Electromagnetic response of disordered superconductors with finite-momentum pairing

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Overview

Background: Finite-momentum pairing with momentum $\vec{q}$ enables photon absorption and optical activity of collective modes.
Questions: 1) How can the electromagnetic response be calculated in a conserving and gauge-invariant way?
2) How does disorder reshape quasiparticle and collective-mode contributions?
Method: Baym–Kadanoff conserving approximation
Results: All BCS-level fluctuations can make non-negligible contributions, demonstrating the necessity of vertex corrections.

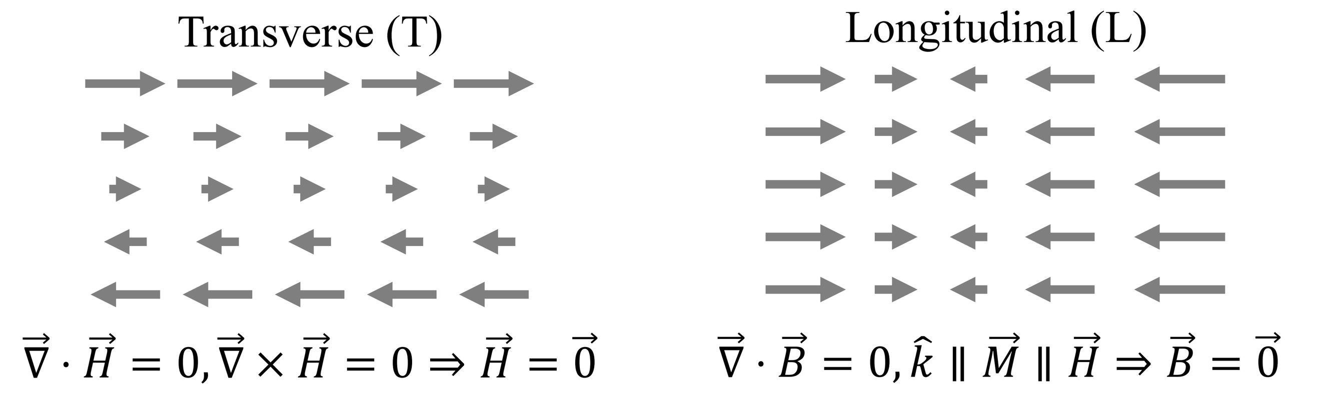
Qualitative Analysis

Symmetry analysis

- Quasiparticle part:
  $\vec{q}=\vec{0}$: electrons satisfy $\vec{v}_{\vec{k}}=-\vec{v}_{-\vec{k}}$, no net current
  finite $\vec{q}$: $\vec{v}_{\vec{k}+\vec{q}}\neq-\vec{v}_{-\vec{k}+\vec{q}}\Longrightarrow$ finite current
- Collective modes part:
  s-wave pairing case: (Similar for general pairing symmetry)
  $\vec{q}=\vec{0}$: Δ is a scalar, its fluctuation couples via $\vec{\nabla}\cdot\vec{A}$
  $\Longrightarrow$ no contribution in transverse gauge $(\vec{\nabla}\cdot\vec{A}=0)$
  finite $\vec{q}$: extra scalar $\vec{q}\cdot\vec{A}$ enables finite current

Magnetization analogy

- Quasiparticle current $\Longrightarrow\vec{M}$, backflow $\Longrightarrow\vec{H}$
  net current $\Longrightarrow\vec{B}\propto(\vec{M}+\vec{H})$



- $\vec{H}=\vec{0}$: no contribution from collective modes
  $\vec{B}=\vec{0}$: exact cancellation of quasiparticle and collective modes

	Galilean invariance	Broken Galilean invariance
$\vec{q}=\vec{0}$	$\vec{M}=\vec{0}\Longrightarrow\vec{H}=\vec{0}\Longrightarrow\vec{B}=\vec{0}$	
$\vec{q}\neq\vec{0}$	$\vec{M}\neq\vec{0}$ and pure L $\Longrightarrow\vec{B}=\vec{0}$	$\vec{M}\neq\vec{0}$ but mixed T and L $\Longrightarrow\vec{H}=-\vec{M}_L\Longrightarrow\vec{B}\propto\vec{M}_T$

Finite $\vec{q}$ enables optical activity of collective mode.
Broken Galilean invariance prevents exact cancellation.

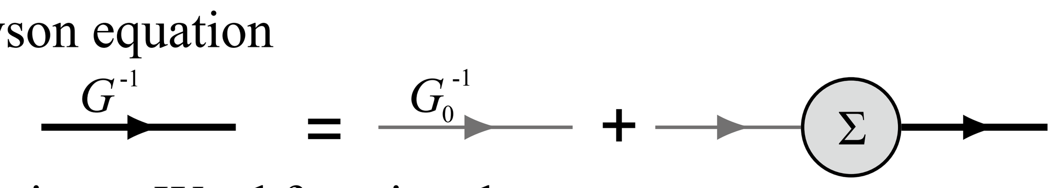
Framework

Baym-Kadanoff approximation

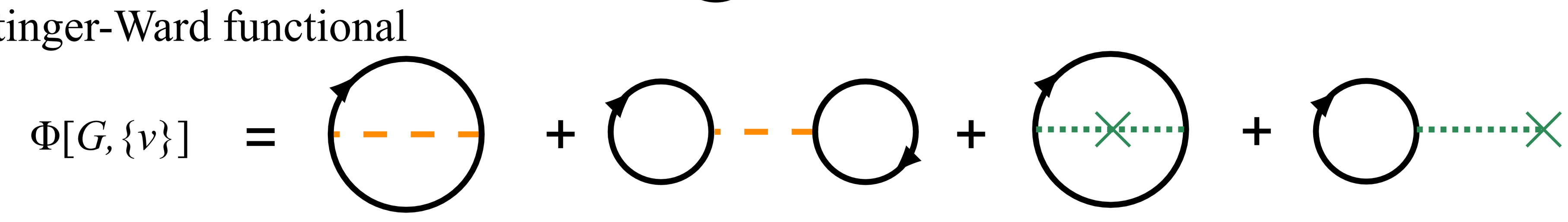
(with self-consistent Hartree-Fock and Born approximations)

H_{int}: effective e-e attractive interaction + static nonmagnetic disorder potential

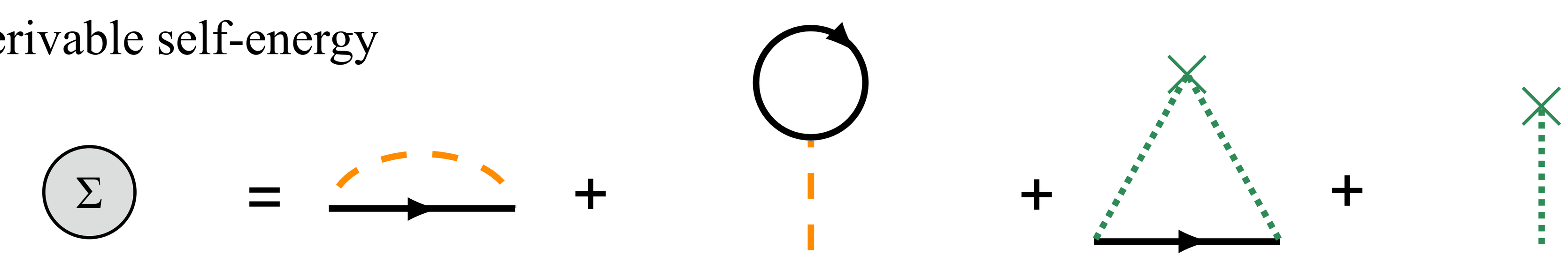
Dyson equation



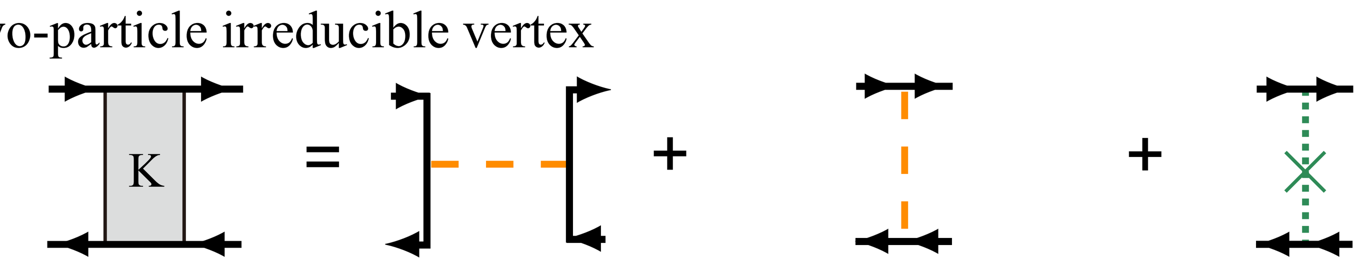
Luttinger-Ward functional



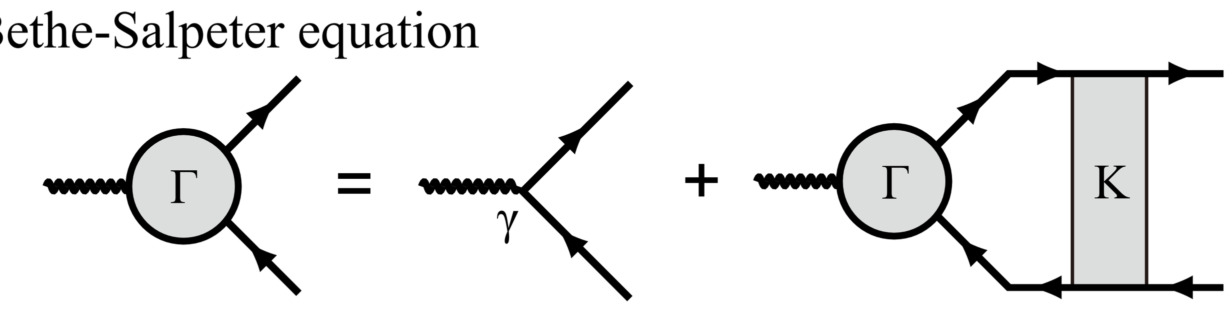
Φ-derivable self-energy



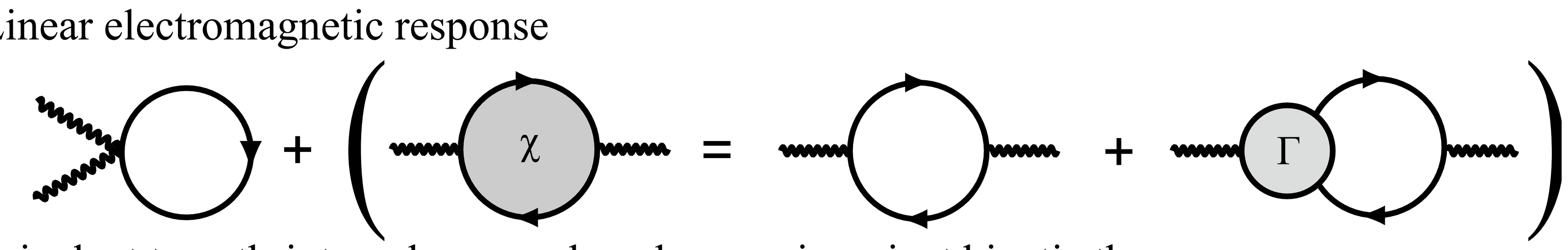
Two-particle irreducible vertex



Bethe-Salpeter equation



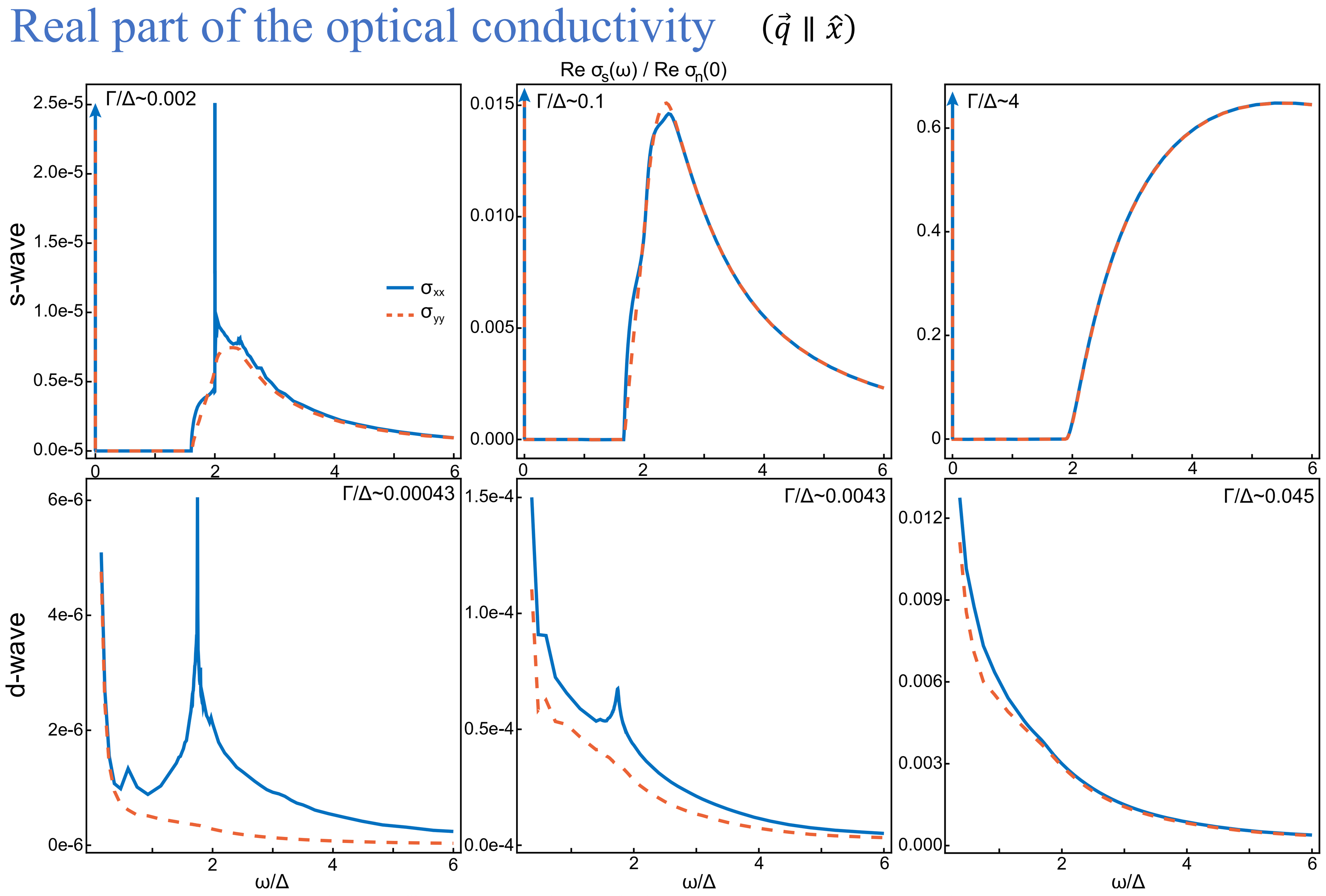
Linear electromagnetic response



- Equivalent to path-integral approach and gauge-invariant kinetic theory.
- Conserving method including all BCS-level fluctuations.
- More complete than consistent-fluctuation of order parameter method.

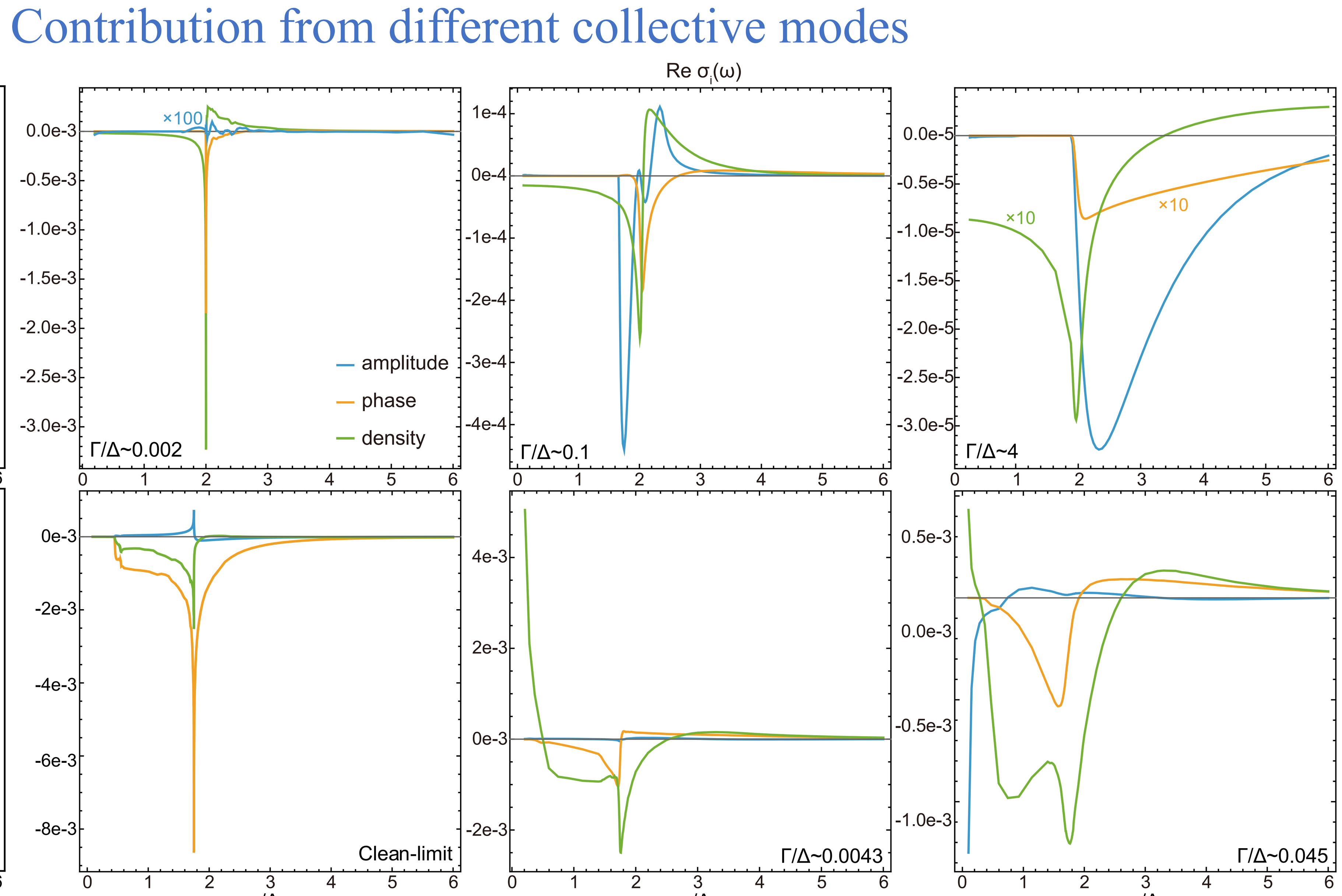
Results

Real part of the optical conductivity ($\vec{q}\parallel\hat{x}$)



- Cleaner case: peaks reflect quasiparticle DOS; strongly anisotropic
- Dirtier case: peaks suppressed; more isotropic; approach the Mattis-Bardeen limit

Contribution from different collective modes



- Amplitude/density contributions grow relative to phase contribution with disorder
- Vertex corrections are important near optical-response peaks

References

[1] PRB 108, 224516 (2023)
[2] arXiv:2410.1867 & arXiv:2501.13722
[3] PRB 112, 094503 (2025)
[4] PRB 106, 144509 (2022) & PRB 109, 064508 (2024)
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[6] PR 117, 648 (1960) & PR 156, 470 (1967)
[7] PR 124, 287 (1961) & PR 127, 1391 (1962)