復旦大學
FUDAN UNIVERSITY

Quantum Entanglement Studies
in Nuclear and Particle Physics

University of
Pittsburgh

Quantum information in DIS process at the EIC

Based on arXiv:2510.23773, PRD(2026), **Kun Cheng**, Tao Han and Sokratis Trifinopoulos

Rich Quantum Information at colliders

- LHC, Belle, BESE, RHIC…
- $t\bar{t}, b\bar{b}, W^{\pm}, Z, q\bar{q}, \Lambda\bar{\Lambda}, \tau^{\pm}\cdots,$

- A near future collider — Electron Ion Collider (EIC).
  - Connections with QCD (nucleon tomography)
  - More control on the initial states (polarization, …)
  - Is the special properties of the EIC relavent to quantum information?

Spin states at colliders

- Depends on inital state, interactions, and kinematics
  - The “unnormalized” spin state of final sate $\mathcal{AB}$

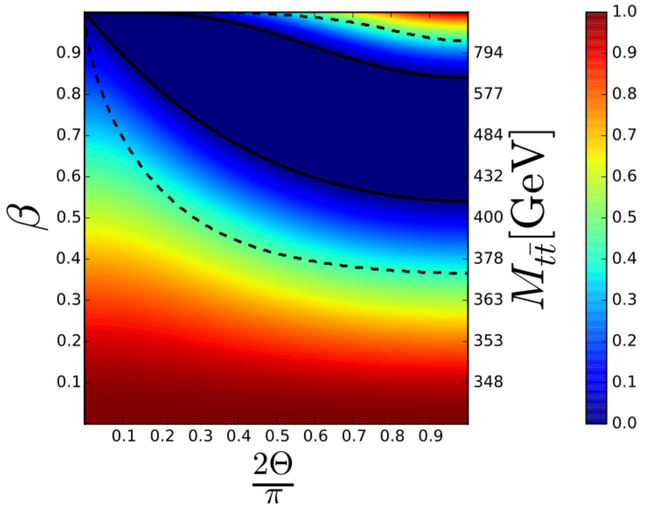
$$|\mathcal{AB}\rangle = \hat{\mathcal{T}}\,|\text{inital}\rangle$$

$$\hat{R}_{\mathcal{AB}}(\mathbf{k}) = \mathcal{M}^{\lambda}_{\alpha\bar{\alpha}}(\mathbf{k})\ \rho^{\text{initial}}_{\lambda\lambda'}\ (\mathcal{M}^{\lambda'}_{\alpha'\bar{\alpha}'}(\mathbf{k}))^{*}\ \ |\alpha\bar{\alpha}\rangle\langle\alpha'\bar{\alpha}'|$$

If the intial state is a superposition/mixture of ① and ②, then final state is a superposition/mixture of the states produced by ① and ②,

- E.g. one of the most studied: $gg \to t\bar{t}$
- E.g. no entanglement for $q\bar{q} \to t\bar{t}$ near threshold:
  both $q_L\bar{q}_R$ and $q_R\bar{q}_L$ produce seperable states, mixture cannot generate entanglement

$$|\psi^{\mathcal{AB}}\rangle = |\psi^{\mathcal{A}}\rangle \otimes |\psi^{\mathcal{B}}\rangle \qquad \rho^{\mathcal{AB}} = \sum_i p_i \rho_i^{\mathcal{A}} \otimes \rho_i^{\mathcal{B}}$$

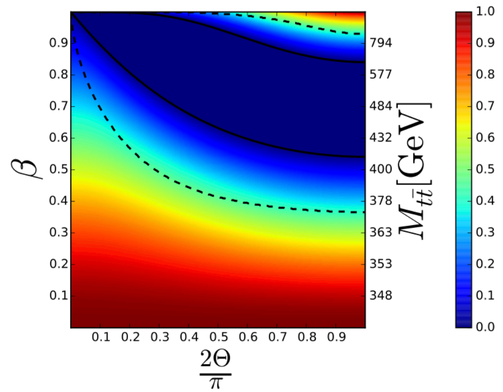


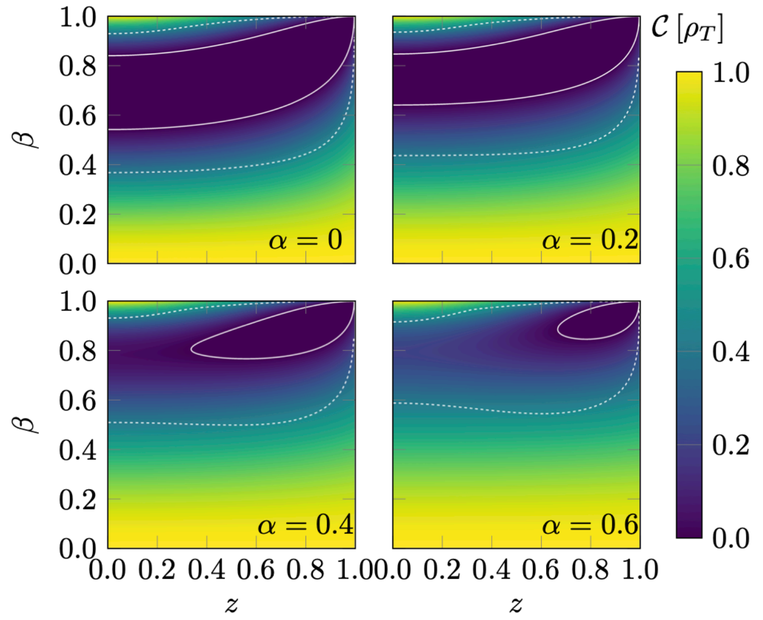
[Afik and de Nova, Quantum (2022)]

Quantum information at the EIC

- At the EIC, $g\gamma/\gamma\gamma \to q\bar{q}$ is a similar process (at lowest order)
  [Qi, Guo, and Xiao, PRD (2026)]
  [Fucilla and Hatta, PRD (2026)]
  [Hatta and Schoenleber ,arXiv:2511.04537]
  [Fucilla, Hatta, and Xiao, arXiv:2604.11697]
  - $gg \to q\bar{q}$, $g\gamma \to q\bar{q}$ and $\gamma\gamma \to q\bar{q}$ produces exactly the same spin state [**KC,** Han and Low, PRD (2025)]
- Something typicial/special
  - Virtuality of photon
  - Ultraperipheral Collisions (UPC)
  - Linear polarization of gluon
- What about the leading process $eq \to eq$?

[Afik and de Nova, Quantum (2022)]





[Qi, Guo, and Xiao, PRD (2026)]

Leading processes at the EIC

- Serves for many major physic focus of the EIC :
  - “3D tomography of the nucleon”
  - PDF, TMD PDF, polarized PDF, tensor charge…

[EIC white paper, arXiv:1212.1701]

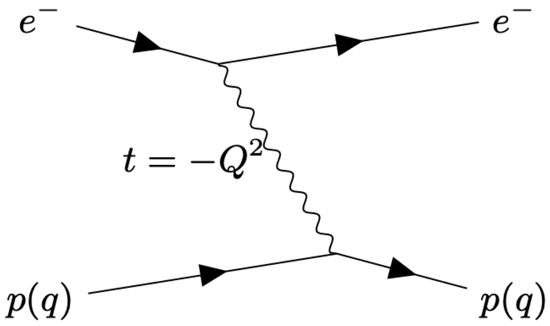


FIG. 1: Elastic e^-p (inelastic e^-q) scattering

- What’s the quantumness of the elastic and deep inelastic scattering process? (in the presence of beam polarization)
  - e.g. entanglement (concurrence $\mathscr{C}$)
  - e.g. nonstabilizerness (magic $\mathscr{M}_2$)

[see yesterday’s talk by Prof. Qun Wang, Hao Zhang and Jinniu Hu]

Leading processes at the EIC

- Helicity is unchanged in t-channel process
  - $|e_\uparrow\rangle \otimes |p_\alpha\rangle \longrightarrow |e_\uparrow\rangle \otimes |p_{\alpha'}\rangle$
  - $|e_\downarrow\rangle \otimes |p_\alpha\rangle \longrightarrow |e_\downarrow\rangle \otimes |p_{\alpha''}\rangle$
- e.g. backward limit, $|e_\uparrow\rangle \to |\uparrow\downarrow\rangle, |e_\downarrow\rangle \to |\downarrow\uparrow\rangle$
  - Unpolarized beams, mixture of $|e_\uparrow\rangle$ and $|e_\downarrow\rangle$

    incoherent mixture of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$
  - Any non-trivial quantum information?

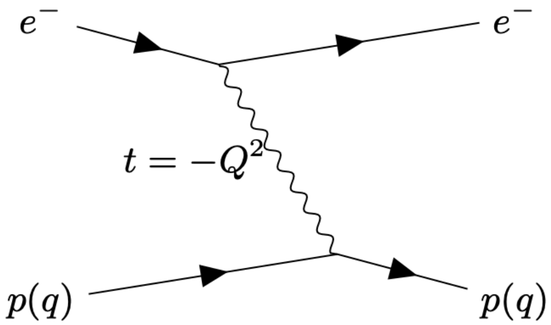


FIG. 1: Elastic $e^- p$ (inelastic $e^- q$) scattering

$$\mathcal{M}^{\alpha\beta}_{\alpha'\beta'} \equiv \mathcal{M}(e^-(\alpha)p(\beta) \to e^-(\alpha')p(\beta'))$$

$$\mathcal{M}^{\uparrow\Uparrow}_{\uparrow\Uparrow} \approx 2Q_p e^2 \frac{s+t}{t} e^{i\phi},$$

$$\mathcal{M}^{\uparrow\Downarrow}_{\uparrow\Downarrow} = \mathcal{M}^{\downarrow\Uparrow}_{\downarrow\Uparrow} \approx 2Q_p e^2 \frac{s}{t},$$

$$\mathcal{M}^{\downarrow\Downarrow}_{\downarrow\Downarrow} \approx 2Q_p e^2 \frac{s+t}{t} e^{-i\phi},$$

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- Helicity is unchanged in t-channel process
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- e.g. backward limit, $|e_\uparrow\rangle \to |\uparrow\downarrow\rangle, |e_\downarrow\rangle \to |\downarrow\uparrow\rangle$
  - Unpolarized beams, mixture of $|e_\uparrow\rangle$ and $|e_\downarrow\rangle$

    incoherent mixture of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$
- **Opportunities at the EIC: transverse polarization**
  - Incoming beam: superposition of $|e_\uparrow\rangle$ and $|e_\downarrow\rangle$
  - Final state: entangled $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

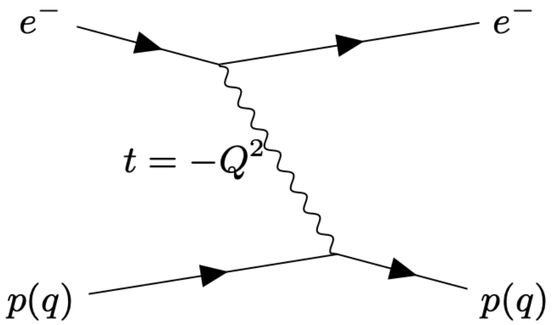
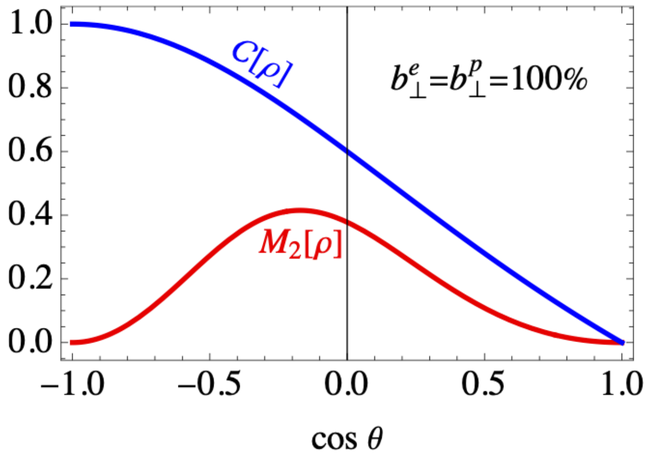


FIG. 1: Elastic e^-p (inelastic e^-q) scattering



[**KC**, Han and Trifinopoulos, arXiv:2510.23773]

Beam polarizations

- Polarizations & Bloch vector $\rho = \frac{I_2 + b_i \sigma_i}{2}, \quad b_i = \langle \sigma_i \rangle$
  - Pure state $|\vec{b}| = 1$: e.g. $\begin{cases} b_z = 1 & \longrightarrow |\psi\rangle = |\uparrow\rangle \\ b_x = 1 & \longrightarrow |\psi\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2} \end{cases}$
  - Mixed state $|\vec{b}| < 1$: e.g. $|\vec{b}| = 10\% \longrightarrow \rho = \underline{10\%}\, |\psi\rangle \langle\psi| + 90\% \frac{I_2}{2}$

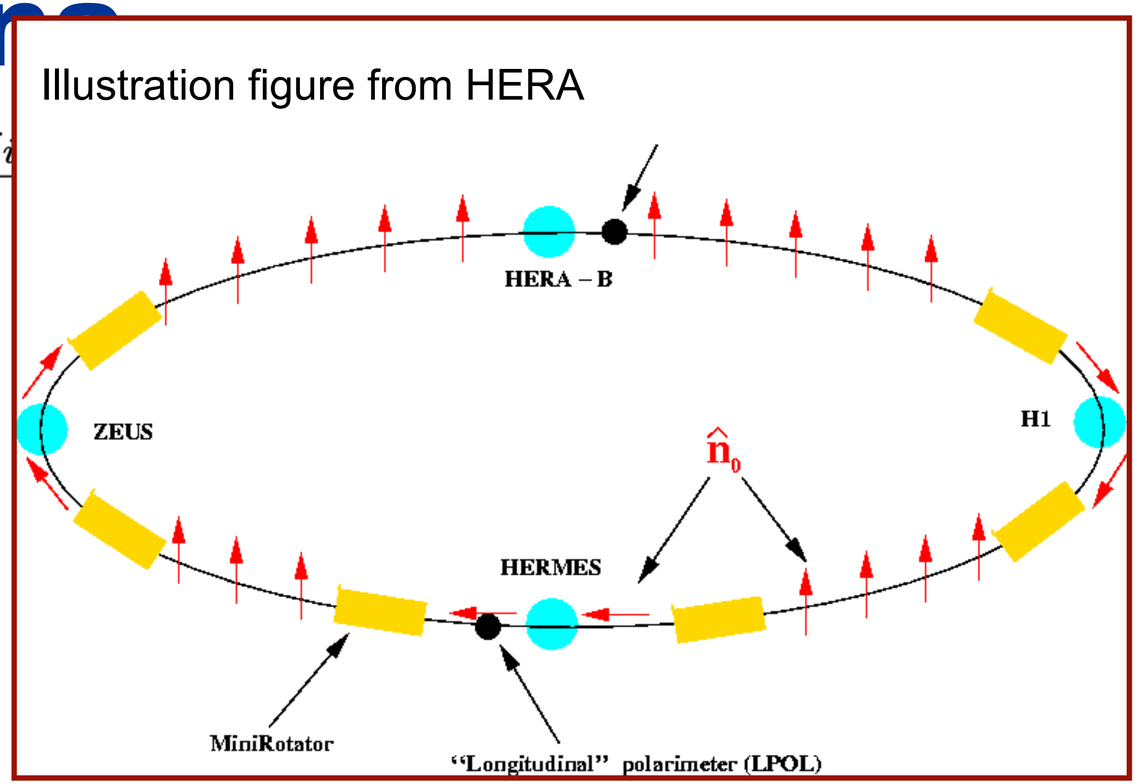
    degree of beam polarization
- Transverse polarization naturally exists
  - Sokolov-Ternov effect: ideally >90%

    Unpolarized $\Longrightarrow$ Transverse (electron spin becoms opposite to magnetic field)
  - Transverse $\Longrightarrow$ Longitudinal

    Transverse polarization is a medium stage even we only want longitudinal polarization

Beam polarization

- Polarizations & Bloch vector $\rho = \frac{I_2 + b_i \sigma_i}{2}$
  - Pure state $|\vec{b}| = 1$: e.g. $\begin{cases} b_z = 1 \longrightarrow \\ b_x = 1 \longrightarrow \end{cases}$
  - Mixed state $|\vec{b}| < 1$: e.g. $|\vec{b}| = 10\% \longrightarrow \rho$



- Transverse polarization naturally exists
  - Sokolov-Ternov effect: ideally >90%

    Unpolarized $\Longrightarrow$ Transverse (electron spin becoms opposite to magnetic field)
  - Transverse $\Longrightarrow$ Longitudinal
  - >70% at EIC for both beams

    [EIC yellow report, arXiv:2103.05419]

    More details in conceptual design report

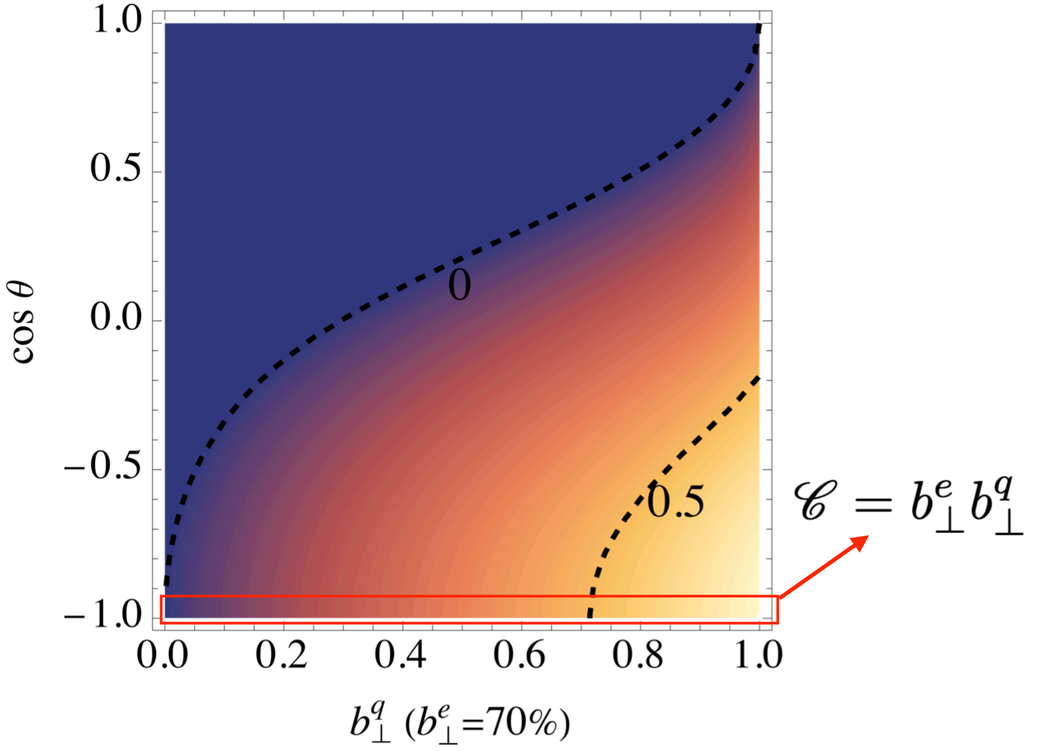
    [www.osti.gov/biblio/1765663/]

DIS process with transverse polarization

- E.g. purely transversely polarized

$$|ep/eq\rangle \propto \left(\frac{s-Q^2}{s}\right)\left(e^{i\phi}\,|\uparrow\uparrow\rangle + e^{-i\phi}\,|\downarrow\downarrow\rangle\right) + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

- For either elastic or inelastic scattering, the entanglement of final state occurs only when:
  - Initial beams are transversely polarized
  - Q^2 is sizable (then mostly DIS process)
- Entanglement increases with:
  - $Q^2 \quad \sim \quad 1-\cos\theta$
  - beam polarization $\vec{b}_\perp$



The dependence of the entanglement on the polarization gives us the opportunity to control the state production by changing the initial state polarization.

Quark polarization: transversity PDF

- Transversity PDF $h_{1,q}(x)$: when proton spin points along transverse direction $+\hat{n}$
  - probability of finding q polarized along $+\hat{n}$ minus the probability of finding q polarized along $-\hat{n}$
- Together with the unpolarized PDF $f_q(x)$, probability of finding q from the proton, we see the relationship between the spin density matrices of quark and proton:

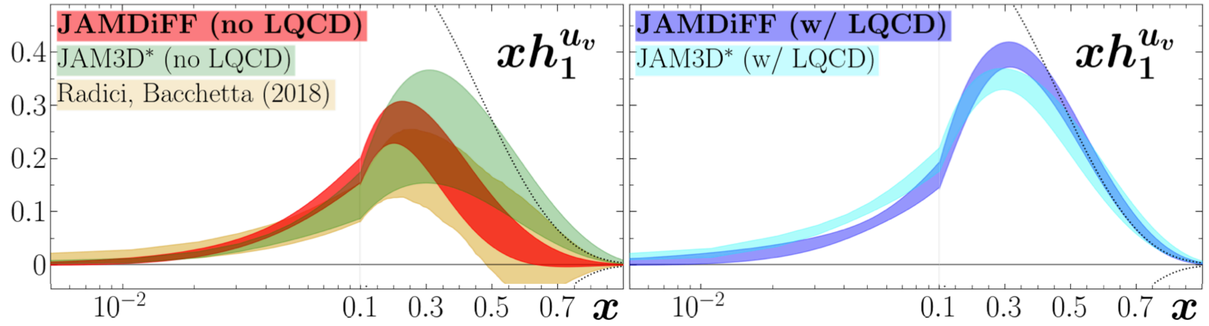
$$\rho^p = \frac{\mathbb{1}_2 + \vec{b}_\perp^p \cdot \vec{\sigma}_\perp}{2}, \quad \rho^q = \frac{\mathbb{1}_2 + \vec{b}_\perp^q \cdot \vec{\sigma}_\perp}{2}, \qquad \vec{b}_\perp^q = \frac{h_{1,q}(x)}{f_q(x)} \vec{b}_\perp^p$$

- Without changing collider settings, we can choose different quark polarizations.

Quark polarization: transversity PDF

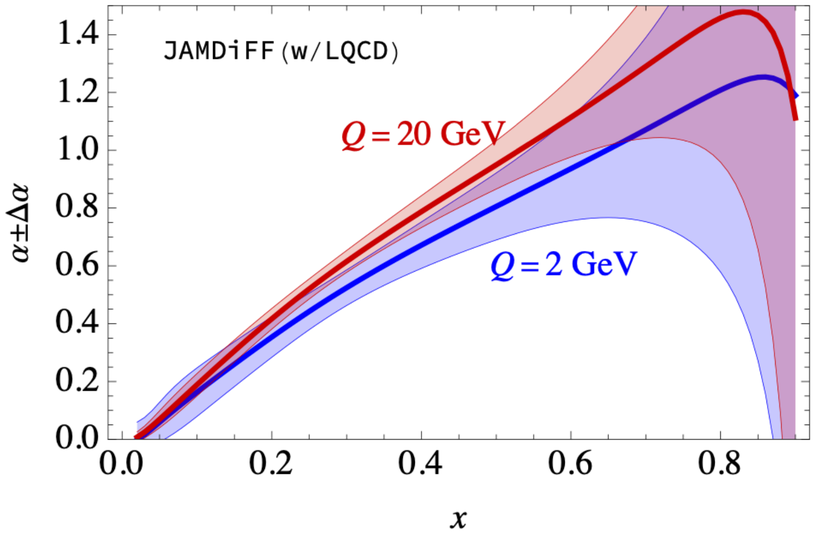
- E.g. $h_1(x)$ from JLab

[Cocuzza et al (JAM Collaboration), PRD (2024)]



- Effective polarization of quarks that participate in the interaction:

$$\vec{b}_\perp^{\langle q\rangle} = \alpha \vec{b}_\perp^{p}, \quad \alpha \equiv \frac{\sum_q e_q^2 h_{1,q}(x)}{\sum_q e_q^2 f_q(x)}$$



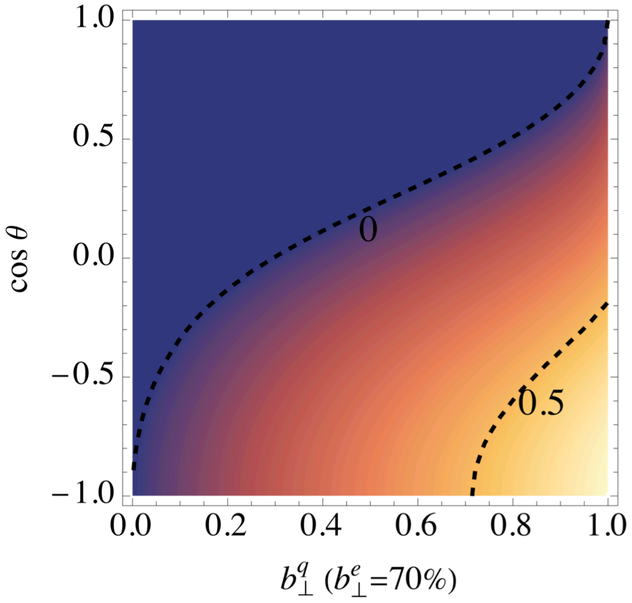
Parton-level vs Hadron-level Kinematics

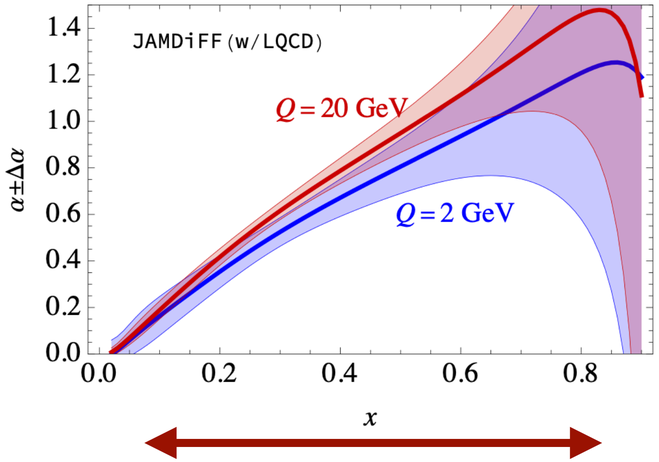
- Controlling/Selecting quark transverse polarization from DIS kinematics

⇓

Control the quantum state production

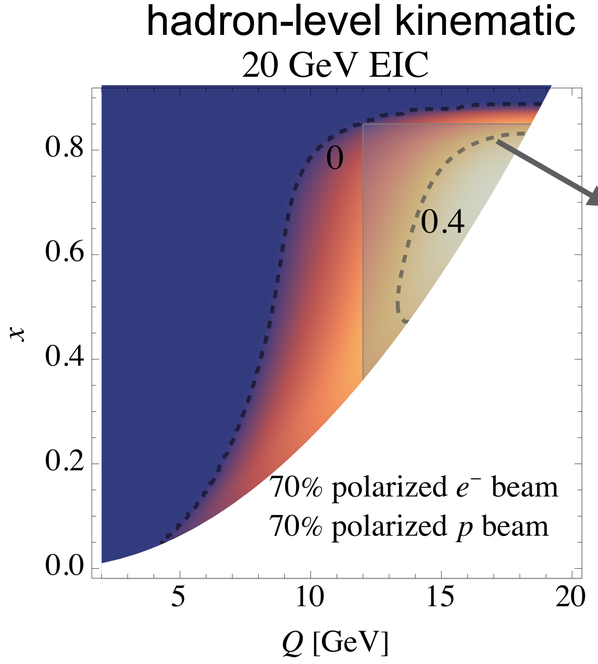
parton-level kinematic





$$\vec{b}_\perp^{\langle q\rangle} = \alpha \vec{b}_\perp^{\,p}, \quad \alpha \equiv \frac{\sum_q e_q^2 h_{1,q}(x)}{\sum_q e_q^2 f_q(x)}$$

hadron-level kinematic



2M event at $10fb^{-1}$

Summary

- In terms of quantum information, transverse polarization of initial states usually makes a qualitative effect compared to longitudinal polarization. [see also Hao Zhang's talk]

- At EIC, there are plenty of ways to select/control initial states.
  - e.g. with the transversity PDF, we show the opportunity to control the state production from the initial state polarization.
  - Beyond leading order, $b_\perp^q$ can arise from unpolarized protons in TMD factorization [EIC white paper, arXiv:1212.1701]

- Spin of quark can be measured. Looking to the future, direct spin measurements of stable particles at colliders would make more information available, and help understanding QCD or NP? [see also Bin Yan's talk]

Quantum entanglement and magic

- Entanglement asks whether the state can be written in a separble form

$$|\psi^{\mathcal{AB}}\rangle = |\psi^{\mathcal{A}}\rangle \otimes |\psi^{\mathcal{B}}\rangle \qquad \rho^{\mathcal{AB}} = \sum_i p_i \rho_i^{\mathcal{A}} \otimes \rho_i^{\mathcal{B}}$$

  - Concurrence, a quantitative measure of entanglement:

$$\mathscr{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) > 0$$

λ_i are eigenvalues of $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$

$$\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$$

$\mathscr{C}(\rho) = 0$ (seperable)

$= 1$ (maximally entangled)

[W. Wootters, PRL (1998)]

- Magic asks whether how different the state is from stabilizer states generated by Clifford gates

$$\mathcal{P}_n = P_1 \otimes P_2 \otimes \ldots \otimes P_n, \qquad P_i \in \{\mathbb{1}, \sigma_1, \sigma_2, \sigma_3\},$$

  - Stabilizer Rényi entropy, a quantitative measure of magic

$$M_k(\rho) = -\frac{1}{1-k} \log_2 \left(\frac{\sum_{P \in \mathcal{P}_n} (\mathrm{Tr}[P\rho])^{2k}}{\sum_{P \in \mathcal{P}_n} (\mathrm{Tr}[P\rho])^2} \right)$$

[Leone, Oliviero, and Hamma, PRL (2022)]

Many ways to infer the quark spin

- Depending on different theory inputs:
  - From production kinematics [**KC**, T. Han and M. Low, PLB (2025)]
  - From Λ hadron spin ($q \to \Lambda_q + X, \Lambda_q \to \dots$) [Afik, Kats, de Nova, Soffer and Uzan, PRD (2025)] [Lin, Liu, Shao, and Wei, JHEP (2025)]

    (some spin fraction of q is carried by Λ_q, nonperturbative theory inputs)

  - From inteference fragmentation functions ($q \to \pi^+\pi^- + X$) [**KC** and Yan, PRL (2025)]

$\epsilon^T \cdot \vec{S}^T$ $\quad R^T = p^T_{\pi^+} - p^T_{\pi^-}$

Backup: Elastic scattering

- Elastic, form factors

$$J^{\mu}_{e,\alpha\alpha'} = \bar{u}_{\alpha'}(p'_e)\gamma^{\mu}u_{\alpha}(p_e), \qquad J^{\mu}_{p,\beta\beta'} = \bar{u}_{\beta'}(p'_p)\Gamma^{\mu}u_{\beta}(p_p),$$

$$\Gamma^{\mu} = \gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m_p}F_2(q^2), \qquad q = p'_p - p_p,\ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}].$$

$$\mathcal{T} = \begin{pmatrix} \mathcal{M}^{\uparrow\Uparrow}_{\uparrow\Uparrow} & \mathcal{M}^{\uparrow\Downarrow}_{\uparrow\Uparrow} & 0 & 0 \\ \mathcal{M}^{\uparrow\Uparrow}_{\uparrow\Downarrow} & \mathcal{M}^{\uparrow\Downarrow}_{\uparrow\Downarrow} & 0 & 0 \\ 0 & 0 & \mathcal{M}^{\downarrow\Uparrow}_{\downarrow\Uparrow} & \mathcal{M}^{\downarrow\Downarrow}_{\downarrow\Uparrow} \\ 0 & 0 & \mathcal{M}^{\downarrow\Uparrow}_{\downarrow\Downarrow} & \mathcal{M}^{\downarrow\Downarrow}_{\downarrow\Downarrow} \end{pmatrix}$$

$$= 2e^2\frac{E_eE_p}{t}F_1(q^2)\begin{pmatrix} e^{i\phi}(1+\beta)(1+c_\theta) & s_\theta\sqrt{1-\beta^2} & 0 & 0 \\ -e^{i\phi}s_\theta\sqrt{1-\beta^2} & 1+3\beta+(1-\beta)c_\theta & 0 & 0 \\ 0 & 0 & 1+3\beta+(1-\beta)c_\theta & e^{-i\phi}s_\theta\sqrt{1-\beta^2} \\ 0 & 0 & s_\theta\sqrt{1-\beta^2} & e^{-i\phi}(1+\beta)(1+c_\theta) \end{pmatrix}$$

$$+ 4e^2\frac{E_eE_p}{t}F_2(q^2)\begin{pmatrix} 0 & -s_\theta\frac{\beta(\beta+1)}{\sqrt{1-\beta^2}} & 0 & 0 \\ e^{i\phi}s_\theta\frac{\beta(\beta+1)}{\sqrt{1-\beta^2}} & 2\beta(1-c_\theta) & 0 & 0 \\ 0 & 0 & 2\beta(1-c_\theta) & -e^{-i\phi}s_\theta\frac{\beta(\beta+1)}{\sqrt{1-\beta^2}} \\ 0 & 0 & s_\theta\frac{\beta(\beta+1)}{\sqrt{1-\beta^2}} & 0 \end{pmatrix}$$