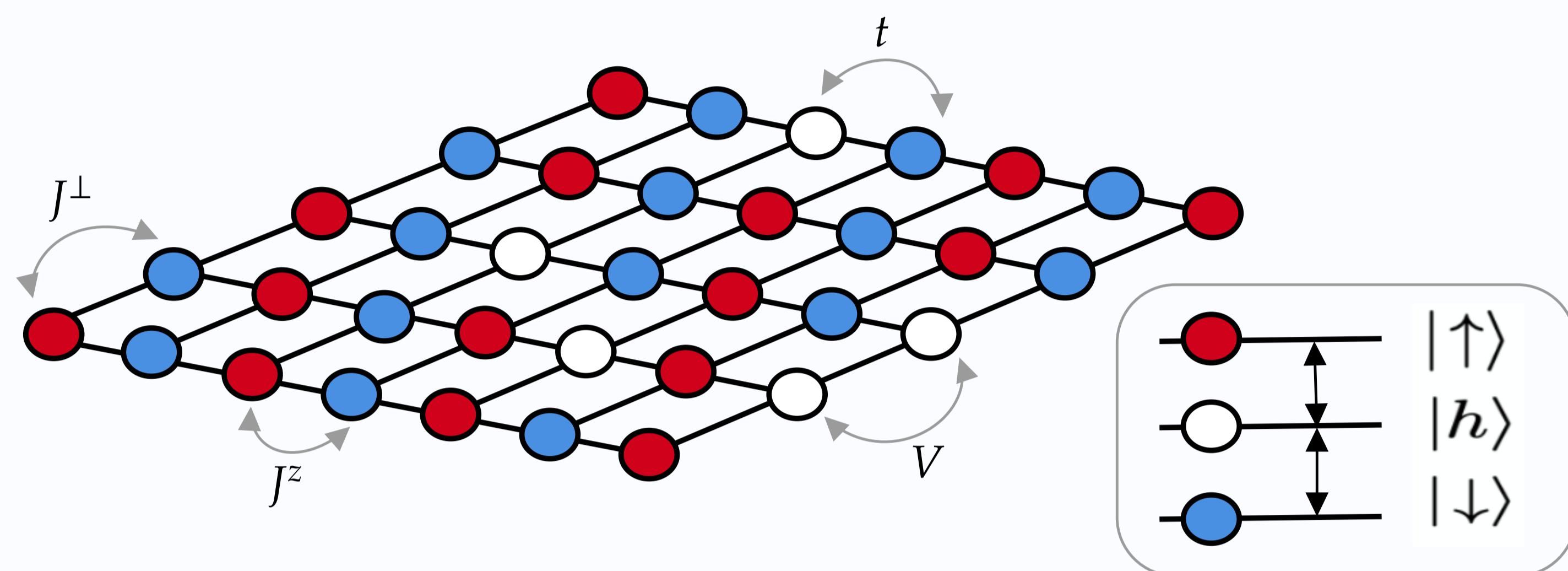
Quantum simulation of doped magnets is a central route toward understanding strongly correlated matter and high- T_c superconductivity. Rydberg tweezer arrays now make this problem experimentally programmable by combining spin exchange, mobile holes, and tunable long-range interactions. Here we study a bosonic t - J - V model recently realized with Rydberg atoms. Large-scale quantum Monte Carlo simulations reveal a rich phase diagram containing an antiferromagnetic (AFM) insulator, a double superfluid (DSF), and an emergent double supersolid (DSS), where crystalline order coexists with two broken $U(1)$ symmetries associated with hole number and total magnetization.

The Bosonic t - J - V Model

The bosonic t - J - V model describes Rydberg atoms on a square tweezer array:

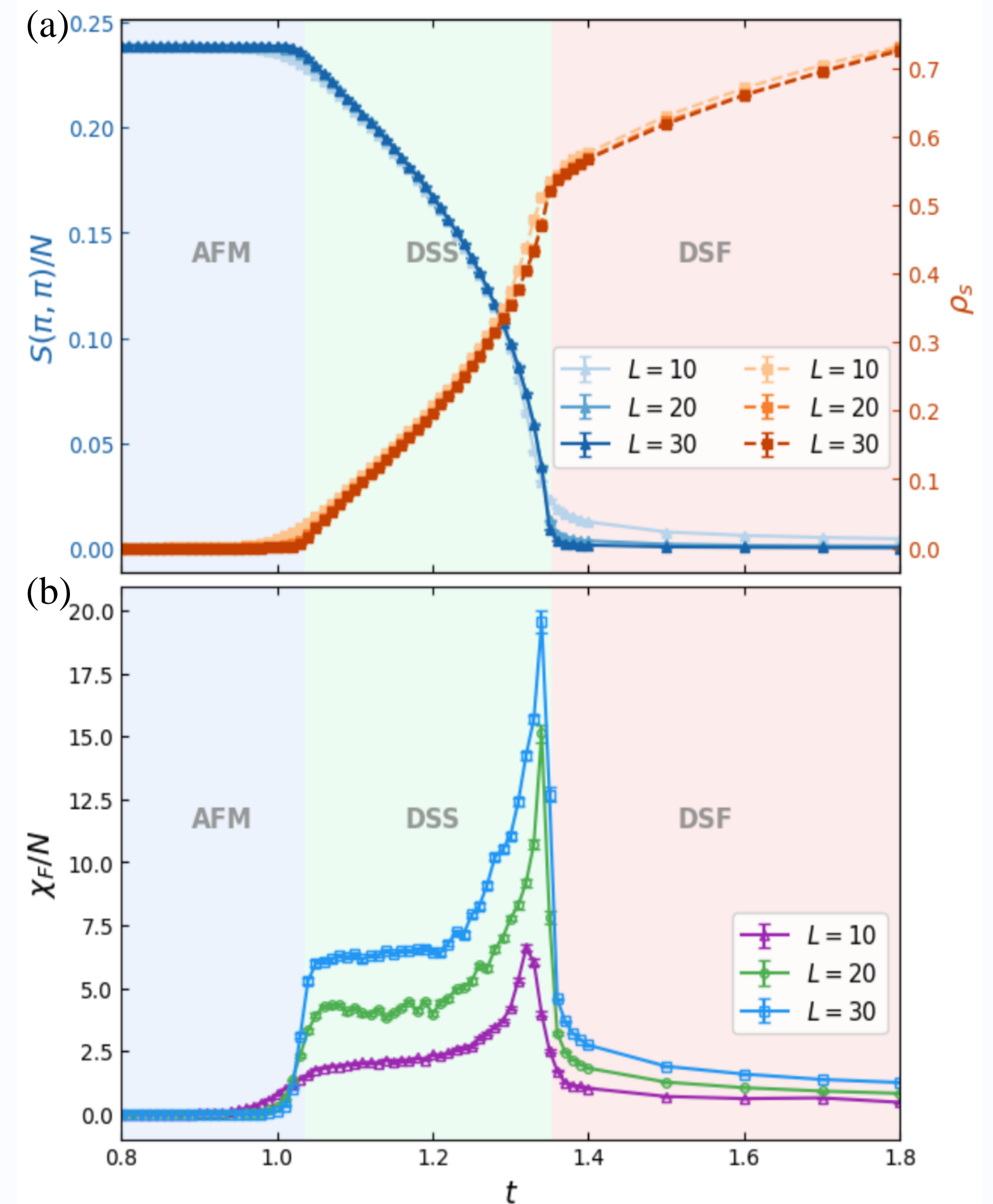
$$\begin{aligned} \hat{H}_{tJV} &= \hat{H}_t + \hat{H}_J + \hat{H}_V, \\ \hat{H}_t &= -\sum_{i,j} \sum_{\sigma=\uparrow,\downarrow} \frac{t_\sigma}{r_{ij}^\alpha} (\hat{a}_{i,\sigma}^\dagger \hat{a}_{j,\sigma}^\dagger \hat{a}_{i,\sigma} \hat{a}_{j,\sigma} + \text{h.c.}), \\ \hat{H}_J &= \sum_{i,j} \frac{1}{r_{ij}^\beta} [J^z \hat{S}_i^z \hat{S}_j^z + \frac{J^\perp}{2} (\hat{S}_i^+ \hat{S}_j^- + \text{h.c.})], \\ \hat{H}_V &= \sum_{i,j} \frac{V}{r_{ij}^\gamma} \hat{n}_i^h \hat{n}_j^h - \mu \sum_j \hat{n}_j^h. \end{aligned} \quad (1)$$

The spin-1/2 and hole states are encoded in three Rydberg levels. Dipolar exchange generates spin-dependent hole tunneling t_σ , while van der Waals interactions produce the diagonal J^z , V and off-diagonal J^\perp couplings. The Schwinger boson operators $\hat{a}_{j,h}^\dagger$ and $\hat{a}_{j,\sigma}^\dagger$ create a hole and a spin σ at site j .



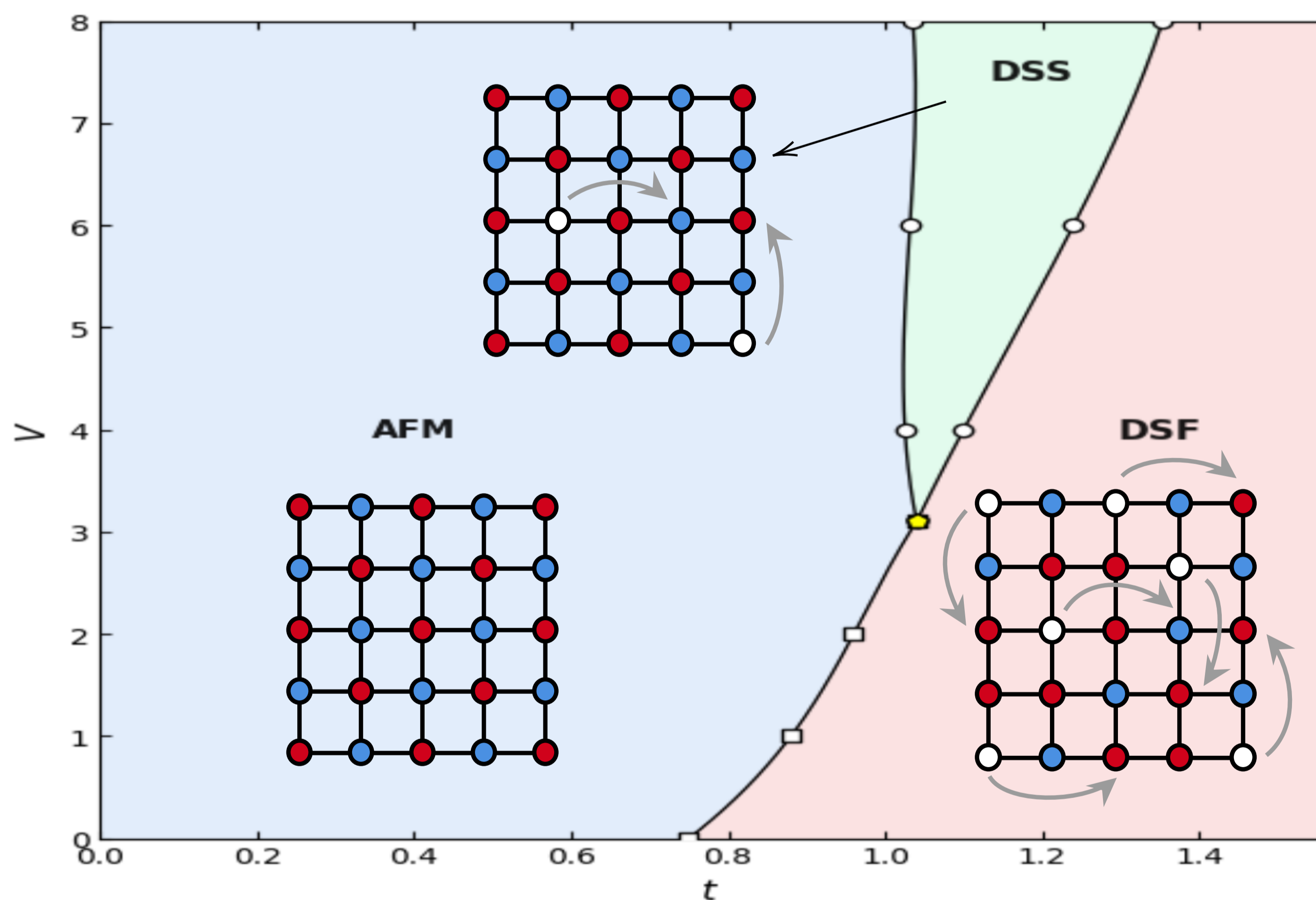
Emergent DSS Phase

- At strong repulsion ($V = 8.0$), the spin structure factor $S(\pi, \pi)/N$ and hole superfluid density ρ_s are simultaneously finite over an intermediate range of t .
- The fidelity susceptibility χ_F exhibits sharp changes at the AFM-DSS and DSS-DSF transitions, in agreement with the order-parameter analysis.



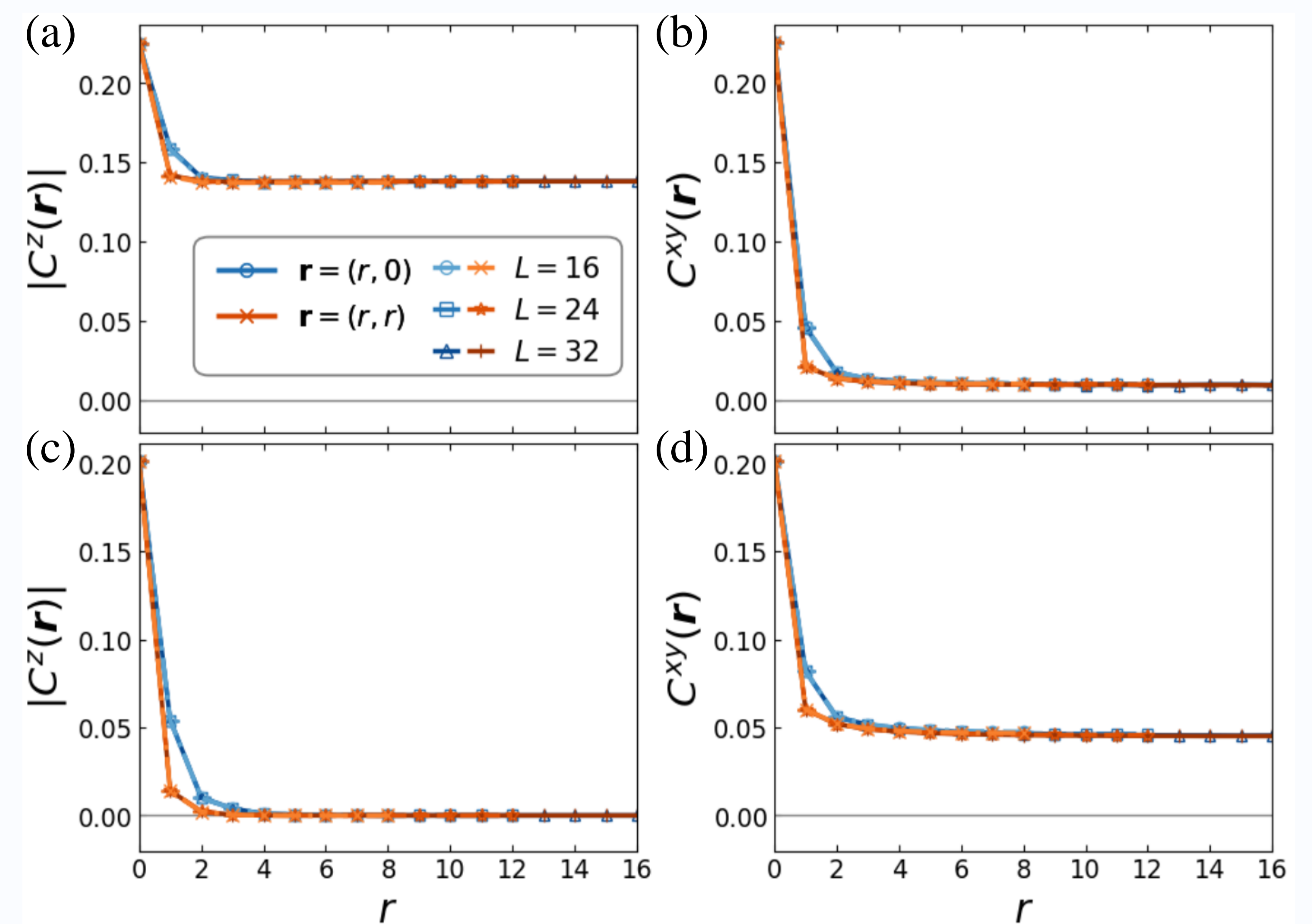
Ground-State Phase Diagram

- Parameters: $J^z = 4$, $J^\perp = -1$, $\mu = 0$, $t_\uparrow = t_\downarrow = t$, and $V \geq 0$.
- Repulsive hole-hole interactions open a stable DSS window between AFM and DSF phases.



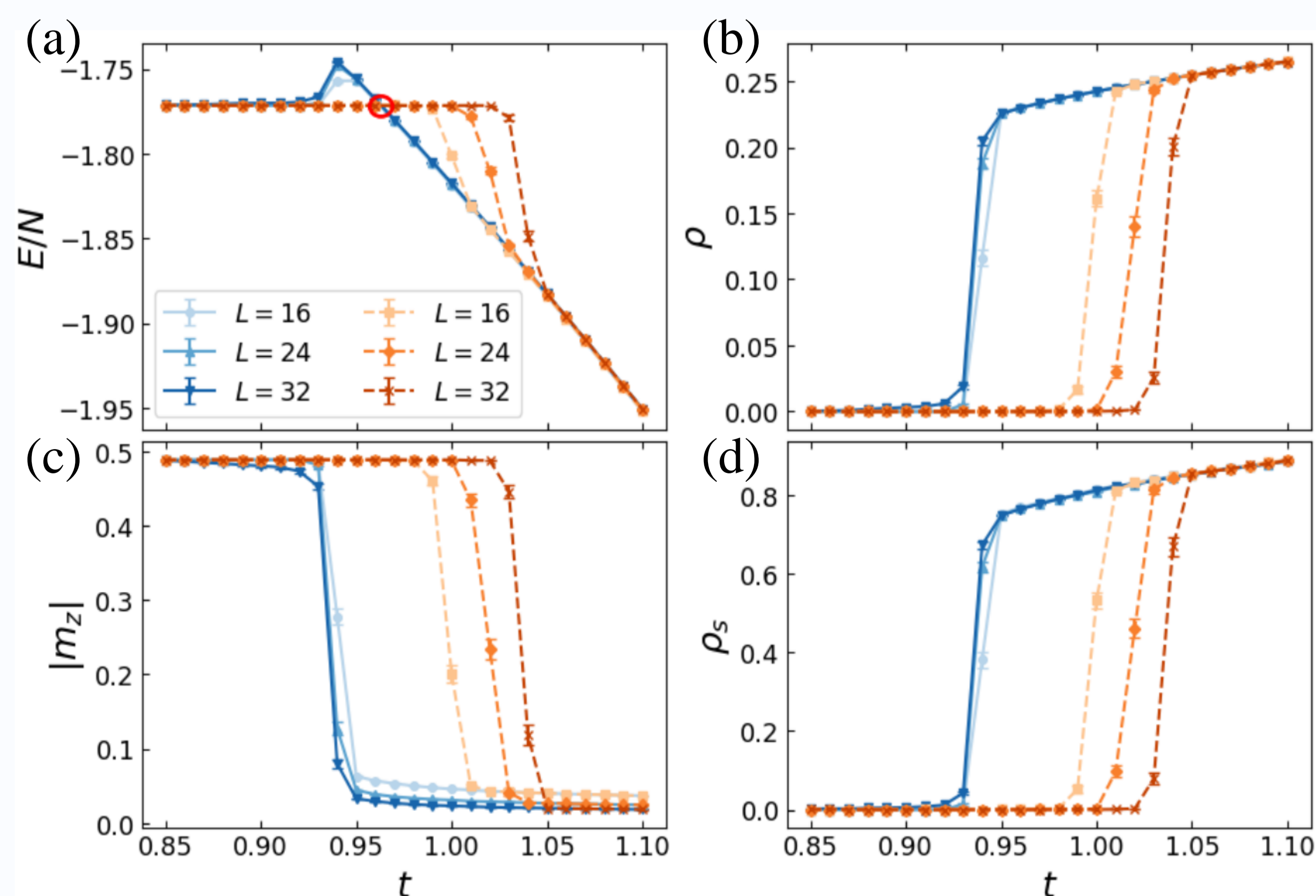
DSS&DSF Spin-Spin Correlations

- Diagonal and off-diagonal spin correlations distinguish DSS [(a), (b)] from DSF [(c), (d)].



AFM-to-DSF: First-Order Transition

- At weak repulsion ($V = 2.0$), the AFM insulator changes directly into the DSF.
- Forward/backward annealing shows hysteresis in energy, hole density, $|m_z|$, and ρ_s .



Finite-Temperature Behavior

- Two-step melting: superfluidity is lost before crystalline order.
- AFM crystalline order is thermally enhanced at intermediate T .

