电磁势作用下介观环中的电子输运

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Jul. 19, 2023

Apply electromagnetic potentials to mesoscopic loops

A few important effects in electron transport

Main characteristics involves in conductance oscillation

Three important themes in twenty century physics

Quantization, symmetry, and phase factor

§ 1 Gauge Transformation of Electronic Wavefunctions

Invariance of gauge transformation, one type of symmetry,

related to electronic wavefunctions gauge, Weyl, 1918

H. Weyl (1885–1955)

Two equivalent schemes for describing

electromagnetic fields

magnetic field $\boldsymbol{B}$ and electric field $\boldsymbol{E}$

or vector potential $\boldsymbol{A}$ and scalar potential φ

Both schemes are related by

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \quad \boldsymbol{E} = -\nabla \varphi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \tag{1}$$

The relation is not one to one correspondence

An arbitrary scalar function $\Lambda(\boldsymbol{r}, t)$ satisfies

$$\boldsymbol{A}' = \boldsymbol{A} + \nabla \Lambda, \quad \varphi' = \varphi - \frac{1}{c}\frac{\partial \Lambda}{\partial t} \tag{2}$$

The new $\boldsymbol{A}'$ and φ' give the same $\boldsymbol{B}$ and $\boldsymbol{E}$

(2) is a gauge transformation, physical quantities invariant

In classical electrodynamics, fields alone are enough

vector and scalar potentials aid to convenience

For same $\boldsymbol{B}$ and $\boldsymbol{E}$, $\boldsymbol{A}$ and φ are not unique

Additional restriction, gauge condition, introduced

Two types of gauges are often used:

Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$ and Lorentz gauge $\nabla \cdot \boldsymbol{A} + \partial\varphi/\partial t = 0$

In quantum theory, the electromagnetic potentials play a more significant role as in the study of Landau levels

Go further to discuss the deep meaning of electromagnetic potentials

The Hamiltonian for a single electron

$$\mathcal{H}=\frac{1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\boldsymbol{A}\right)^2-e\varphi+V(\boldsymbol{r}) \qquad (3)$$

electromagnetic potential $\boldsymbol{A}$, φ, other scattering potential $V(\boldsymbol{r})$

Gauge invariance comes from the fact that wavefunction comprises two parts, i.e., amplitude and phase

To examine the time-dependent Schrödinger equation for an electron

$$i\hbar\frac{\partial}{\partial t}\psi=\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\boldsymbol{A}\right)^2-e\varphi+V(\boldsymbol{r})\right]\psi \tag{4}$$

when the external potentials transformed as (2), the wavefunction ψ correspondingly transformed like

$$\psi'=\psi\mathrm{e}^{-ie\Lambda(\boldsymbol{r},t)/\hbar c} \tag{5}$$

then the Schrödinger equation (4) keeps invariant in its form, except an additional phase factor in the wavefunction

$$\frac{e}{\hbar c}\Lambda(\boldsymbol{r},t)=\frac{e}{\hbar}\int\left(\varphi dt-\frac{\boldsymbol{A}}{c}\cdot dr\right) \tag{6}$$

(2) is called the second, or local, gauge transformation,

as Λ is a spatial function

However, when Λ is a constant, it is called the first, or global, gauge transformation

Electromagnetic potential $(\boldsymbol{A}, \varphi)$ is one of the gauge fields

§ 2 Magnetic Aharonov-Bohm Effect

First, consider only ***A*** there

$\psi(\boldsymbol{r})$ can be related to $\psi_0(\boldsymbol{r})$ with $\boldsymbol{A} = 0$

$$\psi(\boldsymbol{r}) = \psi_0(\boldsymbol{r}) \exp\left(\frac{ie}{\hbar c} \int \boldsymbol{A} \cdot d\boldsymbol{r} \right) \qquad (7)$$

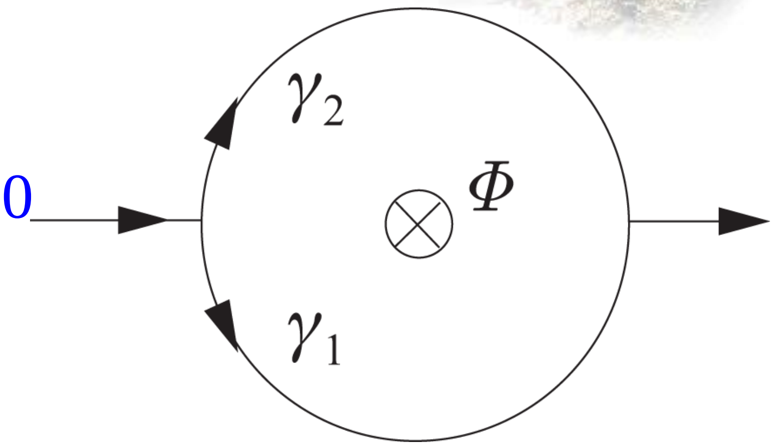

Figure 1 Schematic picture for the Aharonov-Bohm effect.

A can affect electronic behavior, even though there is no classical field ***B*** in the path where electron passed

Aharonov-Bohm (AB) effect, Phys. Rev. **115**, 485 (1959)

Aharonov and Bohm's experimental scheme, Fig. 1

A double slits experiment for an electron beam

$$\psi_1(\boldsymbol{r}) = \psi_0(\boldsymbol{r}) \exp\left(\frac{ie}{\hbar c} \int_{\gamma_1} \boldsymbol{A} \cdot d\boldsymbol{r} \right),$$

$$\psi_2(\boldsymbol{r}) = \psi_0(\boldsymbol{r}) \exp\left(\frac{ie}{\hbar c} \int_{\gamma_2} \boldsymbol{A} \cdot d\boldsymbol{r} \right) \tag{8}$$

The electron density on the screen

$$|\psi_1 + \psi_2|^2 = 2|\psi_0|^2 + 2|\psi_0|^2 \cos\frac{2\pi\Phi}{\phi_0} \tag{9}$$

$$\Phi = \int_{\gamma_1 - \gamma_2} \boldsymbol{A} \cdot d\boldsymbol{r} = \oint \boldsymbol{A} \cdot d\boldsymbol{l} = \iint \boldsymbol{B} \cdot d\boldsymbol{S}$$

is the magnetic flux of the solenoid, and $\phi_0 = hc/e$

(i) no magnetic field, $\Phi = 0$

(9) gives an electron interference pattern

(ii) increase flux, $\Phi \neq 0$

interference fringes move periodically with the period ϕ_0

a quantum effect from modulation of phase factor by $\boldsymbol{A}$

In quantum level, $\boldsymbol{A}$ is a real entity

AB effect displays the geometric phase which comes from

adiabatic cyclic evolution of microscopic particle

M. V. Berry, Proc. Roy. Soc. London, Ser. A **392**, 45 (1984)

AB effect was used to check the quantized frozen magnetic flux in superconducting hollow cylinder

the basic magnetic flux quantum is $hc/2e$ due to Cooper pair

Experimental verification of AB effect in free space

Chambers in 1960 and Tonomura *et al*. in 1982

What will happen in metals?

Is there phase coherence for diffusion motion?

Elastic scattering and inelastic scattering

In mesoscopic metal rings, the AB effect could be observed

Wave nature of an electron gives rise to interference

The resistance of a ring is low or high

for constructive or destructive interference

Two coherent electron beams in medium suffering multi-scatterings

Incident beams at origin

$$\psi_1 = \psi_2 = \psi_0 \exp\left(-\frac{i}{\hbar}Et\right) \qquad (10)$$

In elastic scattering case, E has no change,

but collision phase shifts α_1, α_2 may appear

Wavefunctions on the screen

$$\psi_1(\boldsymbol{r},t)=\psi_0(\boldsymbol{r})\exp\left(-\frac{i}{\hbar}Et+i\alpha_1\right),$$
$$\psi_2(\boldsymbol{r},t)=\psi_0(\boldsymbol{r})\exp\left(-\frac{i}{\hbar}Et+i\alpha_2\right) \tag{11}$$

interference term

$$\psi_1^*\psi_2+\psi_1\psi_2^*=2|\psi_0|^2\cos(\alpha_1-\alpha_2) \tag{12}$$

$(\alpha_1-\alpha_2)$ is related to the elastic scatterers

In inelastic scattering case, E is changed

$$\psi_1(\boldsymbol{r},t)=\psi_0(\boldsymbol{r})\exp\left(-\frac{i}{\hbar}E_1t+i\alpha_1\right),$$

$$\psi_2(\boldsymbol{r},t)=\psi_0(\boldsymbol{r})\exp\left(-\frac{i}{\hbar}E_2t+i\alpha_2\right) \tag{13}$$

interference term

$$\psi_1^*\psi_2+\psi_1\psi_2^*=2|\psi_0|^2\cos\left(\alpha_1-\alpha_2+\frac{E_1-E_2}{\hbar}t\right) \tag{14}$$

time average → zero

Conclusion is only elastic scattering keeps phase coherence

In a small metal loop with a flux Φ, (12) becomes

$$\psi_1^*\psi_2+\psi_1\psi_2^*=2|\psi_0|^2\cos\left(\frac{2\pi\Phi}{\phi_0}+\alpha_1-\alpha_2\right) \tag{15}$$

AB effect in metal rings

Typically $\tau_{\text{in}} = 10^{-11}$ s at 1 K, $v_F = 10^8$ cms^{-1}, $l_{\text{in}} = v_F \cdot \tau_{\text{in}} \approx 10^5$ Å

Distance without losing its phase memory is about 10 μm = 10^5 Å

With the development of micro-fabrication technique

periodic oscillation of resistance in a gold ring enclosing

a magnetic flux, as shown in Fig. 2, R. A. Webb et al. in 1985

A gold ring with an inside diameter of ~ 8000 Å and

a width ~ 400 Å

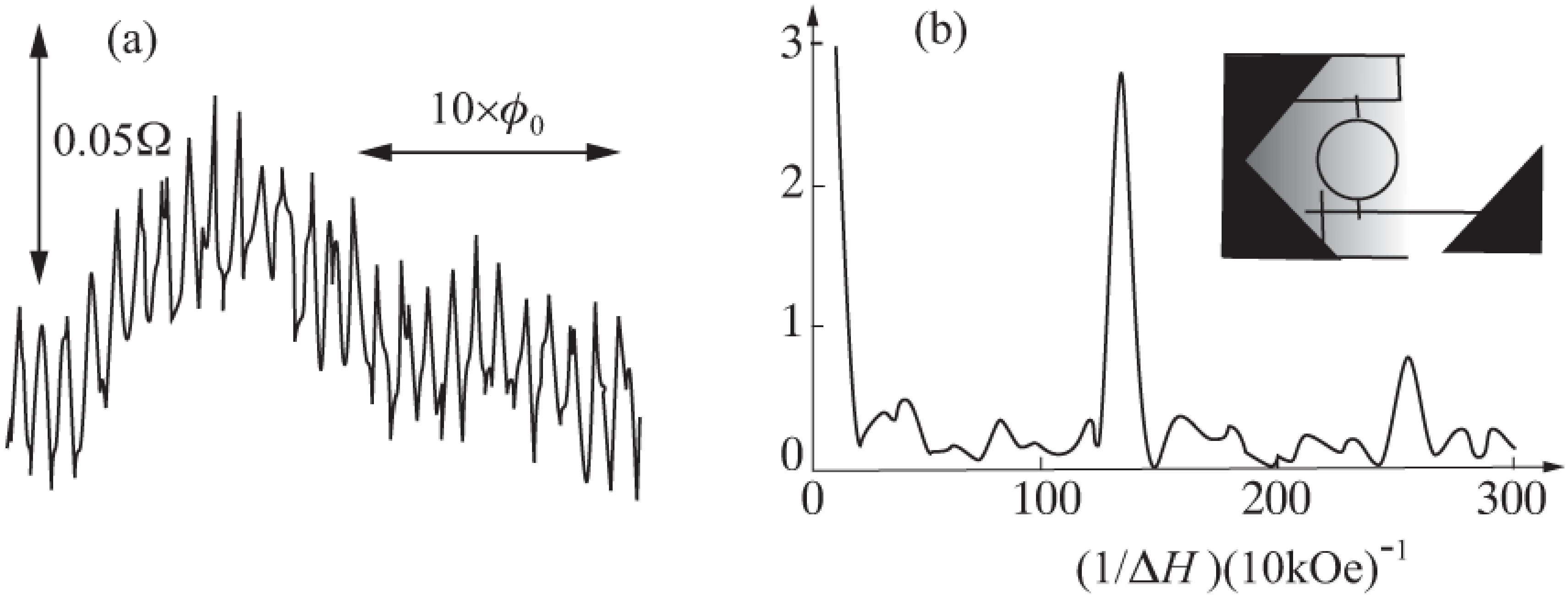

Figure 2 Aharonov-Bohm effect in a gold ring. (a) Oscillation in the magnetoresistance as a function of magnetic field; (b) Fourier transform of the data in (a).

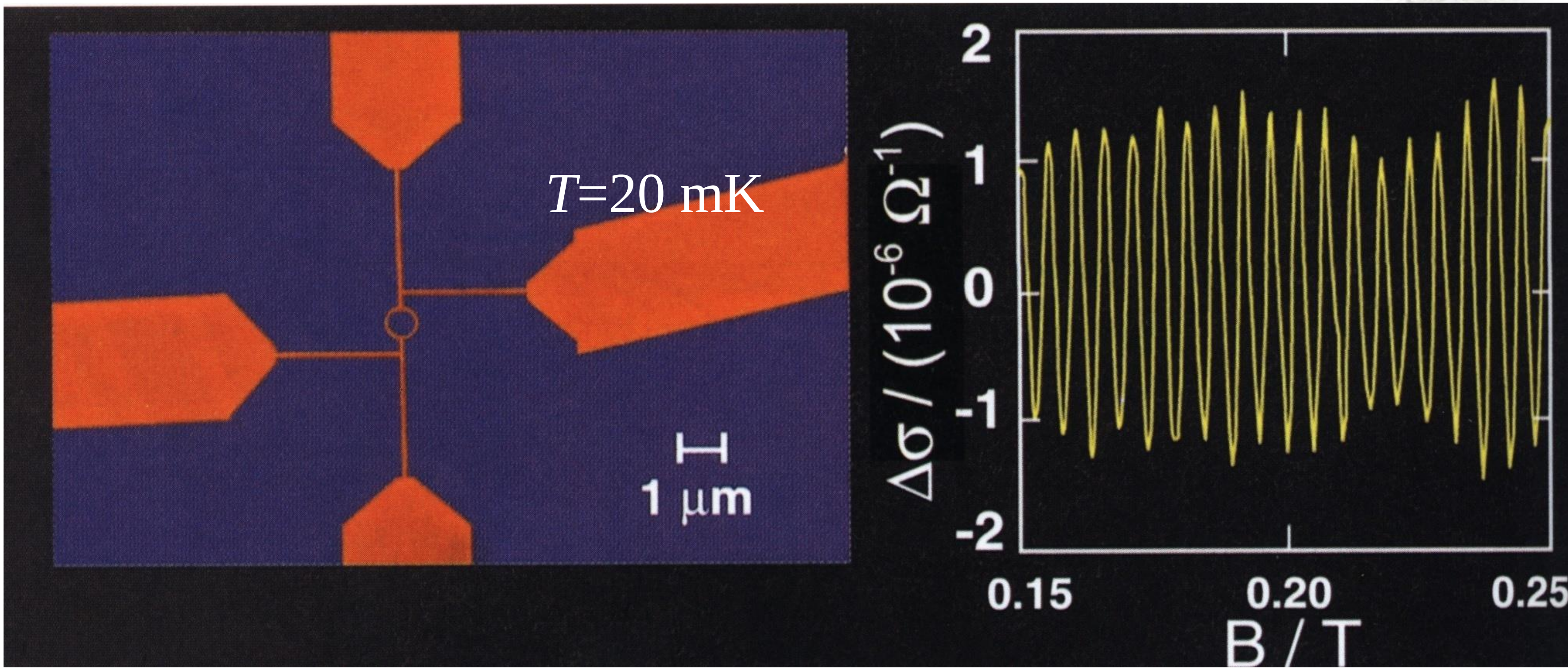

Figure 3 Quantum interference in Cu nanoring by Häussler and Löhneysen.

Aharonov-Bohm Oscillations in Singly Connected Disordered Conductors

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(Received 20 December 2014; published 20 February 2015)

We show that the transport and thermodynamic properties of a *singly connected* disordered conductor exhibit quantum Aharonov-Bohm oscillations as a function of the total magnetic flux through the sample. The oscillations are associated with the interference contribution from a special class of electron trajectories confined to the surface of the sample.

DOI: 10.1103/PhysRevLett.114.076802 PACS numbers: 73.23.-b, 74.78.-w, 74.78.Na

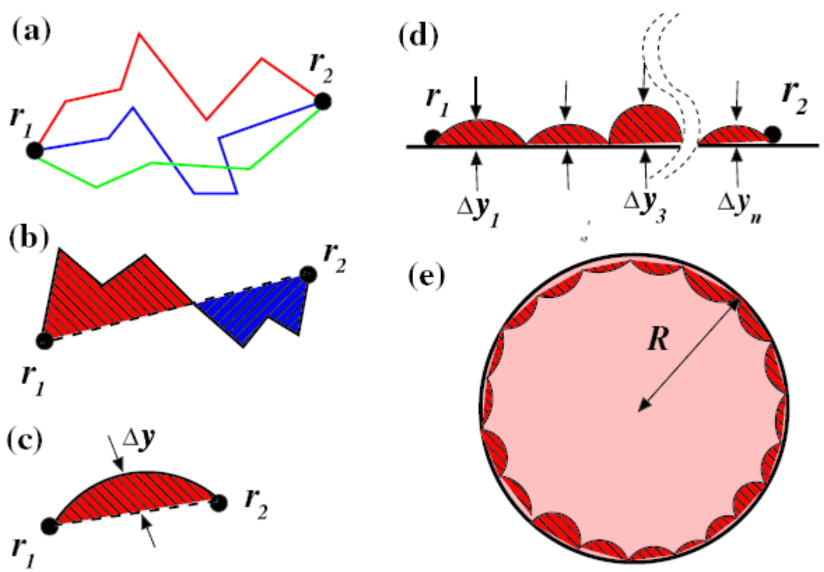

FIG. 1 (color online). (a) Diffusive paths contributing to the interference corrections. (b) Sketch of the directed area swept by a path. The blue and red regions give opposite in sign contributions. (c) Typical path leading to the Gaussian decay of the Cooperon in the bulk. (d) Typical path near the surface. (e) Surface paths leading to AB oscillations. Notice that the straight segment structure of diffusive paths in (a),(b) is not shown in panels (c)–(e).

§ 3 Electrostatic Aharonov-Bohm Effect

The contribution of φ to phase of electronic wavefunction

An electrostatic potential will accumulates a phase

$$\Delta\theta = (e/\hbar)\int^{\tau} \varphi dt \qquad (16)$$

Potential difference for changing the relative phase by 2π is

$$\Delta\varphi = h/e\tau$$

τ is the shorter of the time for electron traveling

through the device and the phase coherence time τ_{in}

The combined influence of electrostatic and magnetic potentials on electron waves

Washburn *et al.* in 1987 investigated the effect of a transverse electric field on the magnetic Aharonov-Bohm oscillations

in Fig. 4, the loop is 0.82 μm on a side and $d \simeq 0.075$ μm thick

A little doubt about it, due to difficulty to apply a well defined voltage difference across the ring

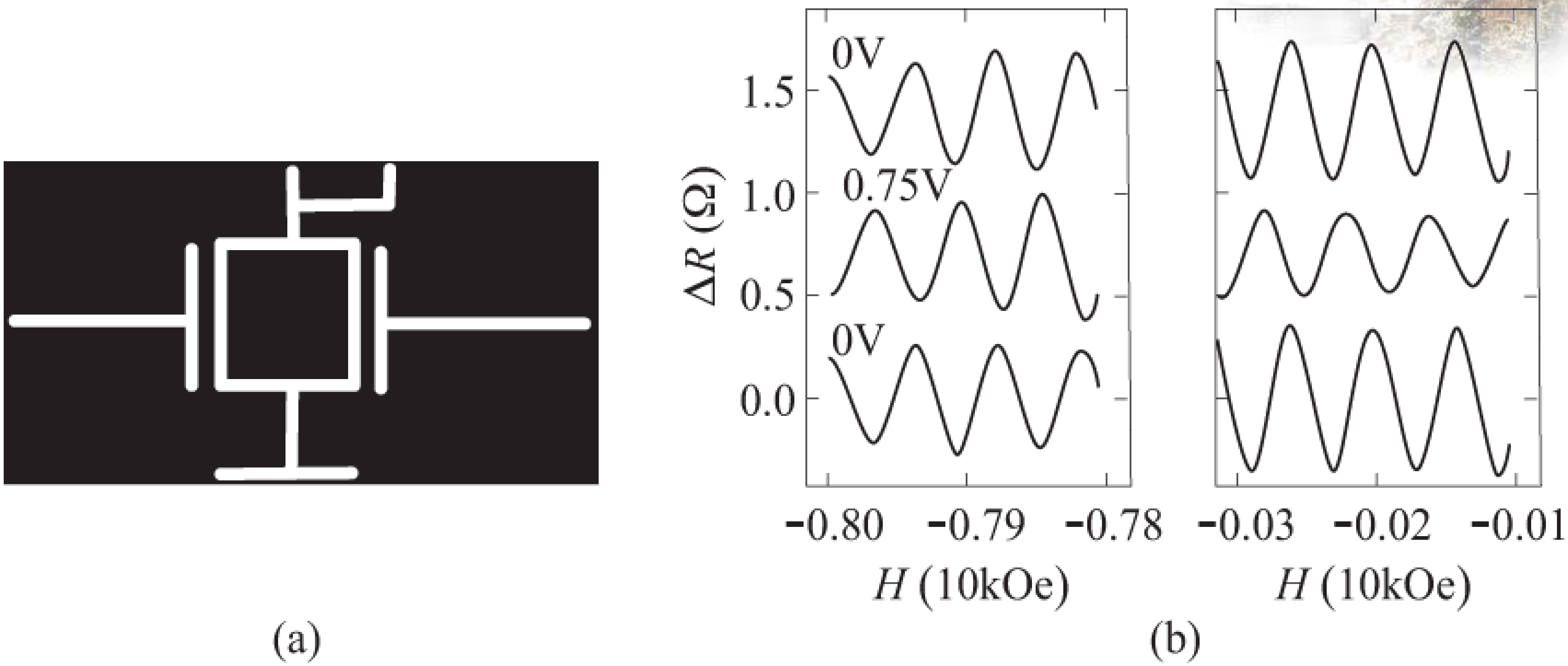

Figure 4 Electrostatic Aharonov-Bohm effect. (a) A square Sb loop placed between two metal electrodes; (b) when an electric field is established between the electrodes the phase of the magnetoresistance oscillations can be changed by 180°.

Later on, a new measurement was made on a metal ring

That was interrupted by two small tunnel junctions

to confirm the electrostatic effect more convincingly

van Oudenaarden *et al*., Nature in 1998

Two periods in the measurement of transport properties:

(i) period of magnetic Aharonov-Bohm effect Φ to hc/e

(ii) period of electrostatic Aharonov-Bohm effect $V L^2/D$ to h/e

V and L are potential difference and distance between

two tunnel barriers, and D is diffusion coefficient

§ 4 Altshuler-Aronov-Spivak Effect

In Fig. 2(b), first peak is from AB oscillation with period hc/e

another oscillation with period $hc/2e$, AAS effect

Weak localization, or backscattering, conductance decreases

Altshuler, Aronov, and Spivak, 1981

Here Φ is enclosed by a loop, the phases along the paths

in positive and negative directions are $\Delta\phi_1$ and $\Delta\phi_2$

the phase difference of two partial waves is $\Delta\phi_1 - \Delta\phi_2$

Round the paths twice, $\Delta\phi_1 - \Delta\phi_2 = 2\Phi$

Interference term with period *hc/2e*

$$\psi_1^*\psi_2 + \psi_1\psi_2^* = 2|\psi_0|^2 \cos\frac{4\pi}{\phi_0} \qquad (17)$$

AAS effect was confirmed by experiment by Sharvins' in 1981

Resistance of a hollow thin-walled metal cylinder, in Fig. 5

In general cases, total electrical resistance *R*(*H*) of

a small two-lead metal loop threaded by a flux *Φ*

$$R(H) = R_0 + R_1\cos\left(\frac{2\pi\Phi}{\phi_0}+\alpha_1\right) + R_2\cos\left(\frac{4\pi\Phi}{\phi_0}+\alpha_2\right) + \ldots \qquad (18)$$

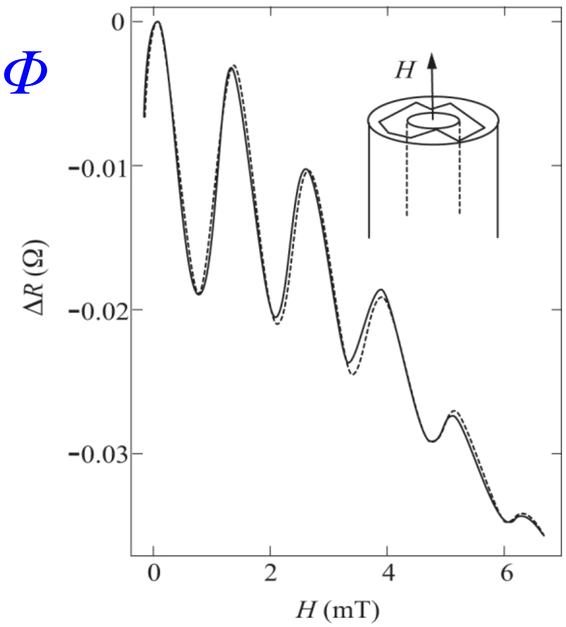

Figure 5 Magnetic field dependence of resistance measured in Li cylinder.

§ 5 Persistent Currents

In

$$|\psi_1+\psi_2|^2=2|\psi_0|^2+2|\psi_0|^2\cos\frac{2\pi\Phi}{\phi_0} \qquad (9)$$

the phase change due to magnetic flux is

$$\Delta\theta=2\pi\Delta\Phi/\phi_0 \qquad (19)$$

The cases for Φ and $\Phi + n\phi_0$ are indistinguishable

The circumference L plays the role of a unit cell

a one-one correspondence between $2\pi\Phi/\phi_0$ and kL

Total energy E will be flux dependent

Stationary Schrödinger equation from (4) with $\varphi=0$

$$\frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx}+\frac{e}{c}A_x\right)^2\psi+V(x)\psi=E\psi \tag{20}$$

$$\oint A_x dx=\iint \boldsymbol{B}\cdot d\boldsymbol{S}=\Phi \to A_x=\Phi/L$$

$V(x)$ is the scattering potential, $V(x)=0$ for an ideal ring

$$\frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx}+\frac{e\Phi}{cL}\right)^2\psi=E\psi \tag{21}$$

a plane wave solution

$$\psi(x)=C\exp(ikx)$$

the periodic condition

$$\psi(x+L)=\psi(x)$$

Wavenumber $k = 2\pi n/L$, eigenenergy in (22) is

$$E = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right)^2\left(n + \frac{\Phi}{\phi_0}\right)^2 \qquad (22)$$

If $V(x)$ is a weak scattering potential, in NFE approximation a one-dimensional energy band structure with gaps is obtained, in Fig. 6

First Brillouin zone $(-\phi_0/2) < \Phi < \phi_0/2$

M. Büttiker et al., Phys. Lett. A **96**, 365 (1983)

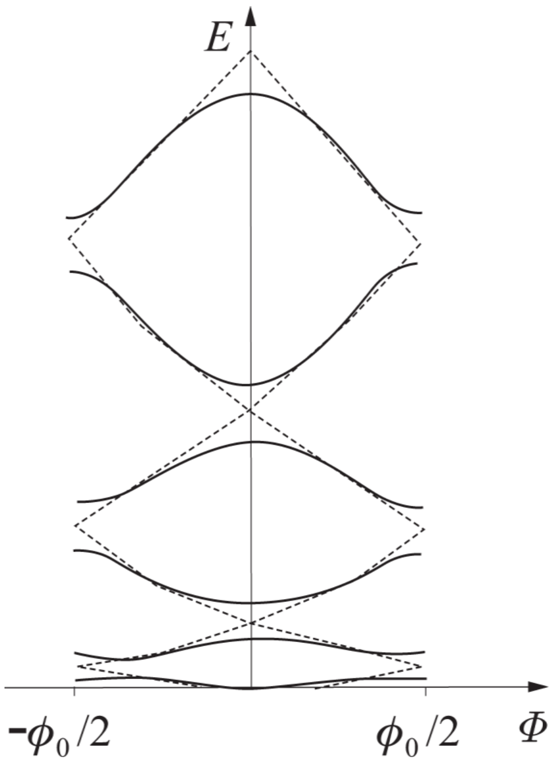

Figure 6 Energy diagram with dependence of flux in metallic ring.

Quantum coherence → persistent currents in mesoscopic rings

Flux-dependent currents, periodic function of ϕ_0

$$I(\Phi) = -\frac{e}{L}\sum_n v_n(\Phi) = -c\sum_n \frac{\partial E_n(\Phi)}{\partial \Phi} \qquad (23)$$

For an isolated ring with N electrons

$$E_\mathrm{F} = \hbar^2 (N\pi)^2 / 2mL^2$$

Different persistent current for odd or even N

Define $I_0 = heN/2mL^2$

$$I(\Phi) = -I_0 \frac{2\Phi}{\phi_0}, \quad -0.5 \le \frac{\Phi}{\phi_0} < 0.5, \qquad N \text{ is odd} \tag{24}$$

$$I(\Phi) = \begin{cases} -I_0\left(1 + \dfrac{2\Phi}{\phi_0}\right), & -0.5 \le \dfrac{\Phi}{\phi_0} < 0 \\ I_0\left(1 - \dfrac{2\Phi}{\phi_0}\right), & 0 \le \dfrac{\Phi}{\phi_0} < 0.5 \end{cases} \qquad N \text{ is even} \tag{25}$$

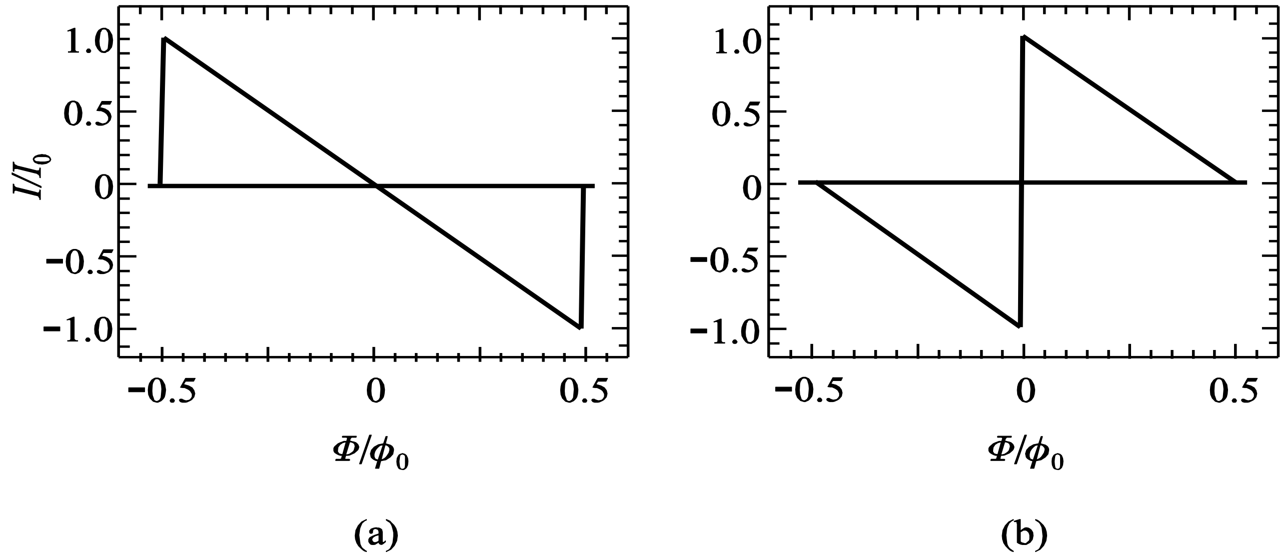

Figure 7 Persistent current in a period of the magnetic flux. The chemical potential is fixed, the number of electrons in the ring is (a) odd and (b) even.

To consider free electrons with an impurity, $V(x) = \gamma\delta(x)$

choose a reduced parameter

$$\gamma^* = \frac{\hbar v_F}{\pi} = \frac{\hbar^2 N}{mL} \qquad (26)$$

weak- and strong-coupling regimes $\gamma \ll \gamma^*$ and $\gamma \gg \gamma^*$

Figure 8, I-Φ characteristics at $T = 0$ for odd N

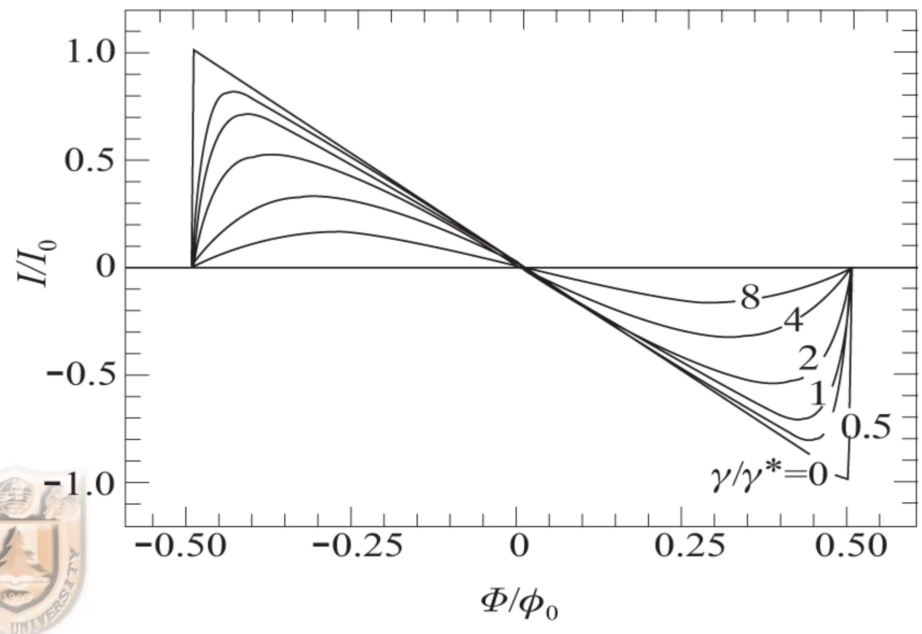

Figure 8 Effect on the persistent current of a single δ-function impurity of strength $\gamma/\gamma^* = 0$, 0.5, 1, 2, 4, and 8 in a ring with an odd number of electrons.

Selected for a Viewpoint in *Physics*
PRL **102,** 136802 (2009) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2009

Persistent Currents in Normal Metal Rings

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(Received 24 October 2008; published 30 March 2009)

The authors have measured the magnetic response of 33 individual cold mesoscopic gold rings, one ring at a time. The response of some sufficiently small rings has a component that is periodic in the flux through the ring and is attributed to a persistent current. Its period is close to h/e, and its sign and amplitude vary between rings. The amplitude distribution agrees well with predictions for the typical h/e current in diffusive rings. The temperature dependence of the amplitude, measured for four rings, is also consistent with theory. These results disagree with previous measurements of three individual metal rings that showed a much larger periodic response than expected. enabled *in situ* measurements of the sensor background. A pa understood anomaly around a zero field are attributed to de

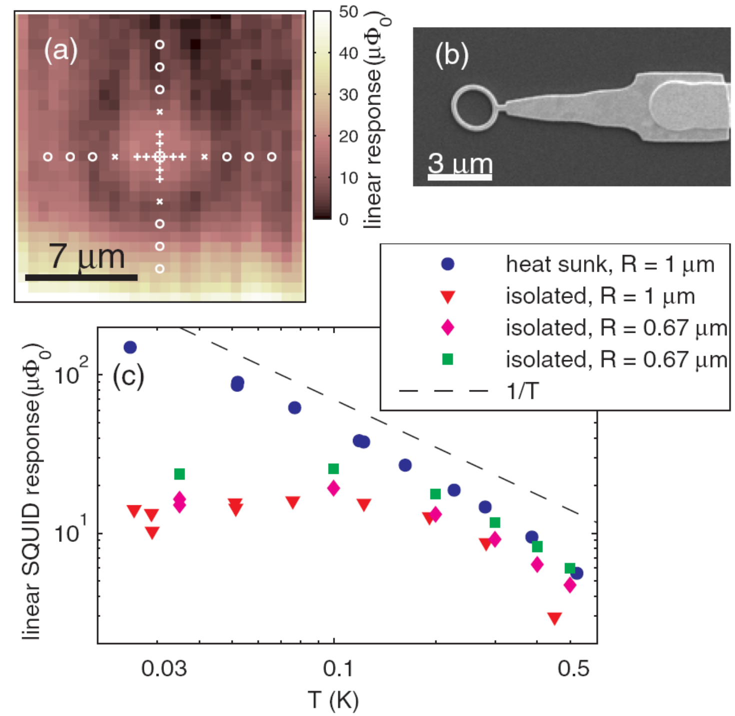

FIG. 1 (color online). (a) Susceptibility scan of an isolated ring used to locate the ring and to determine the indicated measurement positions. Background measurements at positions "○" are subtracted from the data taken at positions "+" to obtain the ring response. (b) Scanning electron micrograph of a heat sunk ring. (c) Temperature dependence of the linear response of one heat sunk and three isolated rings. The data in (a) and (c) reflect the total amplitude of the linear response to a sinusoidal excitation of ± 45 G for (a) and the 0.67 μm rings in (c), and ± 35 G for the 1 μm rings.

For a time-dependent flux Φ, an induced electromotive force

$$U = -\frac{1}{c}\frac{d\Phi}{dt} \qquad (27)$$

In the case U is pure d.c. → Bloch oscillation for the electrons occupying the bands in Fig. 6, the equation of motion

$$-\frac{d\Phi}{dt} = cU \qquad (28)$$

frequency is

$$\omega = \frac{2\pi}{\phi_0}\left|\frac{d\Phi}{dt}\right| = \frac{eU}{\hbar} \qquad (29)$$

§ 6 Simple Summary

A mesoscopic metallic ring applied by electromagnetci potentials shows quantization, phase factor and symmetry, simultaneously, expressing

Three important themes in twenty century physics

Quantization, symmetry, and phase factor

Thank You!