

半轻衰变与 CKM 矩阵元的理论研究

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Overview

I: 半轻衰变的核心问题

II: 一些最新进展

i: $|V_{ub}|$ from $B \rightarrow \pi\pi l^+ \nu_l$

ii: Energy dependent structure of $f_0(980)/[\pi\pi]_S$

III: 总结和展望

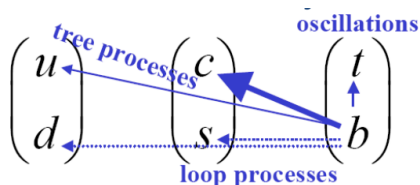
- rotation between the weak interaction eigenstates and the mass eigenstates

$$V_{\text{CKM}} = \begin{pmatrix} \overset{\text{\color{blue}\tiny β衰变}}{V_{ud}} & \overset{\text{\color{brown}\tiny K介子衰变}}{V_{us}} & \overset{\text{\color{blue}\tiny B介子衰变}}{V_{ub}} \\ \overset{\text{\color{green}\tiny D介子衰变}}{V_{cd}} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

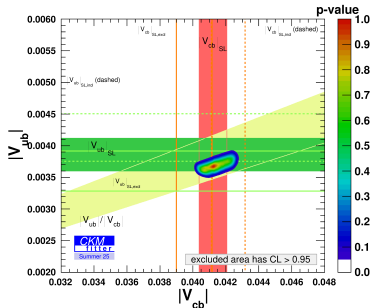
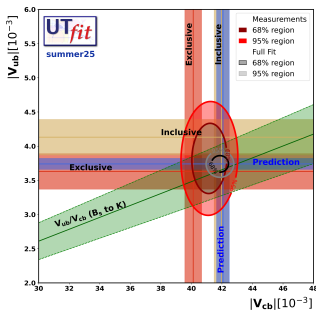
\$K, B\$介子混合/FCNC

- $VV^\dagger = V^\dagger V = I_3$ in the Standard Model
- $VV^\dagger \neq V^\dagger V \neq I_3$ New Physics
- $|V_{ub}|/|V_{cb}|$ contributes to CPV measurement in B decays

- CKM matrix elements are mainly determined via **the charged current processes**, i.e.,
 $b \rightarrow u l^- \bar{\nu}, b \rightarrow c l^- \bar{\nu}, c \rightarrow s l^+ \nu$
- Flavor changing neutral current processes** are sensitive to new physical contributions, i.e.,
 $b \rightarrow s l^+ l^-, b \rightarrow d l^+ l^-$



核心问题: $|V_{cb}|$, $|V_{ub}|$ 疑难

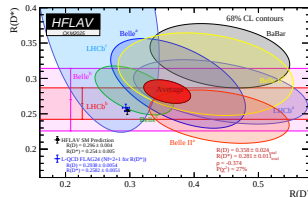


- $|V_{cb}|$ tension** $|V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$: 2.5σ tension between $(42.2 \pm 0.5) \times 10^{-3}$ and $(39.8 \pm 0.6) \times 10^{-3}$ determined via the $B \rightarrow X_c l^- \bar{\nu}$ and $B \rightarrow D^{(*)} l^- \bar{\nu}$ processes, respectively
- $|V_{ub}|$ tension** $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$: 2.5σ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.67 \pm 0.15) \times 10^{-3}$ determined via the $B \rightarrow X_u l^- \bar{\nu}$ and $B \rightarrow \pi l^- \bar{\nu}$ processes, respectively

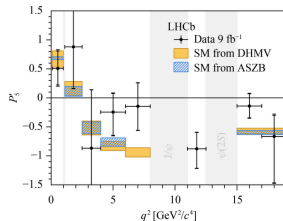
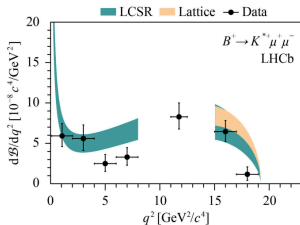
[PDG 2024, 25 update]

核心问题: 反常现象

- **LFU** $R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau^- \bar{\nu})}{\mathcal{B}(B \rightarrow D^{(*)} \mu^- \bar{\nu})}$: exceed the SM predictions by 2.5σ and 2.3σ [HFCPV 2025]
- make the CKM determinations more complicated if confirmed



- **Anomalies** in FCNC processes $B \rightarrow K^* \mu^+ \mu^-$: 3.6σ derivation from SM of $d\mathcal{B}(B \rightarrow K^* \mu^+ \mu^-)/dq^2$ in $q^2 \in [1, 6] \text{ GeV}^2$, 1.9σ derivation from SM of $p'_5 = S_5/\sqrt{F_L(1-F_L)}$ in $q^2 \in [4, 8] \text{ GeV}^2$



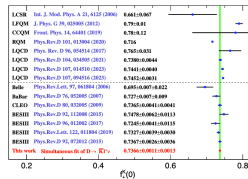
[Heavy Flavour Physics and CP Violation at LHCb: a Ten-Year Review, Front. Phys.18.44601(2023)]

解决方案 在传统过程继续奋斗

[BESIII 2023,24, Fermi Lattice and MILC 23]

- 更精确的测量和格点计算

$|V_{cs}| = 0.975 \pm 0.006$: 0.972 ± 0.007 and 0.984 ± 0.012 measured via the $D \rightarrow K l \nu$ and $D_s \rightarrow \mu^+ \nu_\mu$ processes
 $\sim 1.5\sigma$ [PDG 2024, 25 update] $\leftarrow 3\sigma$ in [PDG 2021]



- 更全面更系统的物理分析方法

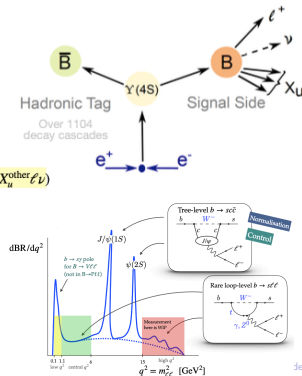
$|V_{ub}|$ result from Belle collaboration with Simultaneous Determination in excl. and incl. processes [Belle PRL131, 211801 (2023)]
 $(3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$ and
 $(3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$

$$\mathcal{B}(B \rightarrow \pi^0 \ell \nu) + \mathcal{B}(B \rightarrow \pi^+ \ell \nu) + \mathcal{B}(B \rightarrow X_u^{\text{other}} \ell \nu) = \mathcal{B}(B \rightarrow X_u \ell \nu)$$

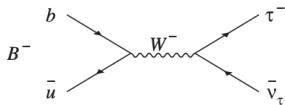
- 精细结构、丰富的 QCD 效应

high order QCD corrections, more structures

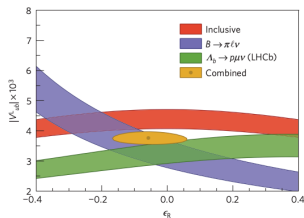
[AK, TM, YMW, JHEP 02 (2013) 010, AK, TM, AAP, YMW, JHEP 09 (2010) 089]



解决方案 寻找新的增长极



- $|V_{ub}|$ from **pure leptonic decay** $(4.11 \pm 0.39) \times 10^{-3}$:
 $|V_{ub}|f_B = 0.72 \pm 0.09$ MeV from Belle, 1.01 ± 0.14 MeV from BABAR,
 0.77 ± 0.12 MeV average [FLAG, PRD 107. 052008 (2023)]



- $|V_{ub}|$ from **baryon decay** see Xiao-kang's talk

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \rightarrow p \mu^- \bar{\nu})}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu})} R_{\text{FF}} = 0.68 \pm 0.07 \downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.079 \pm 0.006 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.97 \pm 0.26) \times 10^{-3}$$

consistent with the determinations in exclusive $B \rightarrow \pi l \bar{\nu}$ decay
 [LHCb, Nature Physics 11. 743-747 (2015)]

- $|V_{ub}|/|V_{cb}|_{\text{high}} = 0.098 \pm 0.008$ via $\mathcal{B}(B_s \rightarrow K^- \mu^+ \nu)/\mathcal{B}(B_s \rightarrow D_s^- \mu^+ \nu)$ [LHCb, PRL 126. 081804 (2021)]
- $|V_{cb}| = (42.3 \pm 1.7) \times 10^{-3}$ from $B_s \rightarrow D_s \mu^+ \nu$ [LHCb, PRD 101.072004 (2020)]
- $|V_{ub}| = 3.58(9) \times 10^{-3}$ from $B_s \rightarrow K l \nu$ [PRD104. 114041 (2021)]
- $d\mathcal{B}(B \rightarrow K^* \mu^+ \mu^-)/dq^2$ and p'_5 in $\Lambda_b^0 \rightarrow \Lambda \mu^+ \mu^-$ [LHCb, JHEP 06 (2015) 115]

- $B \rightarrow \pi\pi l\nu$ 和 $|V_{ub}|$ 测量

[SC, PRD 112 (2025) L111301]

- Energy dependent structure of $f_0(980)/[\pi\pi]_S$

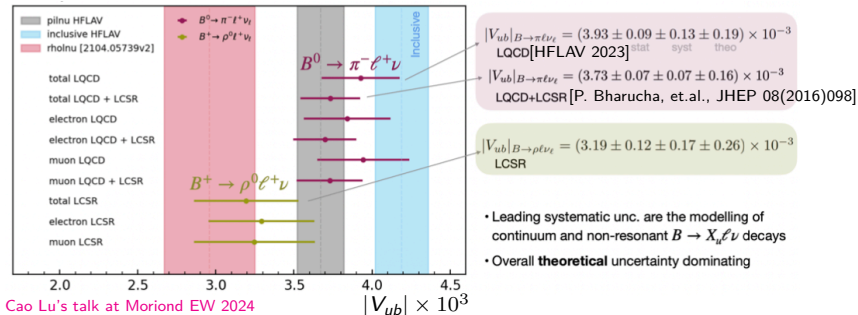
[SC, L.Y. Dai, J.M. Shen and S.L. Zhang, PRD 113 (2026) L031901]

$|V_{ub}|$ from $B \rightarrow \pi\pi l^+\nu$

- 含有激发态粒子的过程也可以提供独立的测量 the same $b \rightarrow u$ transition
- $B \rightarrow V$ transitions updated via B -meson LCSRs [Gao, et.al., PRD 101.074035(2020)]

$$|V_{ub}| \times 10^3 \xrightarrow{B \rightarrow \rho l \nu} 3.05^{+1.34}_{-1.30} |_{\text{th}}^{+0.19}_{-0.20} |_{\text{da}}, \quad \xrightarrow{B \rightarrow \omega l \nu} 2.54^{+1.09}_{-1.05} |_{\text{th}}^{+0.18}_{-0.19} |_{\text{da}}$$

- simultaneous measurements of $d\mathcal{B}/dq^2$ for $B \rightarrow \pi^- l^+ \nu_l$ and $B \rightarrow \rho^0 l^+ \nu_l$ [Belle-II, PRD 111.112009(2025)]



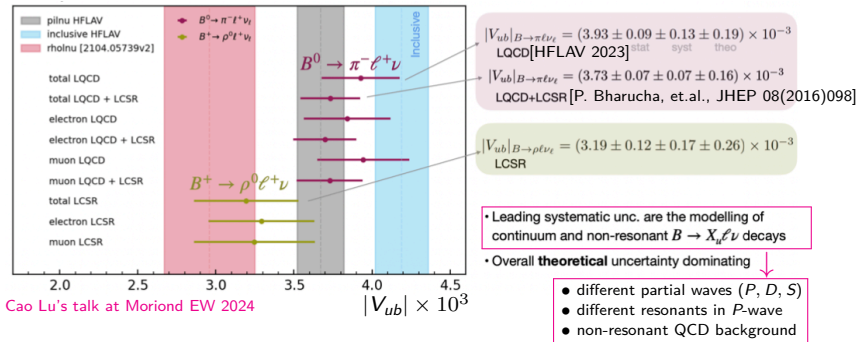
Cao Lu's talk at Moriond EW 2024

$|V_{ub}|$ from $B \rightarrow \pi\pi l^+\nu$

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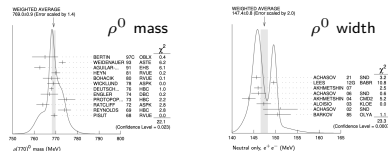
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$|V_{ub}|$ from $B \rightarrow \pi\pi l^+ \nu$

- the $\pi\pi$ spectra in $[0.554, 0.996]$ GeV serves as the **candidate region for ρ**
- ρ is usually treated as a **stable single particle** in the QCD studies of $B \rightarrow \rho$ transitions (LQCD, LCSRs)



- the signal channel in a $B \rightarrow \rho l \nu$ -type decay is $B \rightarrow \pi\pi l \nu$ (B_{l4})
- How to accurately describe the **width effects** of unstable intermediate particles, the contributions and **interference effects** of different partial waves, and the **QCD backgrounds** from non-resonant states
- Dipion LCDAs (2π **DA**s) provide a most general description of $\pi\pi$ spectral

- opportunities for $|V_{cb}|$ from $B \rightarrow D^* l \nu$, $X_c l \bar{\nu}$ and $B \rightarrow K^* l^+ l^-$ anomalies

$$\mathcal{B}(B^+ \rightarrow X_c^0 \ell^+ \nu_\ell) \approx 10.79\%$$

$D^0 \ell^+ \nu_\ell$ 2.31 %	$D^{*0} \ell^+ \nu_\ell$ 5.05 %	$D^{*+0} \ell^+ \nu_\ell + \text{Other}$ 2.38 %	Gap $\sim 1.05\%$
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careful study of $D\pi$, $D\pi\pi$ and $D\pi\pi\pi$ spectral

...or is it even bigger?

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$

$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_\mu \tau q_f(0) | 0 \rangle = \kappa_{ab} k_\mu \int dx e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab,ff}(\mu, u, \zeta, k^2)$$

$$\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(-x) | \rho^-(p) \rangle = f_\rho m_\rho \int_0^1 du e^{i(2u-1)p \cdot x} \left[p_\mu \frac{\epsilon^{(\lambda)} \cdot x}{p \cdot x} \left(\phi_{\parallel}(u, \mu) - \phi_{\perp}^3(u, \mu) \right) + \dots \right]$$

- 2πDAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{l=1}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=0, \text{even}}^{\infty} \sum_{l=1, \text{odd}}^{n+1} B_{n\ell}^{l=1}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{l=0}(u, \zeta, k^2, \mu) = 6u(1-u) \sum_{n=1, \text{odd}}^{\infty} \sum_{l=0, \text{even}}^{n+1} B_{n\ell}^{l=0}(k^2, \mu) C_n^{3/2}(2u-1) C_\ell^{1/2}(2\zeta-1)$$

- Evolution from $4m_\pi^2$ to large k^2 via the Watson theorem of $\pi\pi$ scattering amplitudes

$$B_{n\ell}^l(k^2) = B_{n\ell}^l(0) \text{Exp} \left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^l(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_\ell^l(s)}{s^N(s-k^2-i0)} \right]$$

2πDAs in a wide range of energies is given by δ_ℓ^l and a few subtraction constants

- The subtraction constants of $B_{n\ell}(k^2)$ at low k^2 (around the threshold)

(nℓ)	$B_{n\ell}^{\parallel}(0)$	$c_1^{\parallel, (n\ell)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\parallel}(0)$	$B_{n\ell}^{\perp}(0)$	$c_1^{\perp, (n\ell)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\perp}(0)$
(01)	1	0	1.46 $\rightarrow$ 1.80	1	0	0.68 $\rightarrow$ 0.60
(21)	-0.113 $\rightarrow$ 0.218	-0.340	0.481	0.113 $\rightarrow$ 0.185	-0.538	-0.153
(23)	0.147 $\rightarrow$ -0.038	0	0.368	0.113 $\rightarrow$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [SC 2019, 2023]

- 2πDAs were introduced at leading twist [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_{\mu} \tau q_{f'}(0) | 0 \rangle = \kappa_{ab} k_{\mu} \int dx e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab, ff'}(u, \zeta, k^2)$$

- improved to twist-three level recently [SC, PRD 112, L111301(2025)]

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) q(x) | 0 \rangle = \int du e^{\bar{u} k \cdot x} \frac{ik^2 (k \cdot x)}{2f_{2\pi}^{\perp}} \Phi_{\parallel}^{(s)},$$

$$\langle \pi(k_1) \pi(k_2) | \bar{q}(0) \sigma^{\mu\nu} q(x) | 0 \rangle = -\frac{i}{f_{2\pi}^{\perp}} \int du e^{\bar{u} k \cdot x} \left[\frac{k_{\mu} \bar{k}_{\nu} - k_{\nu} \bar{k}_{\mu}}{2\zeta - 1} \Phi_{\perp} - k^2 \frac{k_{\mu} x_{\nu} - k_{\nu} x_{\mu}}{k \cdot x} \Phi_{\parallel}^{(t)} \right].$$

$B \rightarrow \pi\pi$ form factors

- QCD dynamics of B_{I4} decays is incorporated in $B \rightarrow \pi\pi$ form factors, ~~the $B \rightarrow \rho$ ffs followed by $\rho \rightarrow \pi\pi$ in the cascade decay~~ [X.-W. Kang, et.,al, PRD 89. 053015 (2014), S. Faller, et.,al, PRD 89. 014015 (2014)]

$$\begin{aligned} i\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = & F_\perp(q^2, k^2, \zeta) \frac{2}{\sqrt{k^2}\sqrt{\lambda_B}} i\epsilon_{\nu\alpha\beta\gamma} q^\alpha k^\beta \bar{k}^\gamma \\ & + F_t(q^2, k^2, \zeta) \frac{q_\nu}{\sqrt{q^2}} + F_0(q^2, k^2, \zeta) \frac{2\sqrt{q^2}}{\sqrt{\lambda_B}} \left(k_\nu - \frac{k \cdot q}{q^2} q_\nu\right) \\ & + F_\parallel(q^2, k^2, \zeta) \frac{1}{\sqrt{k^2}} \left(\bar{k}_\nu - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_B} k_\nu + \frac{4k^2(q \cdot \bar{k})}{\lambda_B} q_\nu\right) \end{aligned}$$

- * $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function,
 $q \cdot k = (m_B^2 - q^2 - k^2)/2$, $q \cdot \bar{k} = \sqrt{\lambda}\beta_\pi(k^2) \cos \theta_\pi/2 = \sqrt{\lambda}(2\zeta - 1)$, $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$

- **SU(3) flavor symmetry/breaking** with the intermediate resonance

• [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng, et.al., 2301.00090, PRD 112, 033002 (2025)]

- **QCDF** (QCD factorization) in the large two-pion mass

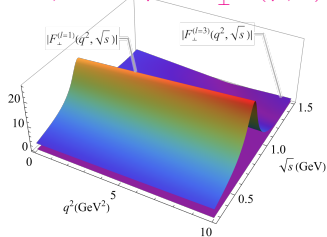
• [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)] $T_I \propto F_{B \rightarrow \pi} \otimes \phi_\pi$,

$B \rightarrow \pi\pi$ form factors

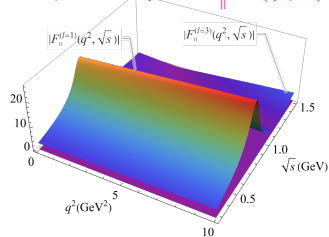
- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
 - [L. Leskovec, et.al, PRL 134.161901 (2025), **Editors' Suggestion**]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
 - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]in the full phase-space by a novel parameterization with unitarity
 - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025), **Editors' Suggestion**]
- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
 - [SC, A. Khodjamirian and J. Virto, JHEP 05(2017)157] **B -meson LCSRs** , [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12(2019)083, 06(2023)034] **$B \rightarrow K\pi$**
 - [C. Hambrock and A. Khodjamirian, NPB 905. 379-390(2016)] **2π DAs LCSRs of $F_{\parallel, \perp}$**
 - [SC, A. Khodjamirian and J. Virto, PRD(R) 96. 051901(2017)] **timelike-helicity FF F_t and F_0**
 - [SC, PRD 99. 053005(2019)] **2π DAs updates and $B \rightarrow [\pi\pi]_{S,P}$ FFs**
 - [SC and J.M Shen, EPJC 6:554(2020), SC and S.L Zhang, EPJC 84:379(2024)] **Pheno**
 - [SC, PRD 112 (2025) L111301] **first study of twist-three 2π DAs and $|V_{ub}|$ extraction this talk**
 - [SC, Dai, Shen and Zhang, PRD 113 (2026) L031901] **$D_s \rightarrow [\pi\pi]_S e\nu$ tiny contribution from $q\bar{q}$ this talk**

$B \rightarrow \pi\pi$ form factors

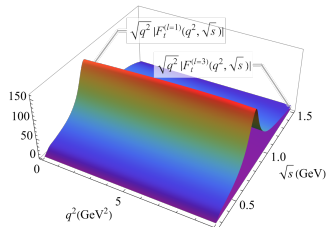
P, F -wave components $F_{\perp}^{l=1,3}(q^2, k^2)$



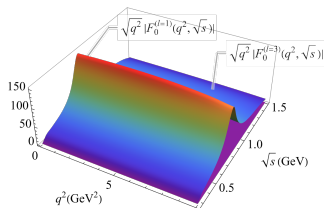
P, F -wave components $F_{\parallel}^{l=1,3}(q^2, k^2)$



[SC, PRD 112. L111301(2025)]

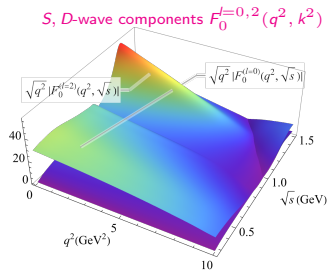
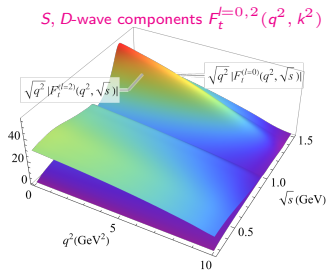


P, F -wave components $F_t^{l=1,3}(q^2, k^2)$



P, F -wave components $F_0^{l=1,3}(q^2, k^2)$

$B \rightarrow \pi\pi$ form factors



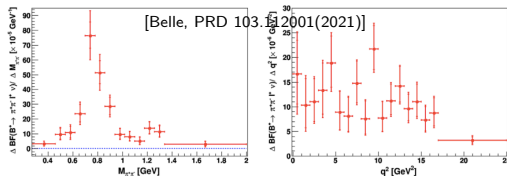
- twist-three contributions vanish in $F_{\perp,\parallel}^{(l=1,3,\dots)}$, while persist in $F_{t,0}^{(l=0,1,\dots)}$, give a significant correction $\sim 40\%$ to leading twist result
- ρ resonance is dominate in $F_{\perp,\parallel,t,0}^{(l=1)}(q^2, k^2)$, F -wave contributions are negligible
- S -wave dominates in the small k^2 regions, D -wave is sizable in the large k^2 and small q^2 regions

$|V_{ub}|$ from $B \rightarrow \pi\pi l^+ \nu$

- 2D partial differential decay width

$$\frac{d^2\Gamma}{dq^2 dk^2} = G_F^2 |V_{ub}|^2 \frac{\beta_\pi \sqrt{\lambda} q^2}{3 (4\pi)^5 m_B^3} \left[(|F_0^{(S)}|^2 + |F_0^{(P)}|^2) + \beta_\pi^2 (|F_{\parallel}^{(P)}|^2 + |F_{\perp}^{(P)}|^2) + \dots \right]$$

- 2D measurement of $\Delta\mathcal{B}^i$ (10^{-5}) in different bins



$\sqrt{s}$	q^2	$\Delta\mathcal{B}^i$	[Belle 2021]
(0.6, 0.9]	[0, 4]	$2.14 \pm 0.23 \pm 0.41$	$2.16^{+0.47}_{-0.42} \pm 0.23$
(0.9, 1.2]	[0, 4]	$0.72 \pm 0.08 \pm 0.13$	$0.70^{+0.32}_{-0.25} \pm 0.20$

take the PDG value: $|V_{ub}| \times 10^3 = 3.82 \pm 0.20 \Rightarrow$

a good consistent in ρ , f_0 regions

$$|V_{ub}| \times 10^3 = 3.84 \pm 0.45|_{\text{Data}} \pm 0.50|_{\text{LCSR}}, \quad 3.77 \pm 0.45|_{\text{Data}} \pm 0.49|_{\text{LCSR}}$$

- simultaneous measurements [Belle II PRD 111.112009 (2025)]

$$10^3 |V_{ub}|_{B \rightarrow \pi, \rho l \nu} = 3.73 \pm 0.10 \pm 0.16, \quad 3.19 \pm 0.22 \pm 0.26.$$

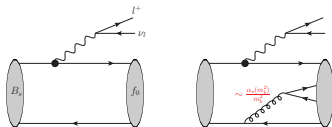
The controversy of $f_0(980)$ structure

- From the quantum theory, it is a superposition of all possible Fock states

$$|f_0(980)\rangle, \quad |\pi\pi\rangle_S = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \dots$$

- Hadron spectroscopy** provides clear evidence for the **complex config**
- The underlying partonic dynamics can be extracted directly from **Semileptonic B, D decays** in terms of **scale-dependent LCDAs**

- color transparency mechanism** in $B_{(s)} \rightarrow f_0 l^+ \nu_l$ decays



* high Fock states' contribution is doubly suppressed by α_s and $\mathcal{O}(1/Q^2)$, FSI is weak

- the mechanism fails in $D_s \rightarrow f_0 l^+ \nu_l$ decays** (the produced $q\bar{q}$ is nonrelativistic)
- while the energy-dependent partonic configurations is a QCD result, the **cascade decay analyses of $D_s(\rightarrow f_0) \rightarrow \pi\pi e\nu$ under $q\bar{q}$ ansatz agrees with data** [SC, Zhang, EPJC 84. 379(2024), Hu, et.al., PRD 112.

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$$\frac{d\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2) g_1 \beta_\pi(s)}{|m_S^2 - s + i(g_1 \beta_\pi(s) + g_2 \beta_K(s))|^2}$$

- * $D_s \rightarrow f_0$ FFs have revealed the scale dependence, the seemingly agreement arises from the Flatté parameterization with unsound use of f_0 LCDAs under the single-meson assumption, a fine-tuned delicate interplay ?

- a model-independent study of the signal channel $D_s \rightarrow [\pi\pi]_S e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{G_F^2 |V_{cs}|^2 \beta_\pi \pi(s) \sqrt{\lambda_{D_s} q^2}}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, s)|^2$$

- * calculate directly the $D_s \rightarrow [\pi\pi]_S$ FFs

- Twist-2 and twist-3 contributions to $D_s \rightarrow \pi\pi, f_0$ FFs at $q^2 = 0$

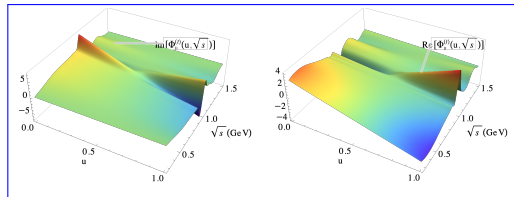
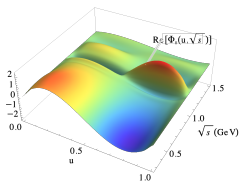
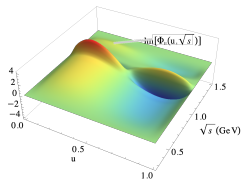
under the $q\bar{q}$ approximation

Form Factors	Twist-2	Twist-3	Total
$\sqrt{q^2} F_0^{(l=0)}(0) = \sqrt{q^2} F_t^{(l=0)}(0)$	$0.20_{-0.02}^{+0.02} - i0.24_{-0.02}^{+0.02}$	$-0.41_{-0.05}^{+0.04} + i0.51_{-0.04}^{+0.02}$	$-0.21_{-0.01}^{+0.02} + i0.27_{-0.02}^{+0.03}$
$\sqrt{q^2} F_0^{(l=2)}(0) = \sqrt{q^2} F_t^{(l=2)}(0)$	$0.27_{-0.02}^{+0.03} + i0.21_{-0.01}^{+0.02}$	$-0.55_{-0.03}^{+0.02} - i0.41_{-0.04}^{+0.05}$	$-0.28_{-0.02}^{+0.02} - i0.20_{-0.01}^{+0.02}$
$f_+(0) = f_0(0)$	$0.20_{-0.05}^{+0.03}$	$0.41_{-0.06}^{+0.04}$	$0.61_{-0.07}^{+0.05}$

destructive

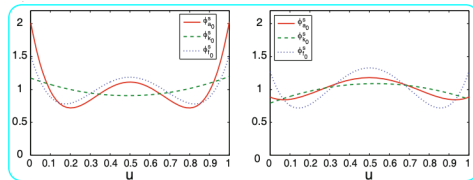
constructive

$2\pi\text{DAs}$ $[\pi\pi]_S$



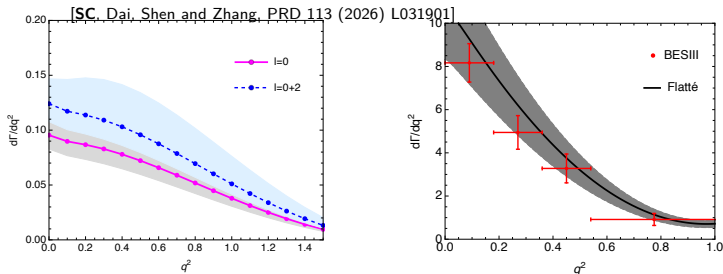
[Lü, et.al., PRD 75. 056001(2007)]

[Han, et.al., EPJA (2013) 49: 78]



- asymmetric twist-3 $2\pi\text{DAs}$ VS symmetric twist-3 f_0 LCDAs
- the asymmetry encoded by **PWE** that the single-meson picture fails to capture
- vanished asymptotic terms in twist-2 $[\pi\pi]_S / f_0$ LCDAs $\Downarrow$
- twist-3 (non-asymptotic twist-2) components dominates in charm (beauty) decays $\Updownarrow$
- energy-dependent picture from the color transparency mechanism

$D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay



$D_s \rightarrow [\pi\pi]_S e^+ \nu_e$	$D_s \rightarrow [f_0 \rightarrow \pi\pi] e^+ \nu_e$ [23]	Data [25]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- $q\bar{q}$ Fock state is not the dominate component of $[\pi\pi]_S / f_0$ in charm decays, consists with the assessment of color transparency mechanism
- go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g, q\bar{q}q\bar{q}$) $\Rightarrow |V_{cs}|$; different in $B \rightarrow f_0 / B \rightarrow \rho$ decays, (nonasymptotic) leading twist dominates

总结和展望

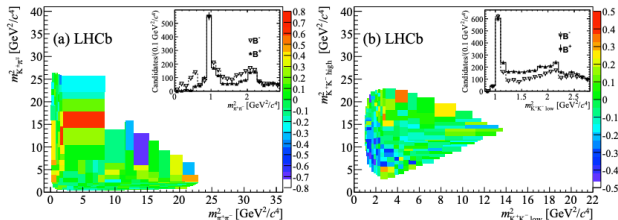
- $|V_{cb}|, |V_{ub}|$ 疑难和 $R_{D^{(*)}}, \rho'_5, \frac{d\Gamma}{dq^2}$ 等反常仍是半轻衰变研究的核心问题。上述问题的根本解决同时依赖于单举和遍举过程的独立研究，传统过程的精细研究和新过程的逐步探索是遍举研究的两条并行道路
- 四体半轻衰变 (H_{l4}) 提供了一个新方案 两介子系统光锥分布振幅
- 当前在 B_{l4} 过程抽取 $|V_{ub}|$ 的误差较大 (10%)，但是精度可以提高
Belle-II 积分亮度未来五年有望达到 $3ab^{-1}$ ，实验精度将提高 3 倍；LCSRs 和 LQCD 对形状因子的联合分析可以将理论误差减小一半
- 级联衰变方案在处理轻标量共振态过程时具有较大的不确定性，直接计算 D_{l4} 过程将是 $|V_{cs}|$ 测量和重味反常检验的一个重要候选 $D \rightarrow \pi l^+ l^-, \pi\pi l^+ l^-$
- H_{l4} 过程的研究正在进入高精度时代 $\gamma^* \rightarrow \pi\pi\gamma$ and $\gamma^*\gamma \rightarrow \pi\pi$

Thank you for your patience.

More opportunities/phenomena of 2π DAs

pion EMFF widely used in the three-body B decays
studied from pQCD and QCDF is the asymptotic
term of 2π DAs [Chai, SC and Ma, PRD 105. 033003 (2022)]

normalized to unit as $\Gamma_{M_1 M_2}^{I=1}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{nI}(s, \mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{nI}(s, \mu) \rightarrow a_n(\mu) \Gamma_{M_1 M_2}^{I=1}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.



$B \rightarrow K\pi\pi, KKK$

[PRL 111.101801(2013) LHCb]

Non-resonant contributions are small ($< 10\%$) in three-body D decays, large/dominate in the penguin dominated three-body B decays, the theoretically unresolved problem