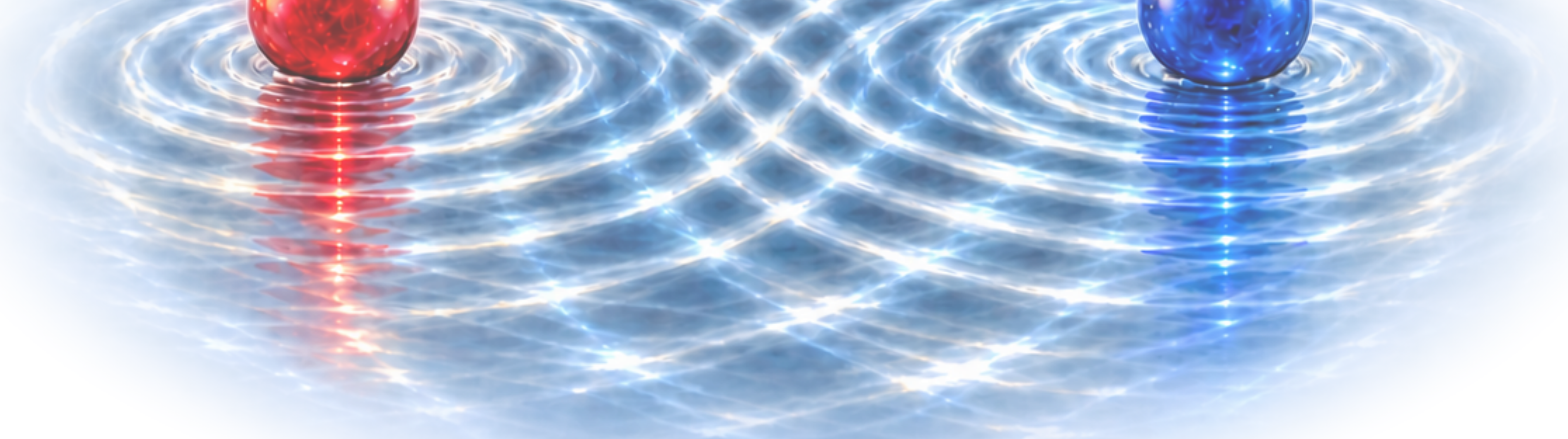
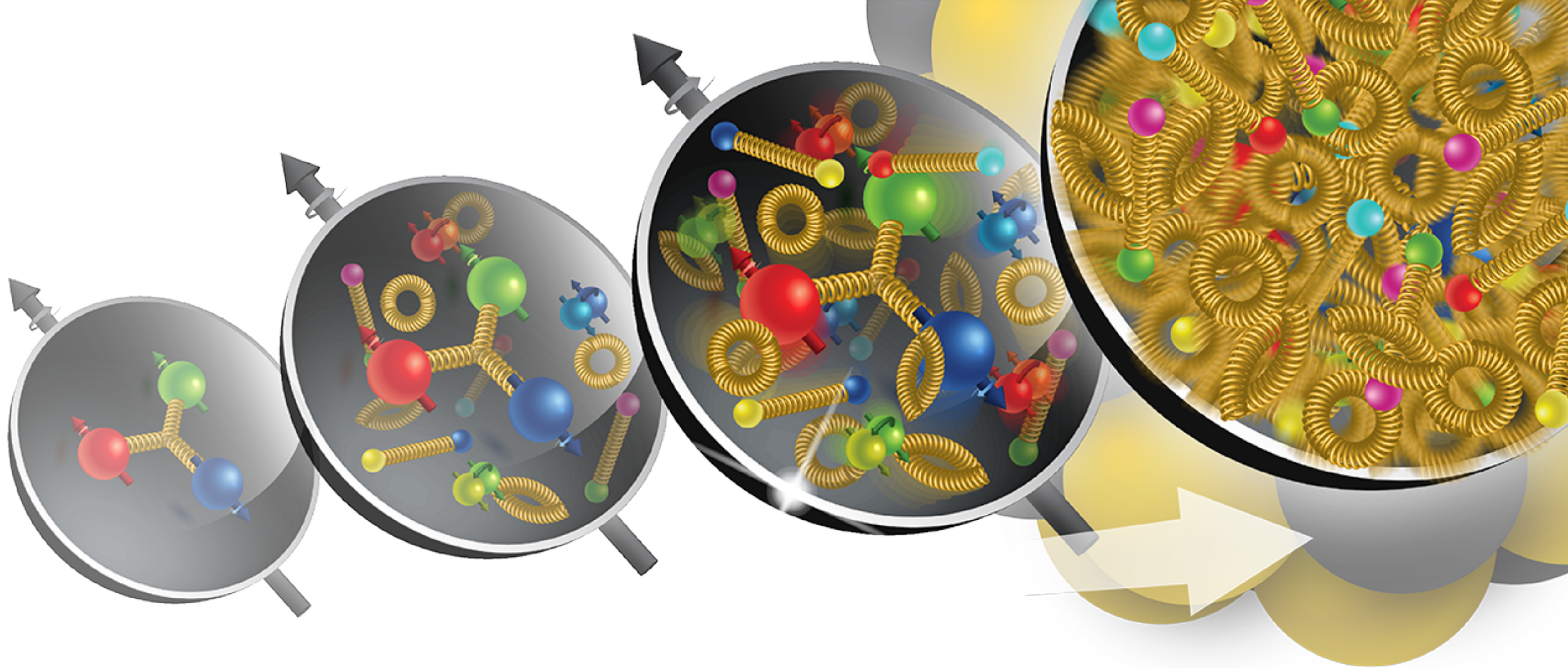




Quantum Entanglement and Quantum Nonlocality at High Energy Colliders

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Quantum Entanglement at High Energy Colliders

The “Verschränkung”



844

SCHRÖDINGER: Die gegenwärtige Situation in der Quantenmechanik.

[Die Natur-
wissenschaften

Die gegenwärtige Situation in der Quantenmechanik.

Von E. SCHRÖDINGER, Oxford.

(Schluß¹.)

Inhaltsübersicht.

- § 11. Die Aufhebung der Verschränkung. Das Ergebnis abhängig vom Willen des Experimentators.
- § 12. Ein Beispiel.
- § 13. Fortsetzung des Beispiels: alle möglichen Messungen sind eindeutig verschränkt.
- § 14. Die Änderung der Verschränkung mit der Zeit. Bedenken gegen die Sonderstellung der Zeit.
- § 15. Naturprinzip oder Rechenkunstgriff?

§ 11. *Die Aufhebung der „Verschränkung“. Das Ergebnis abhängig vom Willen des Experimentators.*

Wir kehren wieder zum allgemeinen Fall der „Verschränkung“ zurück, ohne gerade den besonderen Fall eines Meßvorgangs im Auge zu haben, wie soeben. Die Erwartungskataloge zweier Körper A und B sollen sich durch vorübergehende Wechselwirkung verschränkt haben. Jetzt sollen die Körper wieder getrennt sein. Dann kann ich einen davon, etwa B , hernehmen und meine unter-

kür des Experimentators anheimgestellt. Er braucht nicht besondere Variable auszuwählen, um die Konditionalsätze ausnützen zu können. Er darf sich ruhig einen Plan machen, der ihn zu maximaler Kenntnis von B führen würde, auch wenn er über B gar nichts wüßte. Es kann auch nichts schaden, wenn er diesen Plan zu Ende führt. Wenn er sich nach jeder Messung überlegt, ob er etwa schon am Ziel ist, so nur, um sich weitere überflüssige Arbeit zu ersparen.

Welcher A -Katalog sich solchermaßen indirekt ergibt, hängt selbstverständlich von den Maßzahlen ab, die an B auftreten (bevor die Verschränkung ganz gelöst ist; von den späteren, falls überflüssigerweise weitergemessen wird, nicht mehr). Gesetzt nun, ich hätte in einem bestimmten Fall auf solche Art einen A -Katalog erschlossen. Dann kann ich nachdenken und mir überlegen, ob ich vielleicht einen anderen gefunden haben würde.

Quantum Entanglement

- Quantum entanglement of mixed state

$$\rho_{AB} \stackrel{?}{=} \sum_{i=1}^N p_i \rho_A^{(i)} \otimes \rho_B^{(i)}, \quad \left(\sum_i p_i = 1, p_i > 0 \right)$$

- Bell states $\rho_{\pm} = |\Psi_{\pm}\rangle\langle\Psi_{\pm}| = 2^{-1}(|01\rangle\langle 01| \pm |10\rangle\langle 01| \pm |01\rangle\langle 10| + |10\rangle\langle 10|)$ are entangled (pure) states.
- But their mixing $(\rho_+ + \rho_-)/2 = (|0\rangle\langle 0| \otimes |1\rangle\langle 1| + |1\rangle\langle 1| \otimes |0\rangle\langle 0|)/2$ is **not**!
- Entanglement witness:

*A **functional** which can distinguish a **specific** entangled state from separable ones.*

- Theorem: For every entangled state ρ , there exists a Hermitian operator A such that $\text{Tr}(\rho A) < 0$, and $\text{Tr}(\sigma A) \geq 0$ for all separable states σ .

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The smart mathematicians tell us whether the answer we want exists, but do not tell us how to find it.

Quantum Entanglement

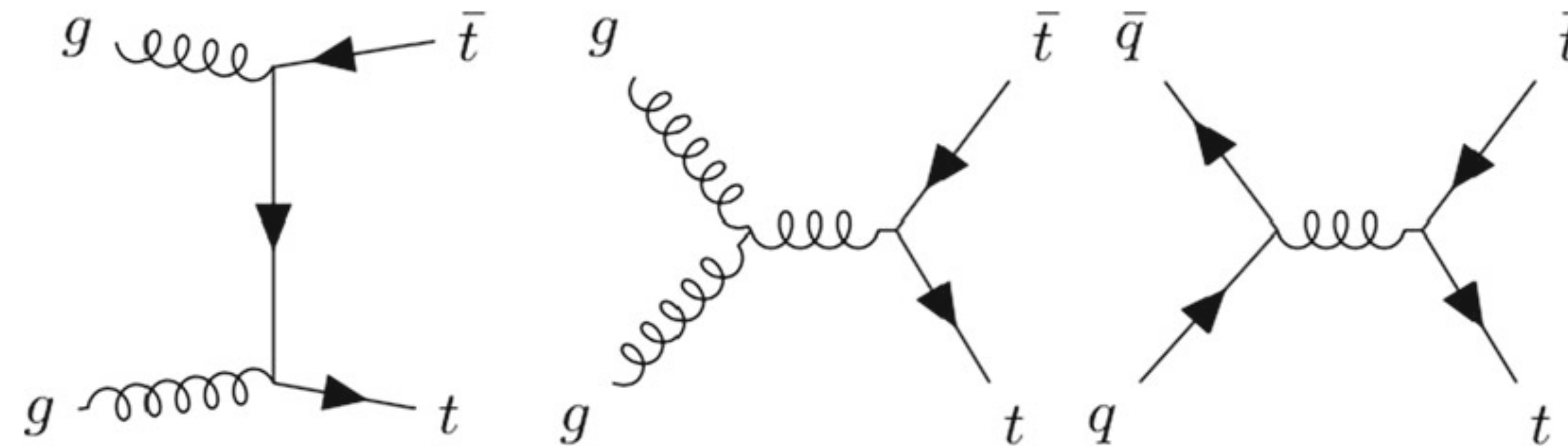
- The final state is a mixed state. How to verify the entanglement in the final state?
- For 2×2 and 2×3 system, it is solved.
- The concurrence: $\mathcal{C}[\hat{\rho}] = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, $0 \leq \mathcal{C}[\hat{\rho}] \leq 1$

$$\lambda_i \in \sigma \left(\sqrt{\sqrt{\hat{\rho}}(\sigma_2 \otimes \sigma_2)\hat{\rho}^*(\sigma_2 \otimes \sigma_2)\sqrt{\hat{\rho}}} \right)$$
- A state is entangled **iff** $\mathcal{C}[\hat{\rho}] > 0$.
- Taking the ttbar pair production as an example, one needs to reconstruct the “density matrix” in some basis in the spin space, e.g., $\{ |t_{\uparrow}\bar{t}_{\uparrow}\rangle, |t_{\uparrow}\bar{t}_{\downarrow}\rangle, |t_{\downarrow}\bar{t}_{\uparrow}\rangle, |t_{\downarrow}\bar{t}_{\downarrow}\rangle \}$.
- Such a density matrix can be decomposed into

$$\rho = \frac{\mathbb{I}_2 \otimes \mathbb{I}_2 + B(t)_i \sigma^i \otimes \mathbb{I}_2 + B(\bar{t})_i \mathbb{I}_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}$$

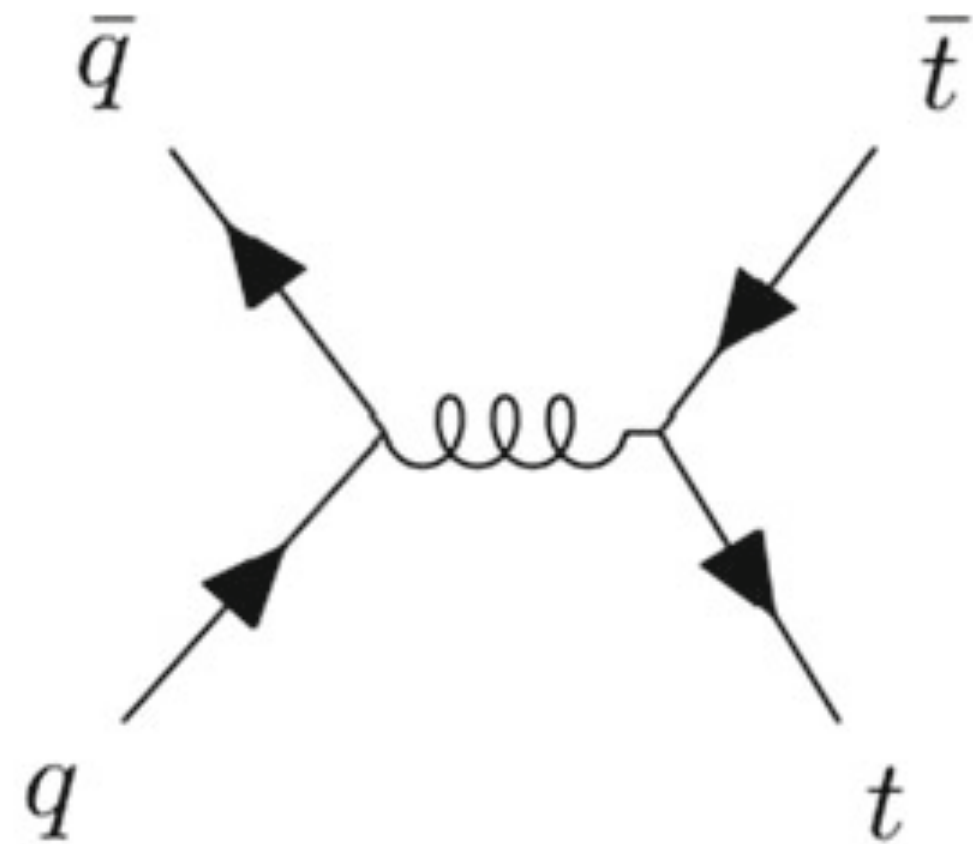
Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider:



Quantum Entanglement at Colliders

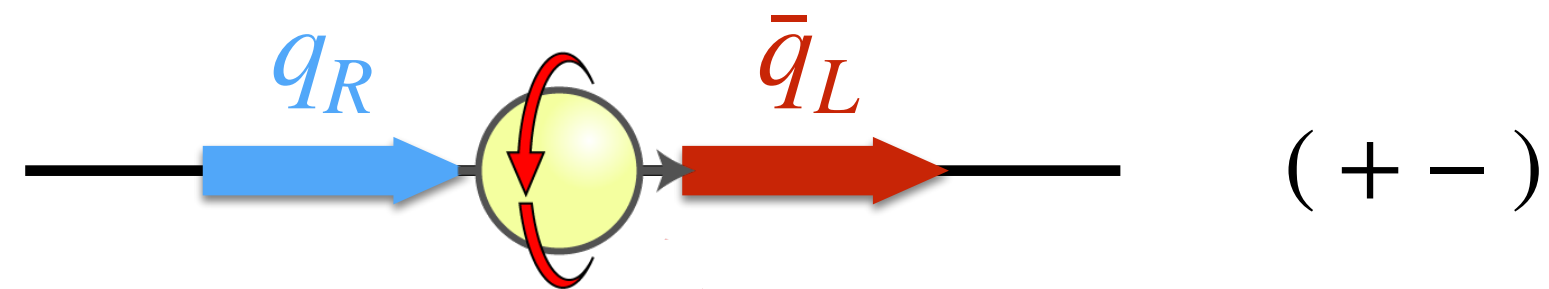
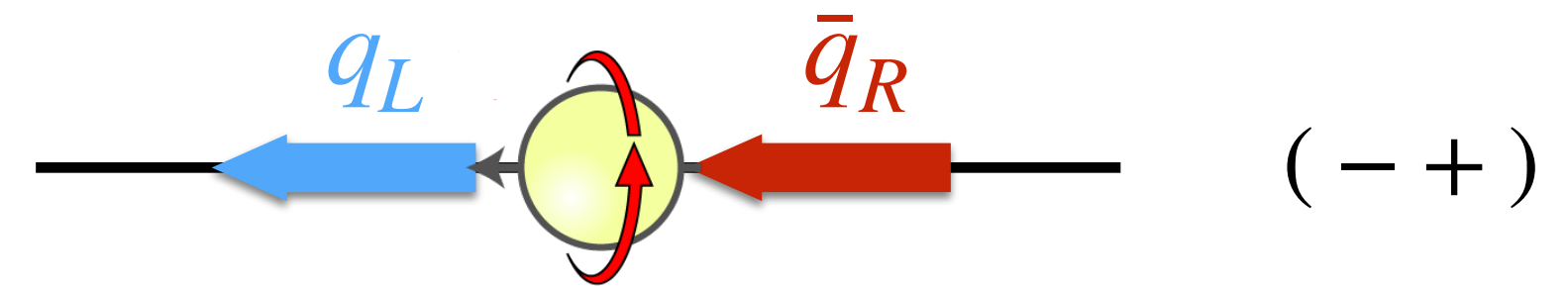
- An example: top-pair production.
- At the Large Hadron Collider:



- s-channel:
 - highly suppressed at high energy region
- quark-antiquark initial state:
 - less in small-x region;
 - boosted c.m. frame at high energy region.

Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider:
 - Entanglement from quark-antiquark initial state

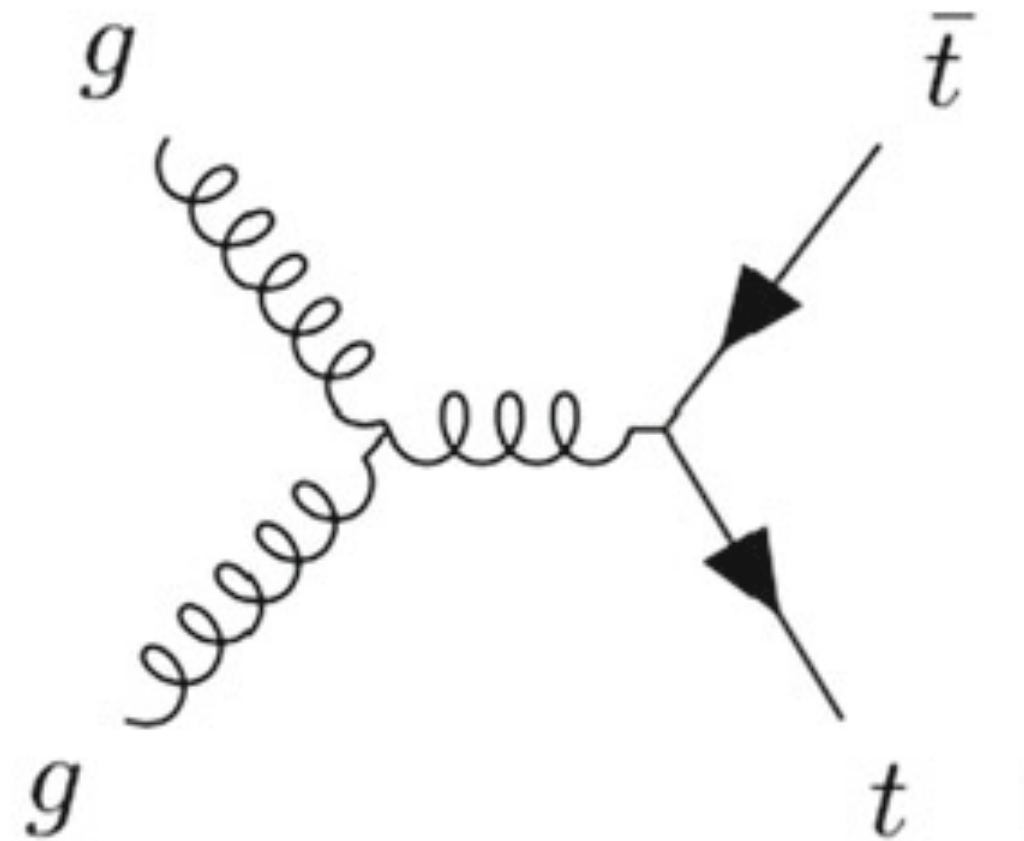


$$|t\bar{t}\rangle \sim (1 \pm \cos \theta) | \uparrow_{\hat{n}} \uparrow_{\hat{n}} \rangle + (1 \mp \cos \theta) | \downarrow_{\hat{n}} \downarrow_{\hat{n}} \rangle + \mathcal{O}(\gamma^{-1})$$

un(longitudinal)polarized initial state fermions, both left- and right-handed interactions

Quantum Entanglement at Colliders

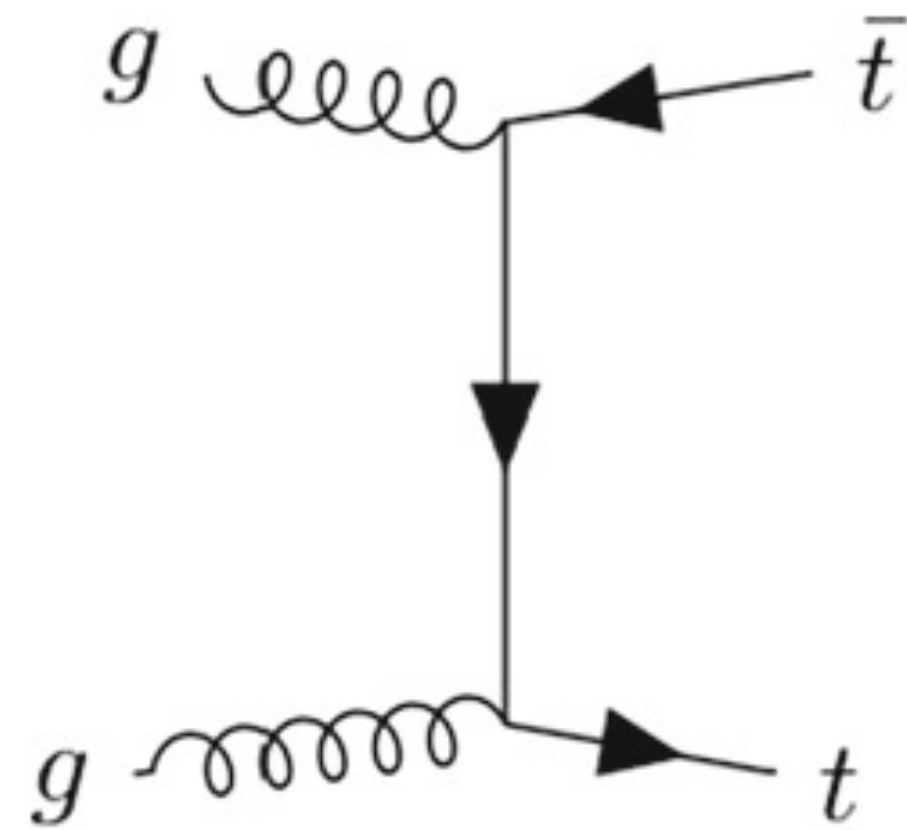
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- s-channel:
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- gluon-gluon initial state:
 - more in small-x region.

Quantum Entanglement at Colliders

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- t-channel:
 - dominant at high energy region
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- An example: top-pair production.
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with another p-wave suppression factor.

Quantum Entanglement at Colliders

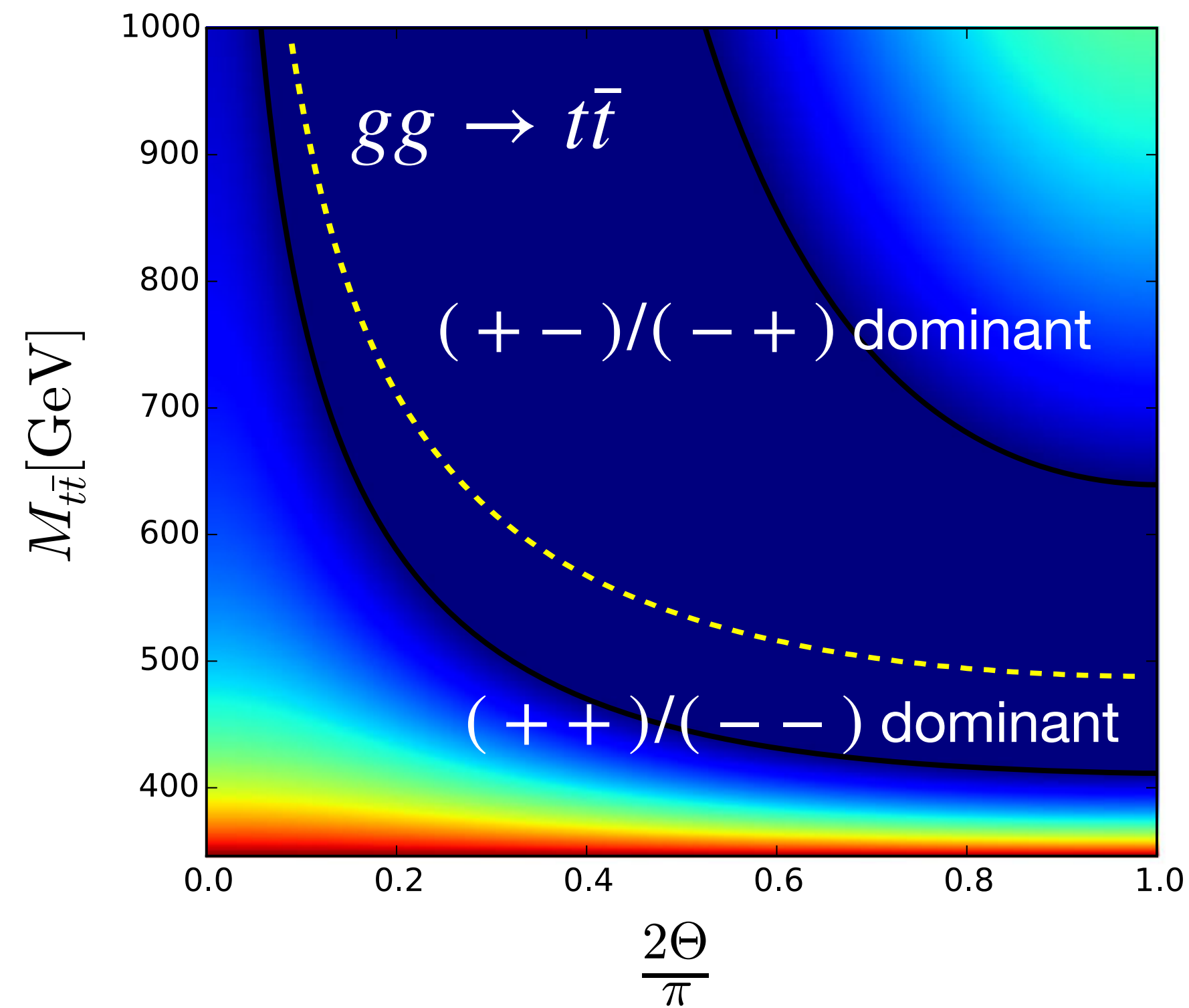
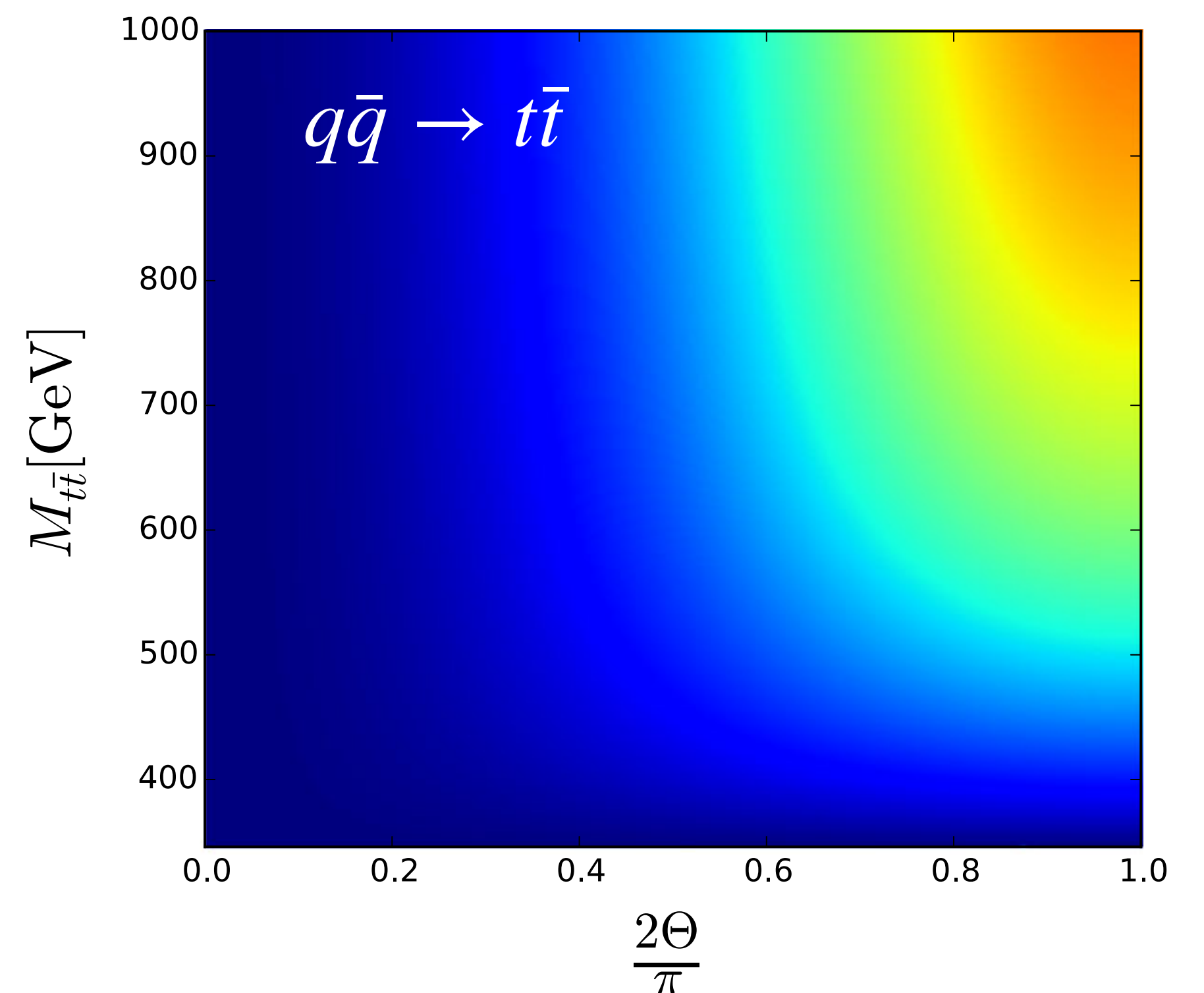
- An example: top-pair production.
- At the Large Hadron Collider:
 - Entanglement from gluon-gluon initial state



$$|t\bar{t}\rangle \sim \underset{\text{s-wave}}{(1 \pm \beta)} | \uparrow_{\hat{n}} \downarrow_{\hat{n}} \rangle - \underset{\text{p-wave}}{(1 \mp \beta)} | \downarrow_{\hat{n}} \uparrow_{\hat{n}} \rangle$$

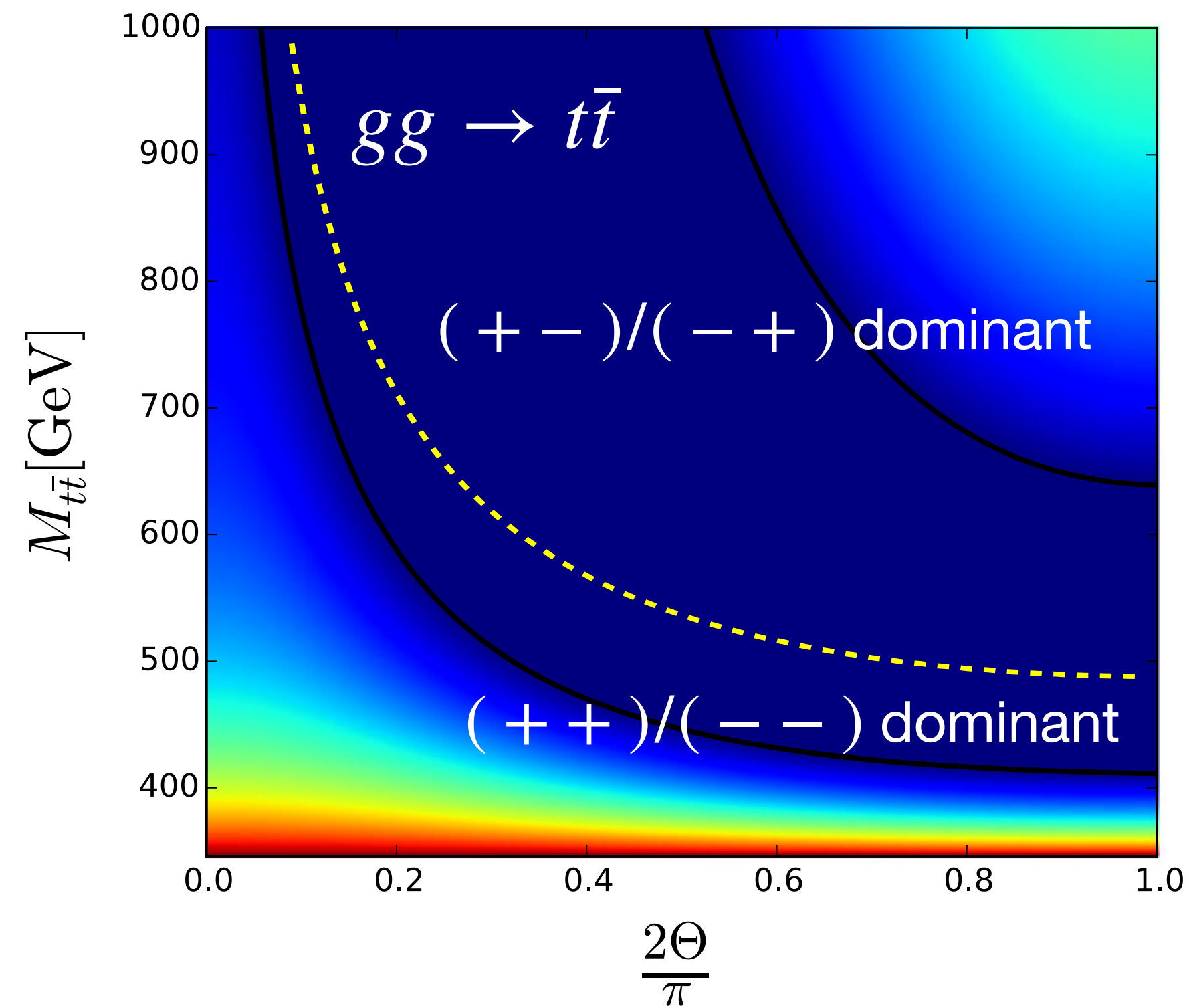
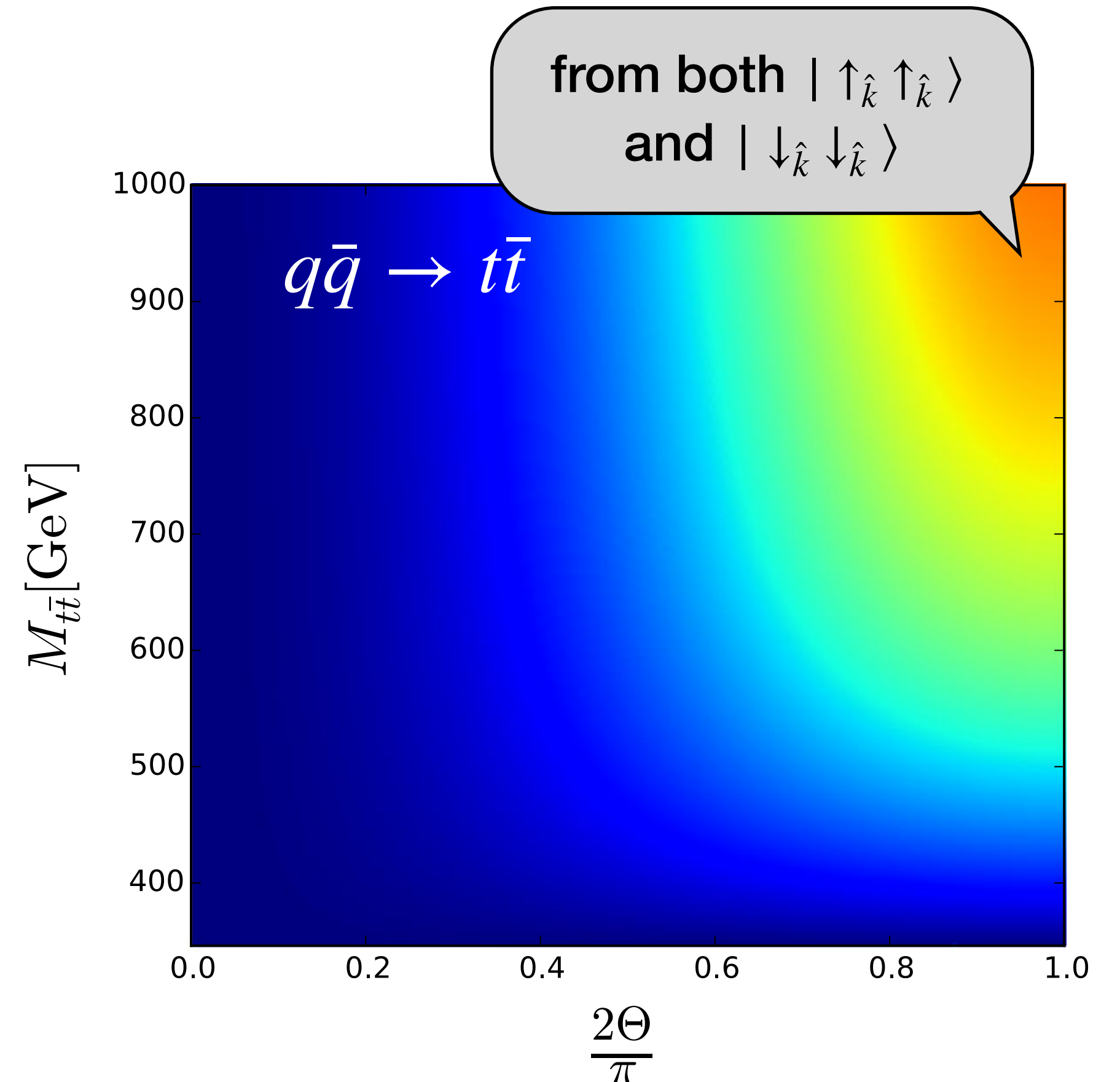
Quantum Entanglement at Colliders

- An example: top-pair production.
- At the Large Hadron Collider: helicity basis



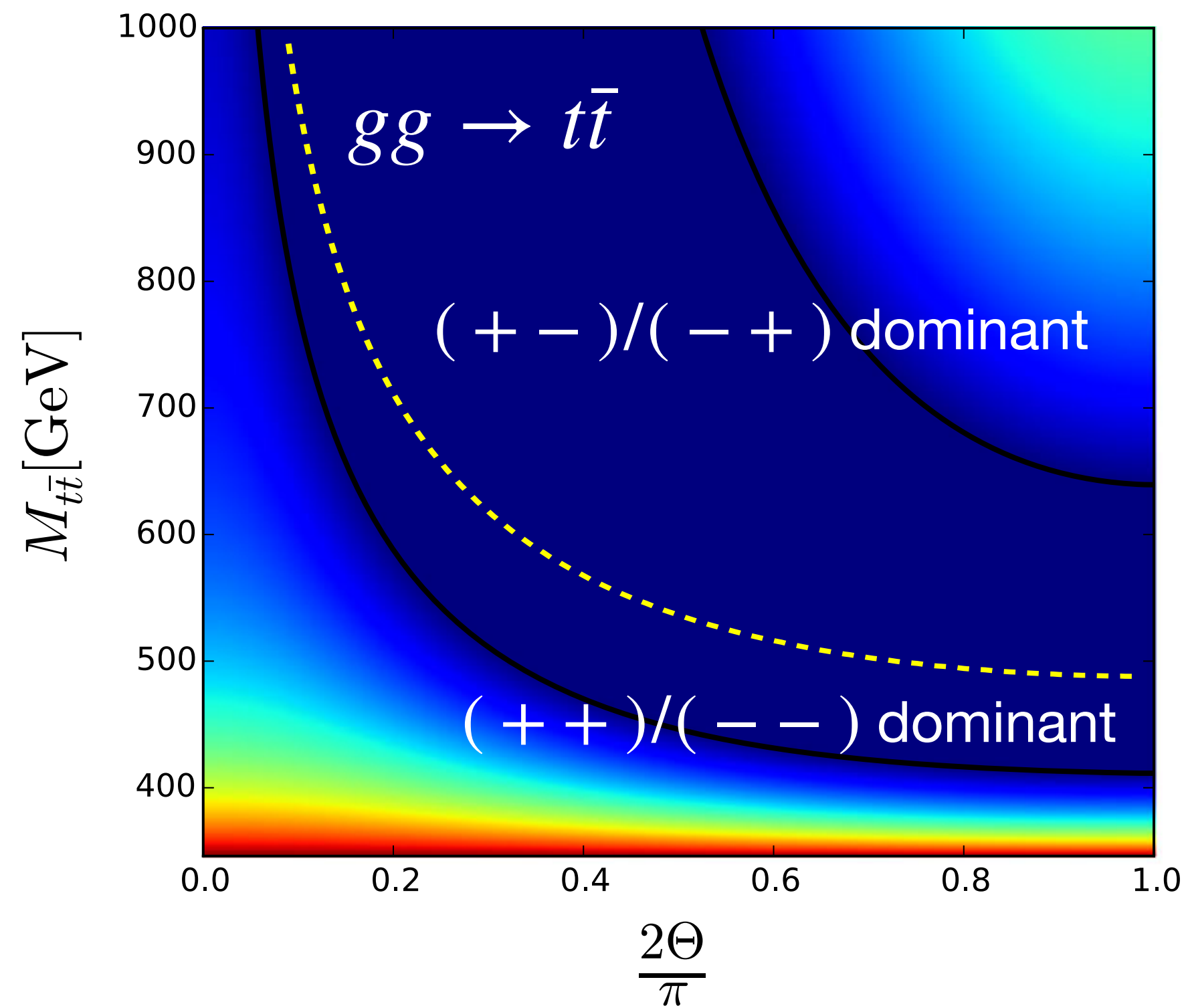
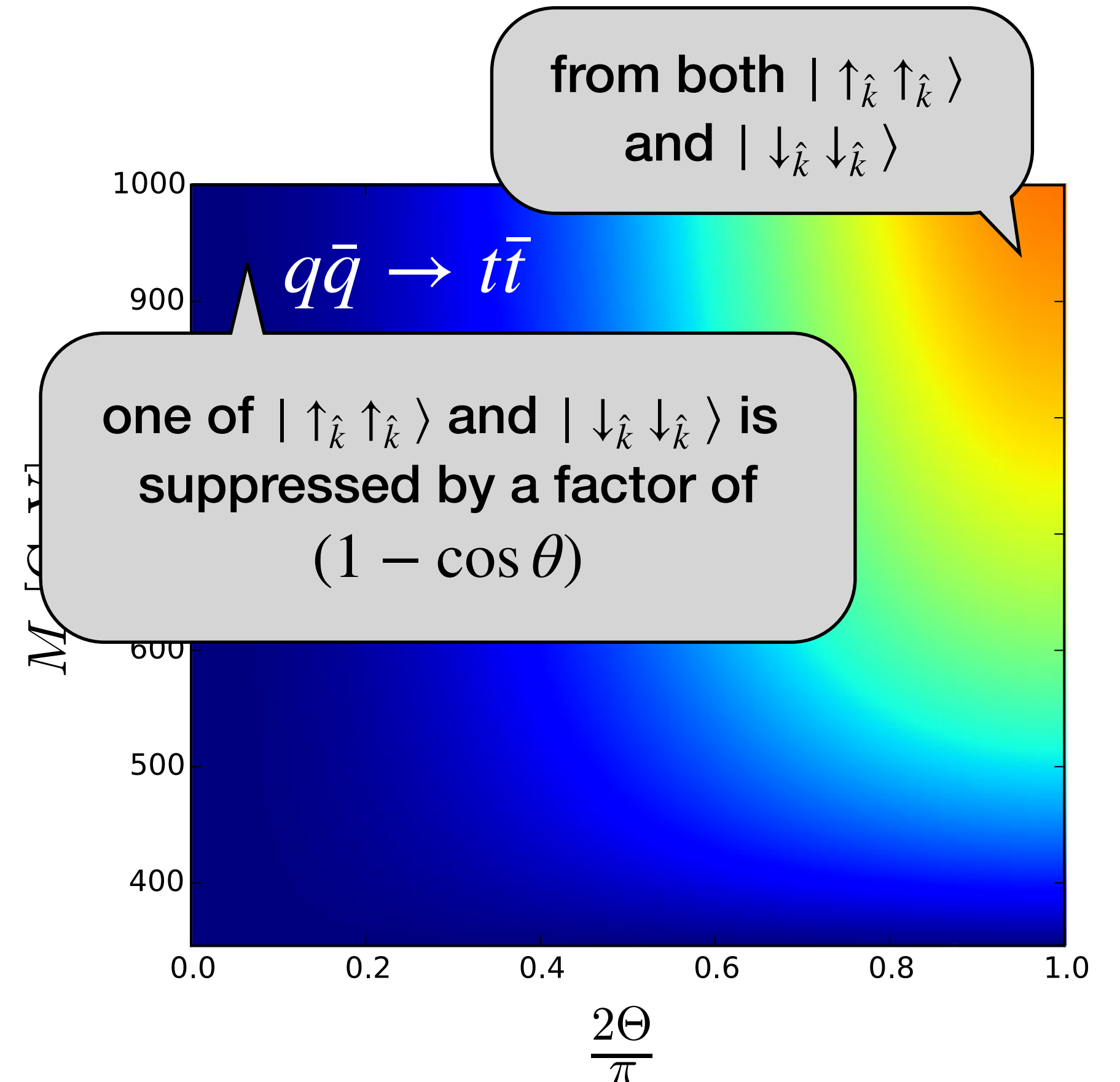
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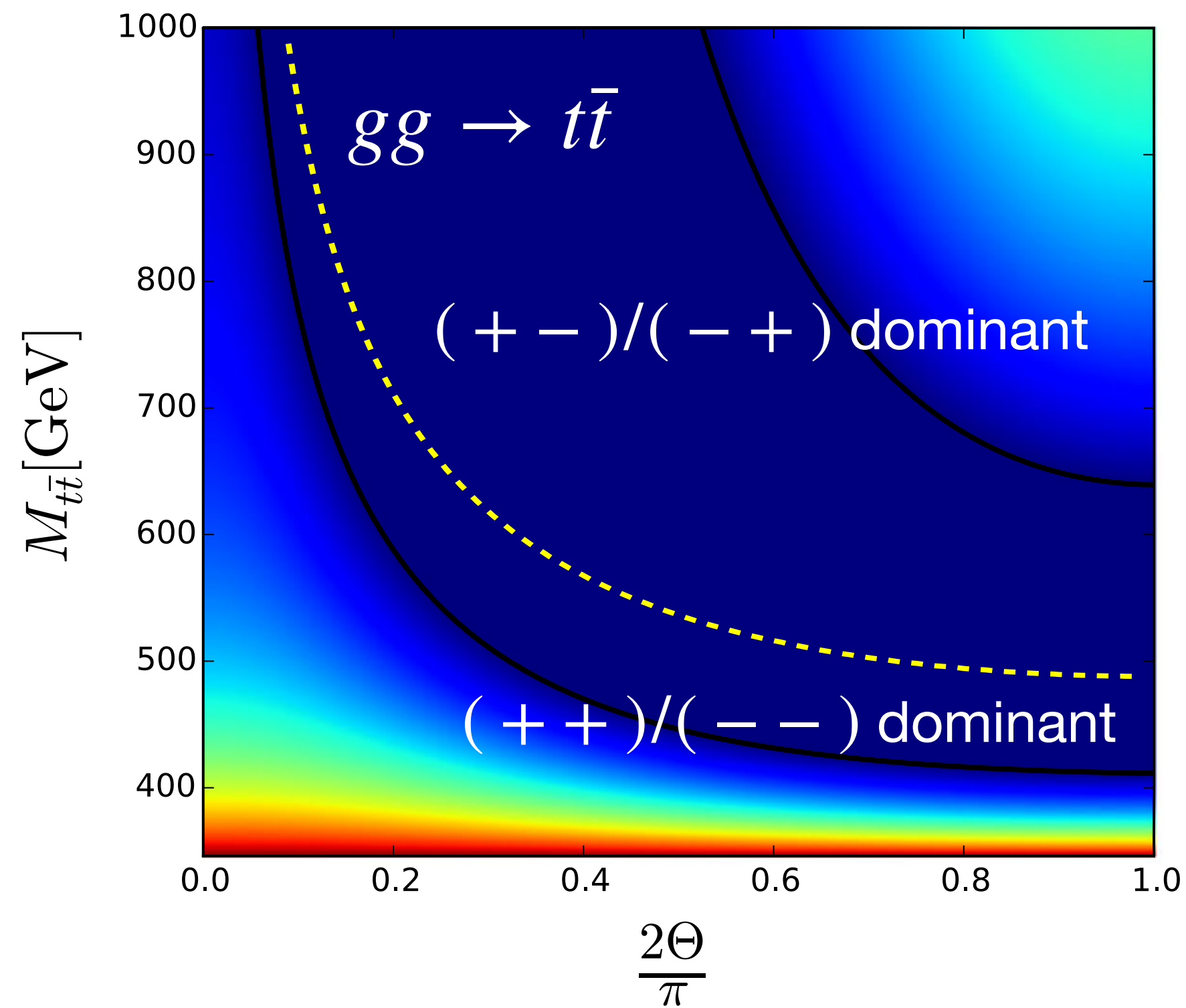
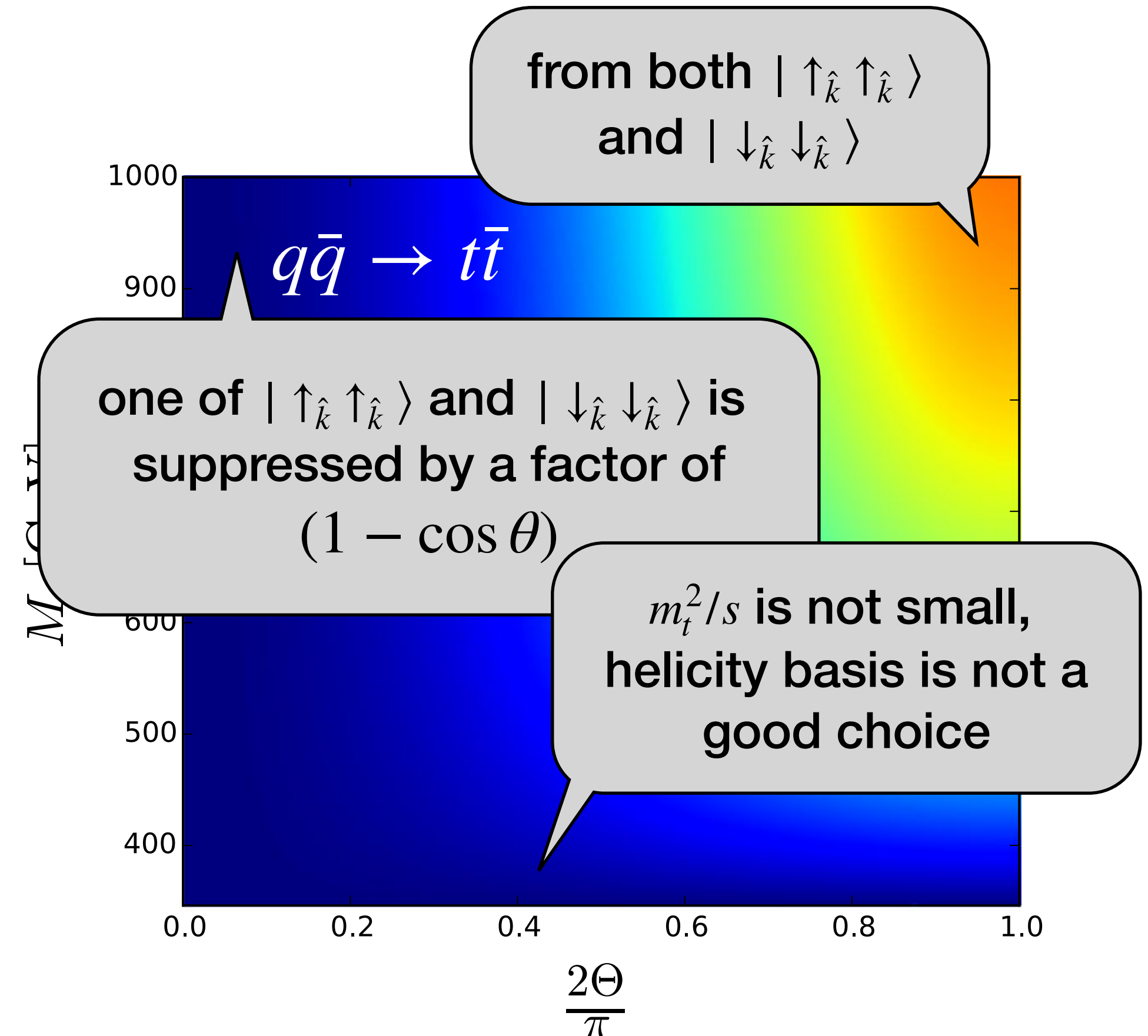
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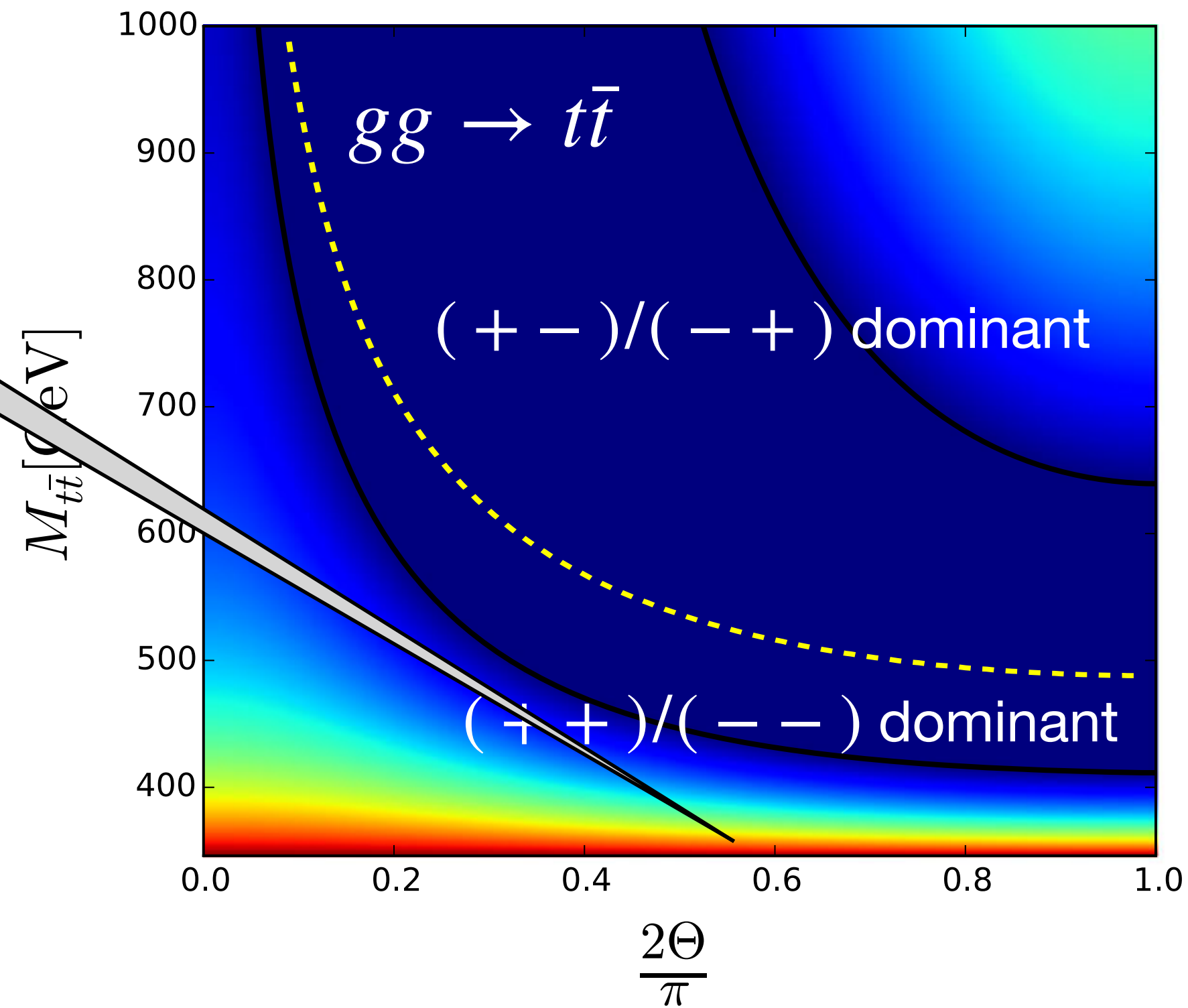
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Quantum Entanglement at Colliders

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The most important contribution to the entanglement at the LHC



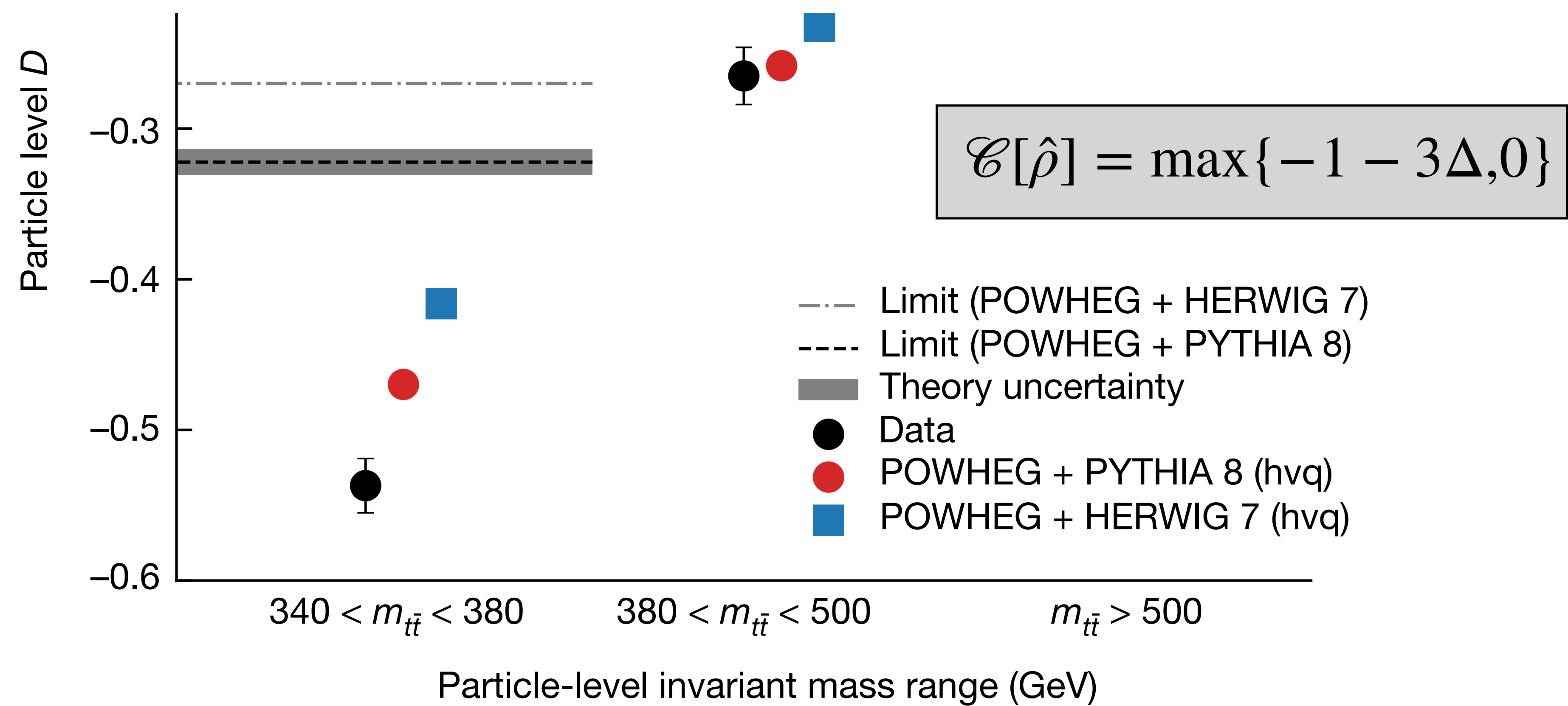
from both $|\uparrow_{\hat{k}} \uparrow_{\hat{k}}\rangle$ and $|\downarrow_{\hat{k}} \downarrow_{\hat{k}}\rangle$

one of $|\uparrow_{\hat{k}} \uparrow_{\hat{k}}\rangle$ and $|\downarrow_{\hat{k}} \downarrow_{\hat{k}}\rangle$ is suppressed by a factor of $(1 - \cos \theta)$

m_t^2/s is not small, helicity basis is not a good choice

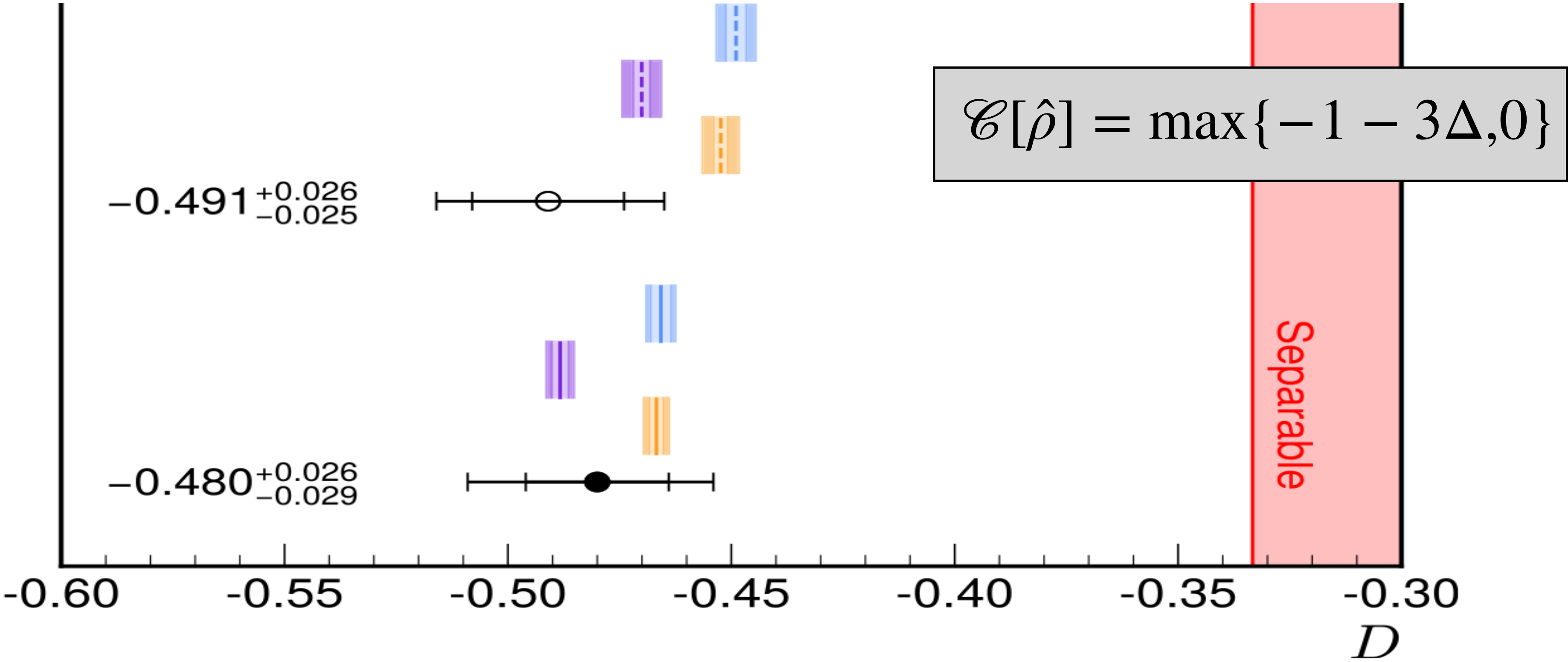
Quantum Entanglement at Colliders

- The result

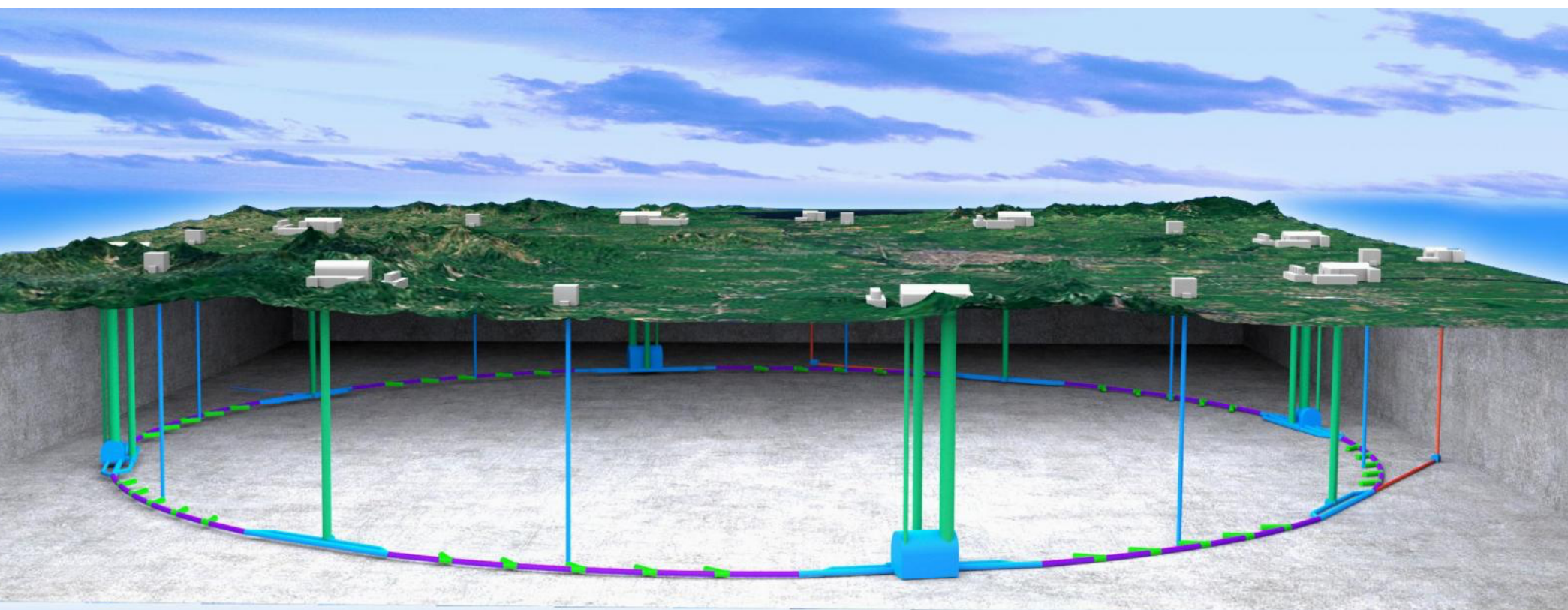
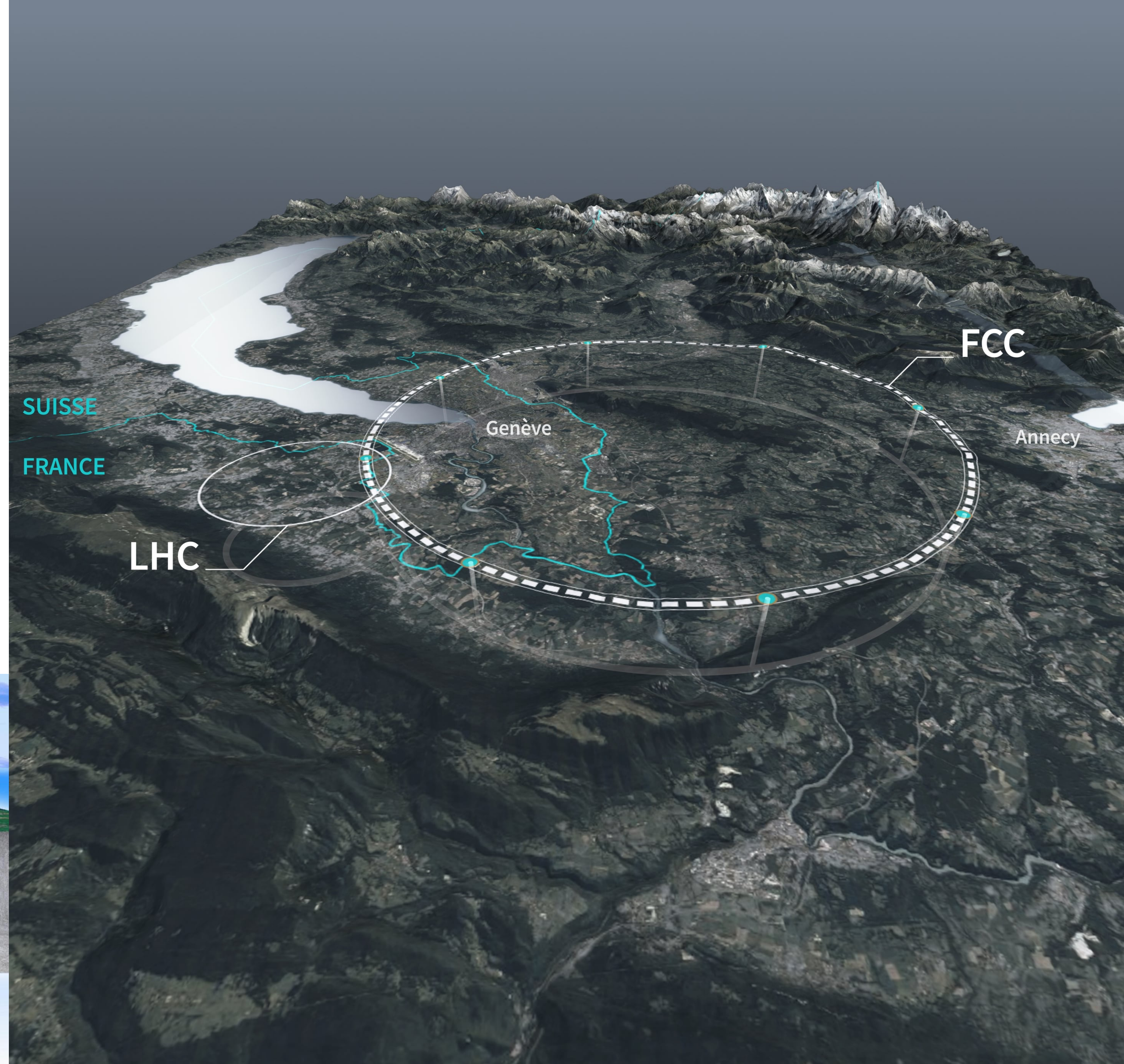


Quantum Entanglement at Colliders

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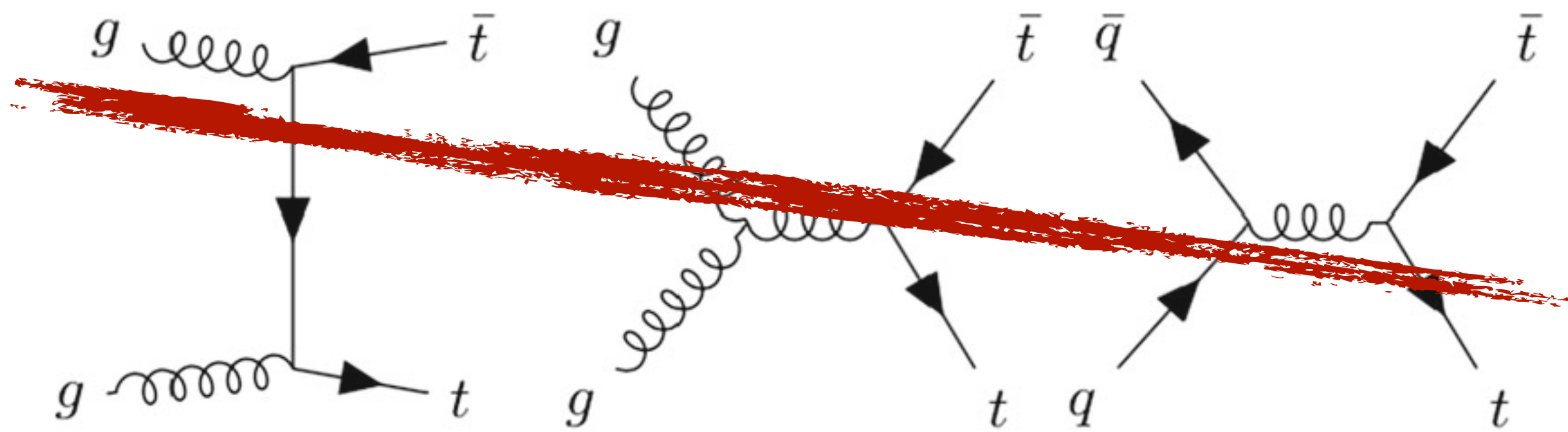
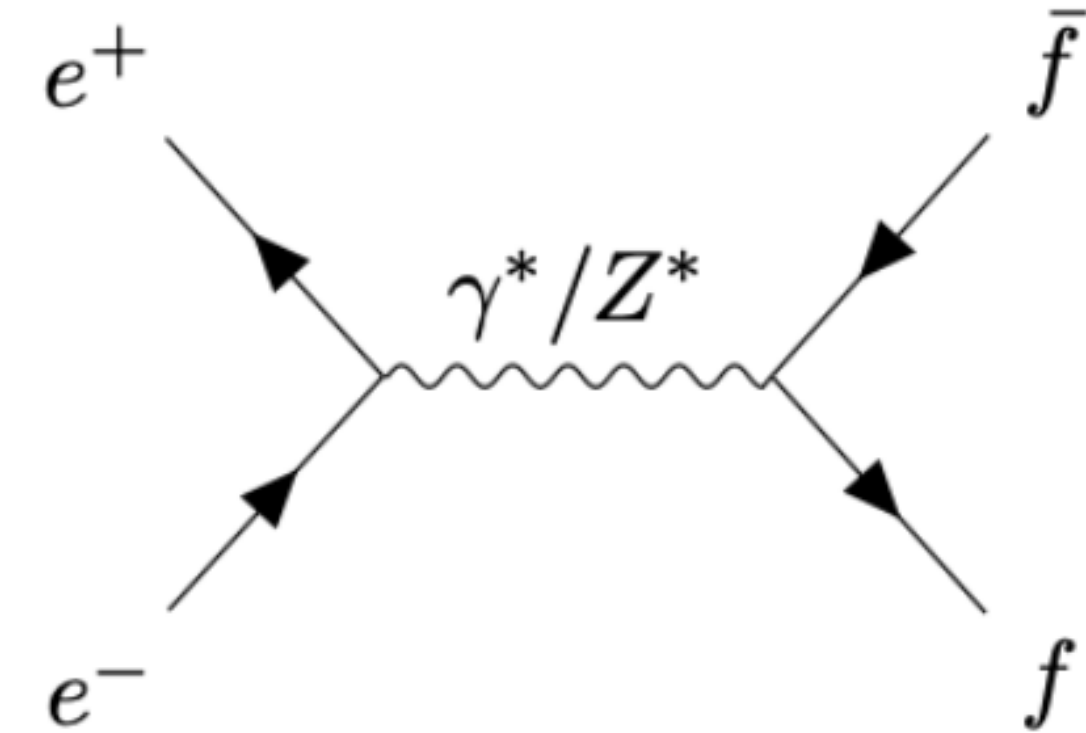


How about Higgs Factory?



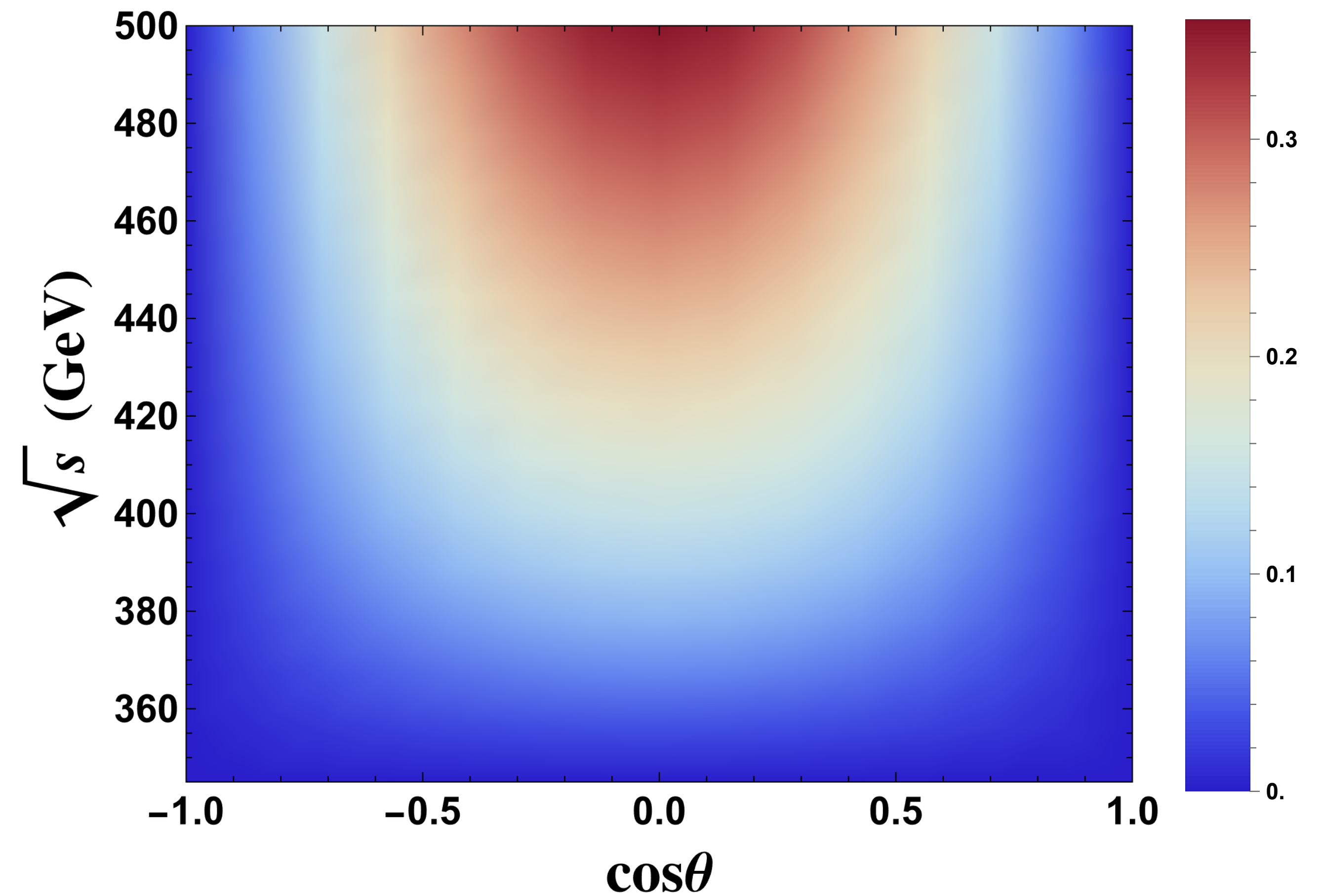
Quantum Entanglement at Colliders

- The advantage at lepton colliders.
 - Simpler and clear contribution:



Quantum Entanglement at Colliders

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 - Simpler and clear contribution:



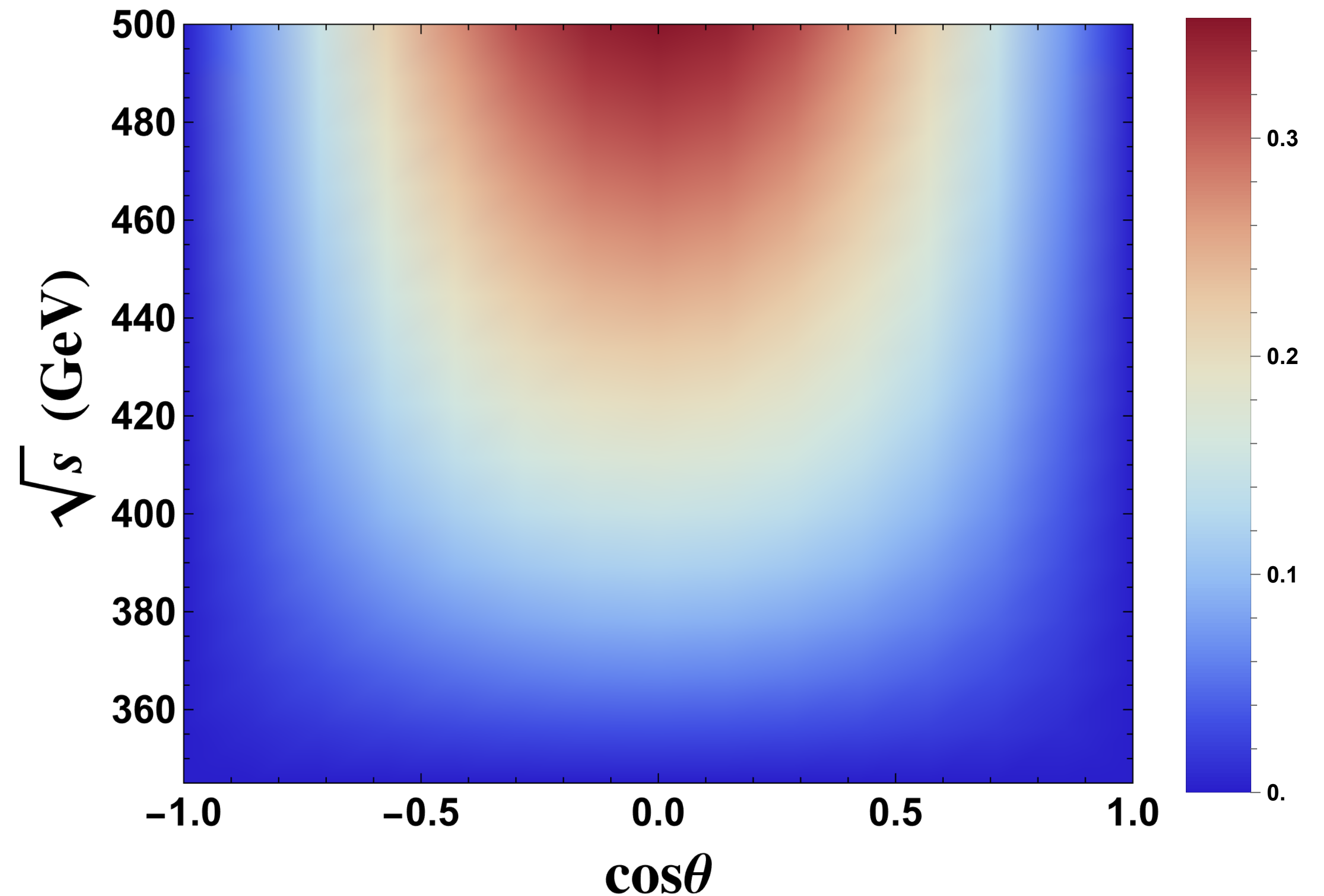
For simplicity and clearly, we do not show the contribution from the Z-boson in this figure, which will shift the distribution to left a little.

Quantum Entanglement at Colliders

- The advantage at lepton colliders.
 - Simpler and clear contribution:

The top-quarks are not energetic enough

The helicity basis is not good enough



For simplicity and clearly, we do not show the contribution from the Z-boson in this figure, which will shift the distribution to left a little.

Quantum Entanglement at Colliders

- It is helpful to use the transverse polarized initial state to increase the degree of the entanglement.
- In this case, the initial state is a Bell state selected by the interaction:

$$\frac{1}{2}(|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle) = \frac{1}{2}(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

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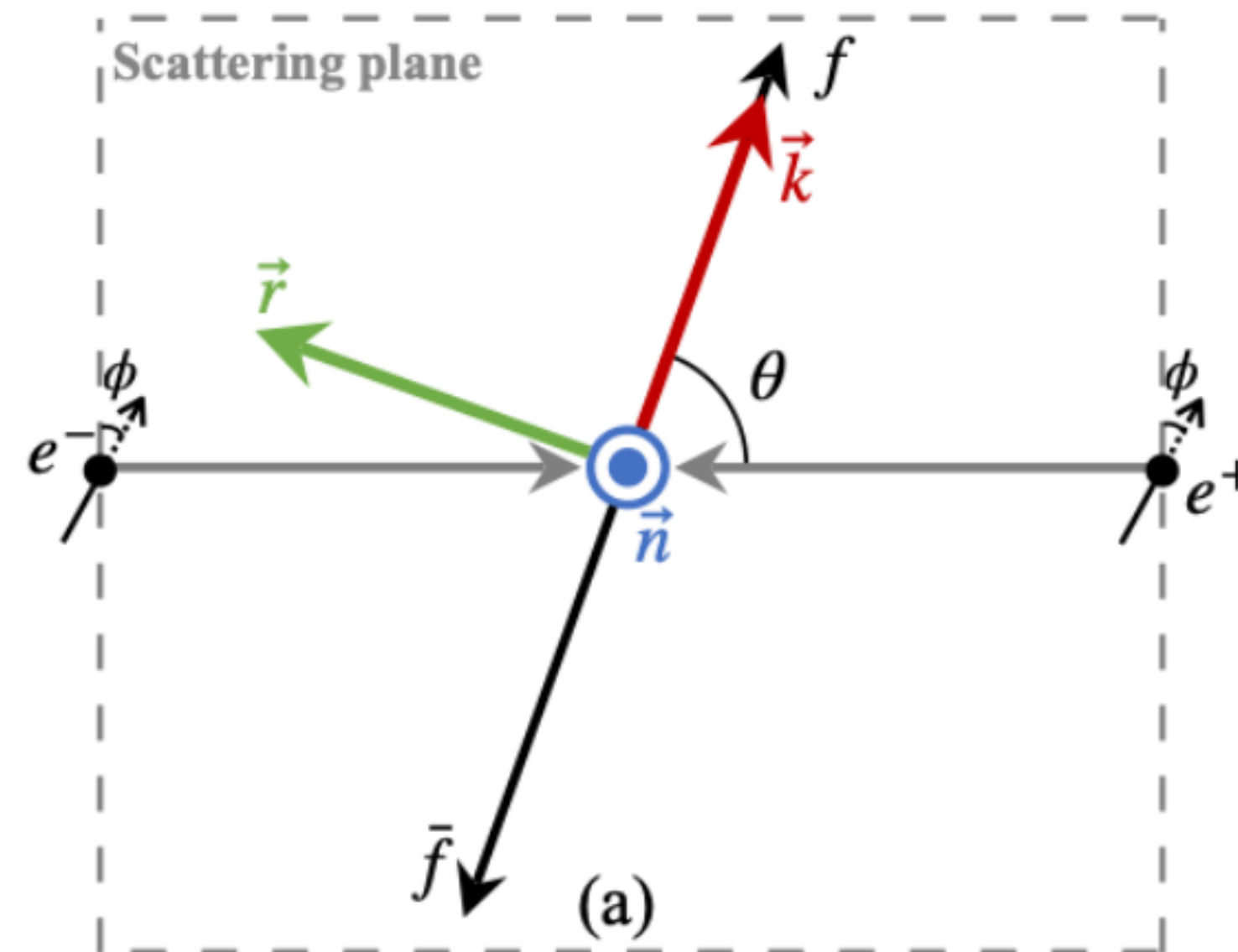
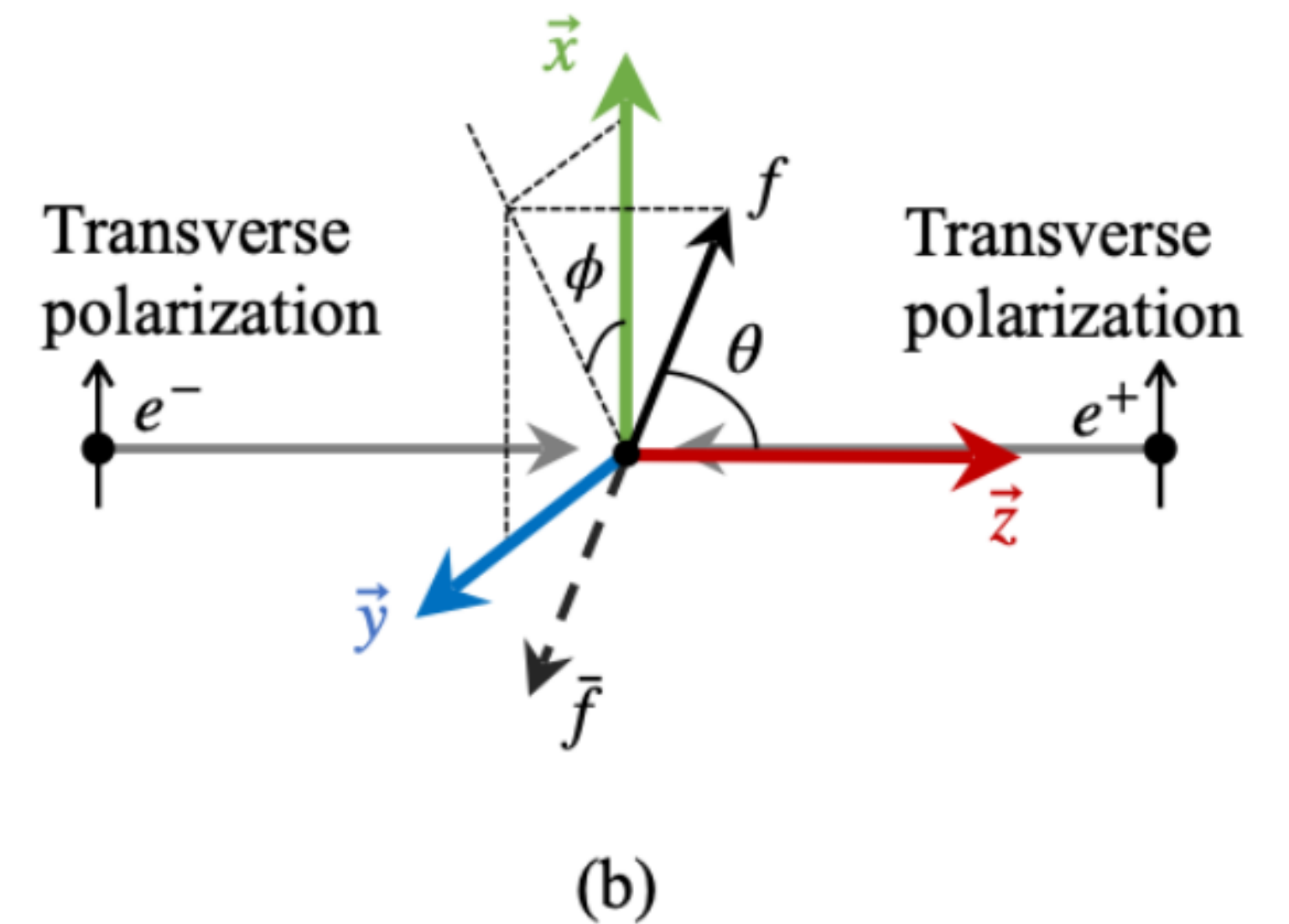
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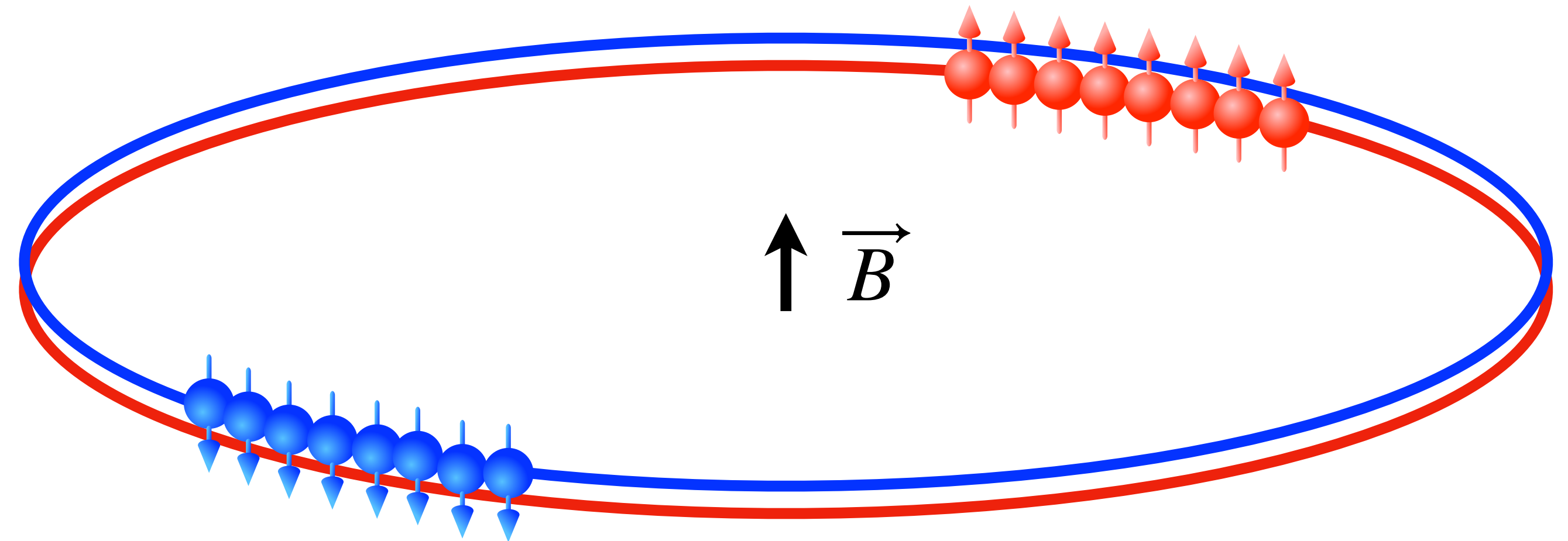
Quantum Entanglement at Colliders

- It is helpful to use the transverse polarized initial state to increase the degree of the entanglement.
- The final state is always the Bell state along the direction $\hat{e}_2 = (\cos \theta \sin \phi, \cos \phi, \sin \theta \sin \phi \sqrt{1 - \beta^2})$ in the helicity basis, which is just $\hat{y} - \left(1 - \sqrt{1 - \beta^2}\right) \hat{k}$.



Quantum Entanglement at Higgs Factory

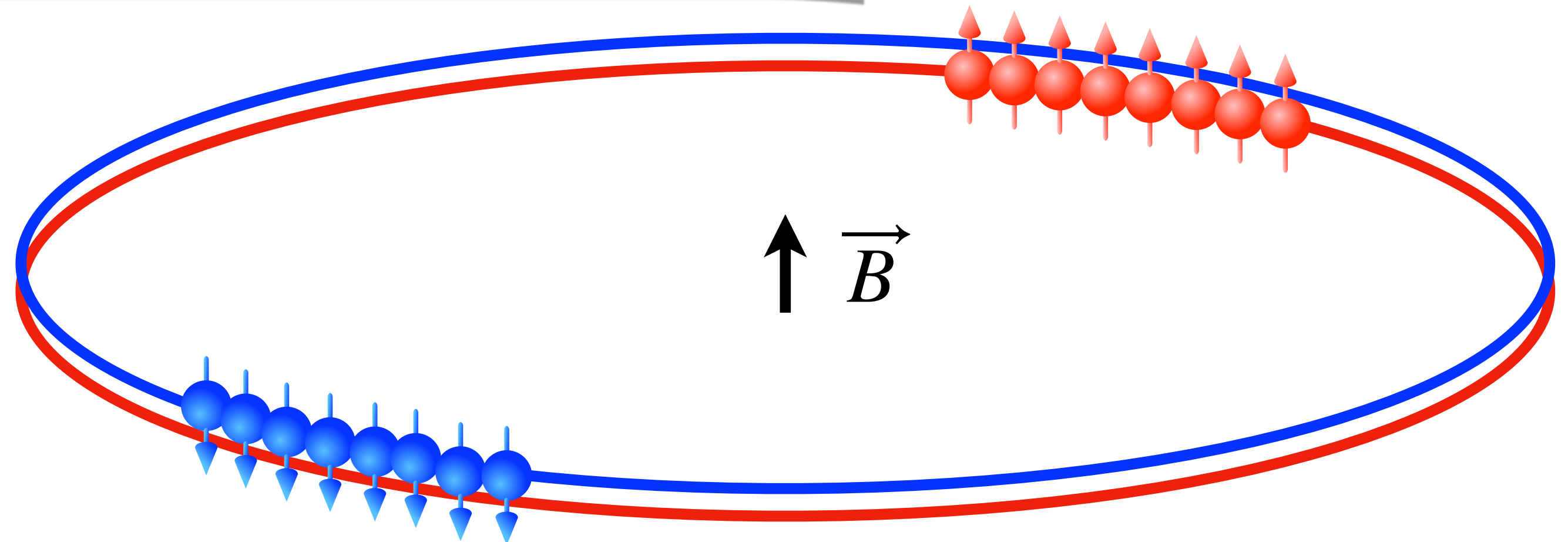
- At Higgs factory, transversely polarized initial state electron-positron beams could be get with the so-called Sokolov-Ternov effects.



Quantum Entanglement at Higgs Factory

- At Higgs factory, transversely polarized initial state electron-positron beams could be get with the so-called Sokolov-Ternov effects.

$$B_x^{e^\pm} \rightarrow 92.4 \% \text{ after several hours}$$



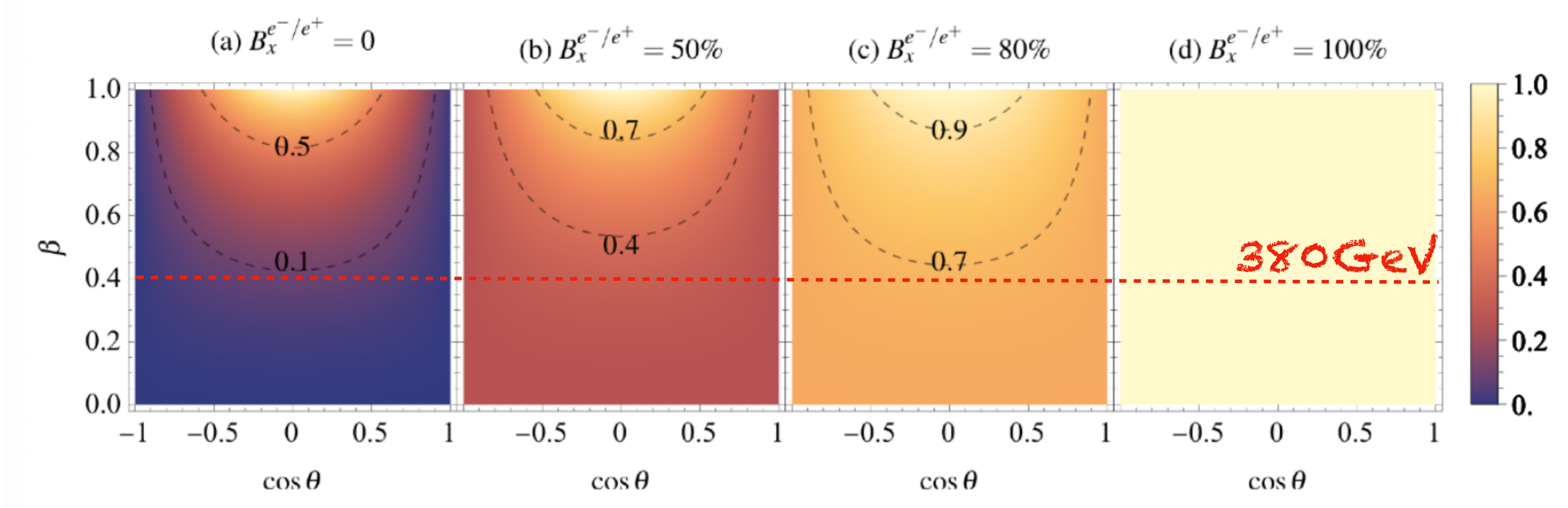
Quantum Entanglement at Higgs Factory

- The final state is maximal entangled!

$$\mathcal{C}[\hat{\rho}] = \sqrt{1 - \frac{4 \left[1 - (B_{e^-} B_{e^+})^2 \right] (1 - \beta^2 \sin^2 \theta)}{(2 - \beta^2 \sin^2 \theta + B_{e^-} B_{e^+} \beta^2 \sin^2 \theta \cos 2\phi)^2}}$$

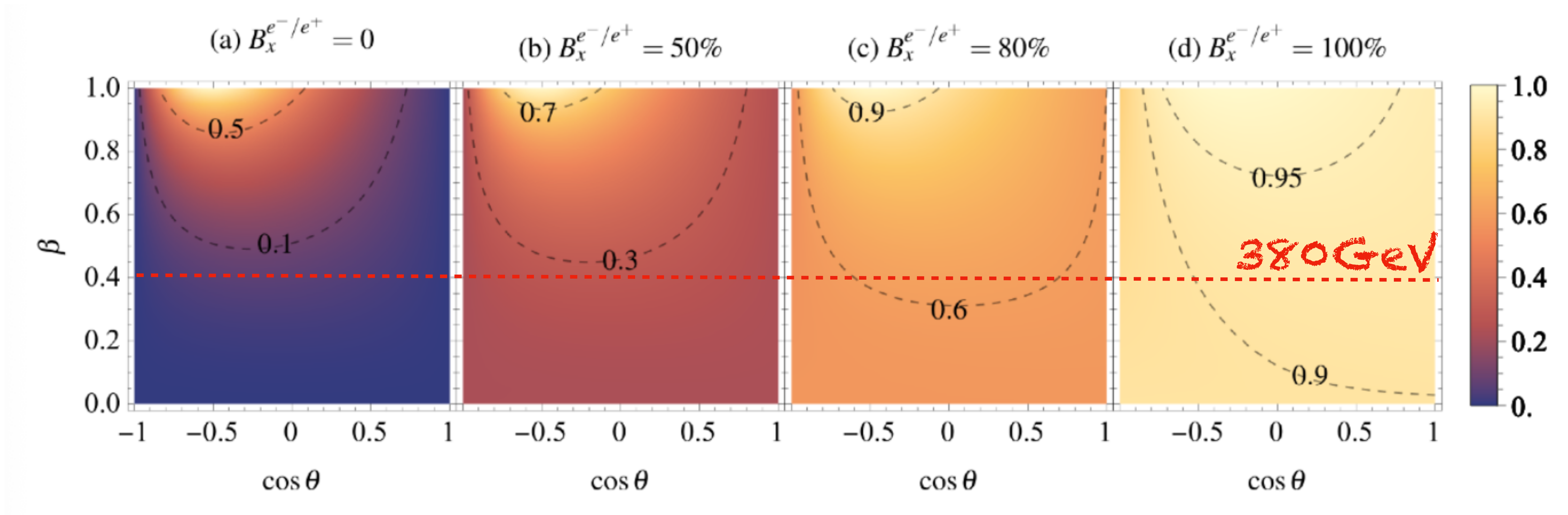
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- It is helpful to use the transverse polarized initial state to increase the degree of the entanglement.



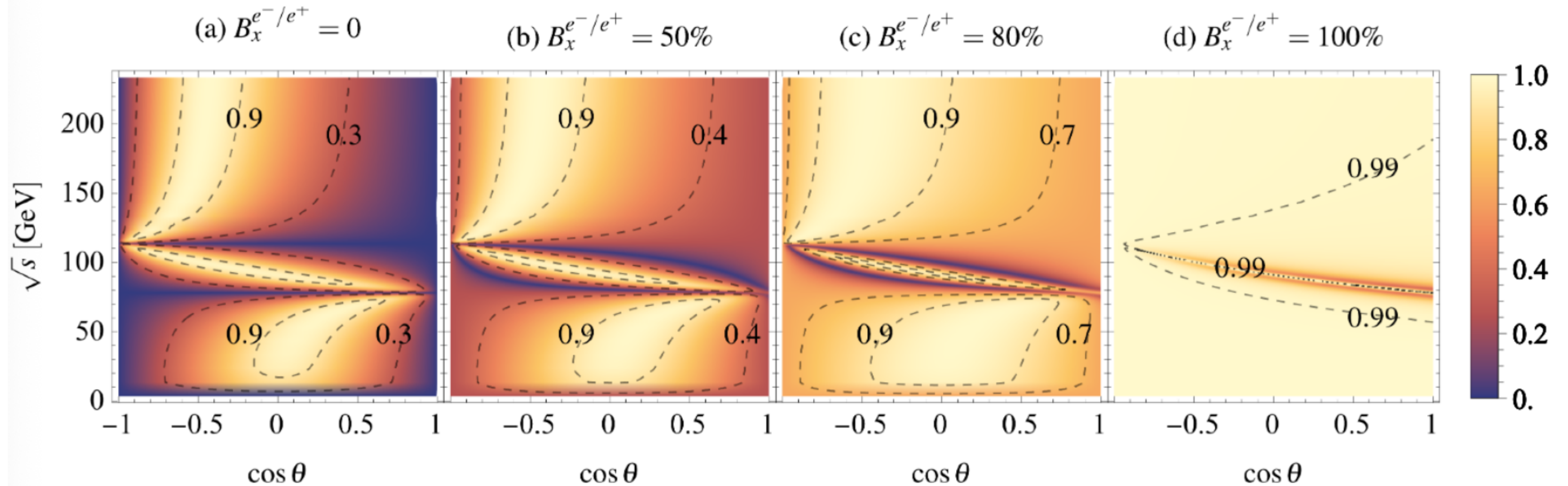
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Quantum Entanglement at Colliders

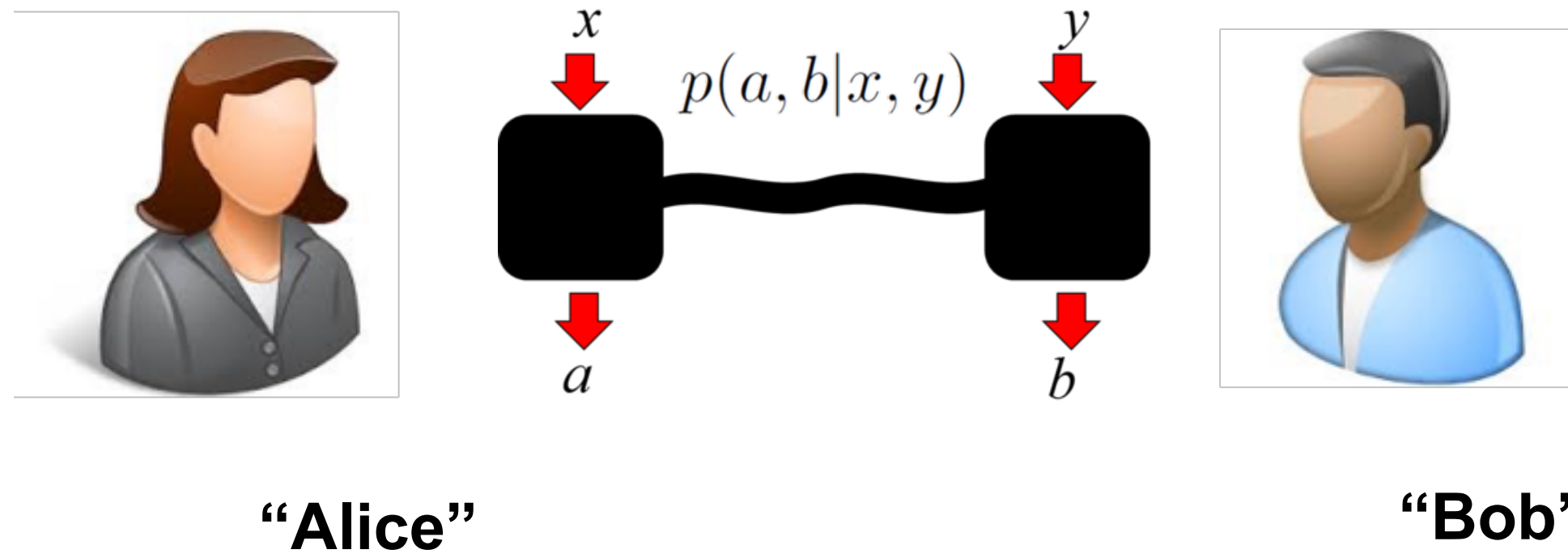
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From Quantum Entanglement to Quantum Nonlocality

The different correlations

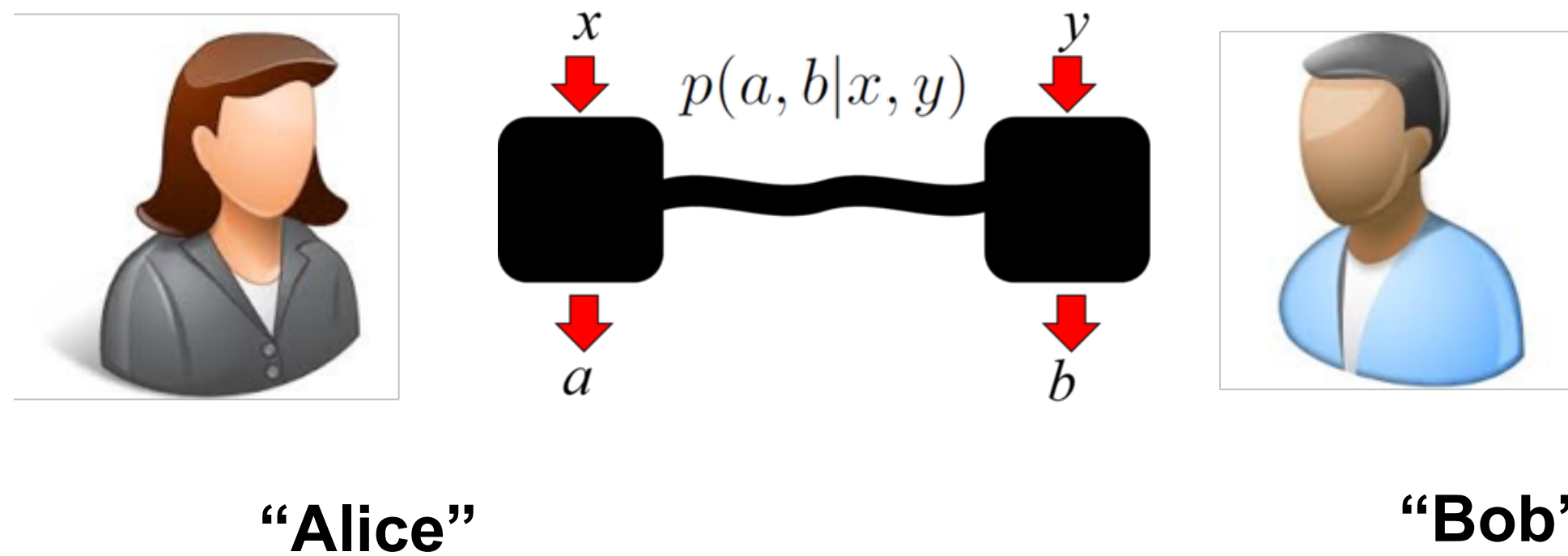


Local correlations

$$p(ab|xy) = \int_{\Lambda} d\lambda q(\lambda) p(a|x, \lambda) p(b|y, \lambda),$$

$$\sum_{a,b} ab \, p(ab|x_1y_1) + \sum_{a,b} ab \, p(ab|x_1y_2) + \sum_{a,b} ab \, p(ab|x_2y_1) - \sum_{a,b} ab \, p(ab|x_2y_2) \leq 2, \quad a, b \in \{+1, -1\}$$

The different correlations



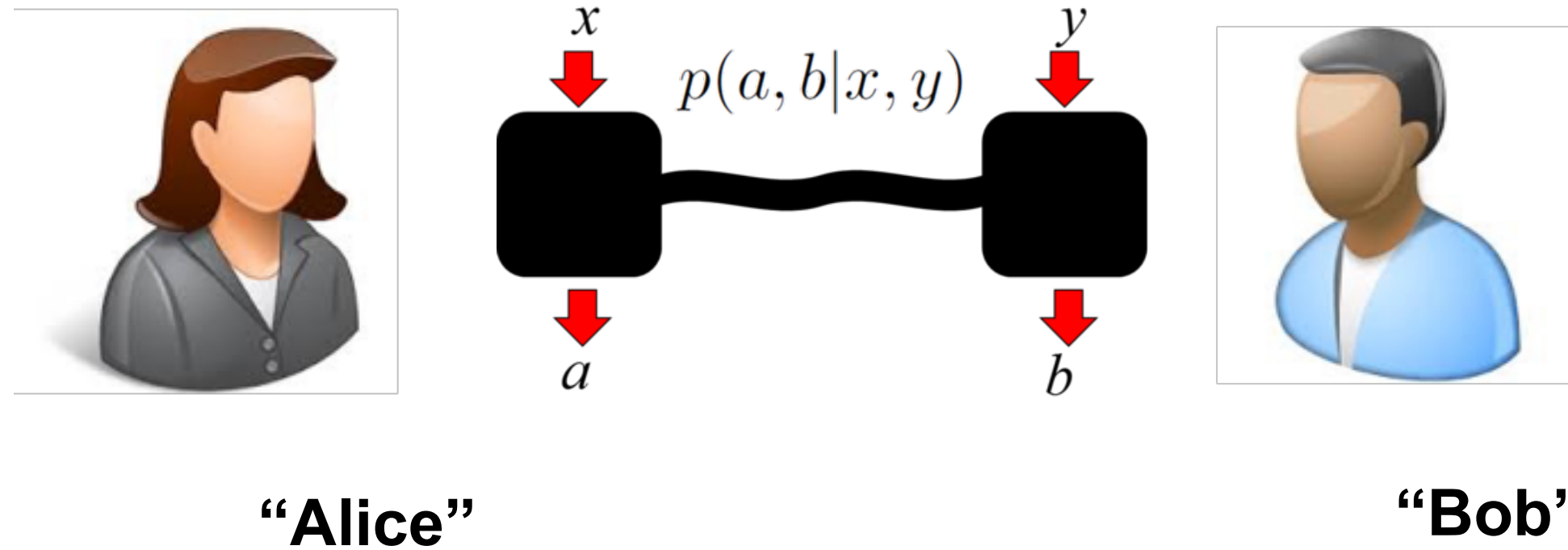
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The hidden variables do not only determine the state of the system, but also determine the result of the measurement **BEFORE** it is measured!

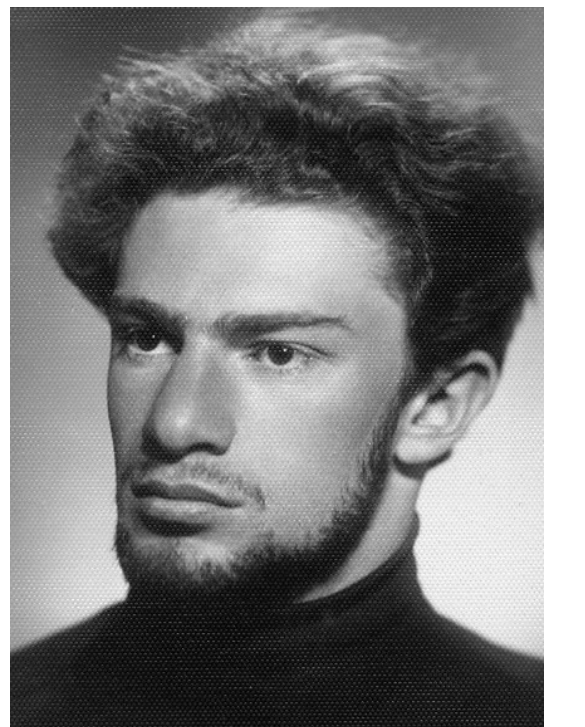
The different correlations



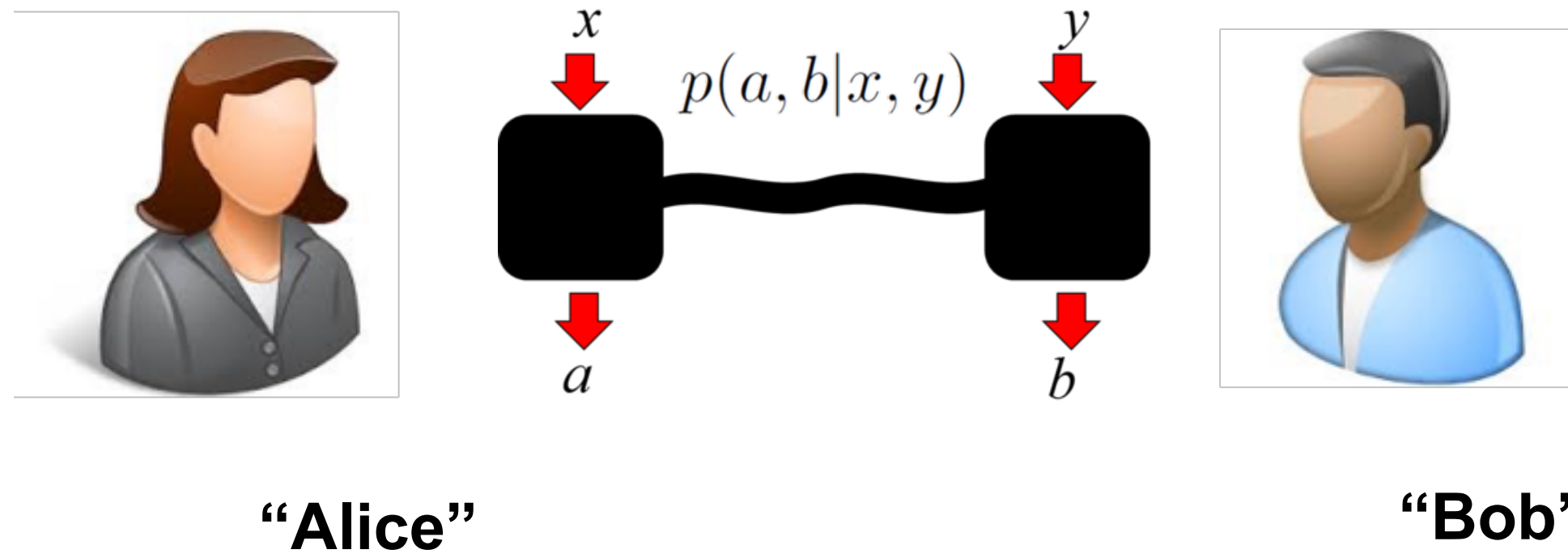
Quantum correlations-II

$$p(ab | xy) = \rho_{AB}(M_{a|x}M_{b|y}), [M_{a|x}, M_{b|y}] = 0$$

$$\|\hat{x}_1\hat{y}_1 + \hat{x}_1\hat{y}_2 + \hat{x}_2\hat{y}_1 - \hat{x}_2\hat{y}_2\|$$



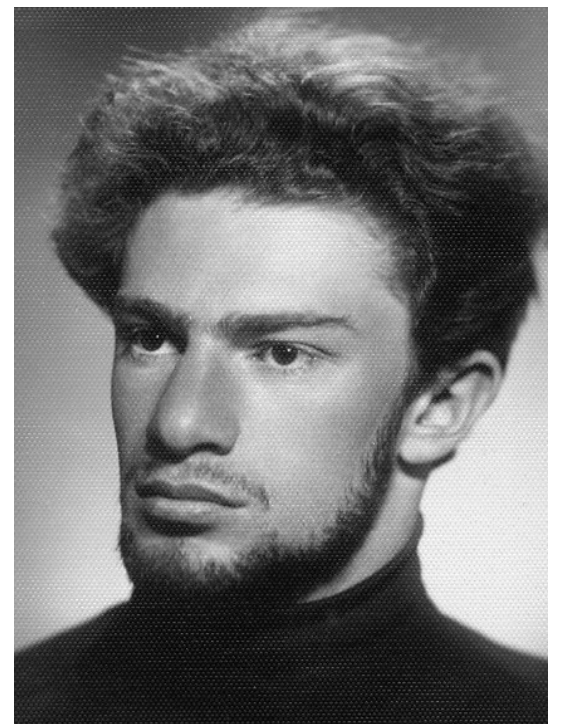
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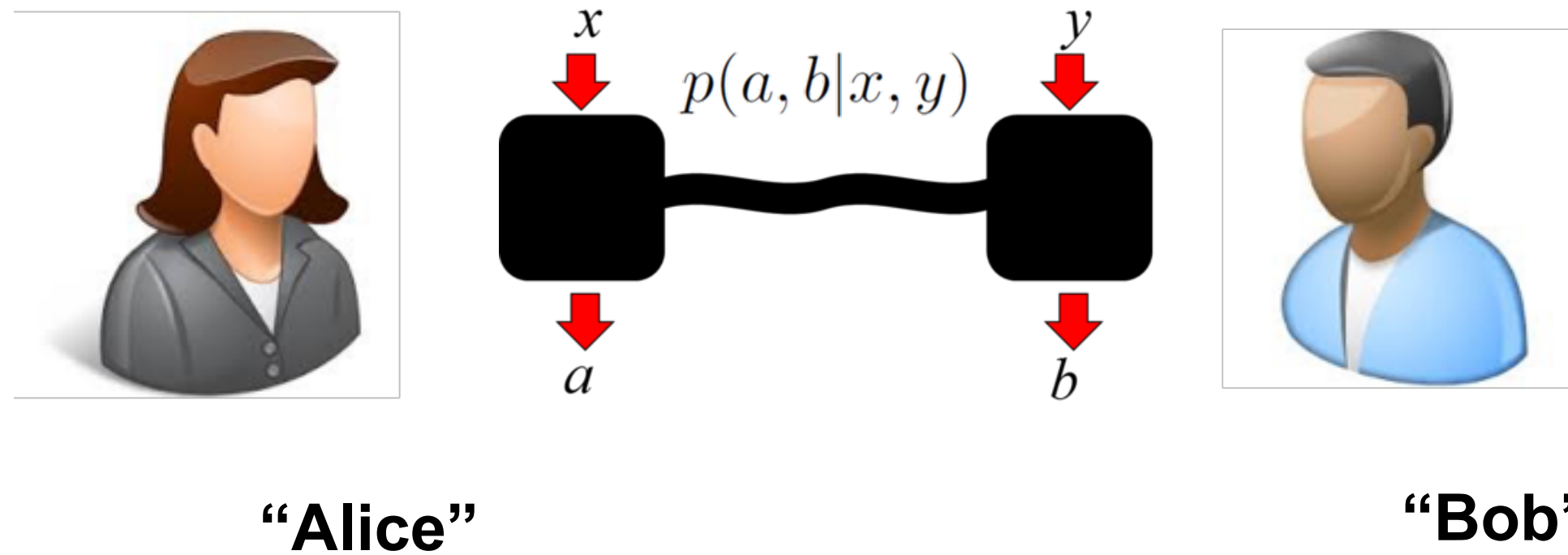
Quantum correlations-II

$$p(ab | xy) = \rho_{AB}(M_{a|x}M_{b|y}), [M_{a|x}, M_{b|y}] = 0$$

$$\begin{aligned}
 \|\hat{x}_1\hat{y}_1 + \hat{x}_1\hat{y}_2 + \hat{x}_2\hat{y}_1 - \hat{x}_2\hat{y}_2\| &= (\hat{x}_1\hat{y}_1\hat{x}_1\hat{y}_1 + \hat{x}_1\hat{y}_1\hat{x}_1\hat{y}_2 + \hat{x}_1\hat{y}_1\hat{x}_2\hat{y}_1 - \hat{x}_1\hat{y}_1\hat{x}_2\hat{y}_2 \\
 &\quad + \hat{x}_1\hat{y}_2\hat{x}_1\hat{y}_1 + \hat{x}_1\hat{y}_2\hat{x}_1\hat{y}_2 + \hat{x}_1\hat{y}_2\hat{x}_2\hat{y}_1 - \hat{x}_1\hat{y}_2\hat{x}_2\hat{y}_2 \\
 &\quad + \hat{x}_2\hat{y}_1\hat{x}_1\hat{y}_1 + \hat{x}_2\hat{y}_1\hat{x}_1\hat{y}_2 + \hat{x}_2\hat{y}_1\hat{x}_2\hat{y}_1 - \hat{x}_2\hat{y}_1\hat{x}_2\hat{y}_2 \\
 &\quad - \hat{x}_2\hat{y}_2\hat{x}_1\hat{y}_1 - \hat{x}_2\hat{y}_2\hat{x}_1\hat{y}_2 - \hat{x}_2\hat{y}_2\hat{x}_2\hat{y}_1 + \hat{x}_2\hat{y}_2\hat{x}_2\hat{y}_2)^{1/2}
 \end{aligned}$$



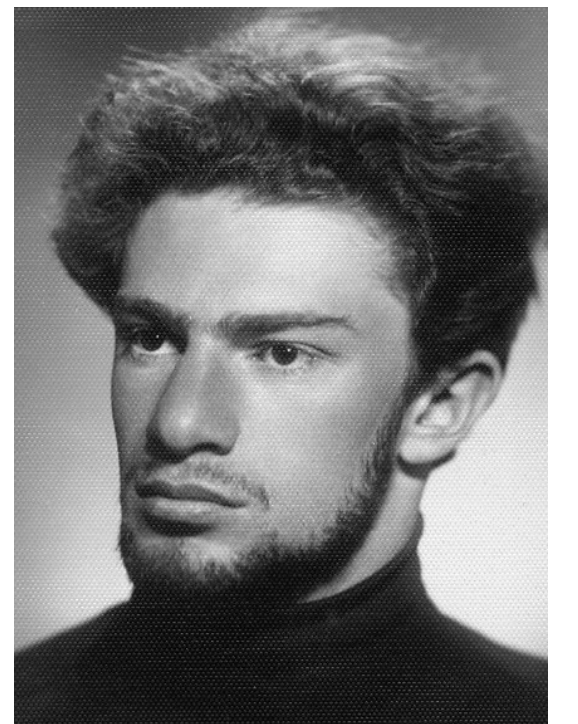
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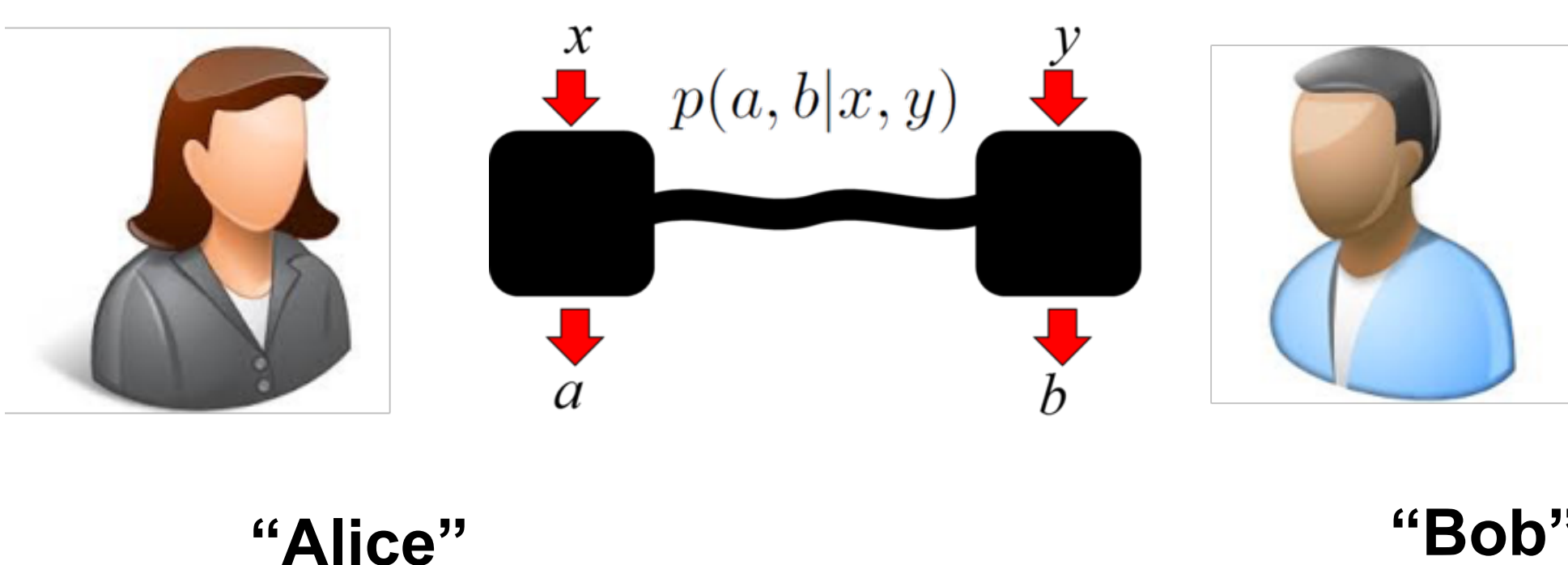
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The different correlations



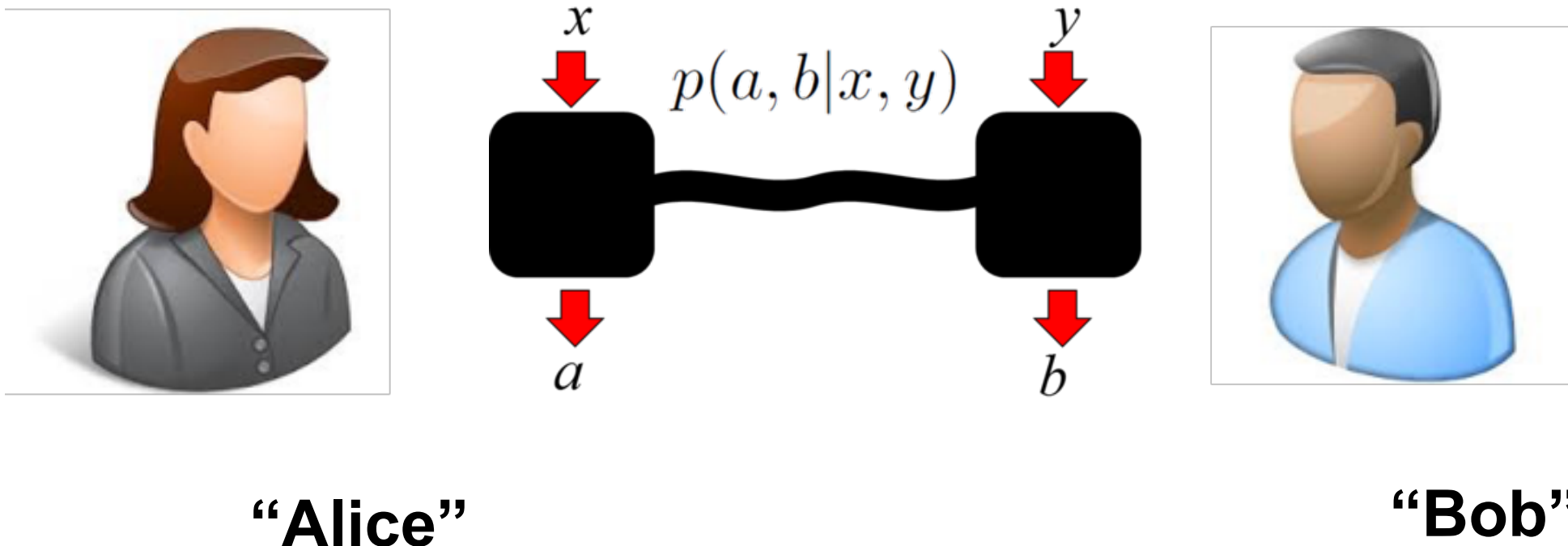
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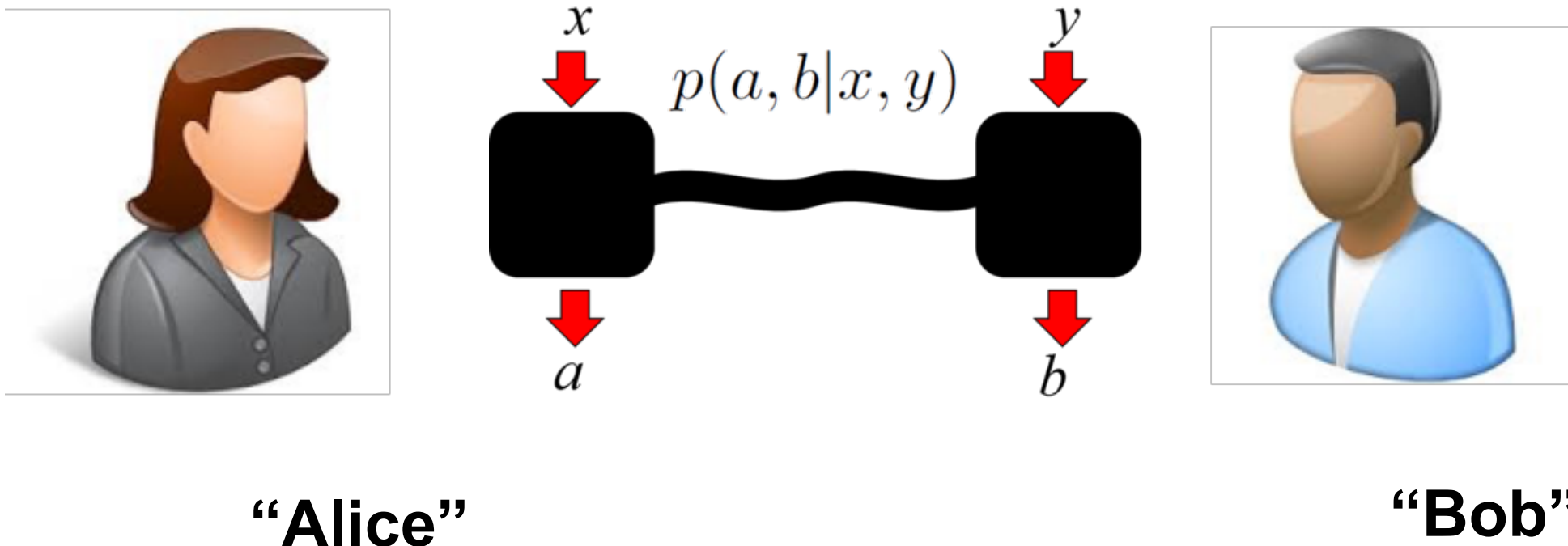
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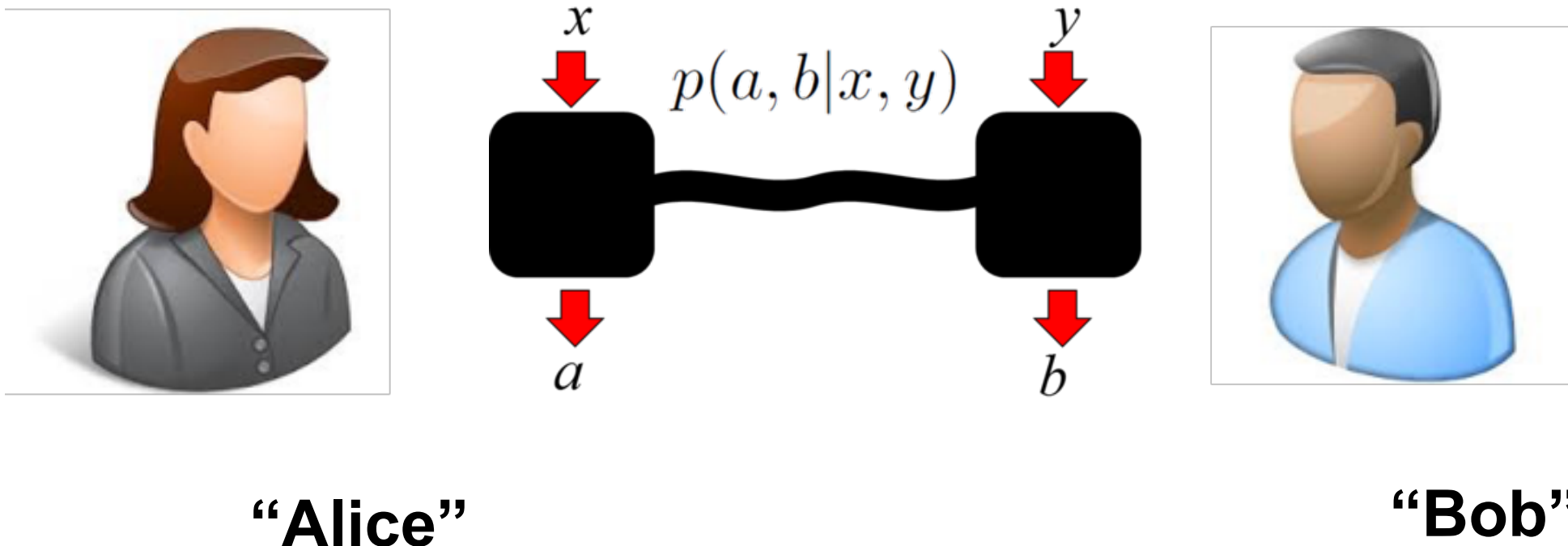
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&\quad - \hat{x}_2\hat{x}_1\hat{y}_2\hat{y}_1 - \cancel{\hat{x}_2\hat{x}_1\hat{y}_1^2} - \cancel{\hat{x}_2^2\hat{y}_2\hat{y}_1} + \hat{x}_1^2\hat{y}_1^2\|)^{1/2}
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The different correlations



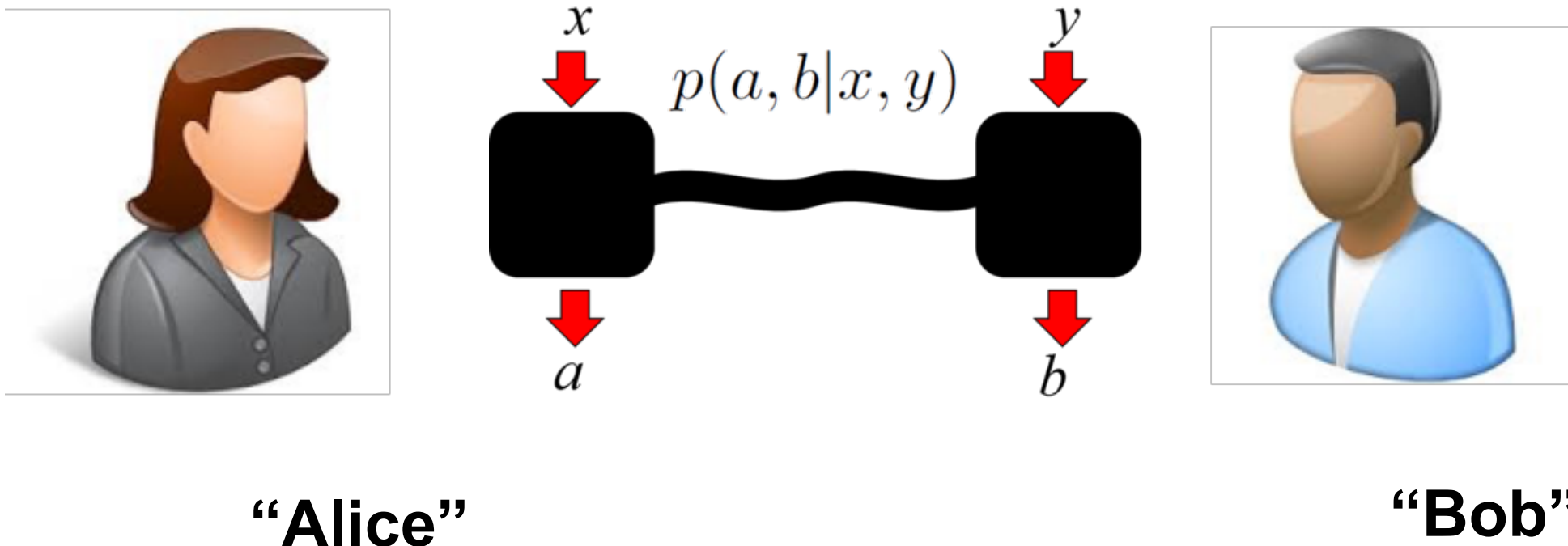
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The different correlations



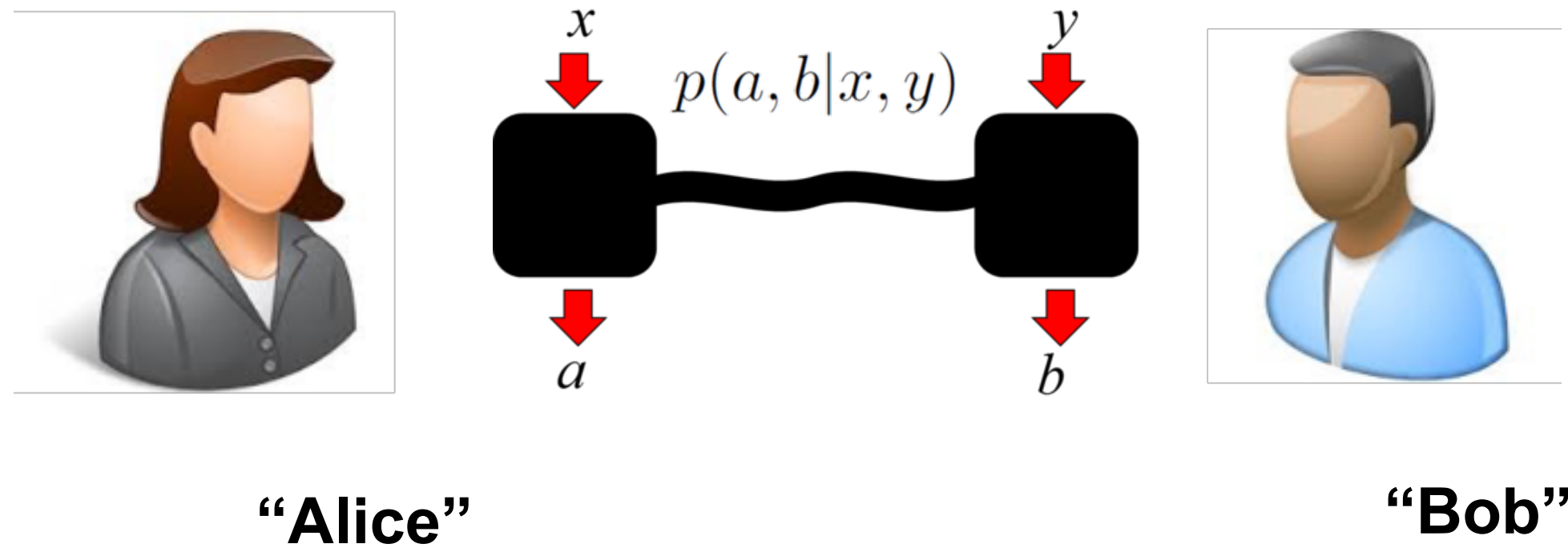
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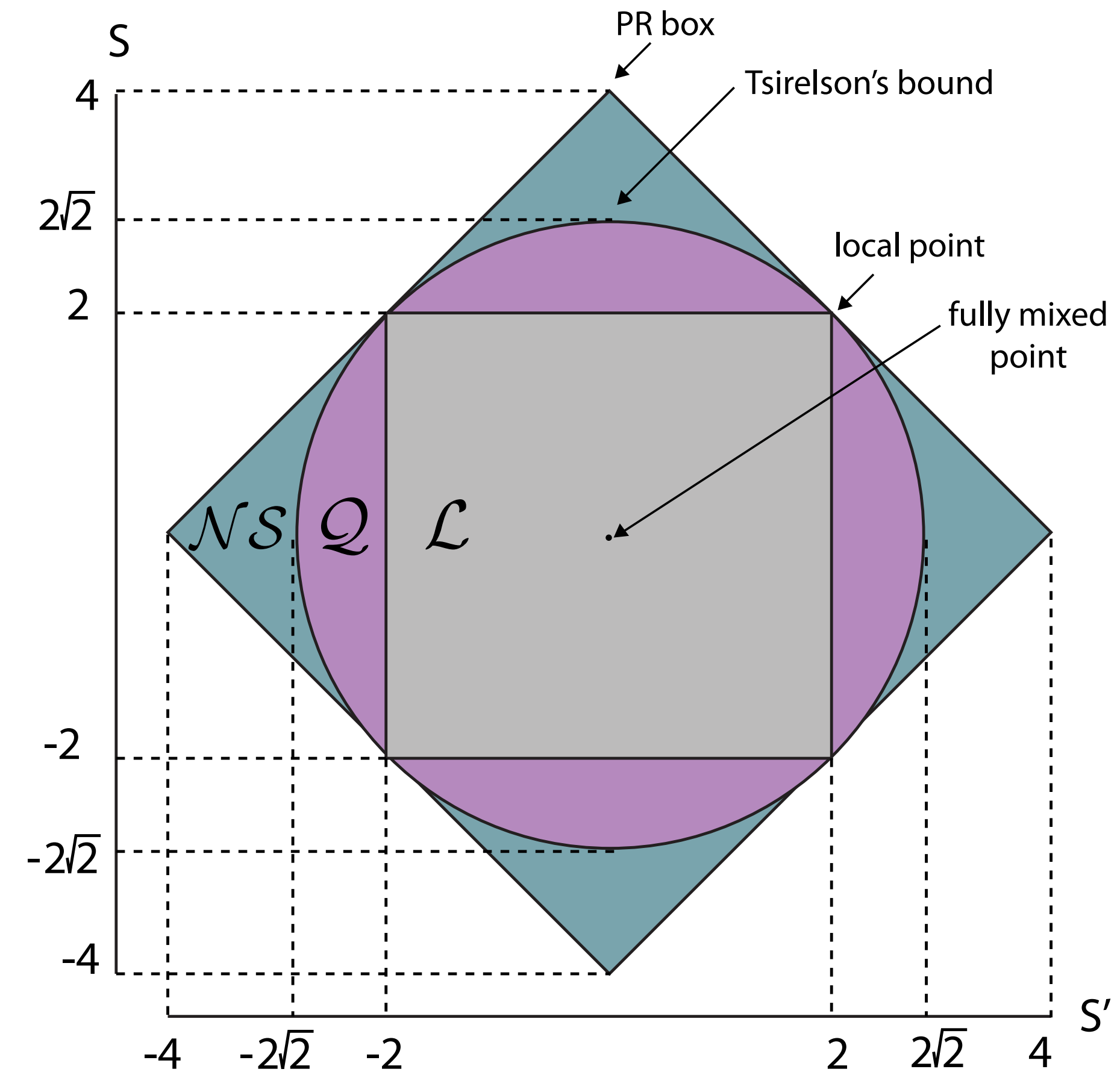
The different correlations



No-signaling correlations

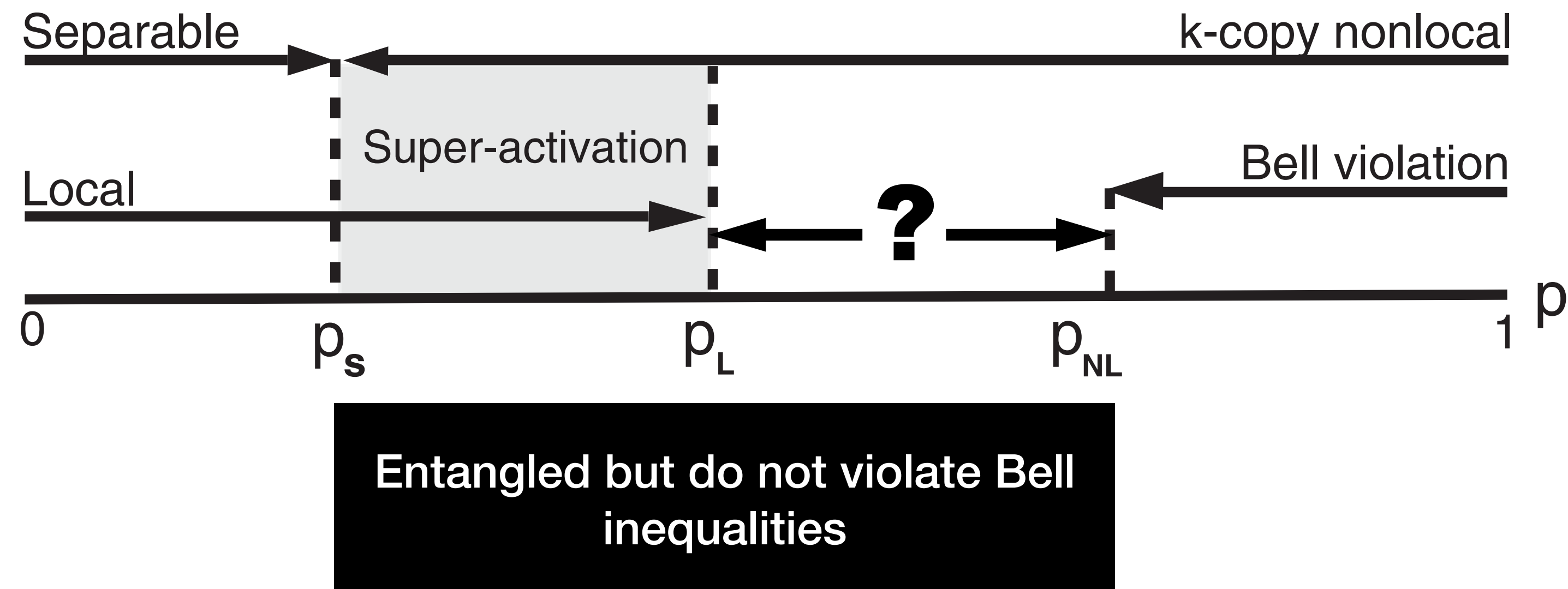
$$\sum_{b=1}^{\Delta} p(ab|xy) = \sum_{b=1}^{\Delta} p(ab|xy'), \quad \text{for all } a, x, y, y',$$

$$\sum_{a=1}^{\Delta} p(ab|xy) = \sum_{a=1}^{\Delta} p(ab|x'y), \quad \text{for all } b, y, x, x'.$$



The Relation between the Different Concepts

- A famous example (the Isotropic state): $\rho_{\text{iso}} = p|\Phi_+\rangle\langle\Phi_+| + (1-p)\frac{\mathbb{1}}{d^2}$,



For the $t\bar{t}$ case ($d = 2$):

$p_s = 1/3$, $p_L = 0.66$ and $p_{NL} = 0.7052$, or, $\mathcal{C}_s = 0$, $\mathcal{C}_L = 0.25$ and $\mathcal{C}_{NL} = 0.3111$, or, $D_s = -1/3$, $D_L = -0.42$ and $D_{NL} = -0.4370$.

WW production at Higgs factory

- Verifying the generalized Bell inequality:

$$\max_{\hat{A}_1, \hat{A}_2, \hat{B}_1, \hat{B}_2} \mathcal{I}_3(\hat{A}_1, \hat{A}_2; \hat{B}_1, \hat{B}_2) > 2$$

Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

$$\begin{aligned} \mathcal{I}_3 \equiv & + \left[P(A_1 = B_1) + P(B_1 = A_2 + 1) \right. \\ & \left. + P(A_2 = B_2) + P(B_2 = A_1) \right] \\ & - \left[P(A_1 = B_1 - 1) + P(B_1 = A_2) \right. \\ & \left. + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1) \right] \end{aligned}$$

WW production at Higgs factory

- The density matrix (some technical details...)

$$\hat{\rho}_{WW} \propto \mathcal{M}(e^+e^- \rightarrow W^+W^-) \hat{\rho}_{e^+e^-} \mathcal{M}(e^+e^- \rightarrow W^+W^-)^\dagger$$

$$\begin{aligned} \hat{\rho}_{WW} = & \frac{1}{9} \hat{I}_9 + \frac{1}{3} d_+^i \hat{S}_i^+ \otimes \hat{I}_3 + \frac{1}{3} d_-^i \hat{I}_3 \otimes \hat{S}_i^- \\ & + \frac{1}{3} q_+^{ij} \hat{S}_{\{ij\}}^+ \otimes \hat{I}_3 + \frac{1}{3} q_-^{ij} \hat{I}_3 \otimes \hat{S}_{\{ij\}}^- \\ & + C_d^{ij} \hat{S}_i^+ \otimes \hat{S}_j^- + C_{d,q}^{i,jk} \hat{S}_i^+ \otimes \hat{S}_{\{jk\}}^- \\ & + C_{q,d}^{ij,k} \hat{S}_{\{ij\}}^+ \otimes \hat{S}_k^- + C_q^{ij,k\ell} \hat{S}_{\{ij\}}^+ \otimes \hat{S}_{\{k\ell\}}^- \end{aligned}$$

WW production at Higgs factory

- The density matrix (some technical details...) is “easy” to calculate
- $\beta \rightarrow 0$?
 - s-channel is p -wave and suppressed by a factor of β .
 - t -channel is purely left-handed current so that the initial state is selected by the (weak) interaction to be a pure state $|e_L^-\rangle \otimes |e_L^+\rangle$.

$$\rho_{W^+} = \begin{pmatrix} \frac{(3 - \cos \theta)}{4} \cos^2 \frac{\theta}{2} & -\frac{(1 - \cos \theta) \sin \theta}{4\sqrt{2}} & -\frac{1}{8} \sin^2 \theta \\ -\frac{(1 - \cos \theta) \sin \theta}{4\sqrt{2}} & \frac{1}{8} (3 + \cos 2\theta) & -\frac{(1 + \cos \theta) \sin \theta}{4\sqrt{2}} \\ -\frac{1}{8} \sin^2 \theta & -\frac{(1 + \cos \theta) \sin \theta}{4\sqrt{2}} & \frac{(3 + \cos \theta)}{4} \sin^2 \frac{\theta}{2} \end{pmatrix}$$

$$= \frac{1}{2} |v_1\rangle \langle v_1| + \frac{1}{2} |v_2\rangle \langle v_2|, \quad S_{vN} = \log 2$$

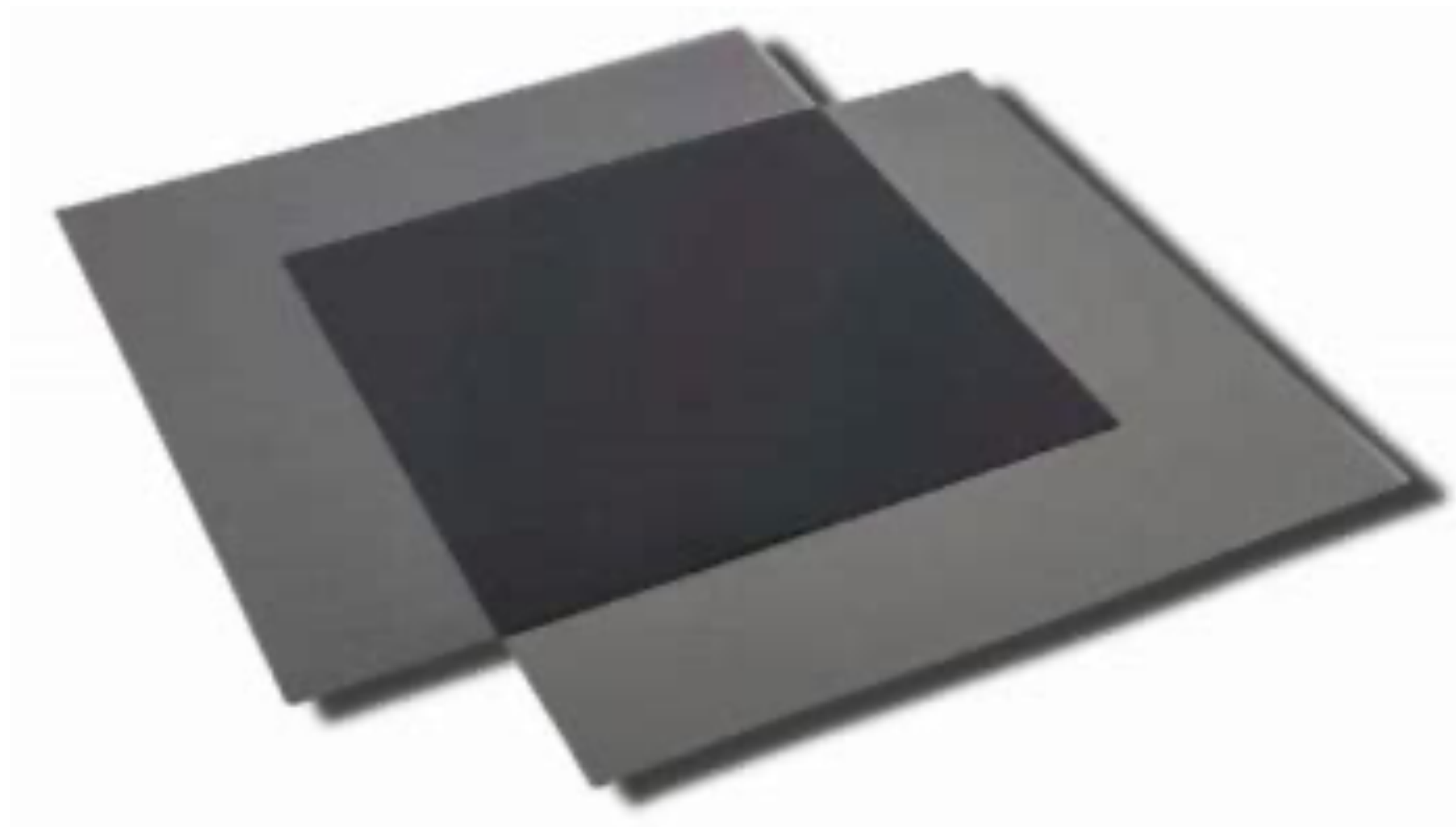
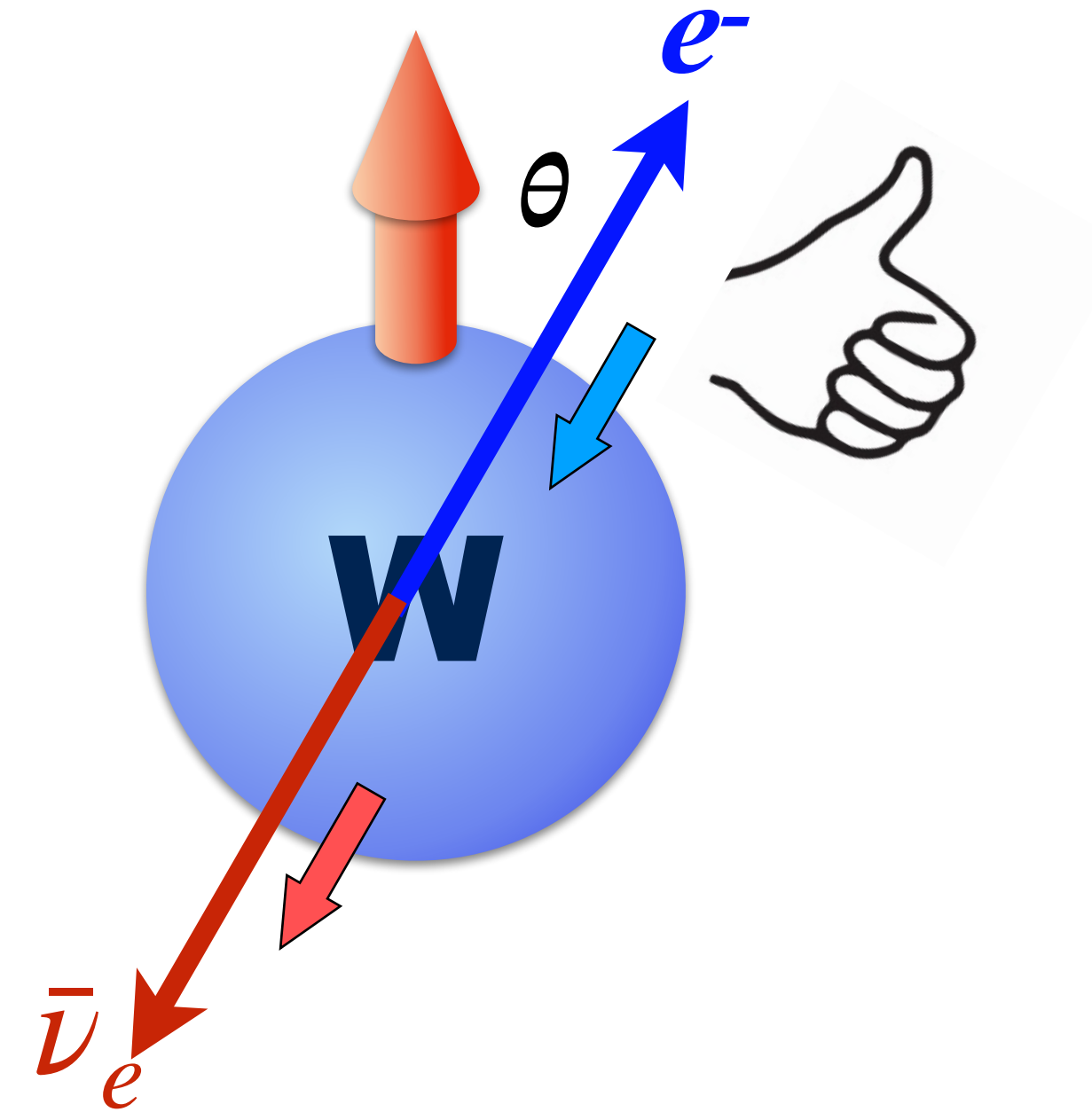
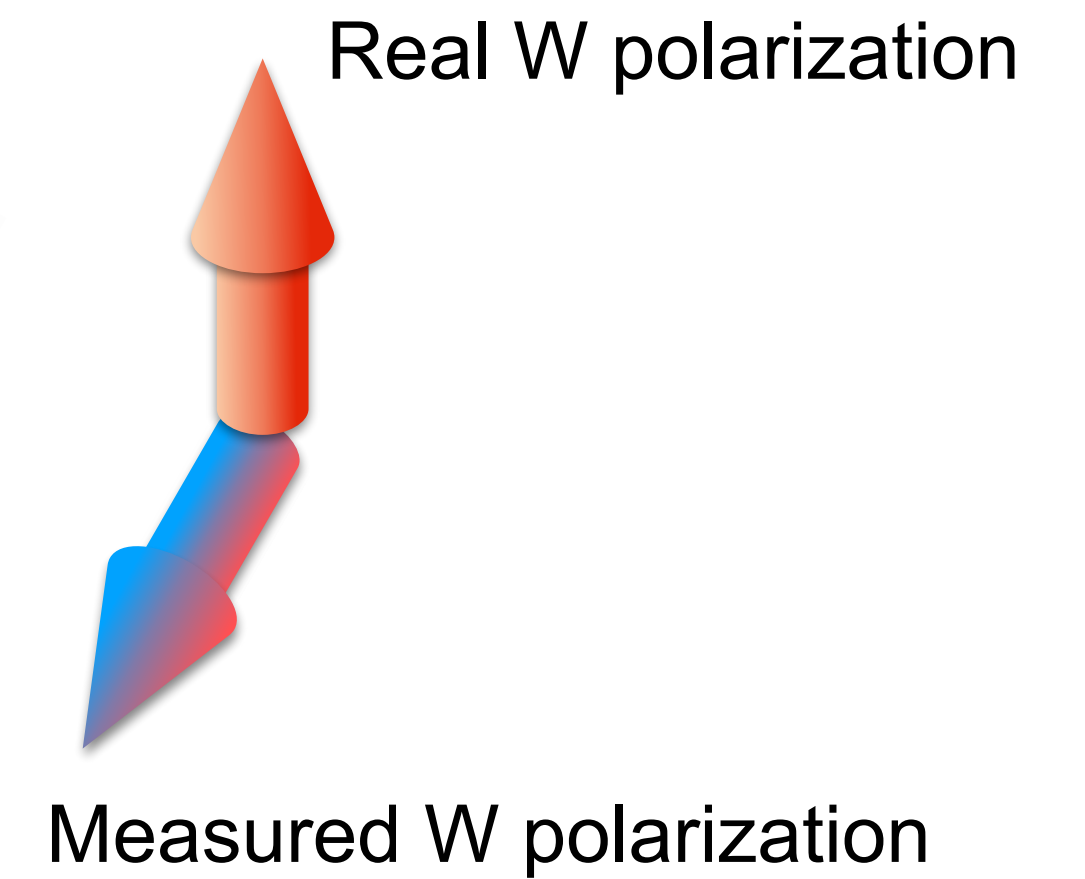
$$|v_1\rangle = \frac{1 + \cos \theta}{\sqrt{3 + \cos 2\theta}} |+\rangle - \frac{1 - \cos \theta}{\sqrt{3 + \cos 2\theta}} |-\rangle,$$

$$|v_2\rangle = -\frac{(1 - \cos \theta) \sin \theta}{\sqrt{2}\sqrt{3 + \cos 2\theta}} |+\rangle + \frac{\sqrt{3 + \cos 2\theta}}{2} |0\rangle - \frac{(1 + \cos \theta) \sin \theta}{\sqrt{2}\sqrt{3 + \cos 2\theta}} |-\rangle.$$

$$|v_3\rangle = \left(\frac{1 - \cos \theta}{2} \right) |+\rangle + \frac{\sin \theta}{\sqrt{2}} |0\rangle + \left(\frac{1 + \cos \theta}{2} \right) |-\rangle$$

WW production at Higgs factory

- “Measuring” the polarization direction of the W boson.



WW production at Higgs factory

- Reconstruct the density matrix and “do” a Bell exp with the density matrix.

$$P(A_1 = B_1) = \sum_{\lambda=-1}^1 \text{Tr}[\hat{\rho}_{WW} \hat{\Pi}_{|A_1=\lambda, B_1=\lambda}],$$

$$d_i = \langle \mathbf{n}_i \rangle, \quad q_{ij} = \frac{5}{2} \langle \mathbf{q}_{ij} \rangle, \quad \mathbf{q}_{ij} \equiv \mathbf{n}_i \mathbf{n}_j - \frac{1}{3} \delta_{ij}$$

$$C_{ij}^d = \langle \mathbf{n}_i^+ \mathbf{n}_j^- \rangle, \quad C_{i,jk}^{dq} = \frac{5}{2} \langle \mathbf{n}_i^+ \mathbf{q}_{jk}^- \rangle,$$

$$C_{ij,k\ell}^q = \frac{25}{4} \langle \mathbf{q}_{ij}^+ \mathbf{q}_{k\ell}^- \rangle, \quad C_{ij,k}^{qd} = \frac{5}{2} \langle \mathbf{q}_{ij}^+ \mathbf{n}_k^- \rangle.$$

WW production at Higgs factory

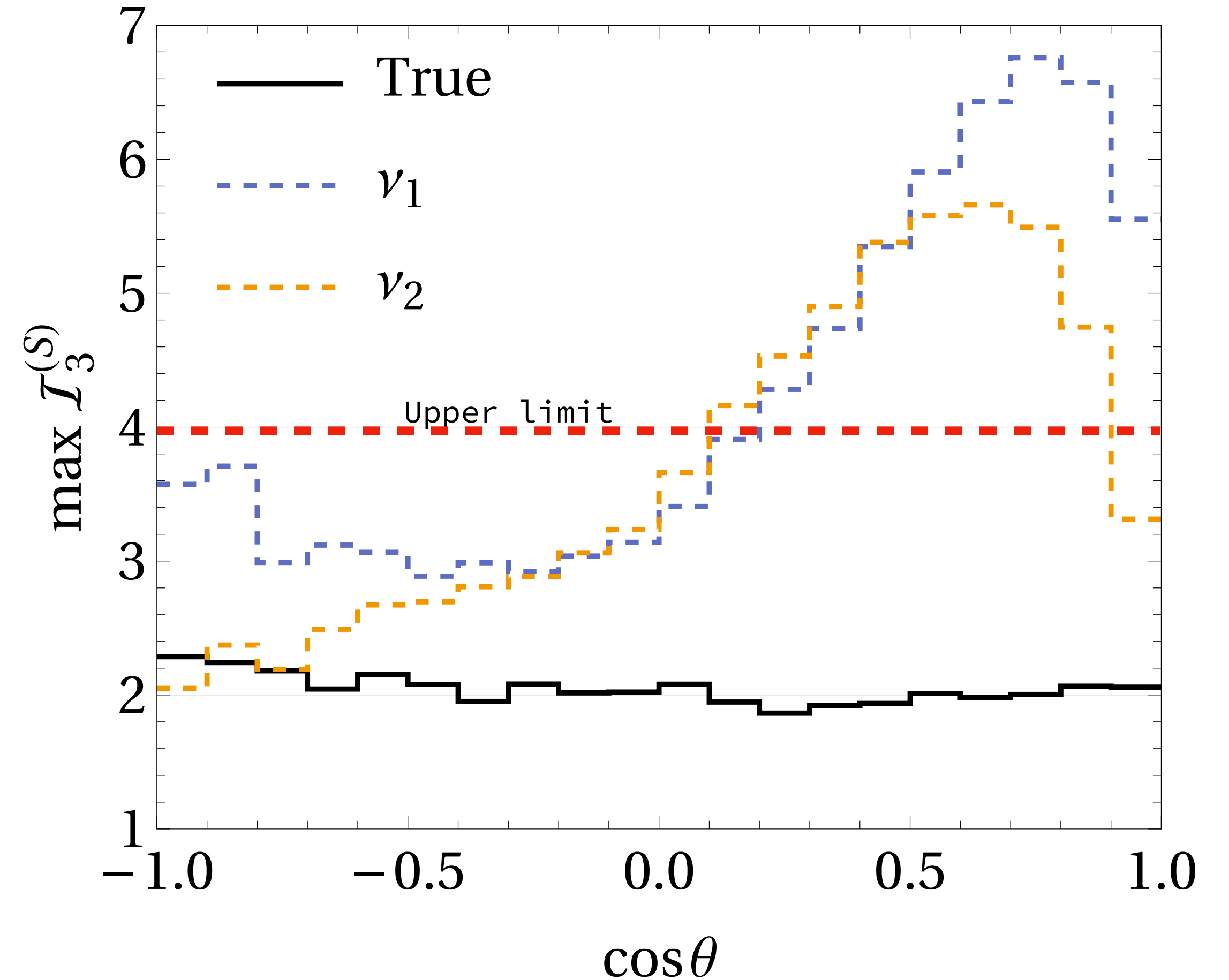
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WW production at Higgs factory

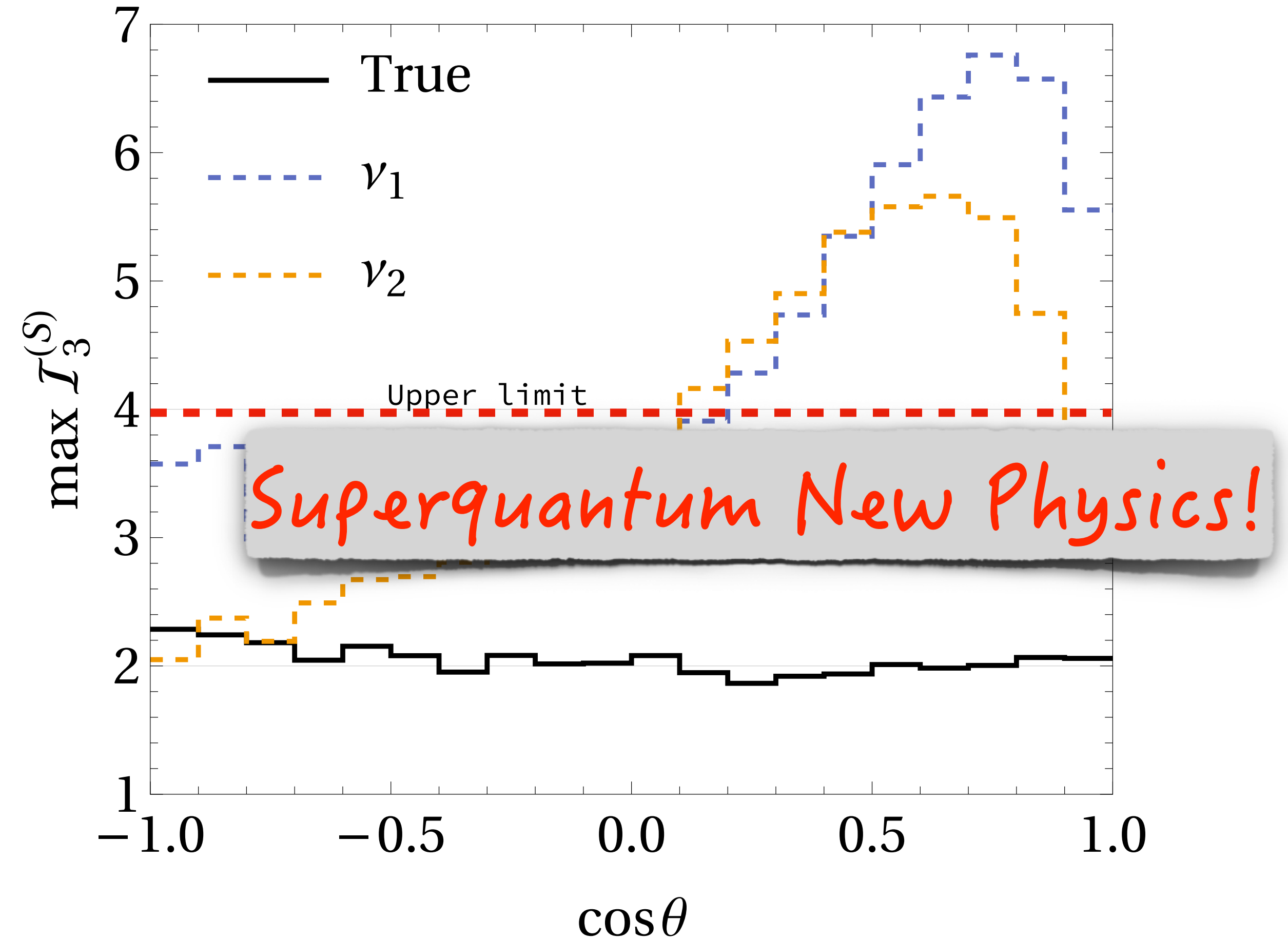
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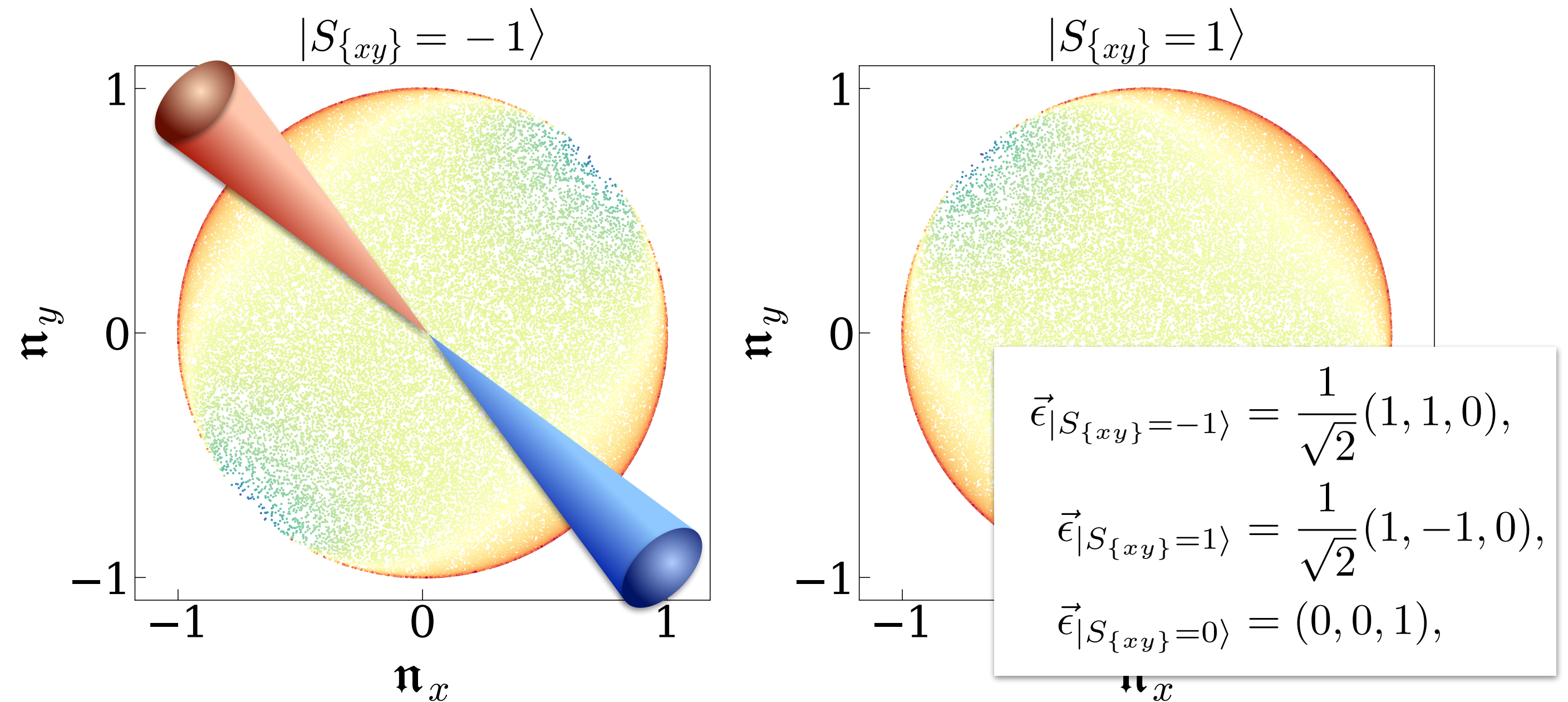
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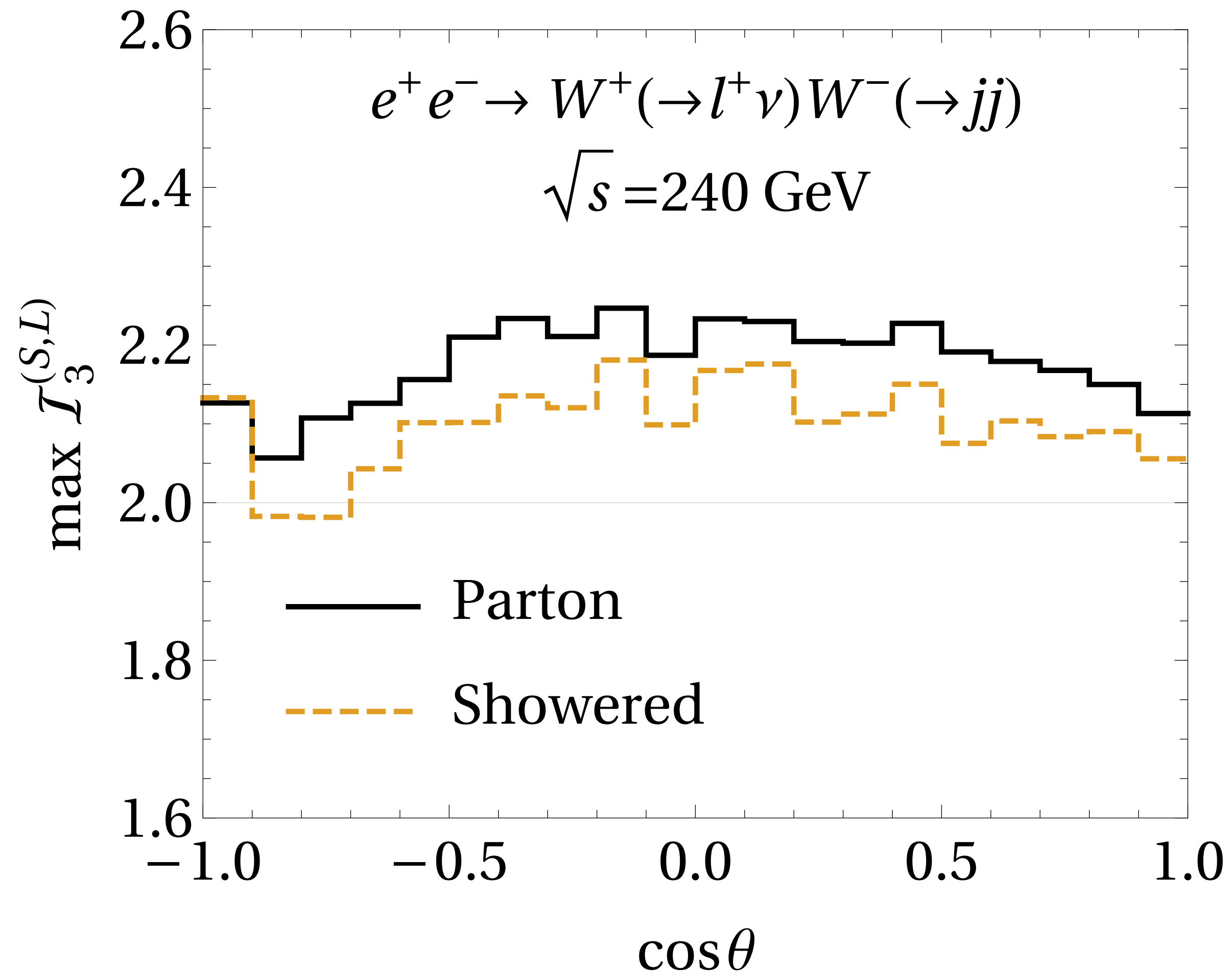
WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization $\rightarrow$ linear polarization. $\hat{\Pi}_{\mathbf{n}} = \hat{I}_3 - \hat{S}_{\mathbf{n}}^2$



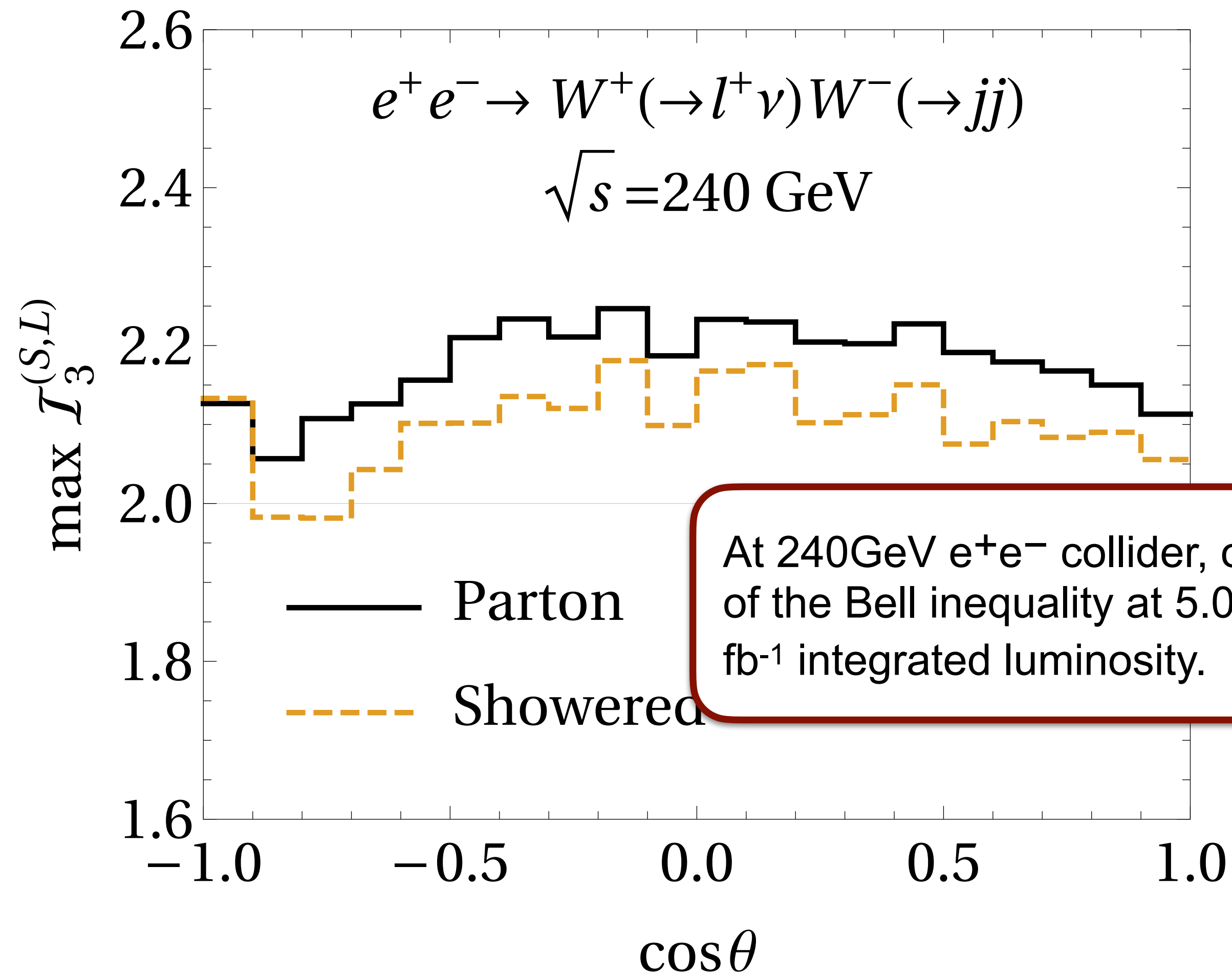
WW production at Higgs factory

- The result



WW production at Higgs factory

- The result



Summary and Discussion

- Recently, a lot of works appear on the topic of quantum entanglement and quantum nonlocality in particle physics. (e.g., $t\bar{t}$, Higgs decays into ZZ , $\Lambda\bar{\Lambda}$, neutrino oscillation, fragmentation, ...)
- More?

The background features a dark blue gradient. Two glowing blue spheres, composed of concentric wireframe lines, are positioned on the left and right sides. A series of bright yellow lines, resembling a ribbon or a complex wave, flows from the left sphere, across the center, and towards the right sphere. The text "Thank you!" is written in a white, elegant script font, centered in the upper half of the image.

Thank you!