Λ spin polarization and feed-down effect in heavy-ion collisions

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Based on: Phys. Rev. C100, 014913 (2019) [arXiv:1905.03120]

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Workshop on "New development of hydrodynamics and its applications in Heavy-Ion Collisions"

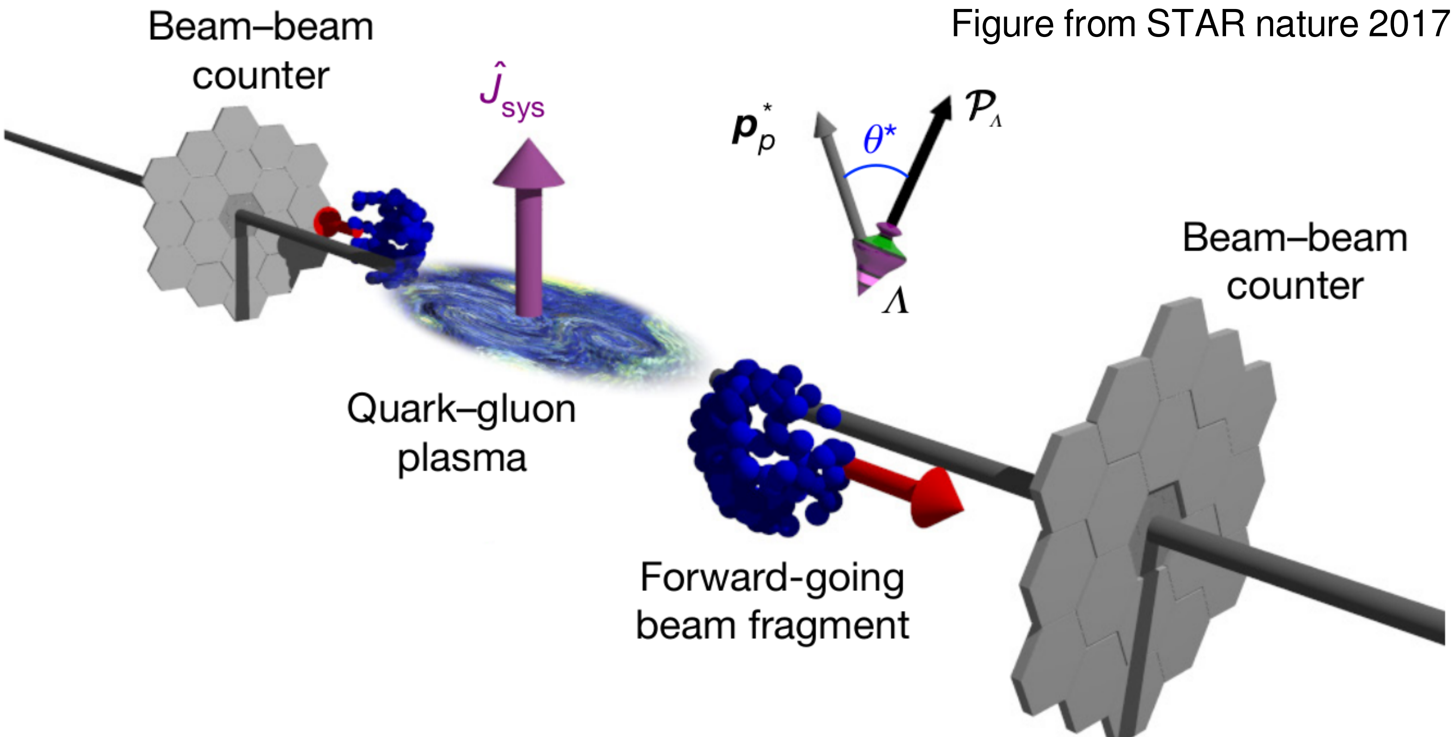
Outline

- Overview on the Λ spin polarization (phenomenology)
  - Global polarization
  - Azimuthal angle dependence
  - The "sign puzzles"
- Feed-down effect
- Summary

Global polarization

- Non-central colliding system carries a huge angular momentum.
- The initial angular momentum creates fluid **voriticty** and leads to the **global spin polarization**.
- Λ polarization is a good probe to vorticity and magnetic field.

Figure from STAR nature 2017

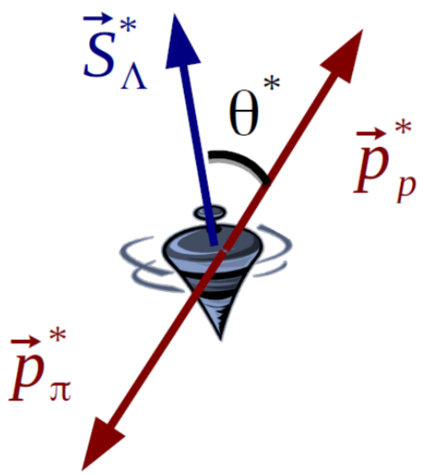


Ref:
Liang, Wang, PRL94, 102301 (2005)
Voloshin, nucl-th/0410089

Why Λ hyperon?

- Λ spin polarization can be measured in its weak decay

$$\Lambda \to \mathrm{p} + \pi^-$$



$$\frac{1}{N}\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_\Lambda \mathbf{P}_\Lambda \cdot \hat{\mathbf{p}}^*) \quad (1)$$

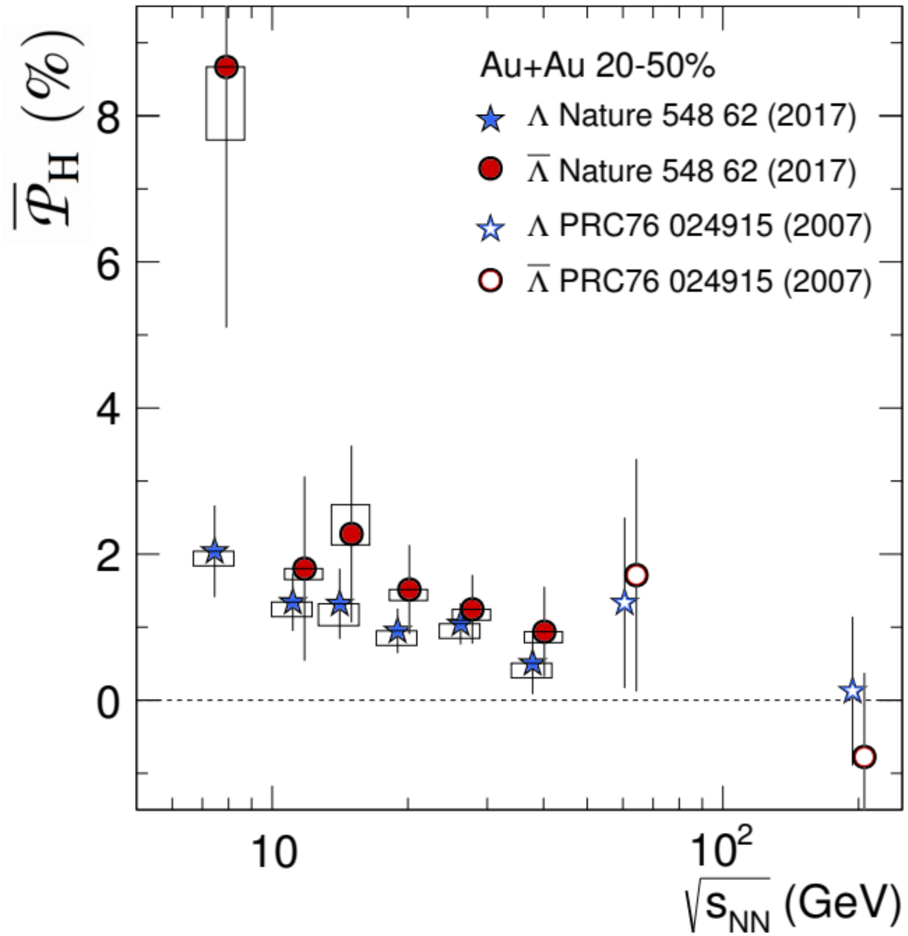
$$\mathbf{P}_\Lambda = \frac{3}{\alpha_\Lambda}\langle \hat{\mathbf{p}}^* \rangle \quad (2)$$

α_Λ: decay parameter.
$\mathbf{P}_\Lambda$: polarization vector of Λ.
$\hat{\mathbf{p}}^*$: unit vector along proton's momentum in the rest frame of Λ.

Global polarization

- RHIC-STAR has observed the global Λ polarization.

STAR Nature 548 62 (2017)



$$\mathbf{P}_\Lambda \simeq \frac{\omega}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T}, \qquad (1)$$

$$\mathbf{P}_{\bar{\Lambda}} \simeq \frac{\omega}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}. \qquad (2)$$

- The most vortical fluid even seen.

$$\omega \approx (9 \pm 1) \times 10^{21} \mathrm{s}^{-1}$$

- Effect of the magnetic field?

$P_{\bar{\Lambda}} > P_\Lambda$

$\rightarrow$ BES II.

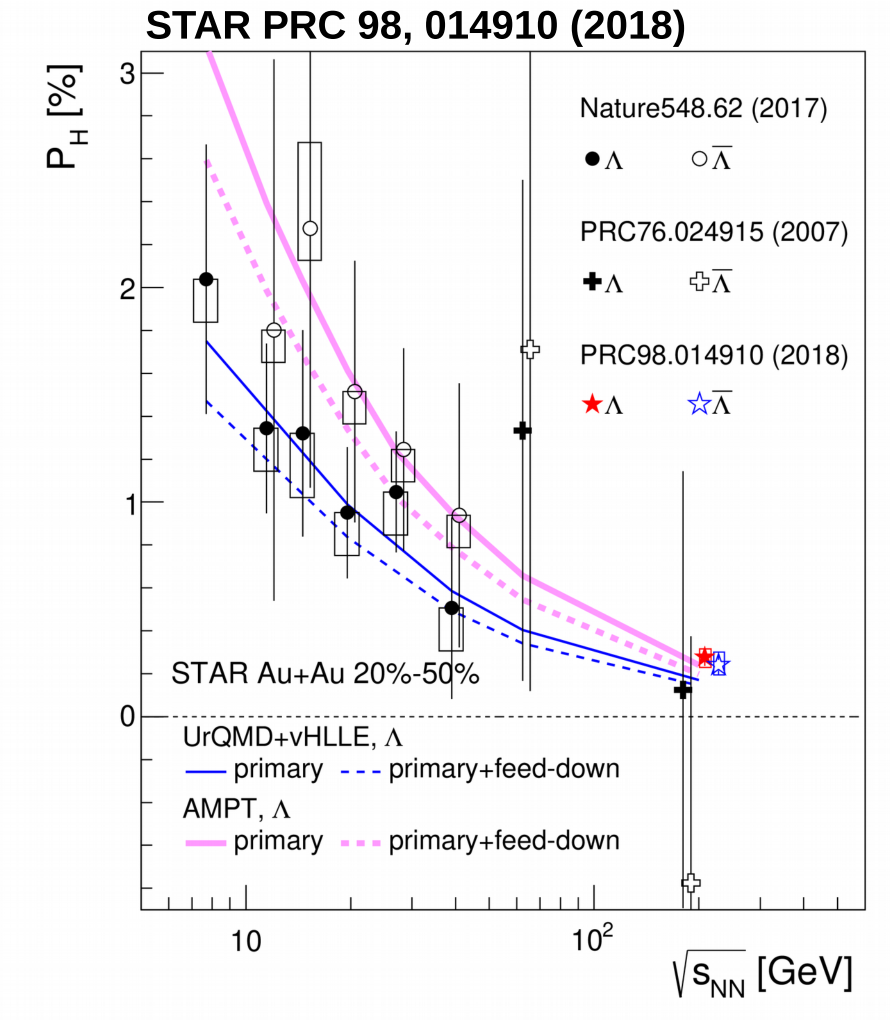
magnetic moment

$\mu_\Lambda \approx -0.613 \mu_N$.

Ref:

Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC95, 054902 (2017)

Global polarization



- Theoretical calculations can well-produce the data.

$$S^\mu(x,p) = -\frac{1}{8m}(1-f)\epsilon^{\mu\nu\rho\sigma} p_\rho \varpi_{\rho\sigma}.$$

$$\varpi_{\mu\upsilon} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu).$$

See Becattini's talk

Ref:
Karpenko, Becattini, EPJC77, 213 (2017)
Xie, Wang, Csernai, PRC95, 031901 (2017)
Li, Pang, Wang, Xia, PRC96, 054908 (2017)
…

Go to the azimuthal-angle dependence

- The **global polarization** is the average value of spin polarization.
  - It reflects the net vorticity, created by initial angular momentum.

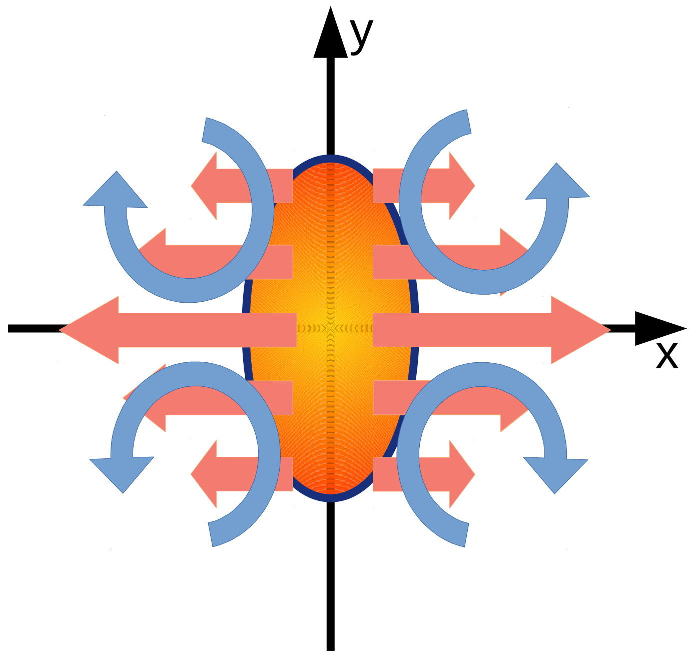
- The azimuthal-angle dependence can tell us more information.
  - Similar to collective flow, we expand the polarization in Fourier series:

$$P_i = P_0^{(i)} + \sum P_n^{(i)} \cos[n(\phi - \Psi_n^{(i)})]$$

$i = x, y, z$: the three compontents of $\mathbf{P} = (P_x, P_y, P_z)$.

(1) Longitudinal polarization

- Besides the initial angular momentum, vorticity can also be created in other ways.
- Transverse velocity can generate vorticity and polarization along the beam direction.



- Azimuthal angle dependence of the polarization in Fourier series:

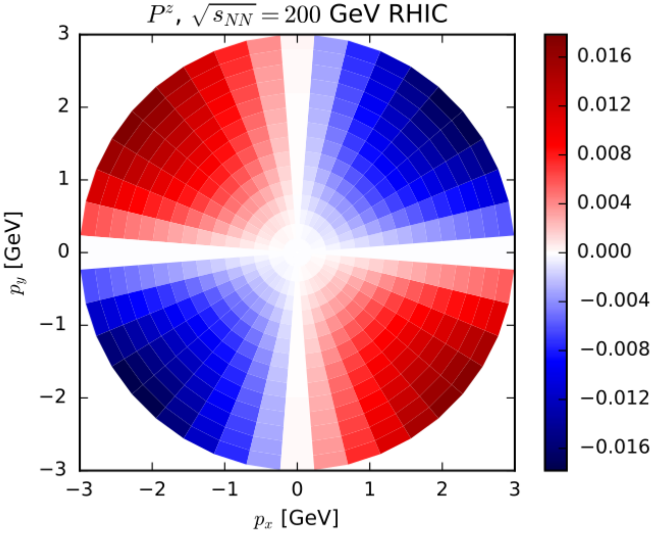
$$P_z(\phi) = f_{2z}\sin(2\phi) + f_{4z}\sin(4\phi) + \ldots$$

Ref:
Becattini, Karpenko, PRL120, 012302 (2018)
Voloshin, 1710.08934

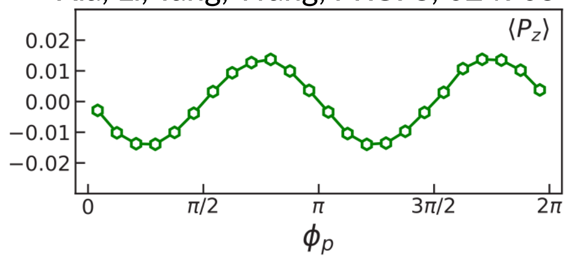
(1) Longitudinal polarization

- However, theoretical calculations predicted **opposite** signs compared to data.

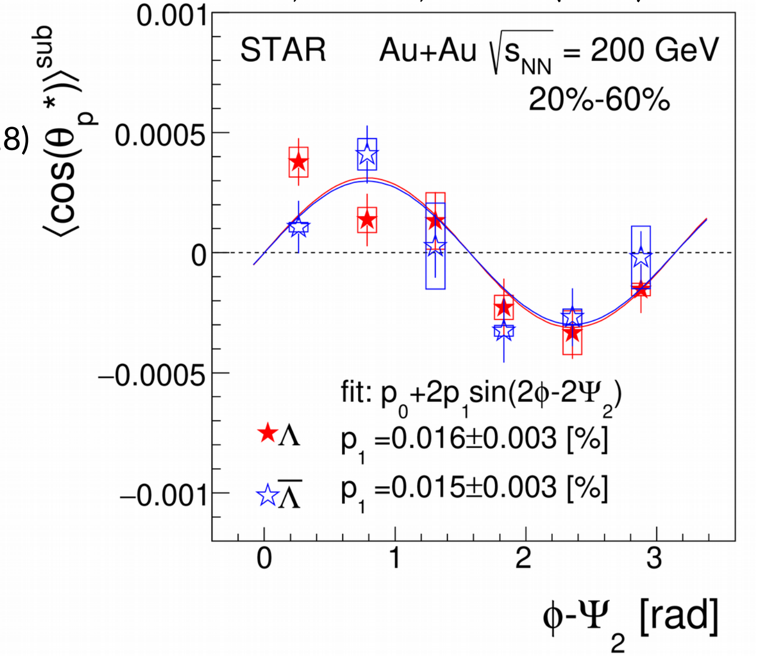
Becattini, Karpenko, PRL120, 012302 (2018)



Xia, Li, Tang, Wang, PRC98, 024905 (2018)



STAR, PRL123, 132301 (2019)



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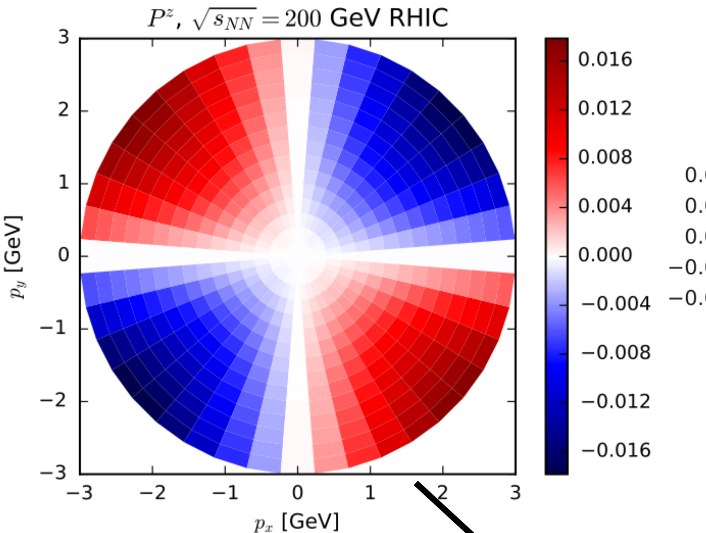

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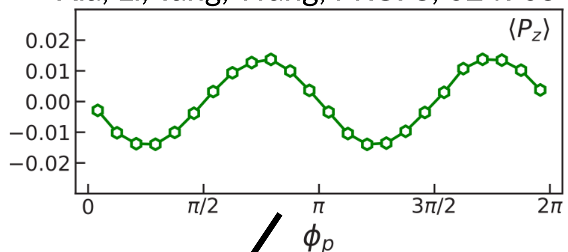
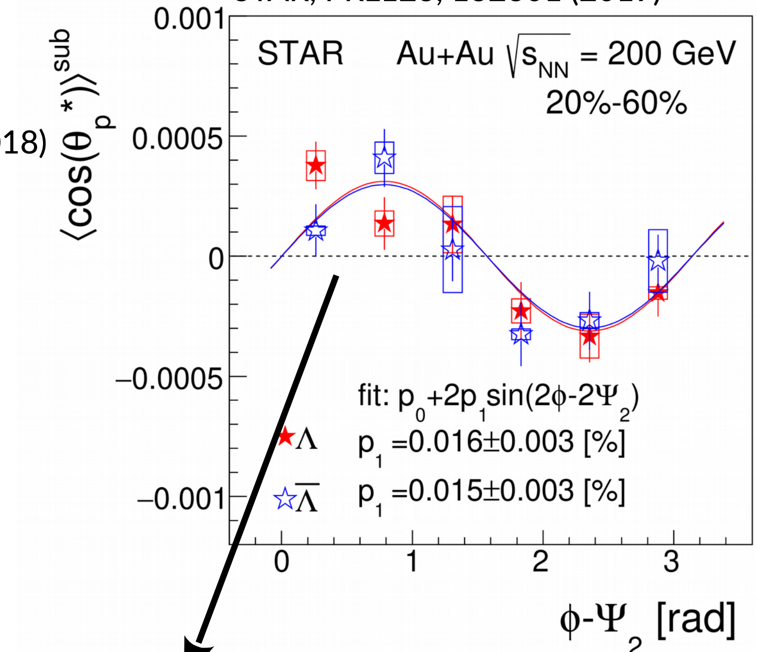


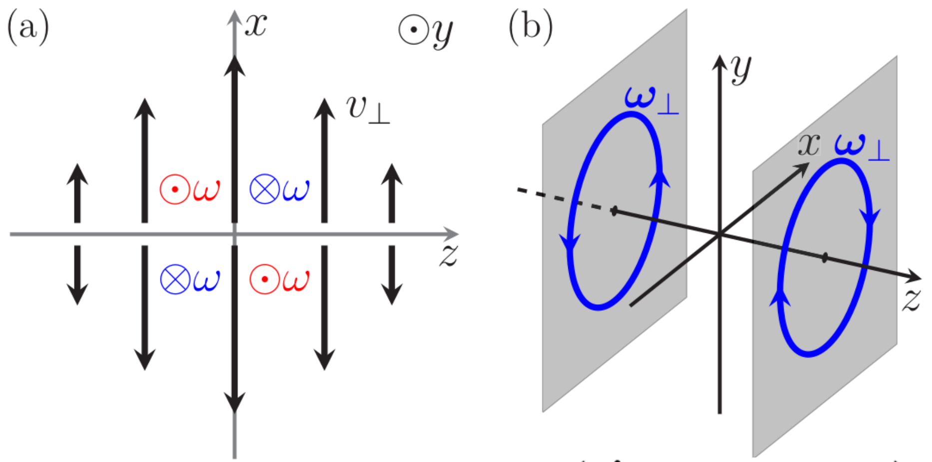
STAR, PRL123, 132301 (2019)

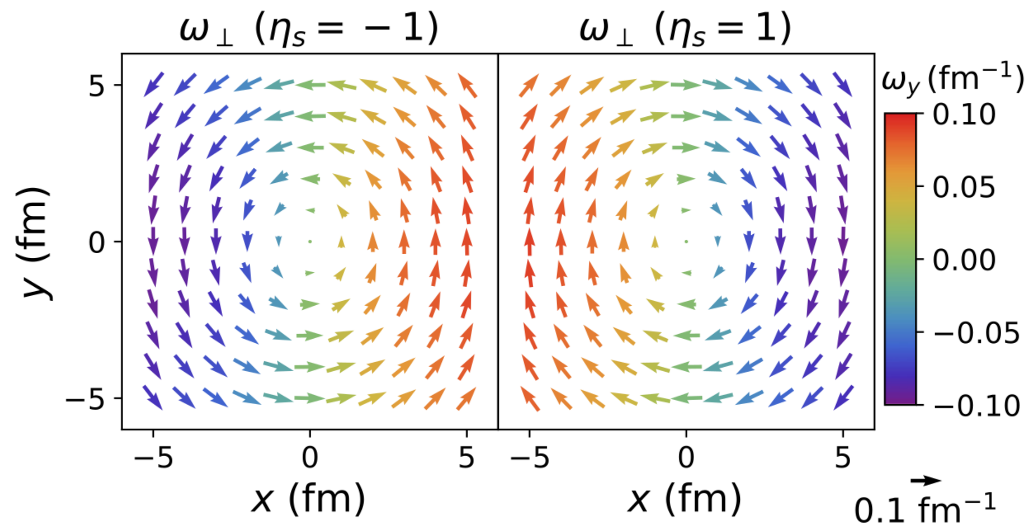


$f_{2z} < 0$ $\qquad\qquad$ $f_{2z} > 0$

$$P_z(\phi) = f_{2z}\sin(2\phi)$$

(2) Transverse polarization along loops





$$\boldsymbol{\omega}_\perp \sim \mathbf{e}_\phi = (\sin\phi, -\cos\phi)$$

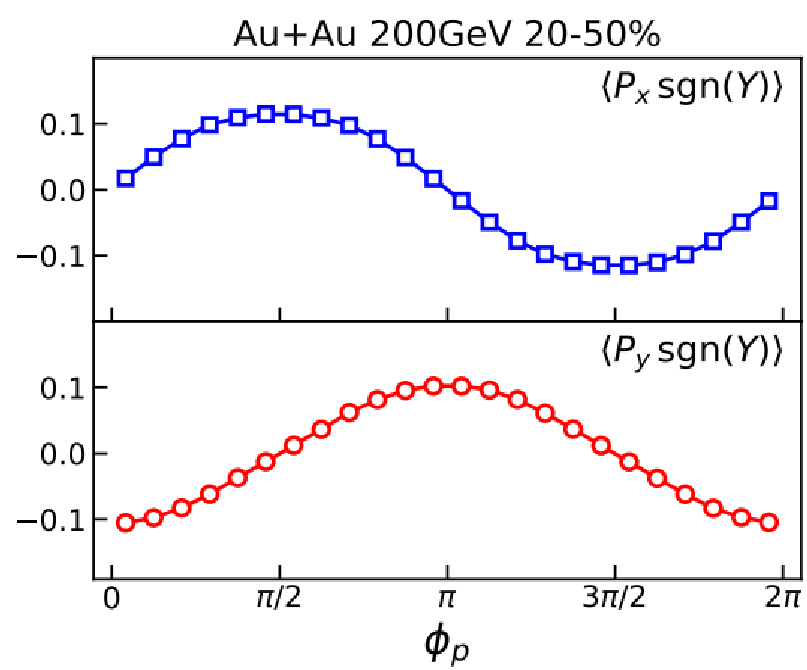
- Transverse expansion of fireball leads to loop-like vorticity and polarization.

$$P_x = \quad f_x \sin\phi,$$

$$P_y = -f_y \cos\phi,$$

f_x and f_y are rapidity-odd

Ref:
Xia, Li, Tang, Wang, PRC98, 024905 (2018)
Wei, Deng, Huang, PRC99, 014905 (2019)

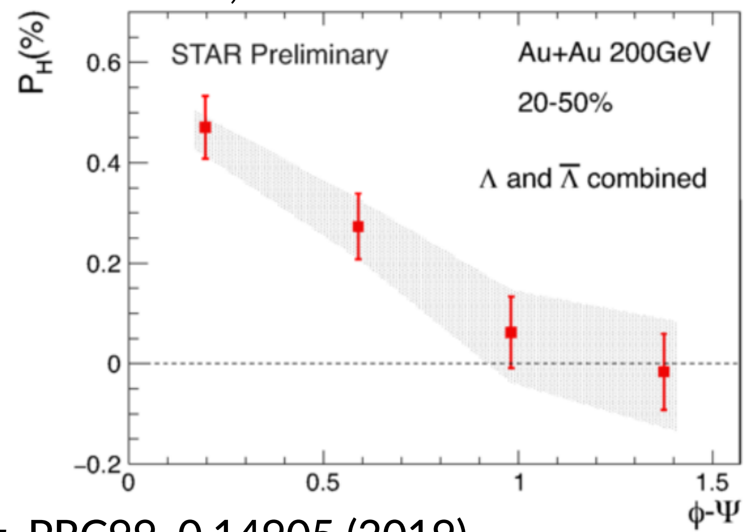


(3) In-plane to out-of-plane difference

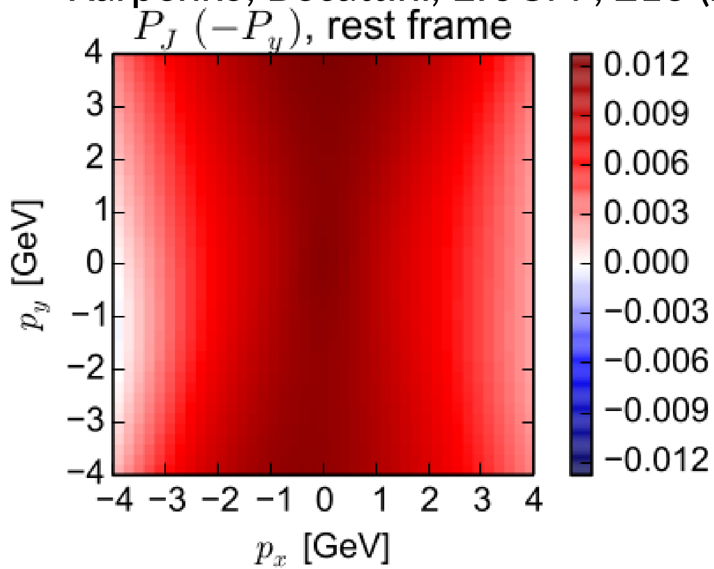
$$P_y = f_0 + f_2 \cos(2\phi)$$

f_0: mean value of the polarization,
f_2: in-plane to out-of-plane difference.

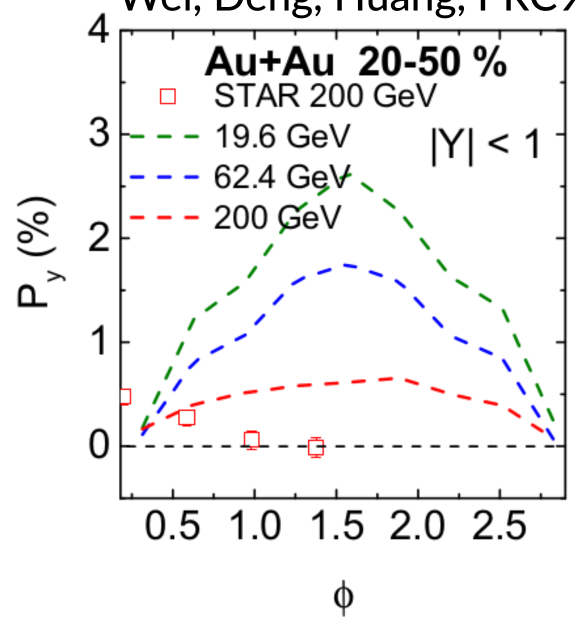
STAR, 1808.10482



Karpenko, Becattini, EPJC77, 213 (2017)

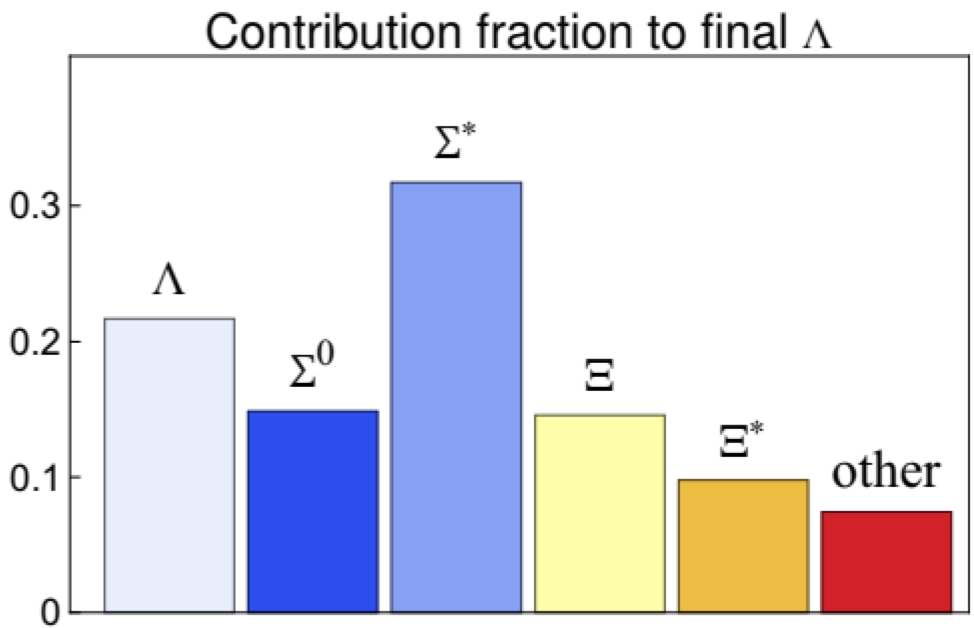


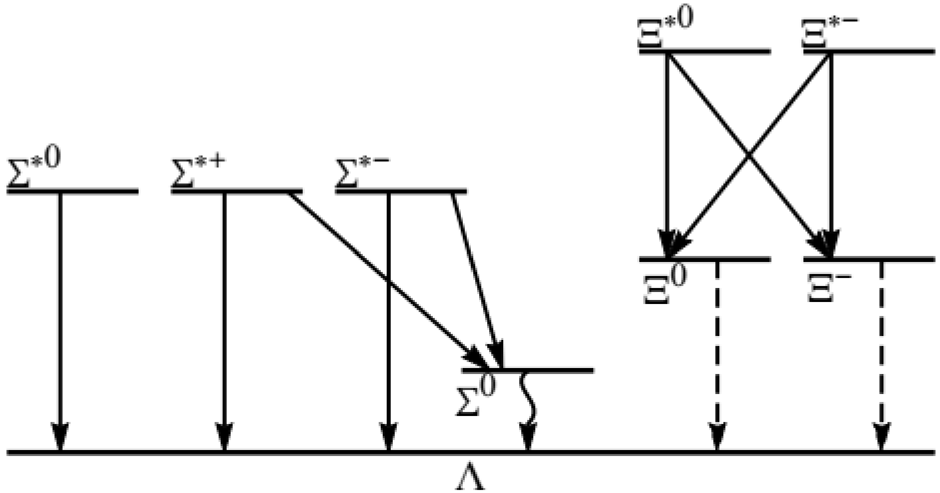
Wei, Deng, Huang, PRC99, 0.14905 (2019)



Feed-down effect

- In the previous theoretical calculations, we count only the **primordial Λ**.
- The experiment data contains the **feed-down** contribution.
- According to thermal model, more than 70% final Λs are created by feed-down decay.





- Parent particles are also polarized, and transfer spin to produced Λ.

Feed-down effect

- To study the feed-down effect to local Λ polarization, we need to answer the following two questions:

Consider a two-body decay where the parent is polarized,

$$P \rightarrow D + X$$

(1) What is the angular distribution of the daughter?

For example, in the weak decay $\Xi \rightarrow \Lambda\pi$:

$$\frac{1}{N}\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_\Xi \mathbf{P}_\Xi \cdot \hat{\mathbf{p}}_\Lambda^*)$$

(2) How does the daughter polarization depend on its momentum direction?

For example, in the EM decay $\Sigma^0 \rightarrow \Lambda\gamma$:

$$\mathbf{P}_\Lambda = -(\mathbf{P}_\Sigma \cdot \hat{\mathbf{p}}_\Lambda^*)\hat{\mathbf{p}}_\Lambda^*$$

The framework

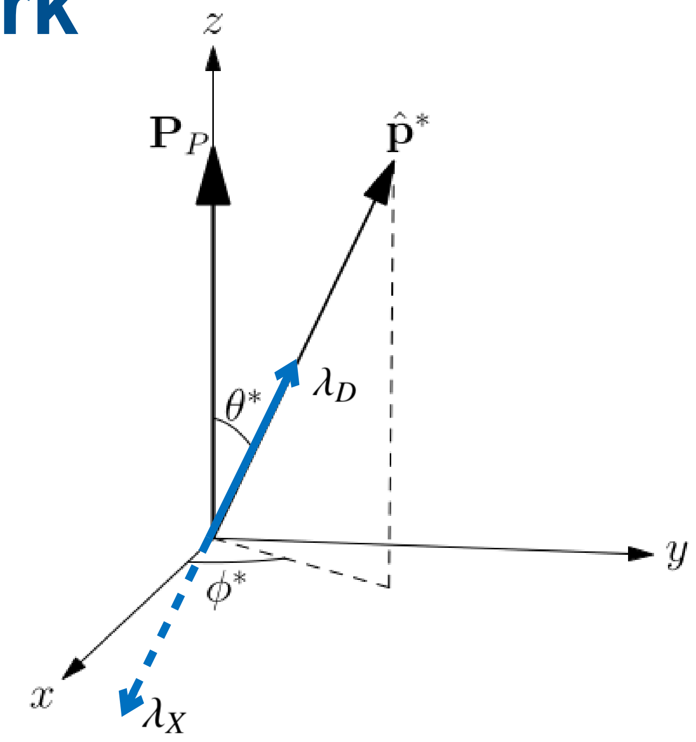
$$P \to D + X$$

parent spin state: $|M_P\rangle$

parent spin density matrix: $\rho^i_{M_P,M'_P}$

final spin state: $|\theta^* \phi^* \lambda_D \lambda_X\rangle$

final spin density matrix: $\rho^f_{\lambda_D\lambda_X;\lambda'_D\lambda'_X}(\theta^*, \phi^*)$



$$\rho^f = H\rho^i H^\dagger$$

$$H_{\lambda_D\lambda_X;M_P} = \sqrt{\frac{2S_P+1}{4\pi}} D^{S_P *}_{M_P;\lambda_D-\lambda_X}(\phi^*, \theta^*, 0) A_{\lambda_D,\lambda_X}$$

Normalization

Wigner D function, angular distribution

Amplitude, select decay channel: s-wave, p-wave, ...

angular distribution: $\frac{dN}{d\Omega^*} = \mathrm{tr}(\rho^f)$

polarization: $\mathbf{P}_D = \mathrm{tr}(\widehat{\mathbf{P}}\rho^f)/\mathrm{tr}(\rho^f)$

Example

Weak decay $\Xi \to \Lambda\pi$ ($\frac{1}{2} \to \frac{1}{2}0$)

$$\rho^i_{M_P;M'_P} = \mathrm{diag}\left(\frac{1+P_P}{2}, \frac{1-P_P}{2}\right)$$

$$\rho^f = H\rho^i H^\dagger$$

$$\rho^D_{\lambda_D;\lambda'_D} = \frac{1}{4\pi}\begin{pmatrix} |A_{1/2}|^2(1+P_P\cos\theta^*) & -A_{1/2}A^*_{-1/2}P_P\sin\theta^* \\ -A^*_{1/2}A_{-1/2}P_P\sin\theta^* & |A_{-1/2}|^2(1-P_P\cos\theta^*) \end{pmatrix}$$

Weak decay is a mixture of s-wave and p-wave:

$$A_{\pm 1/2} = \frac{A_s \pm A_p}{\sqrt{2\left(|A_s|^2+|A_p|^2\right)}},$$

One finally obtains:

$$\frac{1}{N}\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1+\alpha P_P\cos\theta^*), \qquad \mathbf{P}_D = \frac{(\alpha+\mathbf{P}_P\cdot\hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* + \beta(\mathbf{P}_P\times\hat{\mathbf{p}}^*) + \gamma\hat{\mathbf{p}}^*\times(\mathbf{P}_P\times\hat{\mathbf{p}}^*)}{1+\alpha\mathbf{P}_P\cdot\hat{\mathbf{p}}^*}.$$

$$\alpha = \frac{2\mathrm{Re}(A_s^*A_p)}{|A_s|^2+|A_p|^2}, \quad \beta = \frac{2\mathrm{Im}(A_s^*A_p)}{|A_s|^2+|A_p|^2}, \quad \gamma = \frac{|A_s|^2-|A_p|^2}{|A_s|^2+|A_p|^2}.$$

Agree with T.D. Lee and C.N. Yang 1957

Example

$$\rho^D_{\lambda_D;\lambda'_D} = \frac{1}{4\pi}\begin{pmatrix} |A_{1/2}|^2\,(1+P_P\cos\theta^*) & -A_{1/2}A^*_{-1/2}\,P_P\sin\theta^* \\ -A^*_{1/2}A_{-1/2}\,P_P\sin\theta^* & |A_{-1/2}|^2\,(1-P_P\cos\theta^*) \end{pmatrix}$$

$$A_{\pm 1/2} = \frac{A_s \pm A_p}{\sqrt{2\left(|A_s|^2+|A_p|^2\right)}}, \qquad \alpha = \frac{2\mathrm{Re}(A_s^* A_p)}{|A_s|^2+|A_p|^2},\ \beta = \frac{2\mathrm{Im}(A_s^* A_p)}{|A_s|^2+|A_p|^2},\ \gamma = \frac{|A_s|^2-|A_p|^2}{|A_s|^2+|A_p|^2}.$$

$$\mathbf{P}_D = \frac{(\alpha+\mathbf{P}_P\cdot\hat{\mathbf{p}}^*)\,\hat{\mathbf{p}}^* + \beta\,(\mathbf{P}_P\times\hat{\mathbf{p}}^*) + \gamma\hat{\mathbf{p}}^*\times(\mathbf{P}_P\times\hat{\mathbf{p}}^*)}{1+\alpha\mathbf{P}_P\cdot\hat{\mathbf{p}}^*}.$$

For strong decay $\frac{1}{2}^+ \to \frac{1}{2}^+ 0^-$, $A_{1/2} = -A_{-1/2} = A_p$ (parity odd)

$$\alpha = \beta = 0,\ \gamma = -1 \ \Rightarrow\ \mathbf{P}_D = 2(\mathbf{P}_P\cdot\hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \mathbf{P}_P$$

For strong decay $\frac{1}{2}^- \to \frac{1}{2}^+ 0^-$, $A_{1/2} = A_{-1/2} = A_s$ (parity even)

$$\alpha = \beta = 0,\ \gamma = 1 \ \Rightarrow\ \mathbf{P}_D = \mathbf{P}_P$$

All decay channels

TABLE I. Daughter angular distribution and polarization in different decay channels

decay channel	daughter angular distribution	daughter polarization
strong decay $1/2^+ \to 1/2^+ 0^-$	$1/(4\pi)$	$2(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - \mathbf{P}_P$
strong decay $1/2^- \to 1/2^+ 0^-$	$1/(4\pi)$	$\mathbf{P}_P$
strong decay $3/2^+ \to 1/2^+ 0^-$	$3[1 - 2\Delta/3 - (1 - 2\Delta)\cos^2\theta^*]/(8\pi)$	Eq. (1)
strong decay $3/2^- \to 1/2^+ 0^-$	$3[1 - 2\Delta/3 - (1 - 2\Delta)\cos^2\theta^*]/(8\pi)$	Eq. (2)
weak decay $1/2 \to 1/2\ 0$	$(1 + \alpha P_P \cos\theta^*)/(4\pi)$	Eq. (3)
EM decay $1/2^+ \to 1/2^+ 1^-$	$1/(4\pi)$	$-(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^*$

$$\frac{-4\delta(\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* + [1 - 2\delta - (1 - 10\delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2]\mathbf{P}_P}{1 - 2\Delta/3 - (1 - 2\Delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2} \quad (1)$$

$$\frac{2[1 - 4\delta - (1 - 10\delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2](\mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* - [1 - 2\delta - (1 - 10\delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2]\mathbf{P}_P}{1 - 2\Delta/3 - (1 - 2\Delta)(\hat{\mathbf{P}}_P \cdot \hat{\mathbf{p}}^*)^2} \quad (2)$$

$$\frac{(\alpha + \mathbf{P}_P \cdot \hat{\mathbf{p}}^*)\hat{\mathbf{p}}^* + \beta(\mathbf{P}_P \times \hat{\mathbf{p}}^*) + \gamma\hat{\mathbf{p}}^* \times (\mathbf{P}_P \times \hat{\mathbf{p}}^*)}{1 + \alpha\mathbf{P}_P \cdot \hat{\mathbf{p}}^*} \quad (3)$$

$\mathbf{P}_P$: parent polarization
$\hat{\mathbf{p}}^*$: daughter momentum direction
θ^*: angle between $\mathbf{P}_P$ and $\hat{\mathbf{p}}^*$

For parent of spin-3/2,

$$P_P = (\rho_{\frac{3}{2}\frac{3}{2}} - \rho_{-\frac{3}{2}-\frac{3}{2}}) + \frac{1}{3}(\rho_{\frac{1}{2}\frac{1}{2}} - \rho_{-\frac{1}{2}-\frac{1}{2}})$$

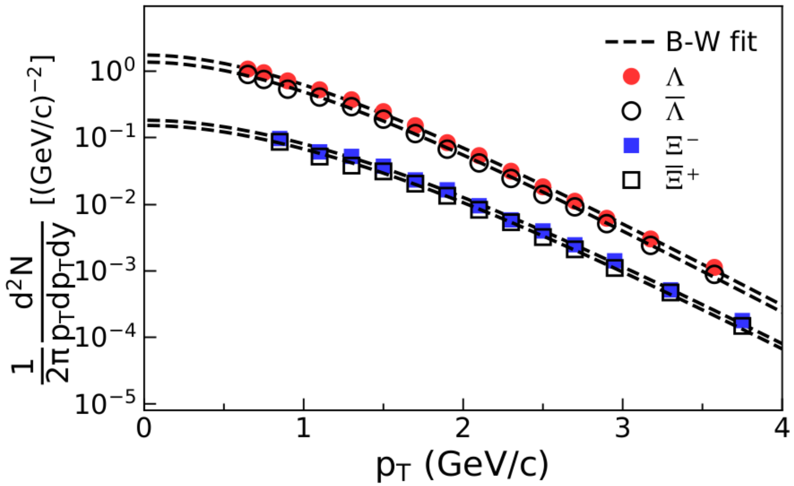
$$\Delta = \rho_{\frac{1}{2}\frac{1}{2}} + \rho_{-\frac{1}{2}-\frac{1}{2}}$$

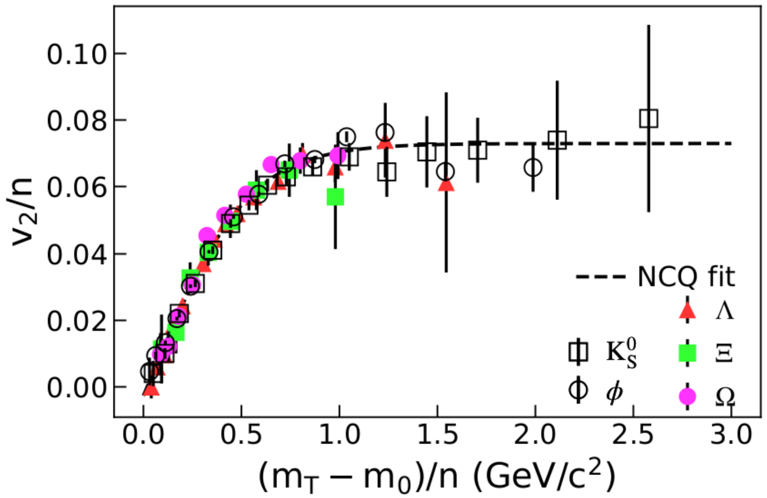
$$\delta = (\rho_{\frac{1}{2}\frac{1}{2}} - \rho_{-\frac{1}{2}-\frac{1}{2}})/(3P_P)$$

Simulation setup

TABLE II. The primordial yield ratio N_i/N_Λ, spin, parity, and decay channels of strange particles

	N_i/N_Λ	spin and parity	decay channel
Λ	1	$1/2^+$	-
$\Lambda(1405)$	0.236	$1/2^-$	$\Sigma^0\pi$
$\Lambda(1520)$	0.265	$3/2^-$	$\Sigma^0\pi$
$\Lambda(1600)$	0.098	$1/2^+$	$\Sigma^0\pi$
$\Lambda(1670)$	0.061	$1/2^-$	$\Sigma^0\pi$, $\Lambda\eta$
$\Lambda(1690)$	0.112	$3/2^-$	$\Sigma^0\pi$
Σ^0	0.686	$1/2^+$	$\Lambda\gamma$
Σ^{*0}	0.533	$3/2^+$	$\Lambda\pi$
Σ^{*+}	0.535	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
Σ^{*-}	0.524	$3/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1660)$	0.068	$1/2^+$	$\Lambda\pi$, $\Sigma^0\pi$
$\Sigma(1670)$	0.125	$3/2^-$	$\Lambda\pi$, $\Sigma^0\pi$
Ξ^0	0.343	$1/2^+$	$\Lambda\pi$
Ξ^-	0.332	$1/2^+$	$\Lambda\pi$
Ξ^{*0}	0.228	$3/2^+$	$\Xi\pi$
Ξ^{*-}	0.224	$3/2^+$	$\Xi\pi$





Simulation setup

We input parent's polarization as functions of azimuthal angle:

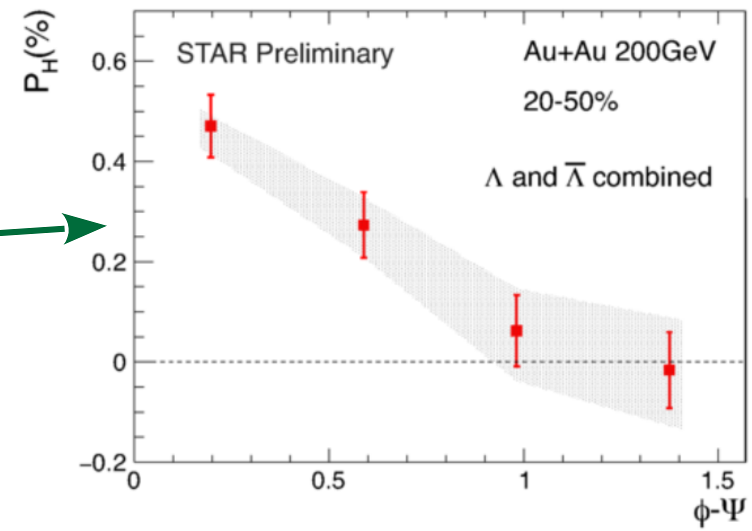
$$\begin{aligned} P_x &= \qquad\qquad f_{1x} \sin\phi, \\ P_y &= f_0 - f_{1y} \cos\phi + f_2 \cos(2\phi), \\ P_z &= \qquad\qquad\qquad\qquad f_z \sin(2\phi). \end{aligned}$$

Simulation setup

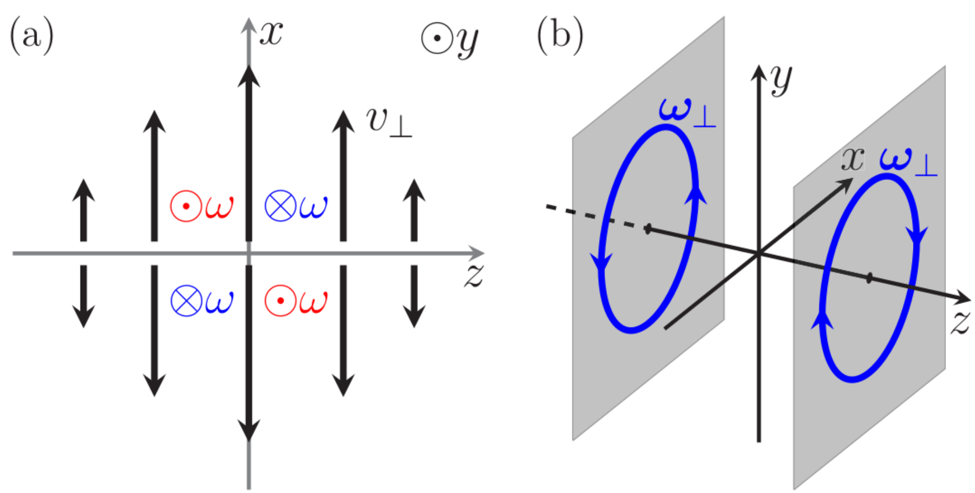
We input parent's polarization as functions of azimuthal angle:

$$P_x = f_{1x}\sin\phi,$$
$$P_y = f_0 - f_{1y}\cos\phi + f_2\cos(2\phi),$$
$$P_z = f_z\sin(2\phi).$$

In-plane to out-of-plane difference

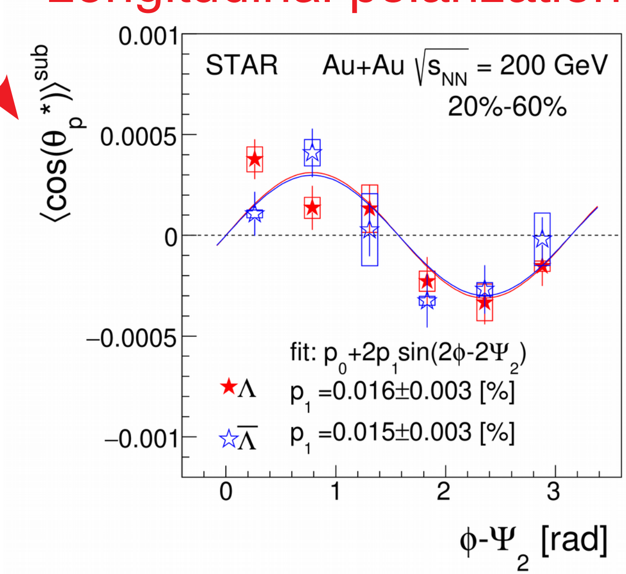


Transverse polarization along loops



$$\omega_\perp \sim \mathbf{e}_\phi = (\sin\phi, -\cos\phi)$$

Longitudinal polarization

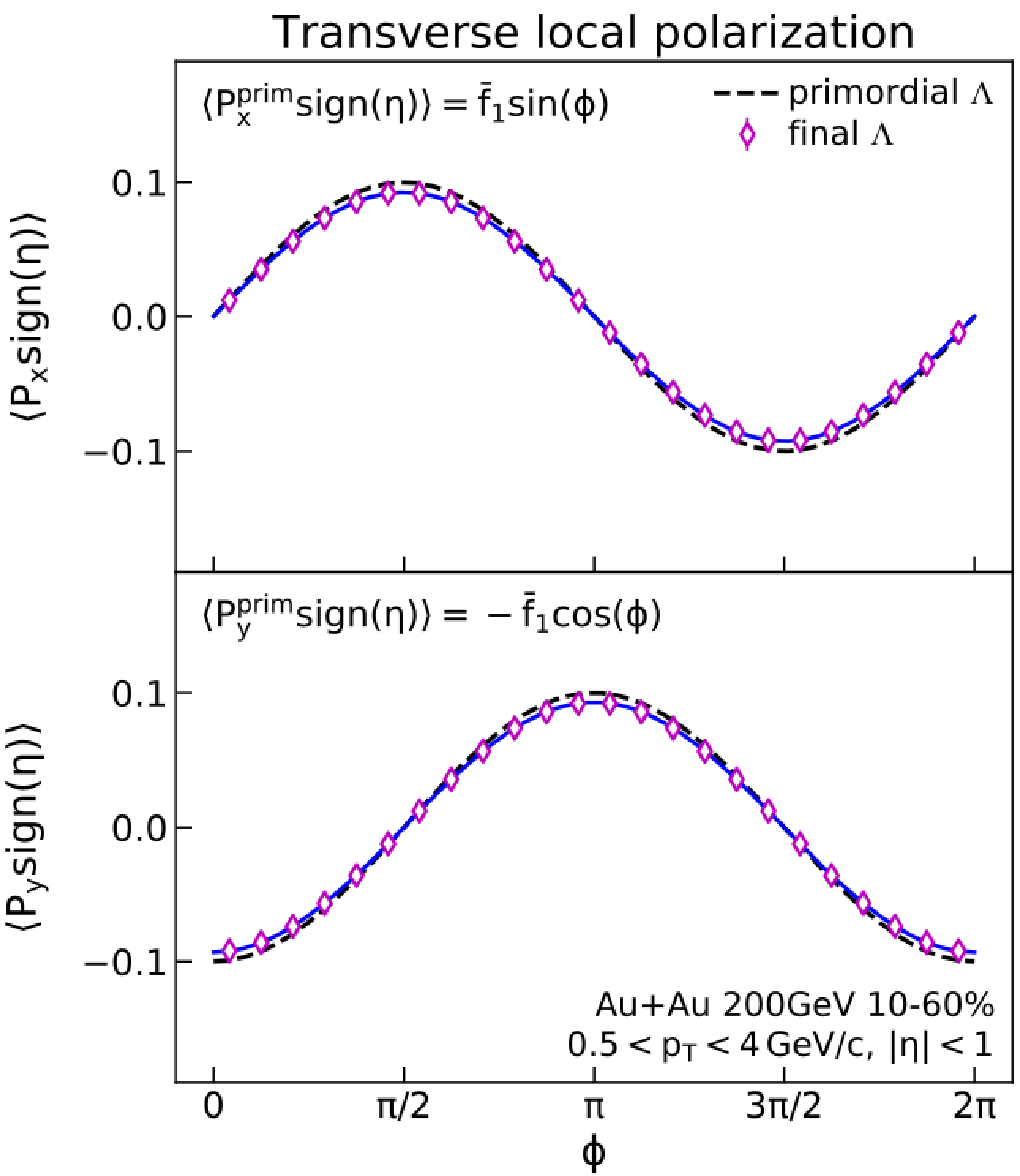


Numerical results

- Transverse polarization along loops

$$P_x = \quad f_x \sin\phi$$
$$P_y = -f_y \cos\phi$$

  - Final Λ polarization is suppressed by ~10%



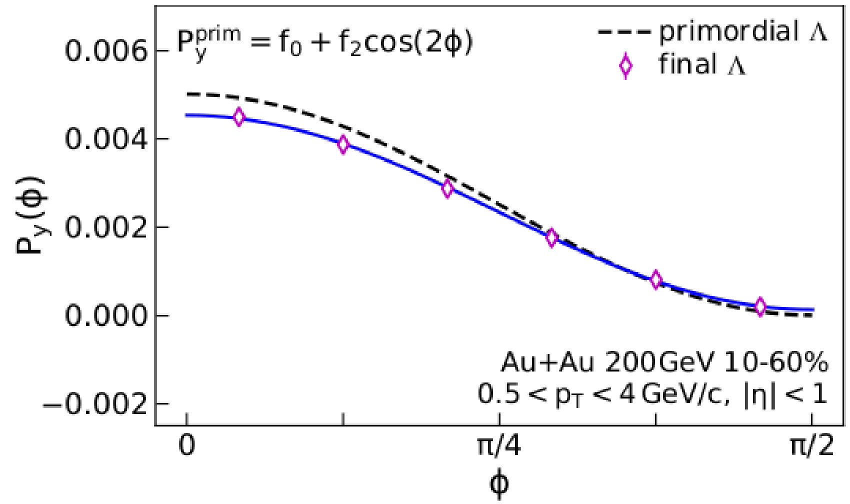
Numerical results

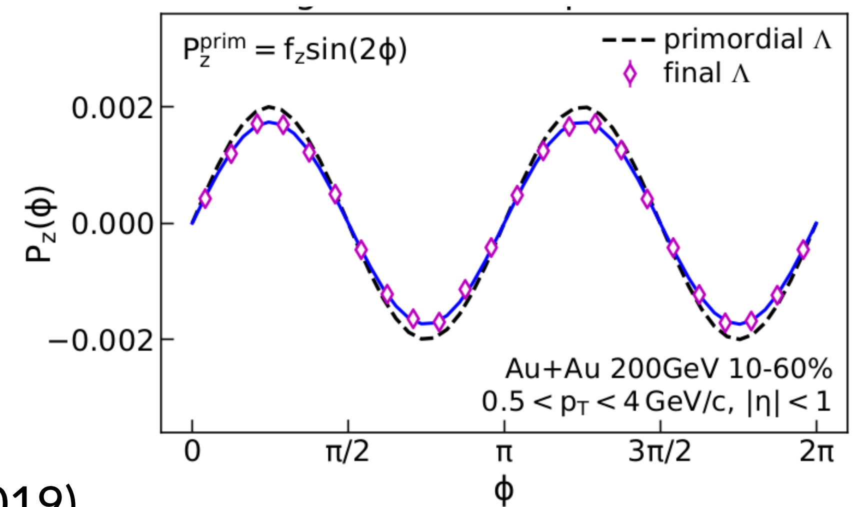
- In-plane to out-of-plane difference

$$P_y = f_0 + f_2 \cos(2\phi)$$

- Longitudinal polarization

$$P_z = f_z \sin(2\phi)$$





- Final Λ polarization is suppressed by ~10%

- The feed-down effect does not flip the sign.

References:
Xia, Li, Huang, Huang, PRC100, 014913 (2019)
Becattini, Cao, Speranza, EPJC79, 741 (2019)

Summary

- Spin polarization can be generated from different sources.
  - Global polarization reflects the net vorticity induced by the initial angular momentum.
  - Fireball's inhomogeneous expansion can also generate the spin polarization locally/collectively.
- We have studied the feed-down effect to the azimuthal-angle dependent polarization.
- The opposite sign between theoretical calculations and the experiment data is not caused by feed-down effect.

Thank you