Reflected Multientropy and Its Holographic Dual

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1. Rényi entropy and holographic dual

For a bipartite pure state, entanglement entropy is obtained from the Rényi entropy

$$S_n(A) = \frac{1}{1-n}\log \mathrm{Tr}\rho_A^n, \qquad S(A) = \lim_{n\to 1} S_n(A).$$

In the Euclidean path integral, ρ_A is prepared by cutting open region A. The trace $\mathrm{Tr}\rho_A^n$ cyclically reconnects these cuts among n replicas. **[1]**

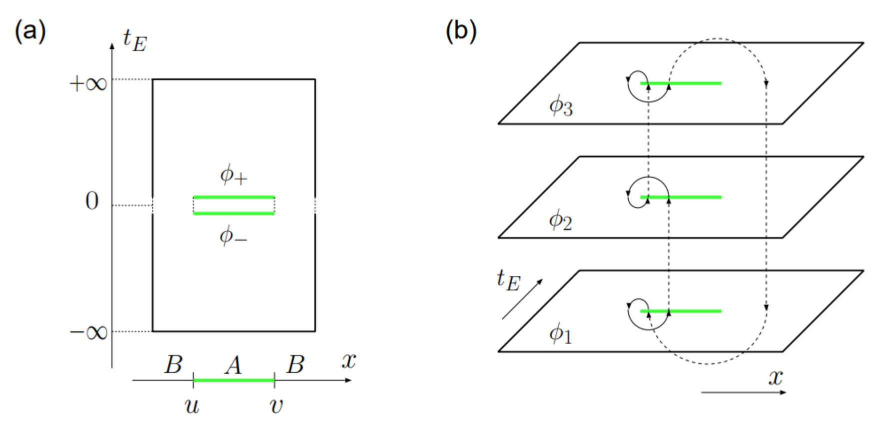

Figure 1: *Path-integral representation of the reduced density matrix and the corresponding n-sheeted Riemann surface. Cyclic identification of the cuts along A implements $\mathrm{Tr}\rho_A^n$; here $n = 3$.*

Holographic dual. In holographic CFTs, this boundary replica construction has a bulk geometric saddle. In the $n \to 1$ limit, the entropy is given by the Ryu–Takayanagi surface

$$S(A) = \frac{\mathrm{Area}(\gamma_A)}{4G_N}, \qquad \partial\gamma_A = \partial A.$$

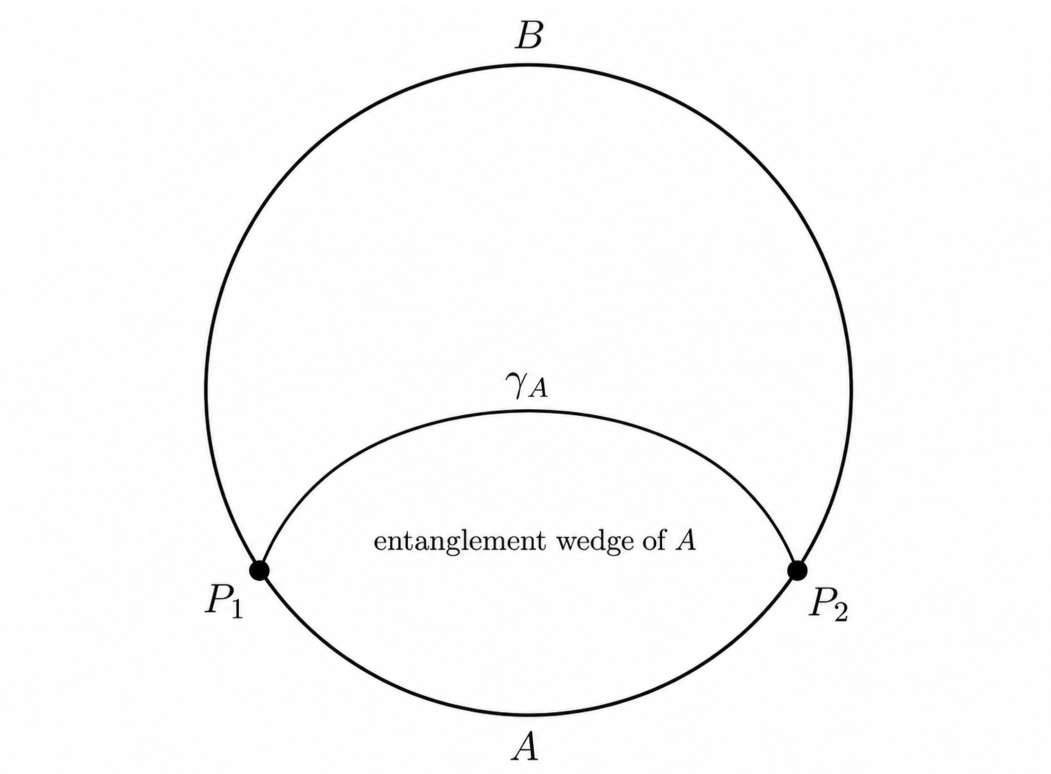

Figure 2: *Holographic dual of entanglement entropy. The interval A is anchored at boundary points P_1, P_2, and its entropy is computed by the bulk geodesic γ_A. The enclosed bulk domain is the entanglement wedge of A.*

2. Multientropy and minimal web

Multientropy generalizes the replica construction by assigning independent permutation patterns to different subsystems. For a q-partite state,

$$S_n^{(q)} = \frac{1}{(1-n)n^{q-2}}\log\left[\frac{Z^{(q)}_{n^{q-1}}}{\left(Z_1^{(q)}\right)^{n^{q-1}}}\right], \qquad S^{(q)} = \lim_{n\to 1} S_n^{(q)}.$$

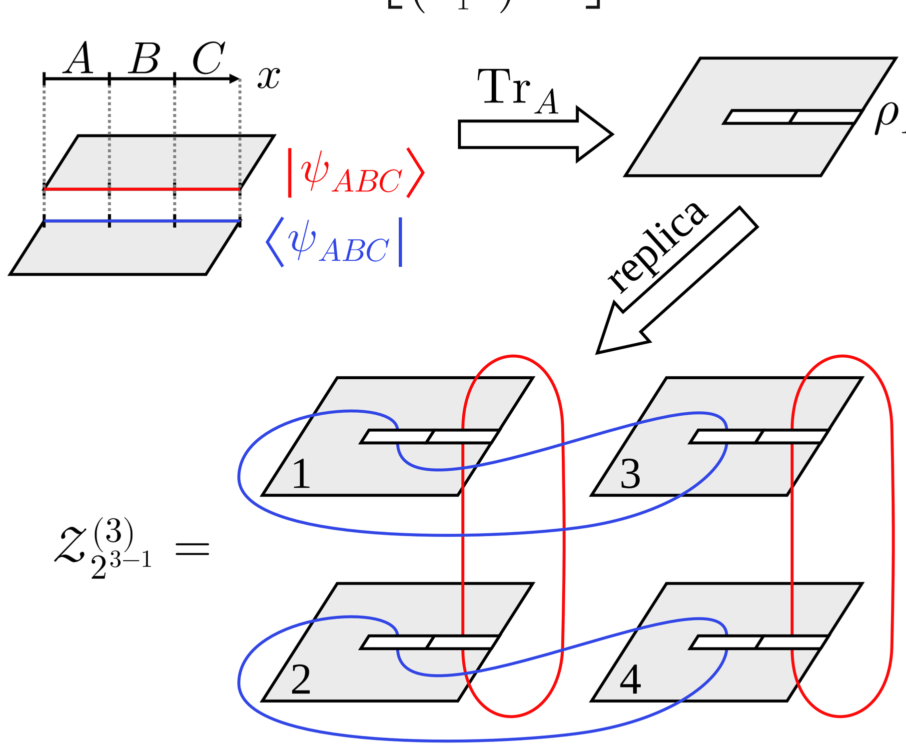

Figure 3: *Tripartite multientropy for $q = 3, n = 2$. The four replicas form a two-dimensional replica pattern: different colors encode independent subsystem permutations.*

For pure holographic states, the RT surface is replaced by a minimal surface web **[2]**.

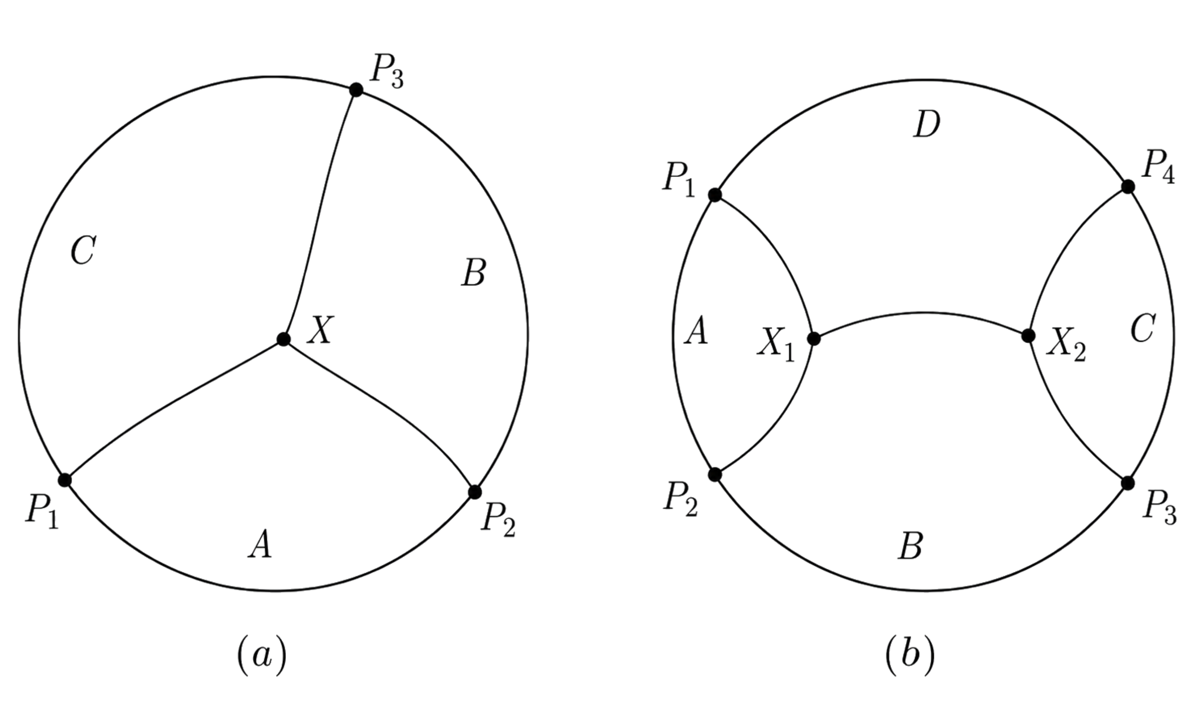

Figure 4: *Minimal-web dual of multientropy. (a) A tripartite web ending on the boundary points P_1, P_2, P_3, with boundary regions A, B, C. (b) A four-partite web with two bulk junctions X_1, X_2, separating the regions A, B, C, D.*

3. Canonical purification

Multientropy is naturally a pure-state construction. To extend it to a mixed state ρ_{ABC}, we use canonical purification **[3]**:

$$|\sqrt{\rho_{ABC}}\rangle \in (\mathcal{H}_A \otimes \mathcal{H}_{A^*}) \otimes (\mathcal{H}_B \otimes \mathcal{H}_{B^*}) \otimes (\mathcal{H}_C \otimes \mathcal{H}_{C^*}).$$

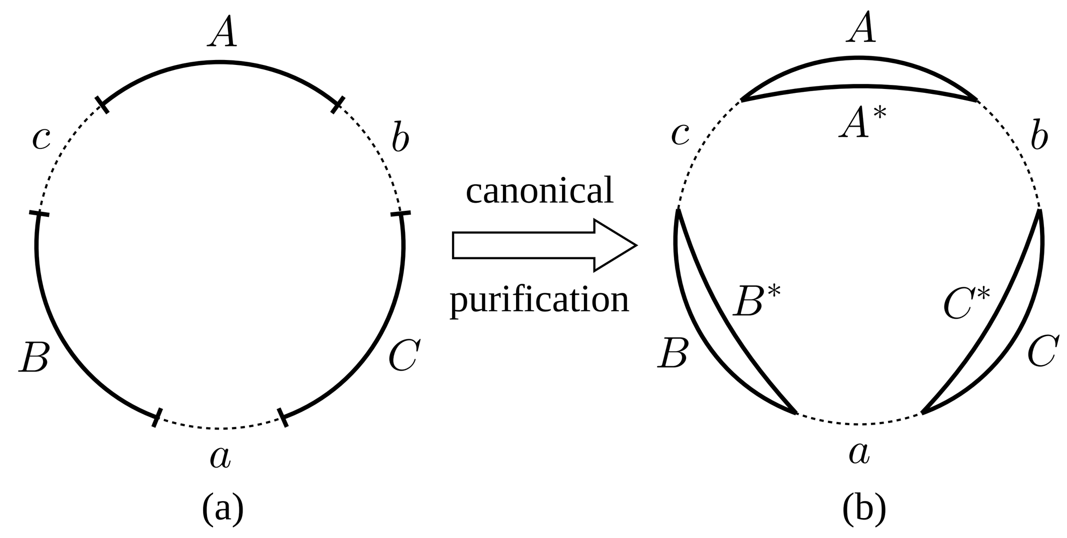

Figure 5: *Canonical purification of ρ_{ABC}. A second copy is glued along the traced-out degrees of freedom, producing the purified state $|\sqrt{\rho_{ABC}}\rangle$.*

4. Reflected multientropy and minimal web

We define the reflected multientropy as the multientropy of the canonically purified state **[4]**:

$$S_R^{(3)}(A:B:C) = S^{(3)}(AA^*:BB^*:CC^*)_{|\sqrt{\rho_{ABC}}\rangle}.$$

It reduces to the reflected entropy for $q = 2$, and to twice the multientropy for pure states.

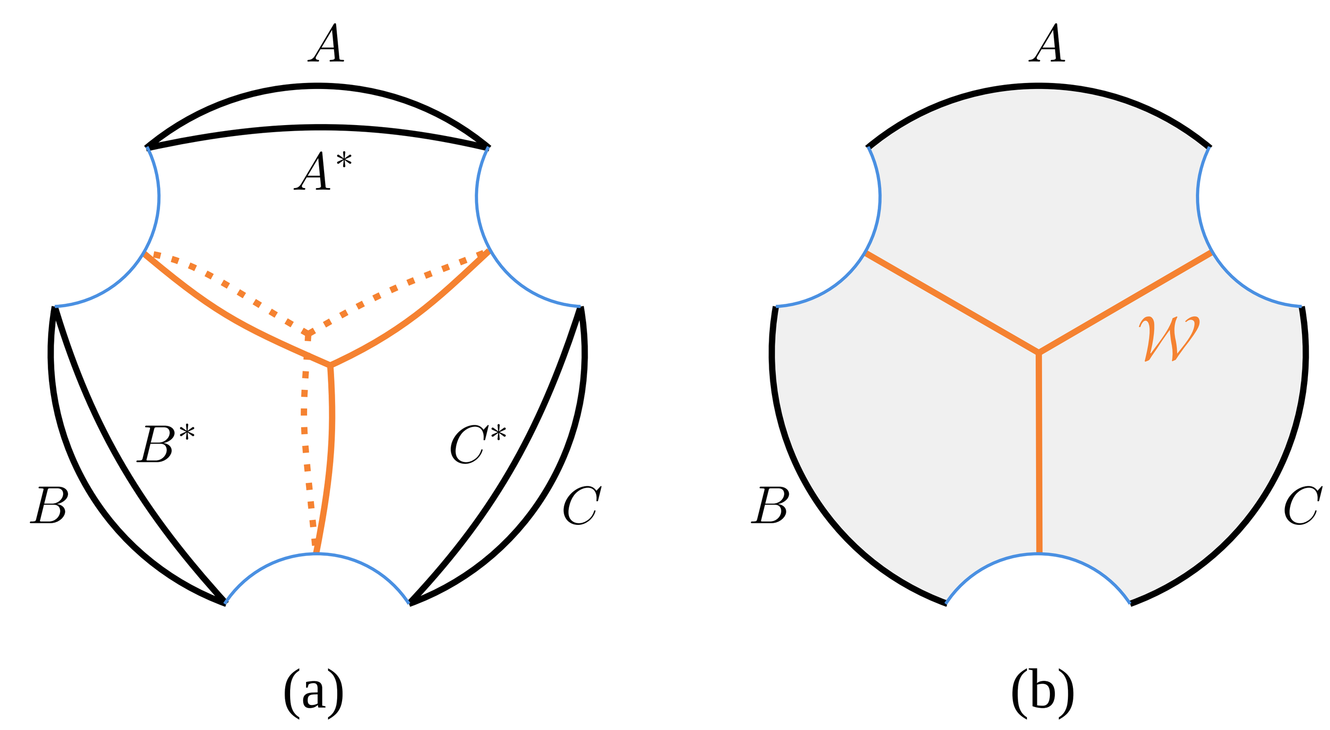

Figure 6: *Holographic canonical purification. The bulk dual is obtained by doubling the entanglement wedge and gluing along the RT surface. In one copy, $S_R^{(3)}$ is computed by a minimal web W anchored on the RT surface.*

The holographic proposal is

$$S_R^{(q)} = \frac{2}{4G_N}\min_W L[W].$$

The factor of two comes from the two copies in the canonical purification, while $L[W]$ is evaluated in a single entanglement wedge.

5. Check in AdS_3/CFT_2

For the tripartite case, the replicated partition function is a six-point twist-operator correlator,

$$Z_{n^{q-1},m} = \langle \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\rangle_{\mathrm{CFT}^{\otimes mn^{q-1}}}.$$

At large central charge, the monodromy result agrees with the holographic web.

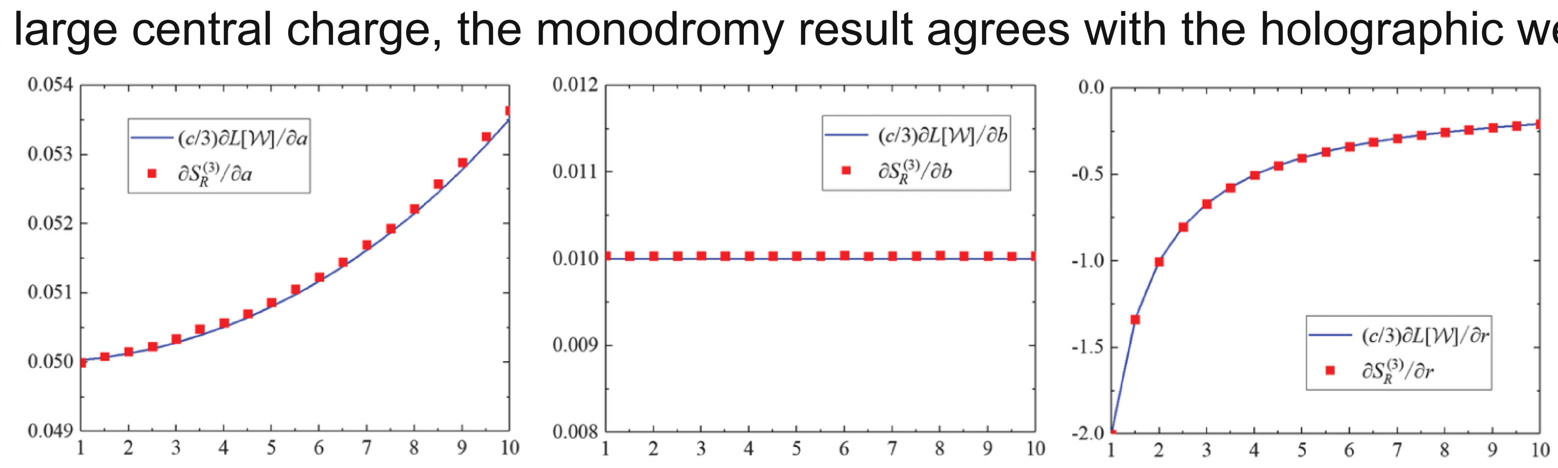

Figure 7: *Zero-temperature check in AdS_3/CFT_2. The derivatives of the CFT result for $S_R^{(3)}$ match the derivatives of the holographic web length $(c/3)L[W]$ with high precision.*

The same comparison can be extended to finite temperature by putting the CFT on a thermal cylinder and evaluating the bulk web in the BTZ background.

References

[1] S. Ryu and T. Takayanagi, "Aspects of holographic entanglement entropy," *JHEP* **08** (2006) 045, arXiv:hep-th/0605073.

[2] A. Gadde, V. Krishna and T. Sharma, "Towards a classification of holographic multi-partite entanglement measures," *JHEP* **08** (2023) 202, arXiv:2304.06082, doi:10.1007/JHEP08(2023)202.

[3] S. Dutta and T. Faulkner, "A canonical purification for the entanglement wedge cross-section," *JHEP* **03** (2021) 178, arXiv:1905.00577, doi:10.1007/JHEP03(2021)178.

[4] M.-K. Yuan, M. Li and Y. Zhou, "Reflected Multientropy and Its Holographic Dual," *Phys. Rev. Lett.* **135** (2025) 091604, doi:10.1103/PhysRevLett.135.091604.