Can Mirror Symmetry Challenge Local Realism? Probing Photon Entanglement from Positronium via Compton Scattering

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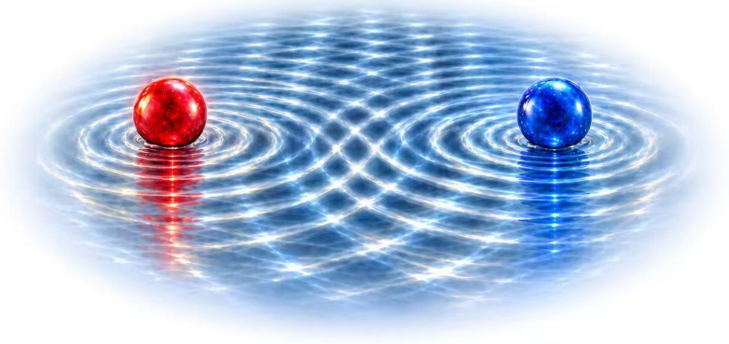
Quantum mechanics vs. local hidden variable theory



- **EPR (1935):** Is the quantum-mechanical description of physical reality complete? Could pre-existing "hidden variables" λ restore locality and realism?
- **Bell (1964):** Any local hidden-variable theory (LHVT) must satisfy conditional factorization

$$P(a, b \mid \lambda) = P(a \mid \lambda)\, P(b \mid \lambda)\,, \tag{1}$$

leading to Bell inequalities for measurable correlation functions.

From p-Ps to Compton polarimetry

- **Pryce–Ward equation (1947):** QFT prediction for the joint angular distribution of the two Compton-scattered photons from annihilation of spin-singlet para-positronium (p-Ps).
- **Bohm & Aharonov (1957):** Annihilation of p-Ps at rest produces two back-to-back 511 keV photons—an ideal physical realization of the EPR scenario.

$$|\Psi\rangle = \frac{|XY\rangle - |YX\rangle}{\sqrt{2}} \,. \tag{2}$$

- No ideal polarization analyzer exists at 511 keV; Compton scattering is the practical polarization filter.

Wu's experimrent and the Kasday–Bell no-go

- **Kasday–Ullman–Wu experimrent (1975):** Symmetric Compton polarimeters and fourfold coincidence plus energy-sum selection. The results measured are in excellent agreement with the quantum-mechanical prediction.
- **Kasday et al., Bell:** Compton scattering is probabilistic. There is no ideal ± 1 polarization outcome, so a naive Bell binning does not violate the Bell inequality.

$$\mathcal{W}(\theta_1, \theta_2, ..., \theta_n) = \int d\lambda_1 \int d\lambda_2 ... \int d\lambda_n G(\lambda_1, \lambda_2, ..., \lambda_n) \times \delta(\lambda_1 - \theta_1)\delta(\lambda_2 - \theta_2)...\delta(\lambda_n - \theta_n) \quad (3)$$

$$G(\lambda_1, \lambda_2, ..., \lambda_n) = \mathcal{W}(\lambda_1, \lambda_2, ..., \lambda_n) \quad (4)$$

- Momentum-only Compton measurements, without extra assumptions, are compatible with local realism; they cannot rule out LHVTs.

Entanglement at the LHC: top-quark pairs

- Entanglement observable for $t\bar{t}$ with leptonic decays:

$$D = -3\langle \cos\phi \rangle \ . \tag{5}$$

  The condition $D < -1/3$ is sufficient to establish entanglement in the $t\bar{t}$ system.

- **ATLAS (2024)**, $\sqrt{s} = 13$ TeV, 140 fb^{-1}, dilepton channel:

$$D = -0.537 \pm 0.002\,(\mathrm{stat.}) \pm 0.019\,(\mathrm{syst.})\ , \tag{6}$$

  more than 5σ away from the no-entanglement case.

- Claimed as the first observation of entanglement in $t\bar{t}$ and the highest-energy entanglement observation to date.

The critique: colliders measure momenta, not spins

- Collider detectors record momenta, not spins. Any extraction of spin correlations from angular distributions may rely on quantum mechanics or quantum field theory, which in turn leads to a circularity problem.
- In both QM and LHVTs (via different routes), for $t\bar{t}$, the angular correlation matrix Q of the decay products and the spin correlation matrix C of the $t\bar{t}$ pair are related by

$$Q = \frac{\alpha\bar{\alpha}}{9} C \,. \tag{7}$$

  To infer C from the measured Q, one must know α.
- Consequently, collider "entanglement observation" is, by itself, a Standard-Model-frame verification of spin correlations, not an LHVT exclusion.

The Question: Can We Exclude LHVTs Without Bell Inequalities?

- A claim of Bell-inequality violation at colliders is logically incompatible with the LHVT realization of the angular distributions in Eqs. (3)–(4).
- **Central question of this talk:**

  Is there a physically motivated criterion—different from Bell inequalities and independent of QM parameters—that can exclude local hidden-variable descriptions of entangled photon pairs?

- The rest of this talk explores one candidate: mirror symmetry of single-photon Compton scattering, combined with angular-momentum conservation and rotation invariance.

QFT calculation

- The polarization state of a photon pair in a pure state can be expressed as

$$|\gamma_1\gamma_2\rangle = \sum_{k_1,k_2=\pm 1} \alpha_{k_1,k_2} |k_1\rangle_{\gamma_1} |k_2\rangle_{\gamma_2} \ . \tag{8}$$

- Two back-to-back photons from p-Ps decay: $\gamma_i : (m_e, 0, 0, \pm m_e)$, $i = 1, 2$.
- Each photon scatters via $\gamma_i + e_i^- \to \gamma_i' + e_i^-$; final-photon kinematics:

$$\gamma_i' : \left(m_e q_i,\ m_e q_i \sin\theta_i \cos\phi_i,\ m_e q_i \sin\theta_i \sin\phi_i,\ m_e q_i \cos\theta_i\right) \tag{9}$$

with $q_i = 1/(2 \mp \cos\theta_i)$.

The entanglement witness: Compton-scattered azimuthal correlations

- **The joint angular distribution**:

$$\mathcal{W}(\theta_1,\theta_2,\phi_1,\phi_2)=\frac{1}{\Gamma}\sum_{j_1,n_1}\sum_{j_2,n_2}\Big|\sum_{k_1,k_2}\alpha_{k_1,k_2}\mathcal{M}^1_{k_1;j_1,m_1,n_1}\mathcal{M}^2_{k_2;j_2,m_2,n_2}\Big|^2 \tag{10}$$

- **Witness observable:**

$$\langle\cos(2\phi_1-2\phi_2)\rangle=C_1\delta\,,\quad C_1=648\frac{(1-\ln 3)^2}{(40-27\ln 3)^2}\,, \tag{11}$$

$$\langle\cos(2\phi_1-2\phi_2)\rangle_{\theta_1,\theta_2}=C_1'(\theta_1,\theta_2)\delta\,, \tag{12}$$

$$C_1'(\theta_1,\theta_2)=\frac{1}{2}\frac{(2-\cos\theta_1)\sin^2\theta_1}{2+(1-\cos\theta_1)^3}\frac{(2+\cos\theta_2)\sin^2\theta_2}{2+(1+\cos\theta_2)^3}\,, \tag{13}$$

$$\delta=\alpha_{-1,-1}\alpha^*_{1,1}+\alpha_{1,1}\alpha^*_{-1,-1}\,. \tag{14}$$

A sufficient condition for entanglement

$$\delta = \alpha_{-1,-1}\alpha^*_{1,1} + \alpha_{1,1}\alpha^*_{-1,-1}$$

- **The normalized correlation observables**:

$$\mathcal{O}_1 = \cos(2\phi_1 - 2\phi_2)/C_1 = \delta\,, \tag{15}$$
$$\mathcal{O}'_1(\theta_1,\theta_2) = \cos(2\phi_1 - 2\phi_2)/C'_1(\theta_1,\theta_2) = \delta\,. \tag{16}$$

- Under the normalization condition of $\alpha_{k,j}$, the predicted range of δ is $[-1,1]$. For separable $\gamma_1\gamma_2$ states, the corresponding range narrows to $[-1/2,1/2]$.
- Any observed $\langle\mathcal{O}_1\rangle$ or $\langle\mathcal{O}'_1(\theta_1,\theta_2)\rangle$ in $[-1,-1/2)\cup(1/2,1]$ provides sufficient evidence for entanglement in the $\gamma_1\gamma_2$ system. This entanglement criterion generalizes straightforwardly from the pure $\gamma_1\gamma_2$ state to the mixed $\gamma_1\gamma_2$ state.

p-Ps $\to \gamma_1\gamma_2$: Production of a Maximally Entangled State

- p-Ps is a spin singlet; at leading order in the non-relativistic QED limit,

$$|\text{p-Ps}\rangle \propto \int \frac{d^3\vec{k}}{(2\pi)^3}\tilde{\psi}(\vec{k})\left(|\tfrac{1}{2};\vec{k}\rangle|\tfrac{1}{2};-\vec{k}\rangle + |-\tfrac{1}{2};\vec{k}\rangle|-\tfrac{1}{2};-\vec{k}\rangle\right) . \quad (17)$$

- Decay amplitudes satisfy exact relations (Clebsch–Gordan protected, not LO artifacts):

$$\mathcal{M}_{1,1} = -\mathcal{M}_{-1,-1} , \qquad \mathcal{M}_{1,-1} = \mathcal{M}_{-1,1} = 0 , \quad (18)$$

$$\to \alpha_{1,1} = -\alpha_{-1,-1} = \frac{1}{\sqrt{2}} , \qquad \alpha_{1,-1} = \alpha_{-1,1} = 0 , \quad (19)$$

$$\to \delta = -1 . \quad (20)$$

- So, the p-Ps photon pair is a maximally entangled Bell state.

Decoherence parameter ρ and the joint distribution

- Without decoherence ($\rho = 0$):

$$\mathcal{W}(\theta_1, \theta_2, \phi_1, \phi_2) = \frac{5184\pi^2}{(40 - 27\ln 3)^2}\left[\frac{2 + (1 - \cos\theta_1)^3}{2 - \cos\theta_1}\frac{2 + (1 + \cos\theta_2)^3}{2 + \cos\theta_2} - \sin^2\theta_1 \sin^2\theta_2 \cos 2(\phi_1 - \phi_2)\right] . \tag{21}$$

- With a decohered fraction ρ (mixed state):

$$\mathcal{W}'(\rho; \theta_1, \theta_2, \phi_1, \phi_2) = \frac{5184\pi^2}{(40 - 27\ln 3)^2}\left[\frac{2 + (1 - \cos\theta_1)^3}{2 - \cos\theta_1}\frac{2 + (1 + \cos\theta_2)^3}{2 + \cos\theta_2} - (1 - \rho)\sin^2\theta_1 \sin^2\theta_2 \cos 2(\phi_1 - \phi_2)\right] . \tag{22}$$

- The coherence of the azimuthal correlation is linearly degraded by ρ; this is directly measurable.

$$\langle \mathcal{O}_1 \rangle = \langle \mathcal{O}_1'(\theta_1, \theta_2) \rangle = -(1 - \rho) . \tag{23}$$

Building an LHVT: Angular-momentum conservation and rotation invariance

- p-Ps has spin 0, so the decay photons carry opposite spin angular momenta:

$$\vec{S}_1 = -\vec{S}_2 \,, \qquad \hat{S}_1 = \hat{S} = (\sin x \cos y,\ \sin x \sin y,\ \cos x) \,. \tag{24}$$

- **Hidden variables:** x and y, with joint probability density

$$G(x,y) \geq 0 \,, \qquad \int_{-1}^{1} d\cos x \int_{0}^{2\pi} dy\, G(x,y) = 1 \,. \tag{25}$$

- **Single-photon response:** for the $y = 0$ reference frame, a normalized distribution $F(x;\theta,\phi)$ with

$$F(x;\theta,\phi) \geq 0 \,, \qquad \int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi\, F(x;\theta,\phi) = 1 \,. \tag{26}$$

- By rotational invariance:

$$F_1(x,y;\theta_1,\phi_1) = F(x;\theta_1,\phi_1 - y) \,, \tag{27}$$

$$F_2(x,y;\theta_2,\phi_2) = F(x;\pi-\theta_2,\pi+y-\phi_2) \,. \tag{28}$$

Building an LHVT

- The LHVT constraint matching QFT:

$$\frac{1}{256\pi^4}\, q_1^2 q_2^2\, \mathcal{W}'(\rho;\theta_1,\theta_2,\phi_1,\phi_2)$$
$$= \int_{-1}^{1} d\cos x \int_{0}^{2\pi} dy\, G(x,y)\, F(x;\theta_1,\phi_1-y)\, F(x;\pi-\theta_2,\pi+y-\phi_2)\,. \tag{29}$$

- Hidden variables beyond x and y?

Naturalness and simplicity

自然哲学的数学原理

〔英〕牛顿　著

赵振江　译

编者的序言

我们把长期期待的新版牛顿的哲学奉献给您，善意的读者，它含有许多修订和增补。这一无与伦比的著作的主要内容，可从所附的目录中得知；增补和改动的内容在作者的序言中已给予指示。剩下要我们增加的是关于这一哲学的方法这方面的东西。

从事物理学研究的人大致可分为三类。其中的一些人给每一类事物赋予特别的且隐蔽的质，然后由此宣称每个物体的行为属于人所不知不识的方式。源自亚里士多德和逍遥学派的经院学派的整个教义基于此。的确他们断言每一种效果起源于物体的特别的性质；但他们没有教我们那些性质从何而来，因此他们什么也没有教。且因为他们全都关心事物的名称而不是事物本身，他们应被认为是发明了哲学谈论，而并未传习哲学。

所以，其他人希望通过抛弃这些无用的混杂的词汇，以辛勤的努力获得称誉。且因此他们以为所有的物质是同质的，在物体上被辨别出来的各种变形起源于构成它们的小部分的非常简单的和极容易理解的相互关系。如果他们不把小部分的原始的相互关系归之于自然所赋予的关系之外的关系，他们建立的从简单事物到更复杂的事物的进程是正确的。但当他们利用自由，随意想象人们所不知道的部分的形状和大小，以及不确定的位置和运动，且甚至虚构隐蔽的流体，它们能非常自由地流入物体的小孔，因为它们具有全能的细微性，且由隐蔽的运动所推动；当他们这样做时，便陷入梦想，忽视了事物的真正构造；当它甚至由最确定无疑的观察也难于发现时，由虚假的猜想寻求更是徒然。那些把假设作为他们推测的基础的人，即使他们之后按照力学的定律极精确地发展，也只是一出传奇，也许优雅而动人，然而不过是认真准备的传奇。

现在剩下的是第三类，也就是那些坦率地承认实验哲学的人。的确可能存在从最简单的原理导出一切事物的原因，但他们不把尚未由现象确定的东西作为原理。在物理学中他们既不虚构，也不接受假设，除非是为了讨论问题的真理性。所以他们发展了双重的方法：分析的和综合的方法。从某些选择的现象用分析法导出自然界的力和更单纯的力的定律，然后由它们通过综合法给出其他现象的构造。这是最佳的哲学方法，是我们无与伦比的作者认潍应优先采用的方法。且独自认为这值得用他的劳作耕耘和点缀。所以对此他给出了最有名的一个例子，即是极幸运地从重力的理论导出了宇宙的系统的解释。其他一些人曾怀疑或想象重力属于物体的普遍特性，但他是第一个且惟一的一个人，他能从现象证明它且把他的出色的研究建立在最牢固的基础之上。

快的校订责任，所有这些我已尽我所能。

剑桥，

1713 年 5 月 12 日

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Mirror Symmetry in QED: A single-photon Compton scattering

- Consider $\gamma_1 + e_1^- \to \gamma_1' + e_1^-$ with incident photon spin pointing along

$$\hat{n} = (\sin x \cos y,\ \sin x \sin y,\ \cos x)\ . \tag{30}$$

- The polarization state $|1\rangle_{\hat{n}}$ is obtained by rotating $|1\rangle_{\hat{z}}$ and dropping the unphysical $|0\rangle$ component:

$$|1\rangle_{\hat{n}} = \frac{1}{\sqrt{2(1+\cos^2 x)}}\left(e^{-iy-iy'}(1+\cos x)|1\rangle_{\hat{z}} + e^{iy-iy'}(1-\cos x)|-1\rangle_{\hat{z}}\right). \tag{31}$$

- The resulting final-photon angular distribution is

$$\mathcal{F}(x,y;\theta_1,\phi_1) = \frac{9}{2\pi(40-27\ln 3)}\left(\frac{2+(1-\cos\theta_1)^3}{(2-\cos\theta_1)^3} + \frac{\sin^2 x}{1+\cos^2 x}\frac{\sin^2\theta_1}{(2-\cos\theta_1)^2}\cos(2(\phi_1-y))\right)\ . \tag{32}$$

Mirror Symmetry in QED: A single-photon Compton scattering

$$\mathcal{F}(x,y;\theta_1,\phi_1) = \frac{9}{2\pi(40-27\ln 3)}\left(\frac{2+(1-\cos\theta_1)^3}{(2-\cos\theta_1)^3} + \frac{\sin^2 x}{1+\cos^2 x}\frac{\sin^2\theta_1}{(2-\cos\theta_1)^2}\cos(2(\phi_1-y))\right)$$

- **Key symmetries** (verified in QED, experimentally well established):
  1. $x \to \pi - x$: same angular distribution;
  2. $\phi_1 \to \phi_1' = d\pi + y - (\phi_1 - y)$ with $d = 0$: **mirror symmetry** with respect to the plane $\mathcal{T}$ spanned by $\hat{z}$ and $\hat{n}$;
  3. $d = 1$: **parity invariance** $\mathcal{F}(x,y;\theta_1,\phi_1) = \mathcal{F}(\pi - x, -y;\theta_1,\pi-\phi_1)$.

Imposing mirror symmetry on the LHVT

- Expand the single-photon response in spherical harmonics:

$$F(x;\theta,\phi) = \frac{1}{4\pi} + \sum_{l\geq 1}\sum_{m=-l}^{l} a_{l,m}(x)\, Y_{l,m}(\theta,\phi)\,. \tag{33}$$

- Spherical harmonics properties:

$$Y^*_{l,m}(\theta,\phi) = (-1)^m Y_{l,-m}(\theta,\phi)\,, \tag{34}$$

$$Y_{l,m}(\theta,-\phi) = (-1)^m Y_{l,-m}(\theta,\phi)\,. \tag{35}$$

- $F(x;\theta,\phi)$ is a real function:

$$a_{l,m}(x) = (-1)^m a^*_{l,-m}(x). \tag{36}$$

- Mirror-symmetry condition:

$$F(x;\theta,\phi) = F(x;\theta,-\phi) \;\rightarrow\; a_{l,m}(x) = (-1)^m a_{l,-m}(x). \tag{37}$$

So, $a_{l,m}(x) \in \mathbb{R}$.

Imposing mirror symmetry on the LHVT

$$\frac{1}{256\pi^4}\, q_1^2 q_2^2\, \mathcal{W}'(\rho;\theta_1,\theta_2,\phi_1,\phi_2)$$
$$= \int_{-1}^{1} d\cos x \int_0^{2\pi} dy\, G(x,y)\, F(x;\theta_1,\phi_1-y)\, F(x;\pi-\theta_2,\pi+y-\phi_2)\,.$$

- **Test observable**:

$$h = \sin^2\theta_1 \sin^2\theta_2 \cos(2\phi_1 - 2\phi_2) \tag{38}$$
$$= \frac{16\pi}{15}\left(Y^*_{2,-2}(\theta_1,\phi_1-y)\, Y^*_{2,-2}(\pi-\theta_2,y+\pi-\phi_2)\right.$$
$$\left. + Y^*_{2,2}(\theta_1,\phi_1-y)\, Y^*_{2,2}(\pi-\theta_2,y+\pi-\phi_2)\right) \tag{39}$$

- Using orthogonality, the LHVT gives

$$\langle h\rangle^{(\text{mirror})}_{\text{LHVT}} = \frac{16\pi}{15}\int_{-1}^{1} d\cos x \int_0^{2\pi} dy\, G(x,y)\left(a^2_{2,-2}(x) + a^2_{2,2}(x)\right)\,. \tag{40}$$

$$\boxed{\langle h\rangle^{(\text{mirror})}_{\text{LHVT}} \geq 0\,.}$$

The QFT prediction

- Using the explicit joint distribution $\mathcal{W}'(\rho;\theta_1,\theta_2,\phi_1,\phi_2)$, the QFT average of the same h is

$$\langle h\rangle_{\mathrm{QFT}} = -288\,(1-\rho)\left(\frac{10-9\ln 3}{40-27\ln 3}\right)^2 . \tag{41}$$

- For any $0 \le \rho < 1$:

$$\langle h\rangle_{\mathrm{QFT}} < 0\,, \qquad \langle h\rangle_{\mathrm{QFT}}\big|_{\rho=0} \approx -0.42\,. \tag{42}$$

- Only the fully decohered case $\rho = 1$ gives $\langle h\rangle_{\mathrm{QFT}} = 0$, consistent with the disappearance of all polarization correlations.

The criterion: Mirror-symmetric LHVTs cannot reproduce QFT predictions

- The two predictions are strictly incompatible:

$$\langle h \rangle_{\text{LHVT}}^{(\text{mirror})} \geq 0 \,, \qquad \langle h \rangle_{\text{QFT}} = -288(1-\rho)\left(\frac{10-9\ln 3}{40-27\ln 3}\right)^2 \leq 0 \,.$$

- Therefore, for any partially entangled p-Ps photon pair ($0 \leq \rho < 1$), no LHVT that respects mirror symmetry can reproduce the QFT angular distribution.
- **Logical structure of the argument:**
  1. Angular-momentum conservation + rotation invariance fix the LHVT ansatz;
  2. QFT single-photon Compton scattering exhibits mirror symmetry—verified experimentally;
  3. Imposing mirror symmetry on the LHVT forces $\langle h \rangle_{\text{LHVT}}^{(\text{mirror})} \geq 0$;
  4. QFT predicts $\langle h \rangle_{\text{QFT}} < 0$ for any $\rho < 1$;
  5. The mirror-symmetric LHVT is excluded **without invoking Bell inequalities** and **without using the spin-analyzing power** α.

Without mirror symmetry, LHVTs can still survive

- If the mirror-symmetry condition is *not* imposed, an explicit LHVT solution exists:

$$G(x,y) = \frac{1}{4\pi}\,, \tag{43}$$

$$F(x;\theta,\phi) = \frac{9}{2\pi(40-27\ln 3)}\Big[\frac{2+(1-\cos\theta)^3}{(2-\cos\theta)^3} + \sqrt{2(1-\rho)}\,\frac{\sin^2\theta}{(2-\cos\theta)^2}\,\cos\big(2\phi+\tfrac{\pi}{2}\big)\Big]\,. \tag{44}$$

- Non-negativity of $F(x;\theta,\phi)$ is guaranteed since

$$\frac{2+(1-\cos\theta)^3}{(2-\cos\theta)^3} > \sqrt{2}\,\frac{\sin^2\theta}{(2-\cos\theta)^2}\,, \qquad \theta\in[0,\pi]\,. \tag{45}$$

Conclusions

- We have derived the full joint angular distribution of Compton-scattered p-Ps photons for a general partially decohered polarization state.
- The normalized observable

$$\langle \mathcal{O}_1 \rangle = \langle \mathcal{O}_1'(\theta_1, \theta_2) \rangle = -(1 - \rho)$$

provides a practical **entanglement witness** and a **decoherence meter** directly from azimuthal correlations.

- The central result: when the physically motivated **mirror symmetry** of single-photon Compton scattering is imposed on the LHVT,

$$\langle h \rangle_{\text{LHVT}}^{(\text{mirror})} \geq 0 \,, \qquad \text{whereas} \qquad \langle h \rangle_{\text{QFT}} < 0 \text{ for any } \rho < 1 \,.$$

This provides a **new, symmetry-based exclusion criterion for LHVTs** independent of Bell inequalities and of any QM parameter like α.

- Without mirror symmetry, LHVTs can still reproduce the distribution—consistent with the Kasday–Bell no-go and with recent collider critiques.