Spin correlation in hyperon-antihyperon systems in e^+e^- annihilation

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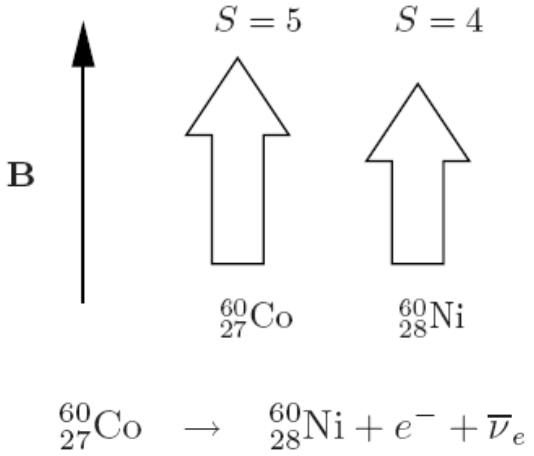
Yang Li, Chen Qian, Cong-Feng Qiao, Si-Hao Wu, Yang-Guang Yang, Wen-Yu Zhang, Xiao-Rong Zhou

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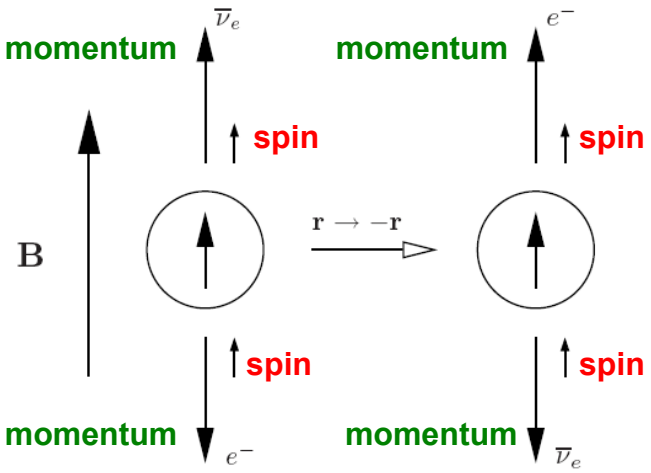
Outline

- **Introduction**
- **Quantum measurement description of hyperon decays**
- **Bell nonlocality, quantum entanglement (concurrence), quantum steering and discord in $Y\overline{Y}$ system in charmonium decays [$e^+e^- \to J/\psi \to Y\overline{Y}$ as an exclusive process]**
- **Summary**

Parity violation in beta decays



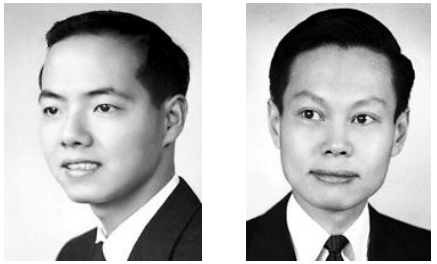
C. S. Wu, et al,
Phys. Rev. 105, 1413 (1957)



√ X



C.S. Wu, 1957



T. D. Lee, C. N. Yang
Nobel Prize, 1957

Lee, Yang, Phys.Rev.104,254(1956)

Experimental result: $\overrightarrow{p_e} \parallel -\vec{B}$
(1) $\overline{\nu}$ **is right-handed (** $\overrightarrow{s_{\overline{\nu}}} \parallel \overrightarrow{p_{\overline{\nu}}}$ **)**
(2) ν **is left-handed (** $\overrightarrow{s_\nu} \parallel -\overrightarrow{p_\nu}$ **)**
(3) Electron in β **decay is left-handed (** $\overrightarrow{s_e} \parallel -\overrightarrow{p_e}$ **)**

By measuring the momentum distribution of the particle one can extract its spin state in weak decays

Parity violation in Hyperon decay

Possible Detection of Parity Nonconservation in Hyperon Decay*

T. D. Lee, J. Steinberger,
Columbia University, New York, New York

and

G. Feinberg, P. K. Kabir, and C. N. Yang,
Institute for Advanced Study, Princeton, New Jersey
(Received May 2, 1957)

RECENTLY, various experiments[1–3] established the nonconservation of parity in β decay, π decay, and μ decay. The purpose of this note is to emphasize that, in view of these developments, experiments on hyperon production and decay in $(\pi+p)$ collisions of the type done by various groups using bubble chambers,[4] seem now to be especially important for a clarification of the following related questions: (i) whether parity conservation is violated in hyperon decays[5] and (ii) whether parity doublets exist.[6]

A detailed analysis concerning the possible detection

Phys.Rev. 106,1367 (1957)

General Partial Wave Analysis of the Decay of a Hyperon of Spin $\frac{1}{2}$

T. D. Lee* and C. N. Yang
Institute for Advanced Study, Princeton, New Jersey
(Received October 22, 1957)

THIS note is to consider the general problem of the decay of a hyperon of spin $\frac{1}{2}$ into a pion and a nucleon under the general assumption of possible violations of parity conservation, charge-conjugation invariance, and time-reversal invariance. The discussion is in essence a partial wave analysis of the decay phenomena and is independent of the dynamics of the decay. *Nonrelativistic approximations are not made on either of the decay products.*

In the reference system in which the hyperon is at rest there are two possible final states of the pion-nucleon system: $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$. Denoting the amplitudes of these two states by A and B, one observes that the decay is physically characterized by *three* real constants specifying the magnitudes and the relative phase between these amplitudes. One of these constants can be taken to be $|A|^2+|B|^2$, and is evidently proportional to the decay probability per unit time. The other two constants are best defined in terms of experimentally measurable quantities. We discuss three types of experiments:

Phys.Rev. 108,1645 (1957)

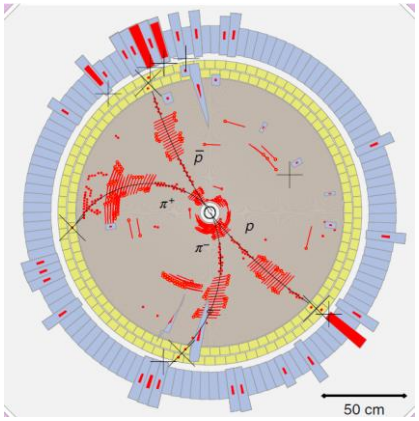
Particle spin as Qubit: Hyperon decay

Spin-1/2 hyperons (Σ, Λ, Ξ) can be regarded as Qubits in particle physics that can be measured through weak decays.

nature physics

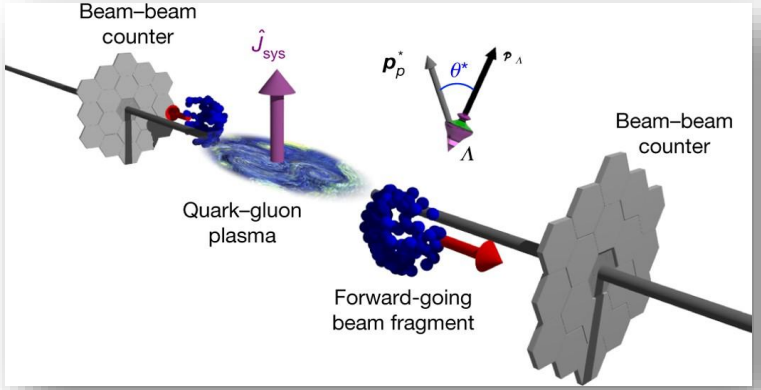
LETTERS

https://doi.org/10.1038/s41567-019-0494-8



Polarization and entanglement in baryon–antibaryon pair production in electron–positron annihilation

The BESIII Collaboration*



Letter | Published: 03 August 2017

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration

Nature **548**, 62–65 (2017) | Cite this article

See talks of Hai-Bo Li and Tao Luo

Hyperon's weak decay can serve as its own polarimeter.

weak decay law

$$\frac{dN}{d\cos\theta} \propto 1 + \alpha \mathcal{P}_B \cos\theta$$

Quantum measurement

The measurement processes in quantum physics are subject to a collection of measurement operators $\{M_m\}$. When we use the density operator ρ to describe a system being measured, the probability of obtaining a certain outcome m is given by

$$P(m) = Tr\left(M_m \rho M_m^\dagger\right) > 0, \qquad \sum_m P(m) = 1$$

where $\{P(m)\}$ are probability distributions for all possible outcomes.

These two conditions lead to constraints on $\{M_m\}$, the positive semi-definiteness and completeness conditions:

$$M_m^\dagger M_m \geq 0, \qquad \sum_m M_m^\dagger M_m = 1$$

Based on the measurement postulate in quantum mechanics, the initial state ρ instantaneously transforms to the state ρ_m after the measurement

$$\rho \to \rho_m = \frac{M_m \rho M_m^\dagger}{P(m)}, \qquad Tr(\rho_m) = 1$$

The resulting quantum states are described by $\{P(m), \rho_m\}$.

Quantum measurement

The post-measurement state is then

Wu, Qian, Yang, Q. Wang, CPL 41,110301 (2024)

$$\sum_m \boldsymbol{P}(\boldsymbol{m})\boldsymbol{\rho}_{\boldsymbol{m}} = \sum_m \boldsymbol{M}_{\boldsymbol{m}}\boldsymbol{\rho}\boldsymbol{M}_{\boldsymbol{m}}^{\dagger}$$

which can be taken as a quantum evolution generated by the measurement. This process is often characterized as a quantum channel

$$\boldsymbol{\rho} \to \mathcal{E}(\boldsymbol{\rho}) = \sum_m \boldsymbol{M}_{\boldsymbol{m}}\boldsymbol{\rho}\boldsymbol{M}_{\boldsymbol{m}}^{\dagger}$$

and the set of $\{\boldsymbol{M}_{\boldsymbol{m}}\}$ is called Kraus operators.

Hyperon decay as quantum measurement

$$P(\boldsymbol{p}) = \mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^{\dagger}\right) = \frac{1}{4\pi}\left(1+\alpha_B \boldsymbol{s}_B\cdot\hat{\boldsymbol{p}}\right), \quad M_{\boldsymbol{p}} = \frac{S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}}{\sqrt{4\pi\left(|S|^2+|P|^2\right)}}$$

$$M_{\boldsymbol{p}}^{\dagger}M_{\boldsymbol{p}} = \frac{1}{4\pi}\left(1+\alpha_B\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right), \quad \int d\Omega_{\boldsymbol{p}} M_{\boldsymbol{p}}^{\dagger}M_{\boldsymbol{p}} = 1$$

Particle spin as Qubit: Hyperon decay

A typical weak decay process for a spin-1/2 hyperon (anti-hyperon) is $B \to B'M$ ($\bar{B} \to \bar{B}'\bar{M}$). In the rest frame of B, the decay amplitude is

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\bar{\psi}_p \gamma^\mu (g_V - g_A \gamma_5) \psi_\Lambda\right] \partial_\mu \phi$$

$$\begin{aligned}\mathcal{A}_{B\to B'M} =& G_F m_M^2 \bar{u}_{B'}(s', \boldsymbol{p})(C_1 - C_2\gamma_5) u_B(s, 0) \\ =& G_F m_M^2 \sqrt{2m_B(E_{B'} + m_{B'})}\, \chi^\dagger_{s'} \left(S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\chi_s\end{aligned}$$

S-wave + P-wave

Dirac spinors for particles

$$u(s, \boldsymbol{p}) = \sqrt{E_p + m} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p + m}\chi_s \end{pmatrix}$$

$$u(s, 0) = \sqrt{2m} \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}$$

spin quantization direction

$$\boldsymbol{s}_B = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$\chi_+ = \begin{pmatrix} e^{-i\phi}\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix} \qquad \chi_- = \begin{pmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

$$(\boldsymbol{\sigma}\cdot\boldsymbol{s}_B)\chi_s = s\chi_s$$

Pauli spinors

Hyperon decay as quantum measurement

Squared amplitude: sum over the daughter spin

Wu, Qian, Yang, Q. Wang, CPL 41,110301 (2024)

$$\sum_{s'}|\mathcal{A}_{B\to B'M}|^2 \propto \chi_s^\dagger\left(S^*+P^*\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\sum_{s'}\chi_{s'}\chi_{s'}^\dagger\left(S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\chi_s$$

$$=\chi_s^\dagger\left(S^*+P^*\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\left(S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\chi_s$$

$$=\mathrm{Tr}\left[\left(S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\rho_B\left(S^*+P^*\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\right]$$

$$\propto\mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger\right)$$

Measurement operator

$$M_{\boldsymbol{p}}=\frac{S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}}{\sqrt{4\pi\left(|S|^2+|P|^2\right)}}$$

We assume that the hyperon is in the spin up state (100% polarization) along the direction $\vec{s}_B$, then the spin density operator

$$\rho_B=\chi_+\chi_+^\dagger$$

$$=\begin{pmatrix}e^{-i\phi}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{pmatrix}\left(e^{i\phi}\cos\frac{\theta}{2},\sin\frac{\theta}{2}\right)$$

$$=\frac{1}{2}(1+\boldsymbol{\sigma}\cdot\boldsymbol{s}_B)$$

$$P(\boldsymbol{p})=\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_{\boldsymbol{p}}}=\mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger\right)$$

$$=C\mathrm{Tr}\left[\left(S^*+P^*\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\left(S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\rho_B\right]$$

$$=\frac{1}{8\pi}\mathrm{Tr}\left[\left(1+\alpha\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)\left(1+\boldsymbol{\sigma}\cdot\boldsymbol{s}_B\right)\right]$$

$$=\frac{1}{4\pi}\left(1+\alpha_B\boldsymbol{s}_B\cdot\hat{\boldsymbol{p}}\right)$$

Decay constant

$$\alpha_B=\frac{2\mathrm{Re}(S^*P)}{|S|^2+|P|^2}$$

$\cos\theta$

Hyperon decay as quantum measurement

According to the quantum measurement postulate, the spin density operator of the daughter baryon can be obtained via the post-measurement state

$$\rho_{B'}(\hat{\boldsymbol{p}}) = \frac{M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger}{\mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger\right)} = \frac{1}{2}\left(1+\boldsymbol{\sigma}\cdot\boldsymbol{s}_{B'}\right) \qquad M_{\boldsymbol{p}} = \frac{S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}}{\sqrt{4\pi\left(|S|^2+|P|^2\right)}}$$

$$\boldsymbol{s}_{B'} = \frac{\alpha_B\hat{\boldsymbol{p}} - \beta_B(\hat{\boldsymbol{p}}\times\boldsymbol{s}_B) + \gamma_B\boldsymbol{s}_B + (1-\gamma_B)(\hat{\boldsymbol{p}}\cdot\boldsymbol{s}_B)\hat{\boldsymbol{p}}}{1+\alpha\boldsymbol{s}_B\cdot\hat{\boldsymbol{p}}}$$

$$\alpha_B = \frac{2\mathrm{Re}(S^*P)}{|S|^2+|P|^2} \qquad \beta_B = \frac{2\mathrm{Im}(S^*P)}{|S|^2+|P|^2} \qquad \gamma_B = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}$$

Wu, Qian, Yang, Q. Wang, CPL 41,110301 (2024)

where $\boldsymbol{s}_{B'}$ is the polarization vector of the daughter baryon in the mother baryon's rest frame as a function of $\hat{\boldsymbol{p}}$, $\boldsymbol{\alpha_B}$, $\boldsymbol{\beta_B}$ and $\boldsymbol{\gamma_B}$ are three real parameters in the weak decay satisfying $\boldsymbol{\alpha_B^2+\beta_B^2+\gamma_B^2=1}$. [T. D. Lee and C. N. Yang, Phys. Rev. 108, 1645 (1957)].

In the hyperon decay process, the daughter baryon may fly in any direction $\hat{\boldsymbol{p}}$ associated with the probability $\boldsymbol{P(\hat{p})}$, which is just the angular distribution of the daughter baryon that can be detected in particle physics experiments.

Hyperon decay as quantum measurement

With the daughter's spin density operator $\rho_{B'}(\hat{p})$, we have an ensemble $\{P(\hat{p}), \rho_{B'}(\hat{p})\}$. According to the generalized measurement postulate, this post-measurement ensemble can be interpreted as a quantum evolution in the channel $\mathcal{E}$ as

$$\begin{aligned}\mathcal{E}(\rho_B) = &\int d\Omega_{\boldsymbol{p}} M_{\boldsymbol{p}} \rho_B M_{\boldsymbol{p}}^{\dagger} = \frac{1}{8\pi}\int d\Omega\,[1+\alpha_B(\hat{\boldsymbol{p}}\cdot\boldsymbol{s}_B)] \\ &+\frac{1}{8\pi}\boldsymbol{\sigma}\cdot\int d\Omega\,[\alpha_B\hat{\boldsymbol{p}}-\beta_B(\hat{\boldsymbol{p}}\times\boldsymbol{s}_B)+\gamma_B\boldsymbol{s}_B+(1-\gamma_B)(\hat{\boldsymbol{p}}\cdot\boldsymbol{s}_B)\hat{\boldsymbol{p}}] \\ =&\frac{1}{2}\left[1+\frac{1}{3}(1+2\gamma_B)\boldsymbol{\sigma}\cdot\boldsymbol{s}_B\right]\end{aligned}$$

Wu, Qian, Yang, Q. Wang, CPL 41,110301 (2024)

The term $(1+2\gamma_B)/3$ just represents the average polarization of the daughter baryon in the rest frame of the mother hyperon.

A chain of decay $B \to B_1 M_1 \to B_2 M_2 M_1$ can be described by the concatenate quantum measurement with joint probability

$$P(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{1}{\Gamma}\frac{d^2\Gamma}{d\Omega_1 d\Omega_2} = \mathrm{Tr}\left[M_{\boldsymbol{p}2}^{1\to2} M_{\boldsymbol{p}1}^{B\to1} \rho_B M_{\boldsymbol{p}1}^{\dagger B\to1} M_{\boldsymbol{p}2}^{\dagger 1\to2}\right]$$

Joint decay of $B\bar{B}$ and spin-spin correlation

The spin state of two spin-1/2 particles such as $B\bar{B}$ is described by spin density operator

$$\rho_{B\bar{B}} = \frac{1}{4}\left(1 + \boldsymbol{s}_B \cdot \boldsymbol{\sigma} \otimes 1 + 1 \otimes \boldsymbol{s}_{\bar{B}} \cdot \boldsymbol{\sigma} + \sum_{ij} C_{ij}\sigma_i \otimes \sigma_j\right)$$

$$\boldsymbol{s}_B = \langle \boldsymbol{\sigma} \otimes 1 \rangle$$

$$\boldsymbol{s}_{\bar{B}} = \langle 1 \otimes \boldsymbol{\sigma} \rangle$$

$$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$$

There are 15 real parameters: $\boldsymbol{s_B}$ (3), $\boldsymbol{s_{\bar{B}}}$ (3), $\boldsymbol{C_{ij}}$ (9).

symmetric and real matrix

The joint Hilbert space associated with spin states of $B\bar{B}$ is denoted as $\mathscr{H}_B \otimes \mathscr{H}_{\bar{B}}$ respectively. The one particle density operator can be obtained by taking the partial trace

$$\rho_B = Tr_{\bar{B}}(\rho_{B\bar{B}}) = \frac{1}{2}(1 + \boldsymbol{s}_B \cdot \boldsymbol{\sigma})$$

$$\rho_{\bar{B}} = Tr_B(\rho_{B\bar{B}}) = \frac{1}{2}(1 + \boldsymbol{s}_{\bar{B}} \cdot \boldsymbol{\sigma})$$

According to the quantum measurement postulate, a joint decay process can be regarded as parallel quantum measurement which gives the joint probability

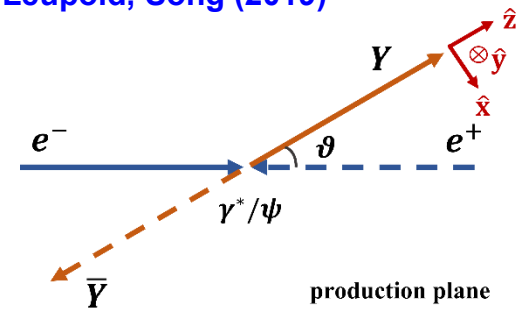
$$P(\boldsymbol{p}, \bar{\boldsymbol{p}}) = Tr\left[\left(M_{\boldsymbol{p}} \otimes M_{\bar{\boldsymbol{p}}}\right)\rho_{B\bar{B}}\left(M_{\boldsymbol{p}}^{\dagger} \otimes M_{\bar{\boldsymbol{p}}}^{\dagger}\right)\right]$$

Spin-spin correlation in $e^+e^- \to J/\psi \to Y\bar{Y}$

We choose the helicity rest frame as

Faldt, Kupsc (2017);
Perotti, Faldt, Kupsc, Leupold, Song (2019)

$$\hat{y} = -\hat{p}_e \times \hat{p}_Y, \quad \hat{z} = \hat{p}_Y, \quad \hat{x} = \hat{y} \times \hat{z}$$



The spin density operator for $Y\bar{Y}$ can be rewritten as

$$\rho_{Y\bar{Y}} = \frac{1}{4}\Theta_{\mu\nu}\sigma_\mu \otimes \sigma_\nu$$

$\mu, \nu = (0, i) = (0, 1, 2, 3)$,
do not distinguish upper or lower indices

$$\sigma_\mu = (1, \boldsymbol{\sigma}), \ \Theta_{i0} = s_{Yi}, \ \Theta_{0j} = s_{\bar{Y}j}, \ \Theta_{ij} = C_{ij}$$

$$\cos\theta = \hat{\mathbf{p}}_e \cdot \hat{\mathbf{p}}_Y$$

In the rest frames of Y and $\bar{Y}$ we have

$$\Theta_{\mu\nu} = \frac{1}{1+\alpha_\psi \cos^2\theta}\begin{bmatrix} 1+\alpha_\psi\cos^2\theta & 0 & \beta_\psi \sin\theta\cos\theta & 0 \\ 0 & \sin^2\theta & 0 & \gamma_\psi \sin\theta\cos\theta \\ \beta_\psi \sin\theta\cos\theta & 0 & -\alpha_\psi \sin^2\theta & 0 \\ 0 & \gamma_\psi \sin\theta\cos\theta & 0 & \alpha_\psi + \cos^2\theta \end{bmatrix}$$

$$\alpha_\psi = \frac{s - 4M^2\,|G_E/G_M|^2}{s + 4M^2\,|G_E/G_M|^2} \in [-1, 1]$$

$$\Delta\Phi = \arg\{G_E/G_M\} \in (-\pi, \pi]$$

$$\beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi), \quad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$$

Local unitary equivalence and X states

Before our investigation of Bell nonlocality, it is convenient to transform the two-qubit state described by $\Theta_{\mu\nu}$ to the X state by swapping the y and z axes and then diagonalizing C_{ij} in Y and $\bar{Y}$'s rest frame

$$\rho_{Y\bar{Y}}^{X} = (U_Y \otimes U_{\bar{Y}})\, \rho_{Y\bar{Y}}\, (U_Y \otimes U_{\bar{Y}})^{\dagger} = \frac{1}{4}\left(1 + a\sigma_z \otimes 1 + 1 \otimes a\sigma_z + \sum_i t_i \sigma_i \otimes \sigma_i\right)$$

Corresponding to

$$\Theta_{\mu\nu}^{X} = \left[\begin{array}{c|ccc} 1 & 0 & 0 & a \\ \hline 0 & t_1 & 0 & 0 \\ 0 & 0 & t_2 & 0 \\ a & 0 & 0 & t_3 \end{array}\right]$$

$$a = \frac{\beta_\psi \sin\theta\cos\theta}{1 + \alpha_\psi \cos^2\theta}$$

$$t_{1,2} = \frac{1 + \alpha_\psi \pm \sqrt{(1 + \alpha_\psi \cos 2\theta)^2 - \beta_\psi^2 \sin^2 2\theta}}{2(1 + \alpha_\psi \cos^2\theta)}$$

$$t_3 = \frac{-\alpha_\psi \sin^2\theta}{1 + \alpha_\psi \cos^2\theta}$$

The states described by $\rho_{Y\bar{Y}}$ and $\rho_{Y\bar{Y}}^{X}$ are said to be local unitary equivalent in the sense that they have same quantum correlation properties such as Bell nonlocality and entanglement.

X state for spin density matrix

Using the definition of the direct product of two 2×2 matrices

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \otimes \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} A_{11}\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} & A_{12}\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \\ A_{21}\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} & A_{22}\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \end{pmatrix}$$

The explicit form of $\rho^{X}_{Y\bar{Y}}$ is

$$\rho^{X}_{Y\bar{Y}} = \frac{1}{4}\left(1 + a\sigma_z \otimes 1 + 1 \otimes a\sigma_z + \sum_i t_i \sigma_i \otimes \sigma_i\right)$$

$$= \frac{1}{4}\begin{pmatrix} 1+2a+t_3 & 0 & 0 & t_1 - t_2 \\ 0 & 1 - t_3 & t_1 + t_2 & 0 \\ 0 & t_1 + t_2 & 1 - t_3 & 0 \\ t_1 - t_2 & 0 & 0 & 1 - 2a + t_3 \end{pmatrix}$$

X state looks like X-shape

Bell-nonlocality in $Y\bar{Y}$ system

The nonlocal property in a quantum entangled system can be tested by the violation of Bell inequality. The most widely used Bell-type inequality is the CHSH inequality

$$|\langle \boldsymbol{a}_1\cdot\boldsymbol{\sigma}\otimes\boldsymbol{b}_1\cdot\boldsymbol{\sigma}\rangle+\langle \boldsymbol{a}_1\cdot\boldsymbol{\sigma}\otimes\boldsymbol{b}_2\cdot\boldsymbol{\sigma}\rangle+\langle \boldsymbol{a}_2\cdot\boldsymbol{\sigma}\otimes\boldsymbol{b}_1\cdot\boldsymbol{\sigma}\rangle-\langle \boldsymbol{a}_2\cdot\boldsymbol{\sigma}\otimes\boldsymbol{b}_2\cdot\boldsymbol{\sigma}\rangle|\leq 2$$

where $\langle O\rangle=\mathrm{Tr}\left[\rho_{Y\bar{Y}}O\right]$ and $\boldsymbol{a}_1,\ \boldsymbol{a}_2,\ \boldsymbol{b}_1,\ \boldsymbol{b}_2$ are four directions (unit vectors) along which the spin polarization is measured. The inequality can be rewritten in a simpler form

$$|\boldsymbol{a}_1\cdot C\cdot(\boldsymbol{b}_1+\boldsymbol{b}_2)+\boldsymbol{a}_2\cdot C\cdot(\boldsymbol{b}_1-\boldsymbol{b}_2)|\leq 2$$

C. Qian, J.-L. Li, A.S. Khan, C.-F. Qiao, PRD(2020)
C. Qian, Y.-G. Yang, C.-F. Qiao, QW, PRA (2021)
S. Wu, C. Qian, QW, X.-R. Zhou, PRD(2024)

with C being the spin correlation matrix C_{ij}. The Bell non-locality is defined as the maximum of the left-hand side for all possible directions

$$\mathcal{B}[\rho]\equiv\max_{\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{b}_1,\boldsymbol{b}_2}|\boldsymbol{a}_1\cdot C\cdot(\boldsymbol{b}_1+\boldsymbol{b}_2)+\boldsymbol{a}_2\cdot C\cdot(\boldsymbol{b}_1-\boldsymbol{b}_2)|$$

Horedecki, et al.,
Phys. Lett. A200, 340 (1995)

$$=2\sqrt{m_1+m_2}$$ ⟹ **m_1 and m_2 are are two largest eigenvalues of C^TC**

Effective method for Bell-CH inequalities

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Class of Bell-Clauser-Horne inequalities for testing quantum nonlocality

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Quantum nonlocality, one of the most important features of quantum mechanics, is normally connected in experiments with the violation of Bell-Clauser-Horne (Bell-CH) inequalities. We propose effective methods for the rearrangement and linear inequality to prove a large variety of Bell-CH inequalities. We also derive a set of Bell-CH inequalities using these methods which can be violated in some quantum entangled states.

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Bell-nonlocality in $Y\bar{Y}$ system

Therefore, the CHSH inequality can be violated iff (if and only if) $m_1 + m_2 > 1$ and the maximum possible violation of the CHSH inequality is the upper bound value $2\sqrt{2}$. We define $m_{12}[\rho] = m_1 + m_2$ to be a measure of the Bell nonlocality.

Since we have put the density operator into the X form, we have

$$C = \mathrm{diag}\{t_1, t_2, t_3\}, \quad C^{\mathrm{T}}C = \mathrm{diag}\{t_1^2, t_2^2, t_3^2\}$$

The measure of the Bell nonlocality reads

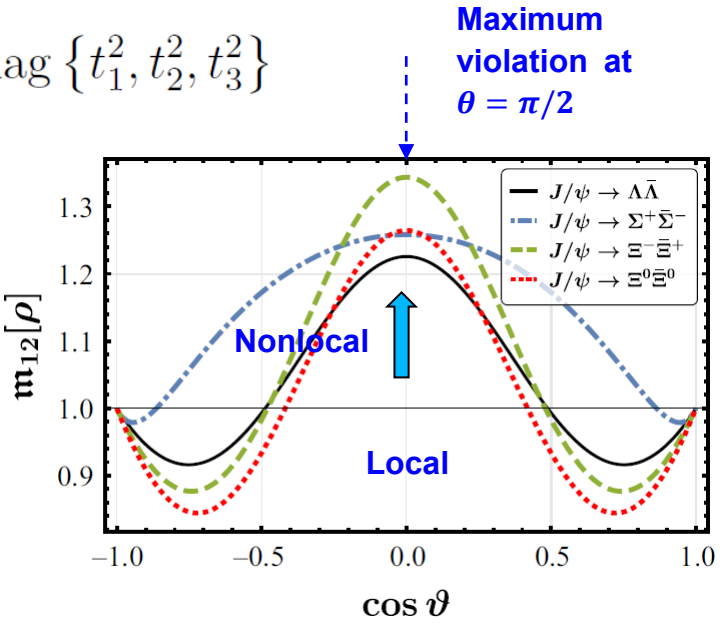
$$m_{12}\left[\rho^X_{Y\bar{Y}}\right] = \begin{cases} t_1^2 + t_2^2, & \alpha_\psi \geq 0 \\ \max\{t_1^2 + t_2^2, t_1^2 + t_3^2\}, & \alpha_\psi < 0 \end{cases}$$

$$\max_\theta \mathcal{B}\left[\rho^X_{Y\bar{Y}}\right] = 2\sqrt{1+\alpha_\psi^2}$$

Maximum violation at $\theta = \pi/2$

$$\theta^* = \arctan\left|\sqrt{2-2\alpha_\psi^2}\frac{\sin\Delta\Phi}{\alpha_\psi}\right|, \quad \text{for } \alpha_\psi \geq 0$$

Non-local range: $(\theta^*, \pi - \theta^*)$



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Quantum entanglement in $Y\bar{Y}$ system

For a bipartite quantum system a state is said to be separable iff the density matrix can be factorized as

$$\rho_{AB} = \sum_k p_k \rho_A^k \otimes \rho_B^k \qquad p_k \geq 0 \text{ and } \sum_k p_k = 1$$

Moreover, the state cannot be decomposed into the above form is called non-separable or entangled.

The Peres-Horodecki criterion provides a sufficient and necessary condition for separability [Peres, Phys. Rev. Lett. 77, 1413 (1996); Horedecki, et al., Phys. Lett. A200, 340 (1995)]: A state ρ_{AB} is separable iff its partial transpose $\rho_{\mathrm{AB}}^{T_B}$ with respect to the second subsystem is positive semi-definite. The Peres-Horodecki criterion is also called Positive Partial Transpose (PPT) criterion.

$$\begin{aligned}\rho_{Y\bar{Y}}^{T_{\bar{Y}}} =& \frac{1}{4}\left(1 + a\sigma_z \otimes 1 + 1 \otimes a\sigma_z + \sum_i t_i \sigma_i \otimes \sigma_i^T\right) \\ =& \frac{1}{4}\left(1 + a\sigma_z \otimes 1 + 1 \otimes a\sigma_z + \sum_i t_i' \sigma_i \otimes \sigma_i\right), \quad t' = (t_1, -t_2, t_3)\end{aligned}$$

Quantum entanglement in $Y\overline{Y}$ system

The concurrence is an entanglement monotone, so it can be regarded as a measure for the entanglement [Wootters, Phys. Rev. Lett. 80, 2245 (1998)]

$$\mathcal{C}[\rho] \equiv \max\{0, \mu_1 - \mu_2 - \mu_3 - \mu_4\}$$

$$\mathcal{C}[\rho] = 0, \quad separable$$
$$\mathcal{C}[\rho] > 0, \quad entangled$$
$$\mathcal{C}[\rho] = 1, \quad maximally\ entangled$$

where μ_i $(i = 1, 2, 3, 4)$ are the eigenvalues (in the decreasing order) of the Hermitain matrix $R(\rho) = \rho\tilde{\rho}$ with $\tilde{\rho} = \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$.

The Peres-Horodecki criterion for a general X state: the state is entangled iff

$$\rho_{22}^X \rho_{33}^X < \left|\rho_{14}^X\right|^2, \text{ or } \rho_{11}^X \rho_{44}^X < \left|\rho_{23}^X\right|^2$$

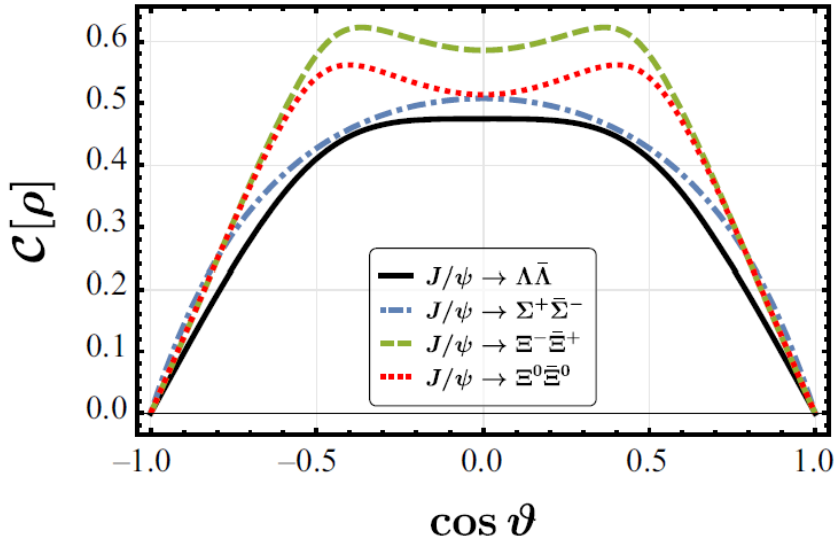
The Wootters' concurrence for the X state is given by

$$\mathcal{C}\left[\rho^X\right] = 2\max\left\{0, |\rho_{14}^X| - \sqrt{\rho_{22}^X \rho_{33}^X}, |\rho_{23}^X| - \sqrt{\rho_{11}^X \rho_{44}^X}\right\}$$
$$= |t_2| = \frac{\left|1 + \alpha_\psi - \sqrt{(1 + \alpha_\psi \cos 2\vartheta)^2 - \beta_\psi^2 \sin^2 2\vartheta}\right|}{2(1 + \alpha_\psi \cos^2 \vartheta)}$$

Quantum entanglement (concurrence)

The concurrence is an entanglement monotone, so it can be regarded as a measure for the entanglement [Wootters, Phys. Rev. Lett. 80, 2245 (1998)]

Wootters's concurrence $C(\rho^X_{Y\bar{Y}})$ for hyperon-antihyperon in $J/\psi \to Y\bar{Y}$ as functions of cosϑ (ϑ is the scattering angle), where $Y = \Lambda, \Sigma^+, \Xi^-, \Xi^0$ and corresponding to curves in black solid, blue dash-dotted, green dashed, and red dotted lines, respectively. The black horizontal line is the entanglement bound. The system is entangled iff $C(\rho) > 0$.



$$\mathcal{C}\left[\rho^X\right] = 2\max\left\{0, |\rho^X_{14}| - \sqrt{\rho^X_{22}\rho^X_{33}}, |\rho^X_{23}| - \sqrt{\rho^X_{11}\rho^X_{44}}\right\}$$
$$= |t_2| = \frac{\left|1 + \alpha_\psi - \sqrt{(1+\alpha_\psi\cos 2\vartheta)^2 - \beta^2_\psi \sin^2 2\vartheta}\right|}{2(1+\alpha_\psi\cos^2\vartheta)}$$

Quantum entanglement in $Y\overline{Y}$ system

The spin density operator for the hyperon-antihyperon system

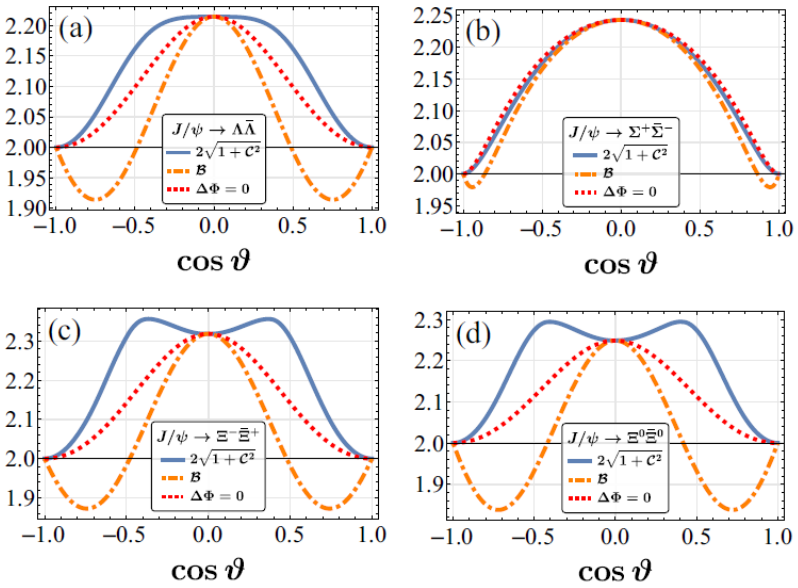
$$\rho^{X}_{Y\bar{Y}} = \frac{1}{4}\left(1 + a\sigma_z \otimes 1 + 1 \otimes a\sigma_z + \sum_i t_i \sigma_i \otimes \sigma_i\right)$$

For a two-qubit density operator ρ with Wootters' concurrence $C[\rho]$, the maximum violation of the CHSH inequality $B[\rho]$ has an upper bound

$$\mathcal{B}[\rho] = 2\sqrt{m_{12}} \leq 2\sqrt{1 + \mathcal{C}^2[\rho]}$$

equality holds at $\theta = \pi/2$

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Bell nonlocality ⊂ entanglement

$2\sqrt{m_{12}}$ **and** $2\sqrt{1+C^2}$

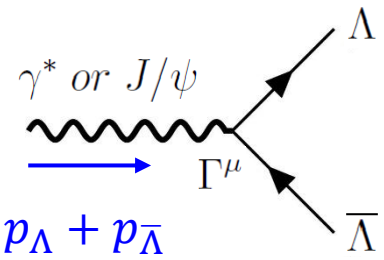
The equality $B[\rho] = 2\sqrt{1 + C^2[\rho]}$ holds at $\theta = \pi/2$. At this transverse scattering angle, Y's and $\overline{Y}$'s polarizations vanish, then $\rho^{X}_{Y\overline{Y}}$ reduces to a very special subclass of the X state: Bell Diagonal State (BDS).

Electromagnetic form factors

The role of time-like electromagnetic form factors in quantum nonlocality and entanglement

$$\Gamma^{\mu} = \gamma^{\mu} F_1(P^2) + i\frac{\sigma^{\mu\nu} P_{\nu}}{2M} F_2(P^2)$$

$$G_E(P^2) = F_1 + \frac{P^2}{4M^2} F_2, \quad G_M(P^2) = F_1 + F_2$$



$$P = p_{\Lambda} + p_{\overline{\Lambda}}$$

The spin polarization of Λ and $\overline{\Lambda}$ is

$$P_{\Lambda} = P_{\overline{\Lambda}} = \frac{\sqrt{1-\alpha_{\psi}^2}\sin\theta\cos\theta}{1+\alpha_{\psi}\cos^2\theta}\sin(\Delta\Phi)$$

$$\alpha_{\psi} = \frac{s - 4M^2\left|G_E/G_M\right|^2}{s + 4M^2\left|G_E/G_M\right|^2} \in [-1, 1]$$

$$\Delta\Phi = \arg\{G_E/G_M\} \in (-\pi, \pi]$$

phase difference between G_E and G_M

If $\Delta\Phi = 0$, $\rho_{Y\bar{Y}}$ is reduced to a BDS form

$$\rho_{Y\bar{Y}}^{\mathrm{BDS}} = \frac{1}{4}\left(1 \otimes 1 + \sum_i t_i \sigma_i \otimes \sigma_i\right)$$

$$m_{12}\left[\rho_{Y\bar{Y}}^{\mathrm{BDS}}\right] = 1 + \mathcal{C}^2\left[\rho_{Y\bar{Y}}^{\mathrm{BDS}}\right] = 1 + \left(\frac{\alpha_{\psi}\sin^2\theta}{1+\alpha_{\psi}\cos^2\theta}\right)^2 \geq 1$$

Bell nonlocality == entanglement at all scattering angles

Same as elementary particle pair production such as $t\bar{t}$ or $\tau^+\tau^-$

[Afik, et.al (2021-23); Ehataht, et.al (2024); T. Han, et.al (2025)]

A conjecture about the compositeness of fermions in pair production

In pair of production fermion-antifermion (spin-1/2) from vector vertex

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For elementary fermions we have

Spin Polarization of fermion/antifermion $= 0$
Bell nonlocality $m_{12}[\rho] = 1 + C^2[\rho]$ Entanglement (concurrence)

For composite fermions we have

Spin Polarization of fermion/antifermion $\neq 0$
Bell nonlocality $m_{12}[\rho] < 1 + C^2[\rho]$ Entanglement (concurrence)

Who ordered that? **We don't know yet.**

Quantum Steering: CJWR Steering Inequality

- **Steering (EPR steering): Alice can "steer" Bob's state iff measurement correlations violate the LHS model -- a nonlocality weaker than Bell nonlocality**
- **Cavalcanti–Jones–Wiseman–Reid (CJWR) three-setting steering inequality:**

$$F_3^{\mathrm{CJWR}} \equiv \frac{1}{\sqrt{3}} \left| \sum_{k=1}^{3} \mathrm{Tr}\left[\rho\left(A_k \otimes B_k\right)\right] \right| \leq 1$$

$$A_k = \mathbf{s}_k \cdot \boldsymbol{\sigma},\ B_k = \mathbf{r}_k \cdot \boldsymbol{\sigma},$$

s_k and r_k $(k = 1,2,3)$ are basis vectors

- **Maximal violation:** $\mathcal{F}_3[\rho] \equiv \max_{\mathbf{s}_k, \mathbf{r}_k} F_3^{\mathrm{CJWR}} = \sqrt{\mathrm{Tr}\left(C^{\mathrm{T}} C\right)}.$
- **For the X-state of YȲ (parameters a, t_1, t_2, t_3):**

$$\mathcal{F}_3\left[\rho_{Y\bar{Y}}\right] = \sqrt{t_1^2 + t_2^2 + t_3^2}$$

$$= \sqrt{1 + 2\left(\frac{\alpha_\psi \sin^2\vartheta}{1 + \alpha_\psi \cos^2\vartheta}\right)^2 - 2\left(\frac{\beta_\psi \sin\vartheta\cos\vartheta}{1 + \alpha_\psi \cos^2\vartheta}\right)^2}$$

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Steering Results in $e^+e^- \to J/\psi \to Y\bar{Y}$

- **All five YȲ systems violate the CJWR inequality within a range around θ = 90° (transverse scattering); F_3 is symmetric under θ → π − θ.**
- **Maximal violation at θ = 90° :**

$$\mathcal{F}_3 = \sqrt{1+2\alpha_\psi^2}$$

- **Steerability range (θ*, π − θ*):**

$$\vartheta^* = \arctan\left|\sqrt{1-\alpha_\psi^2}\frac{\sin\Delta\Phi}{\alpha_\psi}\right|$$

Σ has weaker peak but broader range than Ξ (small ΔΦ).

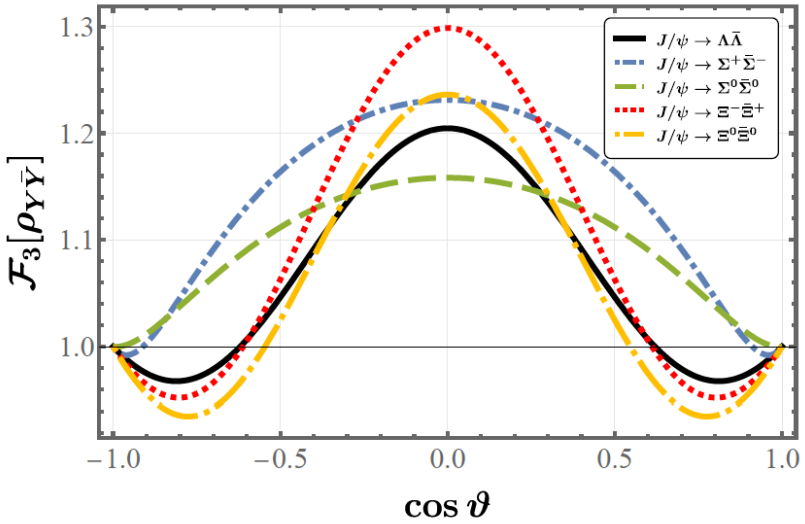


Figure 1. The quantity $\mathcal{F}_3[\rho_{Y\bar{Y}}]$ for the three-setting measurement as functions of $\cos\vartheta$ (ϑ is the scattering angle) in $e^+e^- \to J/\psi \to Y\bar{Y}$ with the black solid, blue dash-dotted, green dashed, red dotted and long yellow dash-dotted lines representing $Y = \Lambda$, Σ^+, Σ^0, Ξ^- and Ξ^0 respectively. The black horizontal line is the steering bound $\mathcal{F}_3 = 1$. The CJWR inequality is violated iff $\mathcal{F}_3 > 1$.

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Quantum Discord: Definition

- **Quantum mutual information is defined as**

$$\mathcal{I}(A:B) \equiv S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \\ = S(\rho_A) - S(\rho_{A|B}),$$

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with von Neumann entropy $S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$.

- **Perform projective measurements $\{\Pi_k\}$ on subsystem B → ensemble $\{p_k, \rho_{A|\Pi_k}\}$; classical correlation = max over all measurements of J(A:B)**

$$\mathcal{D}[\rho_{AB}] \equiv \mathcal{I}(A:B) - \max_{\{\Pi_k\}} \mathcal{J}(A:B) \\ = S(\rho_B) - S(\rho_{AB}) + \min_{\{\Pi_k\}} \sum_k p_k S(\rho_{A|\Pi_k})$$

$$\mathcal{J}(A:B) \equiv S(\rho_A) - \sum_k p_k S(\rho_{A|\Pi_k})$$

- **Discord is non-negative, ≤ 1, and generally asymmetric (measuring A vs. B). For $Y\bar{Y}$ states: CP symmetry makes ρ symmetric. Discord captures quantum correlation even without entanglement -- it is the most general of the four correlations.**

Discord for rank-2 X states and geometric discord

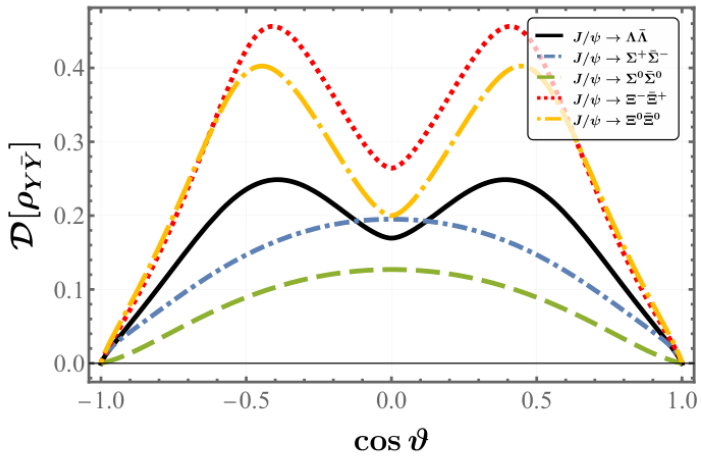


Figure 2. The quantum discord $\mathcal{D}[\rho_{Y\bar{Y}}]$ as functions of $\cos\vartheta$ (ϑ is the scattering angle) in $e^+e^- \to J/\psi \to Y\bar{Y}$ with $Y = \Lambda$, $\Sigma^{+,0}$ and $\Xi^{-,0}$ corresponding to curves in black solid, blue dash-dotted, green dashed, red dotted and long yellow dash-dotted lines respectively. The black horizontal line is $\mathcal{D} = 0$.

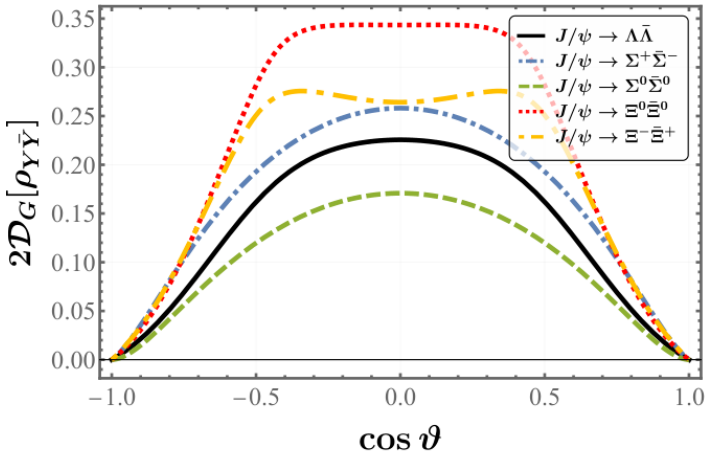


Figure 3. The results for the geometric quantum discord $2\mathcal{D}_G[\rho_{Y\bar{Y}}]$ as functions of $\cos\vartheta$ (ϑ is the scattering angle) in $e^+e^- \to J/\psi \to Y\bar{Y}$ with $Y = \Lambda$, Σ^+, Σ^0, Ξ^0 and Ξ^- corresponding to black solid, blue dash-dotted, green dashed, red dotted and long yellow dash-dotted lines respectively. The black horizontal line is the zero geometric discord $2\mathcal{D}_G = 0$.

$$\mathcal{D}[\rho_{Y\bar{Y}}] = h\left(\frac{1+a}{2}\right) - h\left(\frac{1+t_3}{2}\right) + h\left(\frac{1+\sqrt{t_1+t_3-t_1t_3}}{2}\right)$$

$$h(x) \equiv -x\log_2 x - (1-x)\log_2(1-x)$$

$$\mathcal{D}_G \equiv \min_{\chi\in\Omega_0} \|\rho - \chi\|^2$$

Ω_0: the set of zero discord state

$$\mathcal{D}_G[\rho_{Y\bar{Y}}] = \frac{1}{4}\min\left\{a^2 + t_2^2 + t_3^2, t_1^2 + t_2^2\right\}$$

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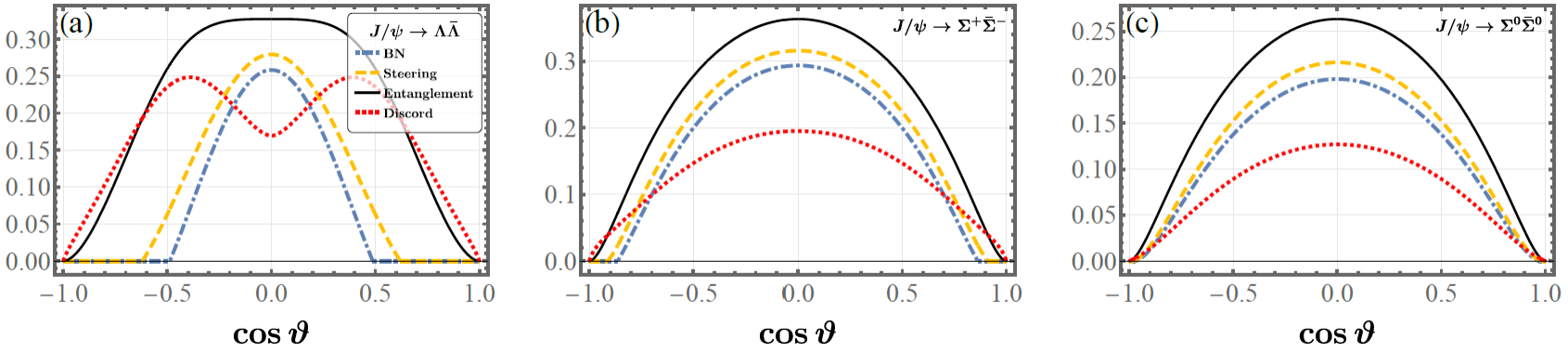
Four measures on a common scale

Bell nonlocality ⊂ steering ⊂ entanglement ⊂ discord

$$\mathscr{B} \equiv \max\left\{0, \frac{\mathcal{B}-2}{2\sqrt{2}-2}\right\} \in [0,1] \qquad \mathscr{S} \equiv \max\left\{0, \frac{\mathcal{F}_3-1}{\sqrt{3}-1}\right\} \in [0,1]$$

$$\mathscr{E} \equiv h\left(\frac{1+\sqrt{1-\mathcal{C}^2[\rho_{Y\bar{Y}}]}}{2}\right) \qquad \mathcal{D}[\rho_{Y\bar{Y}}] = h\left(\frac{1+a}{2}\right) - h\left(\frac{1+t_3}{2}\right) + h\left(\frac{1+\sqrt{t_1+t_3-t_1t_3}}{2}\right)$$

All four plotted measures lie in [0,1]. The hierarchy concerns sets of states, not a numerical ordering of these measures.



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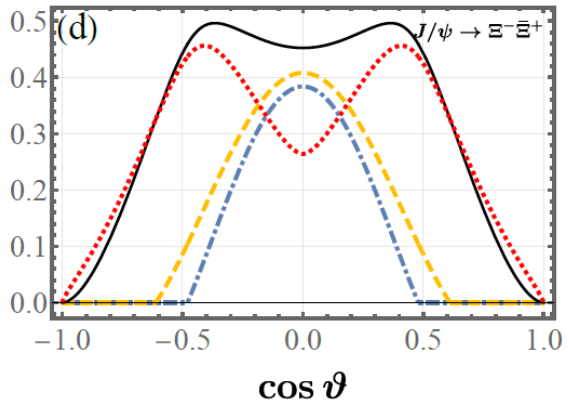
Four measures on a common scale

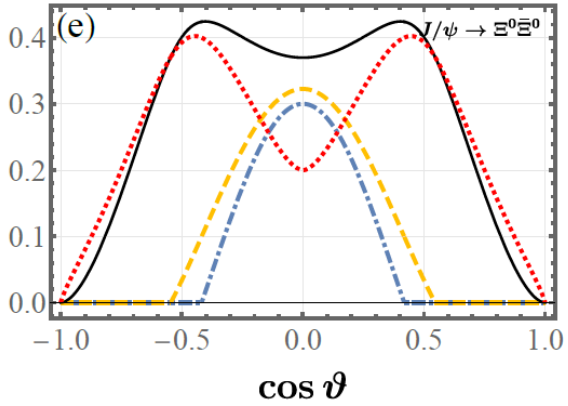
Bell nonlocality ⊂ steering ⊂ entanglement ⊂ discord

$$\mathscr{B} \equiv \max\left\{0, \frac{\mathcal{B}-2}{2\sqrt{2}-2}\right\} \in [0,1] \qquad \mathscr{S} \equiv \max\left\{0, \frac{\mathcal{F}_3-1}{\sqrt{3}-1}\right\} \in [0,1]$$

$$\mathscr{E} \equiv h\left(\frac{1+\sqrt{1-\mathcal{C}^2[\rho_{Y\bar{Y}}]}}{2}\right) \qquad \mathcal{D}[\rho_{Y\bar{Y}}] = h\left(\frac{1+a}{2}\right) - h\left(\frac{1+t_3}{2}\right) + h\left(\frac{1+\sqrt{t_1+t_3-t_1t_3}}{2}\right)$$

All four plotted measures lie in [0,1]. The hierarchy concerns sets of states, not a numerical ordering of these measures.





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The off-center structure is removed without phase

- **A controlled comparison sets $\Delta\Phi = 0$ while keeping a_ψ fixed.**
- **Then we have $a = 0$ and**

$$\mathscr{D}_{\Delta\Phi=0} = 1 - h\left(\frac{1+\alpha_\psi\cos 2\vartheta}{2(1+\alpha_\psi\cos^2\vartheta)}\right),$$

$$\mathscr{E}_{\Delta\Phi=0} = h\left(\frac{1}{2} + \frac{\sqrt{(1+\alpha_\psi)(1+\alpha_\psi\cos 2\vartheta)}}{2(1+\alpha_\psi\cos^2\vartheta)}\right),$$

$$\mathscr{S}_{\Delta\Phi=0} = \frac{\sqrt{1+2\left[\alpha_\psi\sin^2\vartheta/(1+\alpha_\psi\cos^2\vartheta)\right]^2}-1}{\sqrt{3}-1}$$

$$\mathscr{B}_{\Delta\Phi=0} = \frac{\sqrt{1+\left[\alpha_\psi\sin^2\vartheta/(1+\alpha_\psi\cos^2\vartheta)\right]^2}-1}{\sqrt{2}-1}.$$

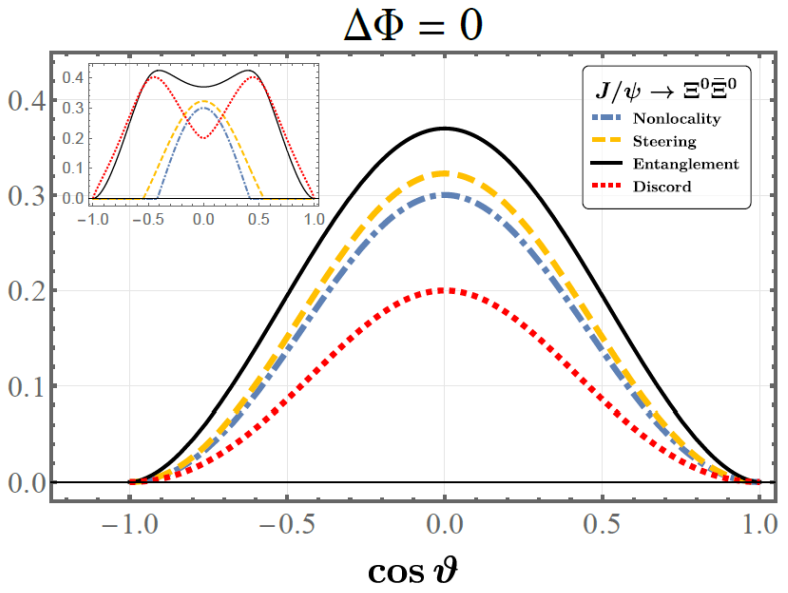


Figure 5. The quantum correlations for $\Xi^0\bar{\Xi}^0$ with $\Delta\Phi = 0$. The subfigure shows the result in Fig. 4 (e) for comparison. The Bell nonlocality $\mathscr{B}$, the steering $\mathscr{S}$, the entanglement $\mathscr{E}$, and the discord $\mathscr{D}$ are shown in blue dot-dashed, yellow dashed, black solid, and red dotted lines respectively.

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Quantum correlations as a probe of compositeness

- **Nonzero ΔΦ produces transverse polarization through the interference of G^E and G^M.**

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$$P_z = \frac{\sqrt{1-a_\psi^2}\,\sin\theta\cos\theta}{1+a_\psi(\cos\theta)^2}\sin\Delta\Phi$$

- **The complex form factors change the relation among the correlation measures.**
- **The paper contrasts these patterns with elementary fermion pair examples that satisfy $B = \sqrt{1+C^2}$.**
- **Proposed interpretation: correlation patterns may reveal internal hadronic structure. This is a conjecture, not an established test of compositeness.**

Spacelike separation of the two decays

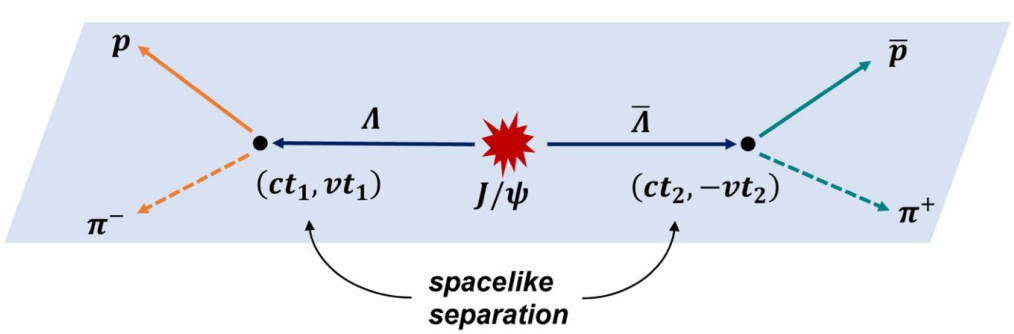
- **The condition for spacelike separation requires**

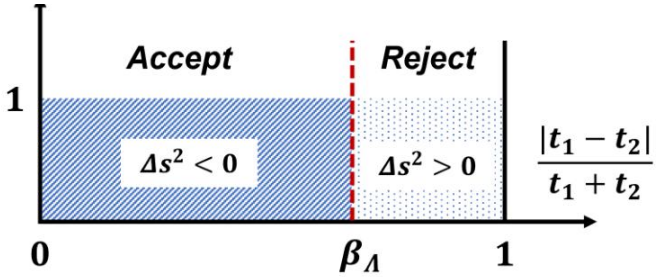
$$\Delta s^2 = c^2(t_1 - t_2)^2 - v^2(t_1 + t_2)^2 < 0,$$

- **Equivalent criterion:**

$$|t_1 - t_2|/(t_1 + t_2) < \beta_\Lambda, \beta_\Lambda = \sqrt{1 - 4M^2/s}$$

- $J/\psi \to \Lambda\bar{\Lambda}$: $\beta_\Lambda \approx 0.69$, **so about 69% of ideal decay pairs are spacelike.**
- **Higher energies improve this fraction. Event yields and other experimental loopholes still matter.**





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Detector materials motivate a decoherence model

- **Longer-lived hyperons can cross the beam pipe and interact with detector materials before decaying.**
- **For $\eta_c \to \Lambda\bar{\Lambda}$, a mean flight distance of 5.1 cm is compared with a 3.15 cm beam-pipe radius.**
- **Phenomenological model:**

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i\,[\mathbf{H},\rho] + \sum_{\ell}\left(\mathbf{L}_\ell \rho \mathbf{L}_\ell^\dagger - \frac{1}{2}\{\mathbf{L}_\ell^\dagger \mathbf{L}_\ell, \rho\}\right), \quad \Longrightarrow \quad \frac{\mathrm{d}\rho_\Lambda}{\mathrm{d}t} = -\frac{\gamma}{2}[\sigma_z,[\sigma_z,\rho_\Lambda]],$$

- **As the Λ traverses the beam pipe or detector materials, it may undergo weak, stochastic interactions with the surrounding medium. These processes typically act as repeated partial "measurements" of their spin projection along the momentum direction.**
- **The flight direction defines z. Dephasing suppresses transverse spin coherence and preserves z populations.**

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Detector materials motivate a decoherence model

The phase damping channel can be equivalently written in the operator-sum (Kraus) representation:

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$$\rho_\Lambda \longmapsto \mathcal{E}(\rho_\Lambda) \equiv \mathbf{K}_0 \rho_\Lambda \mathbf{K}_0 + \mathbf{K}_1 \rho_\Lambda \mathbf{K}_1$$

where the Kraus operators are

$$\mathbf{K}_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\zeta} \end{bmatrix}, \qquad \mathbf{K}_1 = \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{\zeta} \end{bmatrix} \qquad \sqrt{1-\zeta} \equiv e^{-2\gamma t}$$

Acting on the Bloch vector $\vec{B}^+ = (B_x^+, B_y^+, B_z^+)$, the channel yields

$$(B_x^+, B_y^+, B_z^+) \longmapsto (e^{-2\gamma t} B_x^+, e^{-2\gamma t} B_y^+, B_z^+)$$

where transverse components decay with the rate 2γ while the longitudinal component is conserved.

Detector materials motivate a decoherence model

For a two-qubit system, we assume that the antiparticle undergoes the same decoherence process as the particle. The complete two-qubit channel is then given by

$$\rho_{\Lambda\bar{\Lambda}} \mapsto \tilde{\rho}_{\Lambda\bar{\Lambda}} \equiv \mathcal{E} \otimes \mathcal{E}(\rho_{\Lambda\bar{\Lambda}})$$

Since decoherence corresponds to quantum information leaking into the environment, it reduces the correlation between Λ and $\overline{\Lambda}$. We can employ the concurrence as a quantitative measure to illustrate how decoherence degrades the entanglement of the $\Lambda\overline{\Lambda}$ system.

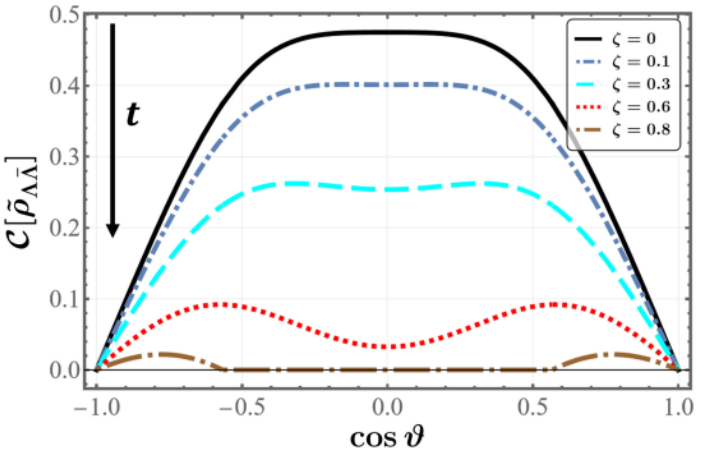


FIG. 8. Entanglement between Λ and $\bar{\Lambda}$ will decrease due to the quantum decoherence effect. The decoherence time t is related to the factor ζ as $\sqrt{1-\zeta} = e^{-2\gamma t}$.

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Spin correlation of hyperon-antihyperon systems in e^+e^- annihilation

Spin correlation of hyperon-antihyperon systems in e^+e^- annihilation

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Abstract

Quantum correlations in high-energy collisions provide a novel perspective on both fundamental physics and hadron structure. We calculate the quantum information observable in spin-1/2 particle-antiparticle systems produced in e^+e^- annihilation. Starting from the two-qubit density operator, we calculate the Bell variable, concurrence and negativity as functions of the scattering angle and collision energy for the $\Lambda\bar{\Lambda}$, $\Sigma^+\bar{\Sigma}^-$, and $\Lambda_c^+\bar{\Lambda}_c^-$ systems in the process $e^+e^- \to B\bar{B}$ at BESIII experiments. Unlike elementary particle-antiparticle systems, the hyperon-antihyperon system exhibits non-vanishing transverse polarization for B and $\bar{B}$ with respect to the production plane, which significantly restrict the kinematic region where the CHSH inequality is violated while leaving the entanglement largely unaffected. We also extend our analysis to the general case of spin-1/2 particle-antiparticle production, and explore the potential of using quantum correlations as probes to hadron structure, particularly to the parton-level entanglement inside heavy-flavored mesons.

$e^+e^- \to \Lambda\bar{\Lambda}$, $\Sigma^+\bar{\Sigma}^-$, $\Lambda_c\bar{\Lambda}_c$
(not through J/ψ)

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Summary

Take-home message:

Spin correlation is a new tool to study quantum properties of particles and sQGP.