Chiral kinetic theory from Landau level basis

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New development of hydrodynamics and its applications in Heavy-Ion Collisions, Fudan University, Shanghai, 2019/10/30-11/2

SL, L.X.Yang, 1909.11514

Outline

- Axial anomaly and manifestations
- Chiral kinetic theory (CKT)
- CKT as hbar expansion and its difficulty at higher order
- Chiral kinetic theory with Landau level basis
- Simple example: longitudinal electric field
- Less obvious example: transverse electric field
- Summary and outlook

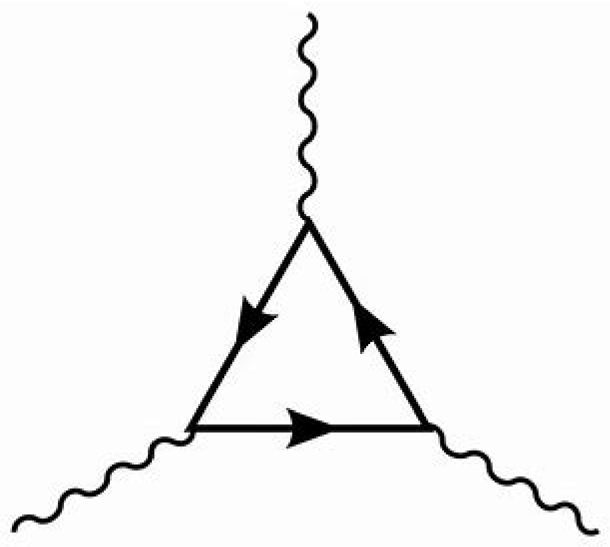
Axial anomaly

Axial symmetry in the chiral (massless) limit

$$\psi \to \psi e^{i\gamma^5 \alpha}$$
$$\partial_\mu j_5^\mu = 0$$

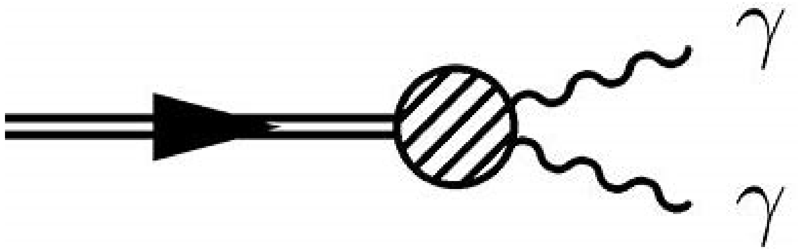
Axial symmetry breaking at quantum level

$$\partial_\mu j_5^\mu = -\frac{q^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G}$$

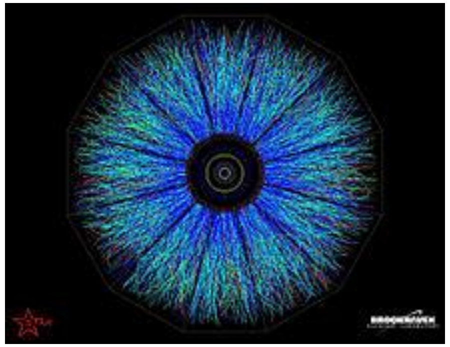


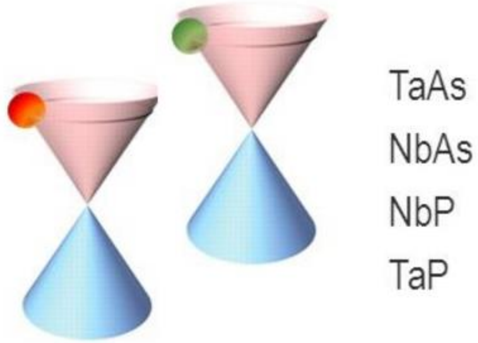
Applications of axial anomaly

In vacuum: pion decay

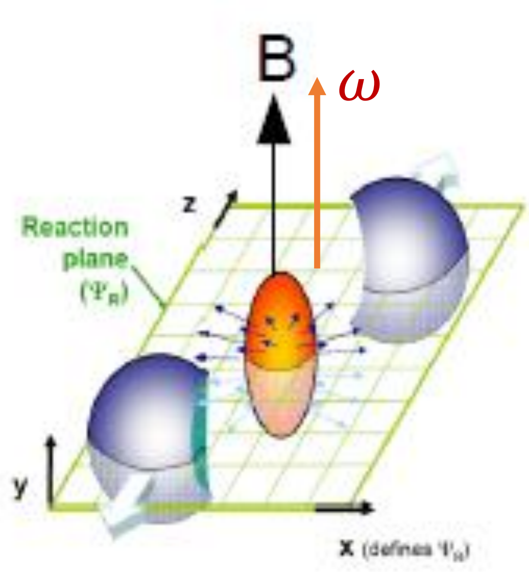


In medium: many manifestations





Anomalous effects in heavy ion collisions



Chiral Magnetic/Separation Effect(CME/CSE)

$$\boldsymbol{j} = C\mu_5 e\boldsymbol{B} \qquad \boldsymbol{j}_5 = C\mu e\boldsymbol{B}$$

Vilenken, PRD 1980
Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008
Metlitski, Zhitnitsky, PRD 2005

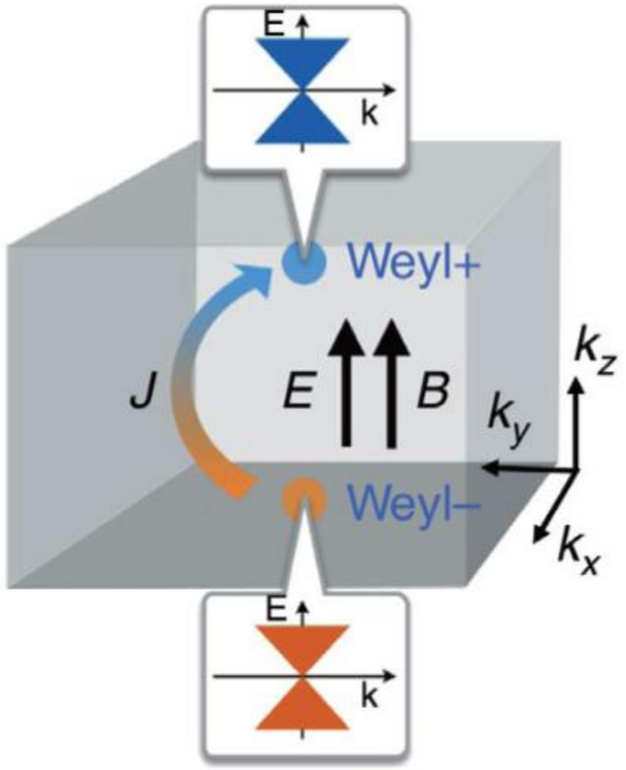
Chiral Vortical Effect(VCVE/ACVE)

$$\boldsymbol{j} = C\mu_5\mu\boldsymbol{\omega} \qquad \boldsymbol{j}_5 = C(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3})\boldsymbol{\omega}$$

Vilenken, PRD 1979
Erdmenger et al, JHEP 2009
Banerjee et al, JHEP 2011
Landsteiner et al, PRL 2011

Anomalous effect in Weyl semimetal

Weyl Semi-metal



Li, Kharzeev et al Nature. Phys. (2016)
Gooth et al Nature. (2017)
Son, Spivak PRB (2013)

A theoretical framework for anomalous effect

CME and CVE are equilibrium phenomena: spin alignment

In real world experiments, CME and CVE involves out-of-equilibrium dynamics

A framework for dynamics of particle with spin: Chiral kinetic theory

Chiral kinetic theory

Chiral kinetic equation

$$\Big[(1+\hbar Q\mathbf{\Omega}\cdot\mathbf{B})\,\partial_t+(\mathbf{v}+\hbar Q\mathbf{E}\times\mathbf{\Omega}+\hbar Q(\mathbf{v}\cdot\mathbf{\Omega})\mathbf{B})\cdot\nabla_{\mathbf{x}}$$
$$+\big(Q\mathbf{E}+Q\mathbf{v}\times\mathbf{B}+\hbar Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}\big)\cdot\nabla_{\mathbf{p}}\Big]f(X,\mathbf{p})=C[f]$$

Berry curvature $$\mathbf{\Omega}_{\mathbf{p}}\equiv\pm\frac{\hat{\mathbf{p}}}{2|\mathbf{p}|^2}$$

reduce to Boltzmann equation when $\boldsymbol{\Omega}_p=0$

$$(\partial_t+\mathbf{v}\cdot\nabla_{\mathbf{x}}+Q(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{p}})\,f(X,\mathbf{p})=C[f]$$

Son, Yamamoto, PRD (2012) PRL (2012)
Stephanov, Yin, PRL (2012)
Gao, Liang, Pu, Q. Wang, X.-N. Wang, PRL (2012), PRD (2014)
Manuel, Torres-Rincol, PRD (2014)
Hidaka, Pu, Yang, PRD (2016)
Huang, Shi, Jiang, Liao, Zhuang PRD (2018)
Liu, Gao, Mameda, Huang, PRD (2019)

CKT as a perturbation theory in hbar

$O(\hbar^0)$: spinless particle $\left(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}} + Q(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{p}}\right) f(X,\mathbf{p}) = C[f]$

mass shell $\delta(p^2)$

$O(\hbar)$: particle with magnetic moment + Berry curvature

$$\Big[(1 + \hbar Q \mathbf{\Omega}\cdot\mathbf{B})\,\partial_t + (\mathbf{v} + \hbar Q \mathbf{E}\times\mathbf{\Omega} + \hbar Q(\mathbf{v}\cdot\mathbf{\Omega})\mathbf{B})\cdot\nabla_{\mathbf{x}}$$
$$+ \Big(Q\mathbf{E} + Q\mathbf{v}\times\mathbf{B} + \hbar Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}\Big)\cdot\nabla_{\mathbf{p}}\Big] f(X,\mathbf{p}) = C[f]$$

modified mass shell $\delta(p^2 + Q\boldsymbol{B}\cdot\boldsymbol{p}/p_0)$

valid when $\hbar E, \hbar B \ll p^2, \hbar\partial_X \ll p$

Difficulty of CKT at higher order

$O(\hbar^0)$: spinless particle $\left(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}} + Q(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{p}}\right) f(X,\mathbf{p}) = C[f]$

$O(\hbar)$: particle with magnetic moment + Berry curvature

$$\Big[\left(1+\hbar Q\boldsymbol{\Omega}\cdot\mathbf{B}\right)\partial_t + \left(\mathbf{v}+\hbar Q\mathbf{E}\times\boldsymbol{\Omega}+\hbar Q(\mathbf{v}\cdot\boldsymbol{\Omega})\mathbf{B}\right)\cdot\nabla_{\mathbf{x}}$$
$$+\Big(Q\mathbf{E}+Q\mathbf{v}\times\mathbf{B}+\hbar Q^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}\Big)\cdot\nabla_{\mathbf{p}}\Big] f(X,\mathbf{p}) = C[f]$$

$O(\hbar^2)$: simple quasi-particle picture lost

$\delta(p^2) \nrightarrow \delta(\tilde{p}^2)$

$$p_\mu G^\mu_{(0)}\left[f\delta(\tilde{p}^2)\right] + \frac{\hbar s}{2}\mathbf{G}^{(0)}\cdot\left\{\frac{1}{p_0}\mathbf{G}^{(0)}\times\left[\mathbf{p}f\delta(\tilde{p}^2)\right]\right\} + \hbar^2 C(f) = 0$$

$C(f)$: off-shell effect

Gao, Liang, Q. Wang, X.N. Wang PRD (2018)
Wang, Guo, Shi, Zhuang PRD (2019)

Two alternative formulations

free particle basis: subject to weak electromagnetic field $\hbar B \ll \boldsymbol{p}^2$

with increasing B, CKT more IR singular, Landau quantization effect becomes relevant.

Landau level basis: subject to strong magnetic field $\hbar B \gg \boldsymbol{p}^2$

with increasing B, lower Landau levels truncation more accurate

Landau level basis

Wigner function at constant B for right-handed chiral fermion

$$\widetilde{W}(X,P)=\int\frac{d^4X'}{(2\pi)^4}\exp(-ip\cdot X')\left\langle\psi\left(X-\frac{1}{2}X'\right)U\left(A,X+\frac{1}{2}X',X-\frac{1}{2}X'\right)\psi^\dagger\left(X-\frac{1}{2}X'\right)\right\rangle$$

Example: lowest Landau level

$$\widetilde{W}(P)=f_+(p_0)\delta\left(p_0-E_p\right)\theta(p_3)W_+^{(0)}(p)+f_-(p_0)\delta\left(p_0+E_p\right)\theta(-p_3)W_-^{(0)}(p)]$$

$$W_r^{(0)}(P)=\frac{2}{(2\pi)^3}\exp\left(-\frac{p_T^2}{eB}\right)\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\qquad r=+/-$$

transverse size: $\frac{1}{\sqrt{eB}}$

LLL spin: aligns with B

LLL dispersion: 1+1D

Sheng, Rischke, Wang, Vasak EPJA (2018)

CKT with Landau level basis from perturbation

$$W(X,P)=\int\frac{d^4X'}{(2\pi)^4}\exp(-ip\cdot X')\left\langle\psi\left(X-\frac{1}{2}X'\right)U\left(A,X+\frac{1}{2}X',X-\frac{1}{2}X'\right)\psi^{\dagger}\left(X-\frac{1}{2}X'\right)\right\rangle$$

$$A_\mu\to A_\mu(B)+a_\mu(e,b)$$

$$F_{\mu\nu}\to F_{\mu\nu}(B)+f_{\mu\nu}(e,b)$$

⇧ background ⇧ perturbation

$$\left(\frac{\hbar}{2}\Delta_\mu-ip_\mu\right)\sigma^\mu W=0$$
$$\left(\frac{\hbar}{2}\Delta_\mu+ip_\mu\right)W\sigma^\mu=0$$

$$\Delta_\mu=\partial_\mu-\frac{\partial}{\partial p_\nu}(F_{\mu\nu}+f_{\mu\nu})$$

Valid to all order in background $\hbar$B and O($\hbar\partial_X$) and O($\hbar f$)

CKT with LL basis for components

W Hermitian $$W = \frac{1}{2}(F \cdot 1 + j_i \sigma^i)$$

$$J_0(X) = \int d^4 P F(X,P) \qquad J_i(X) = \int d^4 P j_i(X,P)$$

$$\begin{cases} \Delta_0 F + \Delta_i j_i = 0 \\ \Delta_0 j_i + \Delta_i F - 2\epsilon^{ijk} p_j j_k = 0 \\ p_0 F - p_i j_i = 0 \\ -p_0 j_i + p_i F + \frac{1}{2}\epsilon^{ijk} \Delta_j j_k = 0 \end{cases}$$

Transport equations

Constraint equations

Automatically satisfied by all LL states with stationary&homogeneous distribution function

longitudinal conductivity from $O(\hbar E)$ perturbation

LLL state background: only time and longitudinal components nonvanishing

$$j_3 = \frac{p_3}{p_0} F \propto f(p_0)\delta(p_0 - p_3)\exp(-\frac{p_T^2}{eB}) \qquad j_1 = j_2 = 0$$

For homogeneous $\mathrm{E} \parallel B$, only distribution function changes

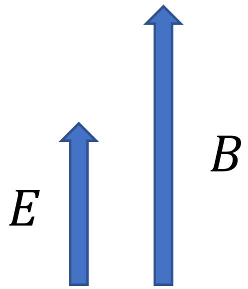
$$\Delta_0 F + \Delta_i j_i = 0$$

$$\Downarrow$$

$$\frac{\partial f_\pm}{\partial t} + \dot{z}\frac{\partial f_\pm}{\partial z} + \dot{p_z}\frac{\partial f_\pm}{\partial p_z} = C[f_\pm, f_g]$$

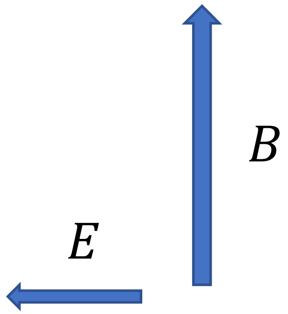
Hattori, Li, Satow, Yee PRD (2018)

Longitudinal & transverse conductivities



Longitudinal motion classical, system evolves within LL basis

Hattori, Satow, PRD 2016
Hattori, Li, Satow, Yee PRD 2018
Fukushima, Hidaka, PRL 2018; 1906.02683



Transverse motion quantum, system evolves beyond LL basis

transverse conductivities from $O(\hbar E)$ perturbation

LLL state background: only time and longitudinal components nonvanishing

$$j_3 = \frac{p_3}{p_0} F \propto f(p_0)\delta(p_0 - p_3)\exp(-\frac{p_T^2}{eB}) \qquad j_1 = j_2 = 0$$

For homogeneous $E \perp B$, need to solve for components

$$\Delta_0 F + \Delta_i\, j_i = -\frac{\delta F}{\tau}$$

$$\Delta_i F + \Delta_0\, j_i - 2\epsilon^{ijk} p_j\, j_k = -\frac{\delta j_i}{\tau}$$

relaxation time approximation

$$\delta j_i = E_i \delta j_{\parallel} + \epsilon^{ij} E_j \delta j_{\perp}$$

transverse components excited

$$\delta j_{\parallel} = \frac{4 p_z \tau F - \left(2B\tau + \frac{1}{\tau}\right)\frac{\partial F}{\partial p_0}}{\left(2B\tau + \frac{1}{\tau}\right)^2 + 4p_z^2},$$

$$\delta j_{\perp} = \frac{2p_z \frac{\partial F}{\partial p_0} + 2\left(2B\tau + \frac{1}{\tau}\right)\tau F}{\left(2B\tau + \frac{1}{\tau}\right)^2 + 4p_z^2}$$

$$\frac{\partial F}{\partial p_0} \propto \delta'(p_0 - p_z)$$

Transverse conductivities

$eB \gg T \& \mu \& 1/\tau$

$$\delta \vec{J} = \frac{e\vec{E}}{8\pi^2\tau}\left[1 + \frac{1}{eB}\left(\frac{\mu^2}{2} + \frac{\pi^2 T^2}{6} - \frac{1}{2\tau^2}\right)\right] - \frac{e^2\mu\vec{B}\times\vec{E}}{4\pi^2 eB}$$

$$e\delta\mathbf{J} = \sigma_{\perp}\mathbf{E} + \sigma_H \frac{\mathbf{B}\times\mathbf{E}}{B}$$

$\sigma_{\perp} \sim O(1/\tau_{\perp})$ compare to $\sigma_{\parallel} \sim O(\tau_{\parallel} B)$ Hattori, Li, Satow, Yee PRD (2018)

$\sigma_H \sim O(\mu B^0)$ compare to weak B limit $\sigma_H \sim O(\mu\tau_0^2 B)$ Gorbar et al PRD (2016)

frequency dependence from $O(\hbar E)$ and $O(\hbar \partial_t)$ perturbations

$$\vec{E} \;\propto\; e^{-i\omega t}$$

$$\tau \to \tau_\omega \equiv \frac{\tau}{1 - i\omega\tau}$$ effective relaxation time

$$\delta\vec{J} = \frac{e\vec{E}}{8\pi^2\tau_\omega}\left[1 + \frac{1}{eB}\left(\frac{\mu^2}{2} + \frac{\pi^2T^2}{6} - \frac{1}{2\tau_\omega^2}\right)\right] - \frac{e^2\mu\vec{B}\times\vec{E}}{4\pi^2 eB}$$

$$\sigma_\perp = \frac{e^2}{8\pi^2\tau_\omega}\left[1 + \frac{1}{eB}\left(\frac{\mu^2}{2} + \frac{\pi^2T^2}{6} - \frac{1}{2\tau_\omega^2}\right)\right]$$

$$\sigma_H = -\frac{e^2\mu}{4\pi^2}$$

Enhancement of transverse conductivity at large ω!

Parameters of QGP

$$\sigma_{\perp} = \sum_f N_c Q_f^2 \frac{e^2}{8\pi^2\tau_\omega}\left[1 + \frac{1}{e|Q_f|B}\left(\frac{\mu^2}{2} + \frac{\pi^2 T^2}{6} - \frac{1}{2\tau_\omega^2}\right)\right]$$

$$\sigma_H = -\sum_f N_c Q_f^2 \frac{e^2\mu}{4\pi^2}$$

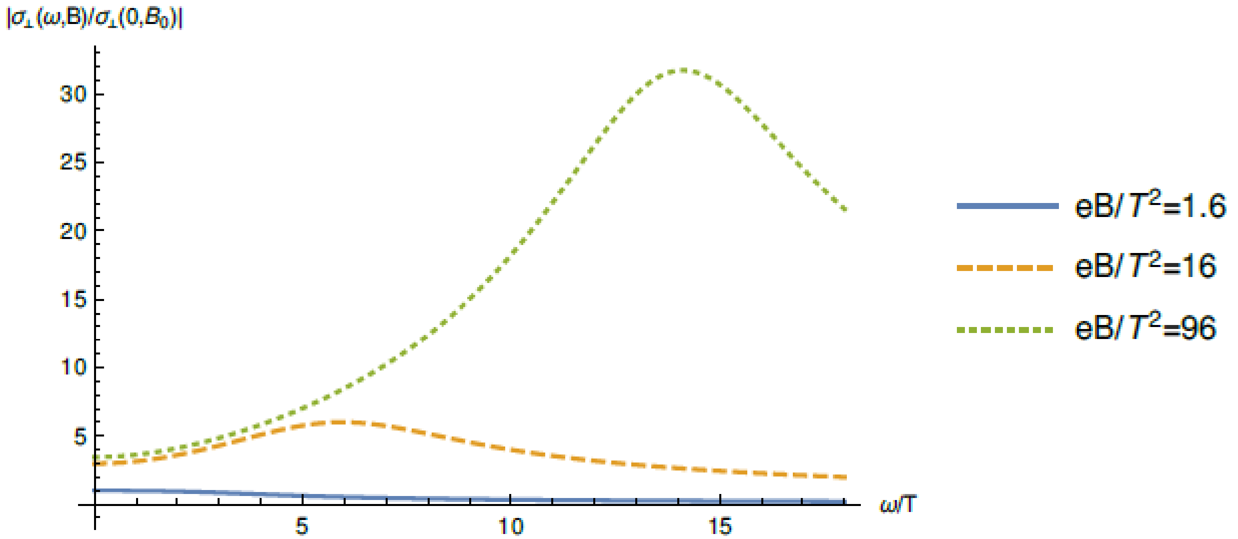
Estimate parameters in HIC

$eB \sim m_\pi^2 - 10m_\pi^2,$
T=350MeV
$\omega \sim 0.2\text{fm} - 1\text{fm}$ from lifetime of B

$$\frac{eB}{T^2}\sim 0.2 - 1.6$$

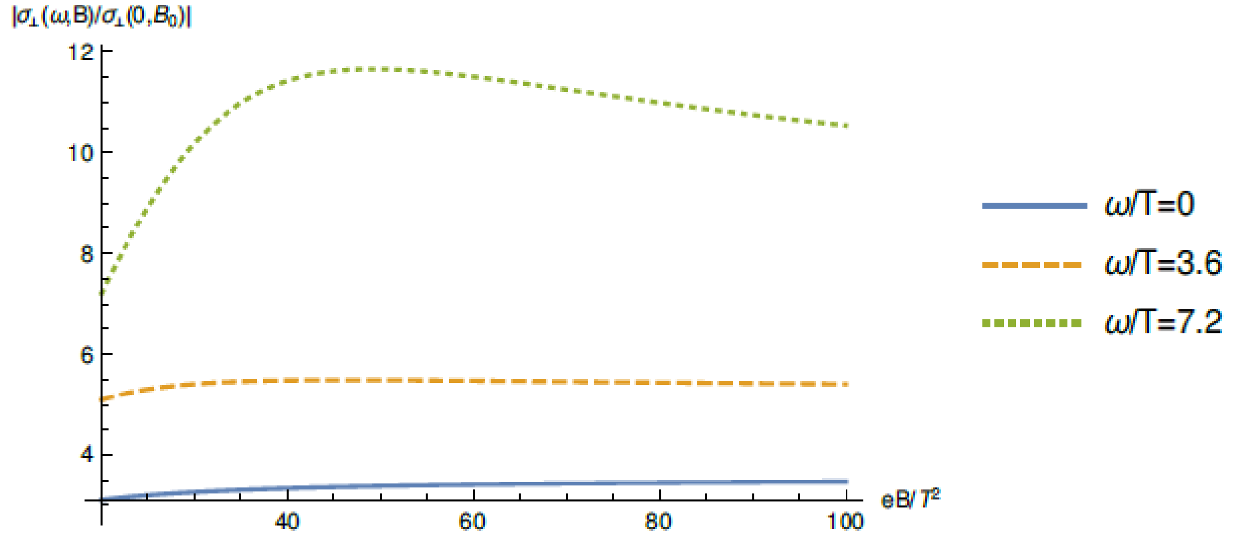
higher LL to be included

ω and B dependence of $\sigma_\perp$



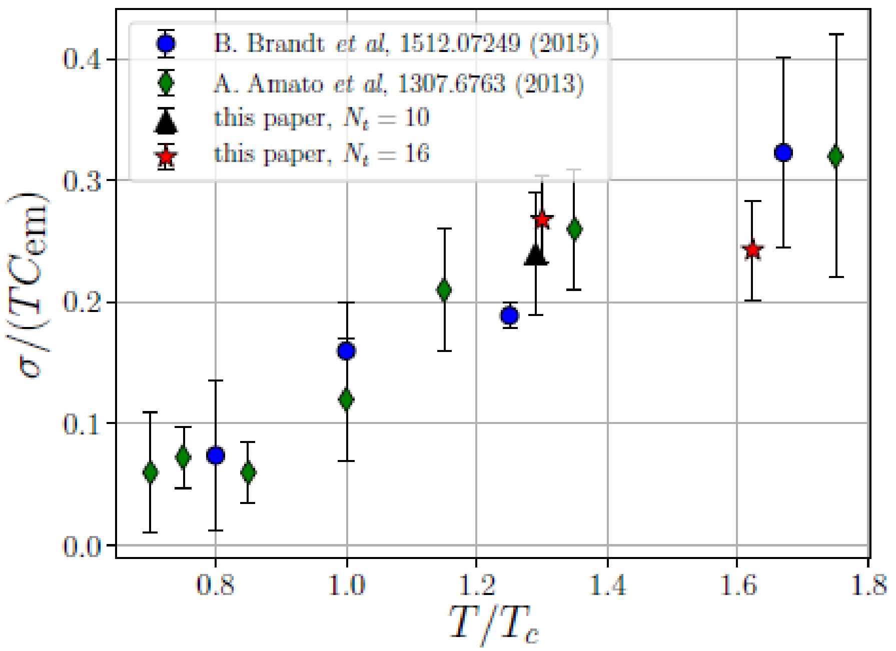
Assuming constant τ

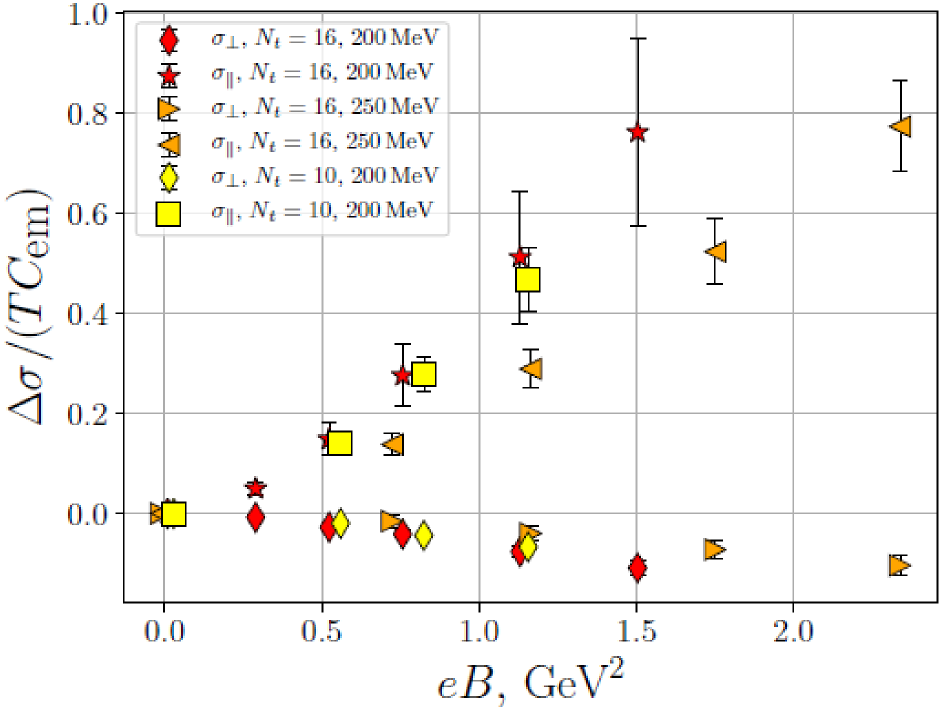
Dropping due to violation of $B \gg \omega^2$
Inclusion of HLL needed for QGP



Saturation at large B

B-dependence of τ?





Kotov et al, 1910.08516

Transverse conductivity decreases with increasing B: suggest $\tau(B)$
Microscopic computation of $\tau(B)$?

Summary

- CKT with Landau level basis
- Application: CKT with homogeneous E field
- Transverse conductivity O$(1/\tau)$ at large B field
- Transverse conductivity enhancement at high frequency

Outlook

- Higher Landau levels contribution
- Self-energy of photon in magnetized QGP

Thank you!

Solution to δj_i

$$\delta j_1 = \frac{4p_3\tau F - \left(2eB\tau + \frac{1}{\tau}\right)\frac{\partial F}{\partial p_0}}{\left(2eB\tau + \frac{1}{\tau}\right)^2 + 4p_3^2}$$

$$\delta j_2 = \frac{-2p_3\frac{\partial F}{\partial p_0} - 2\left(2eB\tau + \frac{1}{\tau}\right)\tau F}{\left(2eB\tau + \frac{1}{\tau}\right)^2 + 4p_3^2}$$

$B \gg p_3^2, 1/\tau^2$ →

$$\delta j_1 = \frac{-\frac{\partial F}{\partial p_0}}{2eB\tau}$$

$$\delta j_2 = \frac{-2\tau F}{2eB\tau}$$