Polarization effects
in heavy-ion collisions

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New development of hydrodynamics and its applications in Heavy-Ion Collisions,
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Outline

- **Introduction**
- **Test of different relativistic vorticities in longitudinal polarization with (3+1)D hydro**
- **A microscopic model for global polarization through spin-orbit couplings in particle scatterings (a non-equilibrium model for polarization)**
- **Summary**

Introduction

Global OAM and Magnetic field in HIC

- **Huge global orbital angular momenta are produced**

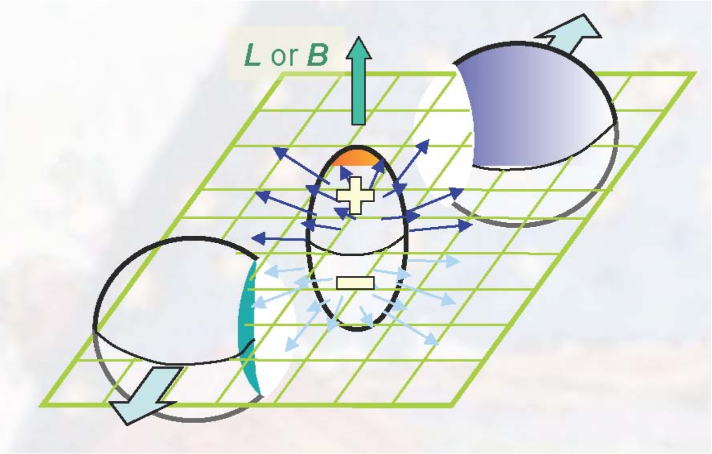
$$\mathrm{L} \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$\mathrm{B} \sim m_\pi^2 \sim 10^{18}\ \mathrm{Gauss}$$

- **How are orbital angular momenta transferred to the matter created?**

- **How is spin coupled to local vorticity in a fluid?**



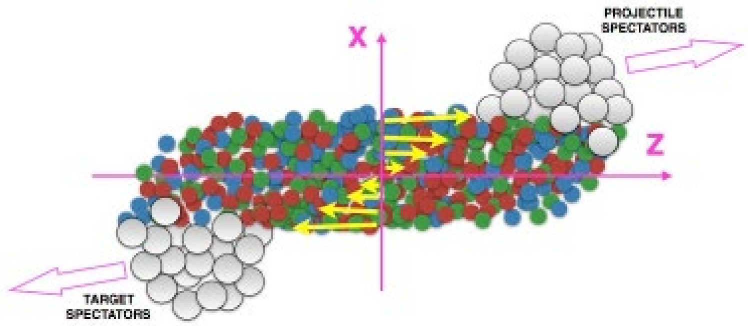


Figure taken from
Becattini et al, 1610.02506

Theoretical models and proposals: early works on global polarization in HIC

With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

- **Polarizations of Λ hyperons and vector mesons through spin-orbital coupling in HIC from global OAM**
- **-- Liang and Wang, PRL 94,102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]**
- **-- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]**

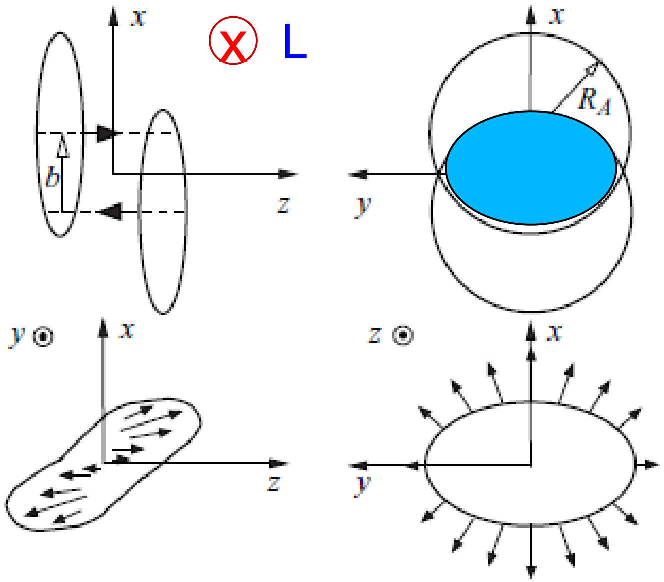
- **Polarized secondary particles in un-polarized high energy hadron-hadron collisions**
- **-- Voloshin, nucl-th/0410089**

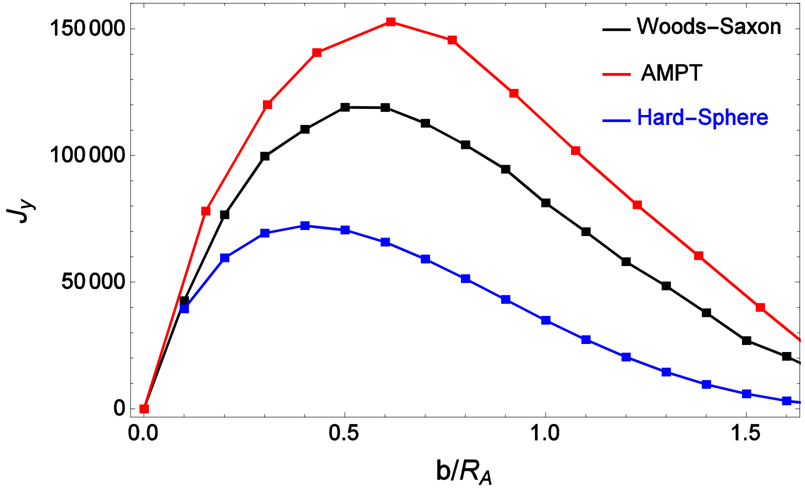
- **Polarization as probe to vorticity in HIC**
- **-- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]**

- **Statistical model for relativistic spinning particles**
- **-- Becattini, Piccinini, Annals Phys. 323, 2452 (2008) [0710.5694]**

Global OAM in HIC

- **Non-central collisions produce global orbital angular momentum**

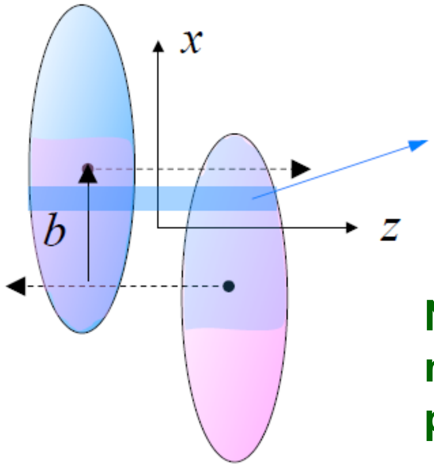




$$L_y = -p_{\text{in}} \int x\,dx \left(\frac{dN^P_{\text{part}}}{dx} - \frac{dN^T_{\text{part}}}{dx} \right)$$

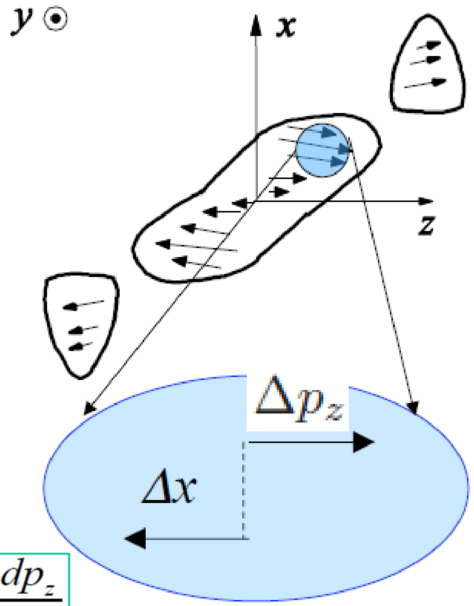
Liang & Wang, PRL 94, 102301(2005); PLB 629, 20(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Huang, Huovinen, Wang,PRC 84,054910(2011); Jiang, Lin, Liao, PRC 94,044910(2016); Deng,Huang, PRC 93,064907(2016); many others ……

Global OAM in HIC



$$\frac{dN^{P,T}_{part}}{dx}$$

Number of participant nucleons per unit x in projectile or target



Collective longitudinal momentum per produced parton

$$p_z(x,b) = \frac{\sqrt{s}}{2c(s)} \frac{\frac{dN^P_{\rm part}}{dx} - \frac{dN^T_{\rm part}}{dx}}{\frac{dN^P_{\rm part}}{dx} + \frac{dN^T_{\rm part}}{dx}}$$

$$\boxed{\Delta p_z = \Delta x \frac{dp_z}{dx}}$$

$$\begin{aligned} L_y &= -\Delta x \Delta p_z \\ &= -(\Delta x)^2 \frac{dp_z}{dx} \sim \beta\omega_y \end{aligned}$$

Liang & Wang (2005); Gao, et al. (2008); Betz, Gyulassy, Torrieri (2007); Jiang, Lin, Liao (2016); Deng, Huang (2016); many others ……

Theoretical models for global polarization

- **Spin-orbit coupling or microscopic models**
- **[Liang and Wang, PRL 94,102301(2005); Gao, Chen, Deng, Liang, QW, Wang, PRC 77, 044902(2008); Zhang, Fang, QW, Wang, arXiv:1904.09152.]**

- **Statistical-hydro models**
- **[Zubarev (1979); Weert (1982); Becattini et al. (2012-2015); Hayat, et al. (2015); Floerchinger (2016).]**

- **Spin hydrodynamic model**
- **[Florkowski,Friman,Jaiswal,Ryblewski,Speranza (2017-2018); Montenegro,Tinti,Torrieri (2017-2019); Hattori,Hongo,Huang,Matsuo,Taya (2019)]**

- **Kinetic theory for massive fermions with Wigner functions**
- **[Early works: Heinz (1983); Vasak, Gyulassy and Elze (1987); Elze, Gyulassy, Vasak (1986); Zhuang, Heinz (1996).]**
- **[Recent developments: Fang, Pang, QW, Wang (2016); Weickgenannt, Sheng, Speranza, QW, Rischke (2019); Gao, Liang (2019); Wang, Guo, Shi, Zhuang (2019); Hattori, Hidaka, Yang (2019).]**

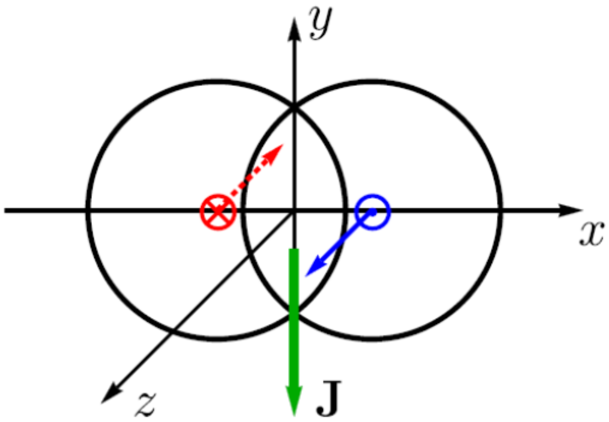
Comparison with data for global polarization

Global polarization of Λ from AMPT

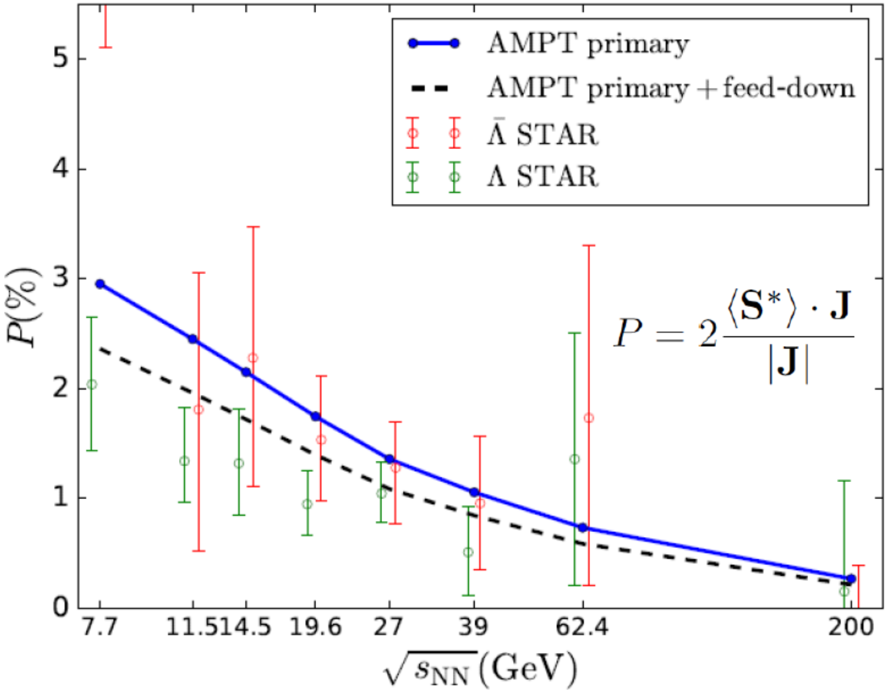
- **Polarization of Λ: average over events with |η|<1**

$$\langle \mathbf{S}^* \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathbf{S}^*(x, p)$$

$$P = 2\frac{\langle \mathbf{S}^* \rangle \cdot \mathbf{J}}{|\mathbf{J}|}$$



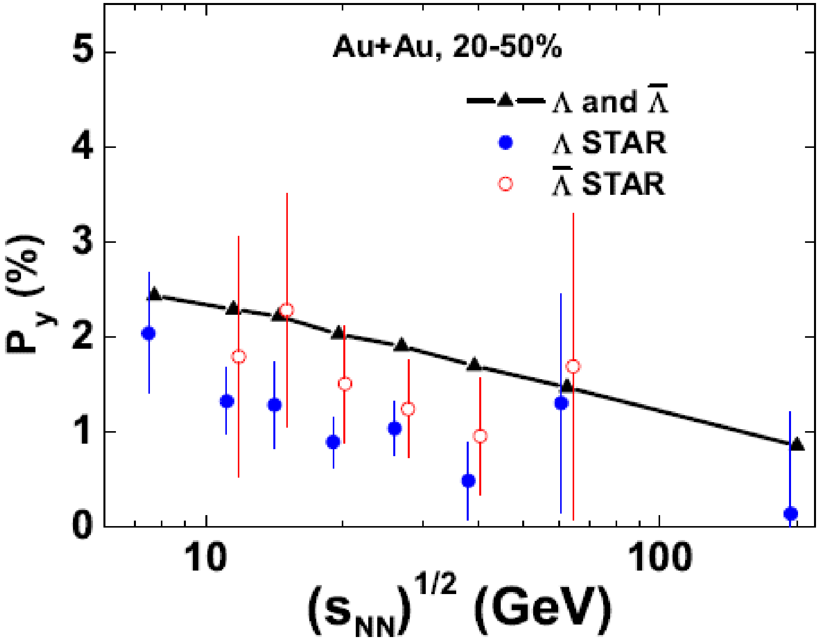
Au+Au, 20%-50%, with feed-down



Li, Pang, QW, Xia, PRC96,054908(2017)

Global polarization of Λ from Chiral Kinetic approach

- **Chiral kinetic approach+ AMPT model**
- **Spin polarizations of quarks and antiquarks**
- **Quarks and antiquarks are converted to hadrons via the coalescence Model**

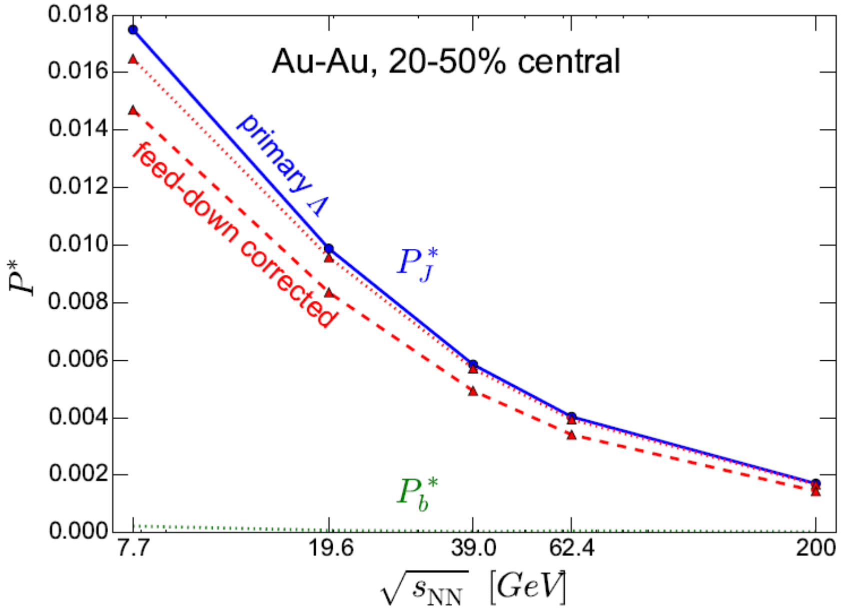


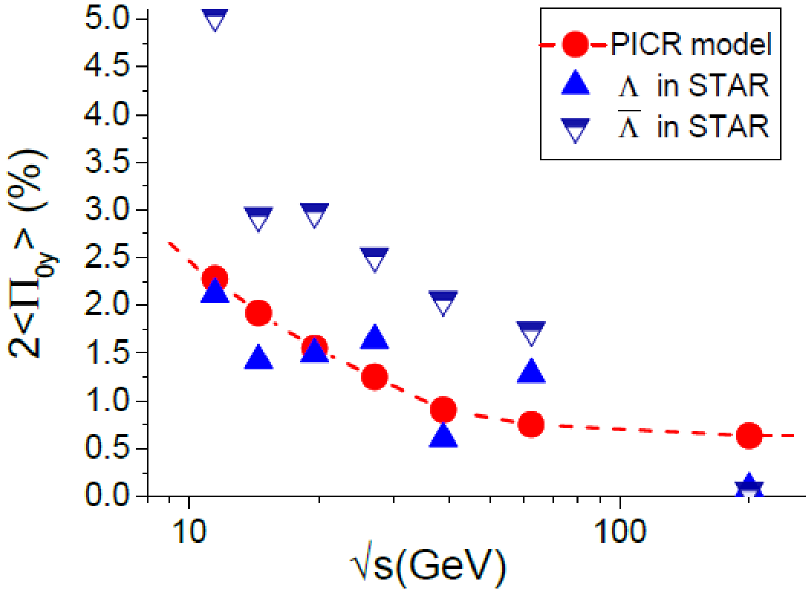
Sun, Ko, PRC96, 024906(2017)

Chiral kinetic approach:

Son, Yamamoto, PRL 109 (2012) 181602;
Stephanov, Yin, PRL 109 (2012) 162001;
Chen, Pu, QW, Wang, PRL 110 (2013) 262301;
Mueller, Venugopalan, PRD 96 (2017) 016023.

Global polarization of Λ from other methods



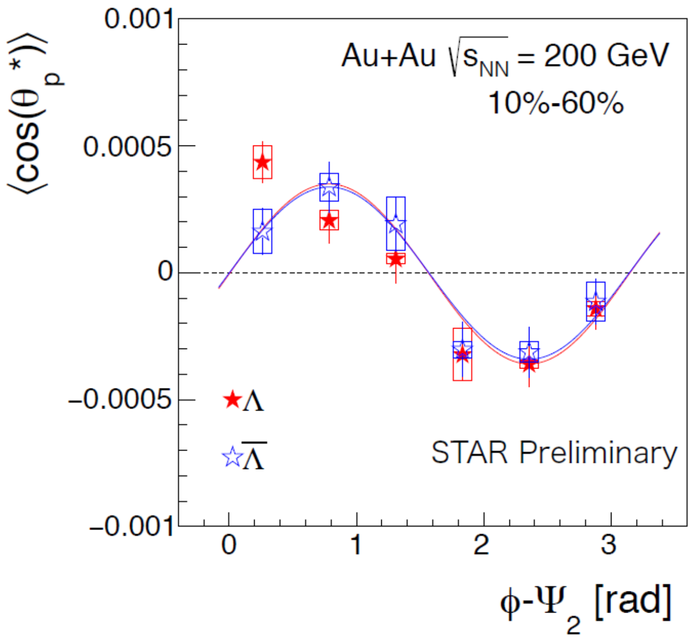


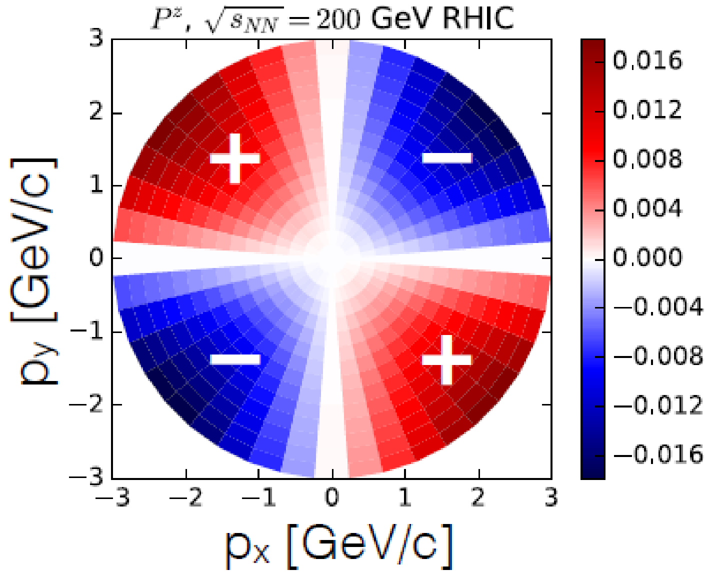
Karpenko, Becattini, EPJC 77,213(2017)
UrQMD + vHLLE hydro

Xie,Wang,Csernai,PRC 95,031901(2017)
PICR hydro

Sign problem in polarization along the beam direction

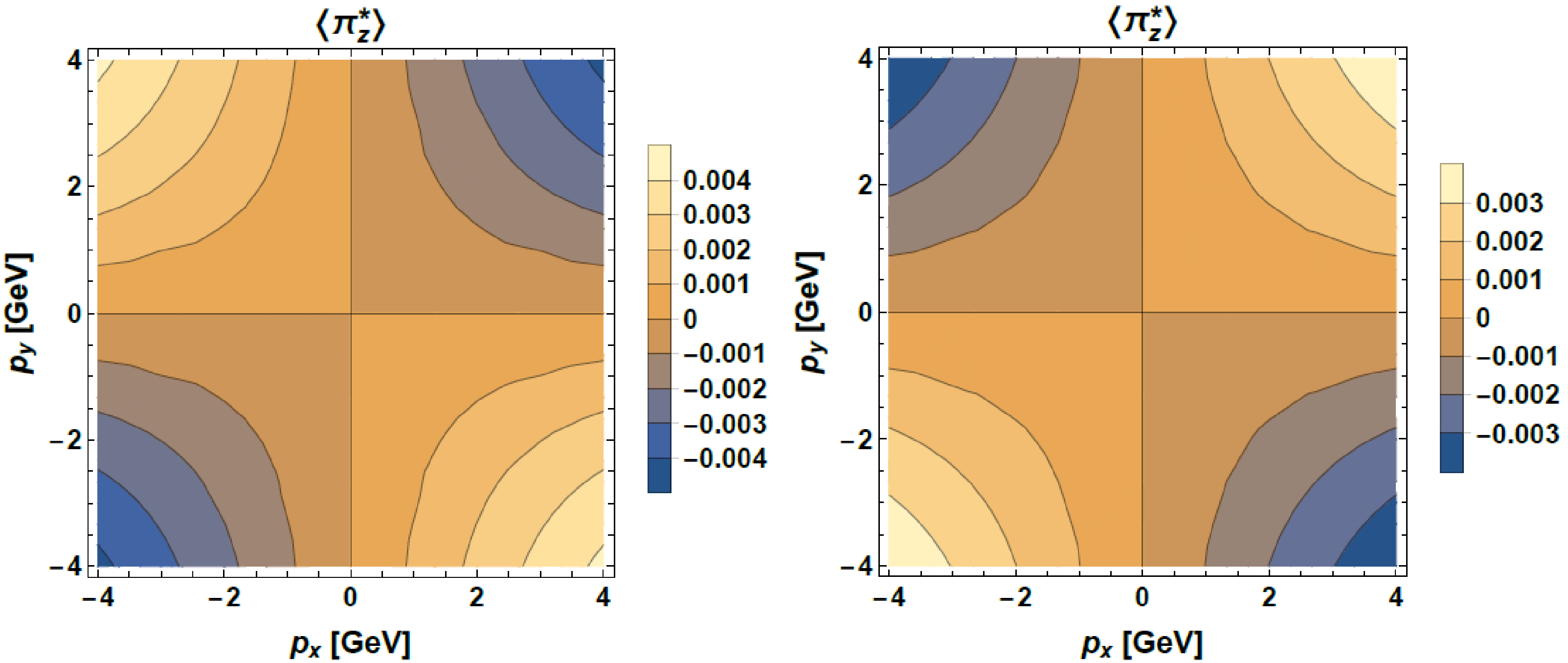
Polarization along the beam direction





- **Sin(2ϕ) structure as expected from the elliptic flow**
- **Opposite sign to the hydrodynamic model and transport model (AMPT)** **[Hydro model: Becattini, Karpenko (2018); Transport model (AMPT): Xie, Li, Tang, Wang (2018)]. Not from resonance decays [Xia,Li,Huang,Huang (2019); Becattini,Cao,Speranza (2019)]**
- **Same sign in chiral kinetic approach [Sun, Ko (2019)]**
- **Same sign in a simple phenomenological model [Voloshin (2017/2018)]**

Polarization along the beam direction



- **Left: the spin polarization is defined by the thermal vorticity. Right: projected spatial thermal vorticity in the Lab frame.**
- **[Florkowski, Kumar, Ryblewski, Mazeliauskas (1904.00002)]**

$$\omega_{\mu\nu} = \varpi_{\alpha\beta}\bar{\Delta}^{\alpha}_{\ \mu}\bar{\Delta}^{\beta}_{\ \nu}$$

$$\bar{\Delta}^{\mu\nu} = g^{\mu\nu} - u^{\mu}_{\rm LAB}u^{\nu}_{\rm LAB}$$

$$u^{\mu}_{\rm LAB} = (1,0,0,0)$$

Spin chemical potential

- **Normal hydrodynamics**
- **-- Energy and momentum conservation: T and u^{μ}**
- **-- Baryon number conservation: μ_B**

- **Including spin into hydrodynamics**
  -- Angular momentum conservation: $\omega^{\mu\nu}$ (spin chemical potential)
  [Becattini, Florkowski, Speranza (2018); Florkowski, Ryblewski, Kumar (2018)]

- **Ambiguity of localization of energy and spin densities**
  -- pseudo-gauge transformations: $T^{\mu\nu}$ and $S^{\lambda,\mu\nu} \Longrightarrow T'^{\mu\nu}$ and $S'^{\lambda,\mu\nu}$
  -- Belinfante construction: $T^{\mu\nu}_{\text{Bel}} = T^{\nu\mu}_{\text{Bel}} \qquad S^{\lambda,\mu\nu}_{\text{Bel}} = 0$

- **In general, the information about spin cannot be encoded in the form of OAM, since OAM can be eliminated by Lorentz transformation**

- **Generally $\omega^{\mu\nu} \neq -\frac{1}{2}(\partial^{\mu}\beta^{\nu} - \partial^{\nu}\beta^{\mu})$ unless in global equilibrium**

Talk by: K. Fukushima

Different relativistic vorticities

- **Different relativistic vorticities:**
- **Kinematic** $$\omega_{\mu\nu}^{(K)} = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu) = \varepsilon_\nu u_\mu - \varepsilon_\mu u_\nu + \boxed{\epsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta}$$
- **Non-Relativistic** $$\omega_{\mu\nu}^{(\mathrm{NR})} = \epsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta$$
- **T-vorticity** $$\begin{aligned} \omega_{\mu\nu}^{(T)} &= -\frac{1}{2}[\partial_\mu (T u_\nu) - \partial_\nu (T u_\mu)] \\ &= T\omega_{\mu\nu}^{(K)} + \frac{1}{2}(u_\mu \partial_\nu T - u_\nu \partial_\mu T) \\ &\equiv T\omega_{\mu\nu}^{(K)} + \omega_{\mu\nu}^{(T)}(T), \end{aligned}$$
- **Thermal** $$\begin{aligned} \omega_{\mu\nu}^{(\mathrm{th})} &= -\frac{1}{2}[\partial_\mu (\beta u_\nu) - \partial_\nu (\beta u_\mu)] \\ &= \frac{1}{T}\omega_{\mu\nu}^{(K)} - \frac{1}{2T^2}(u_\mu \partial_\nu T - u_\nu \partial_\mu T) \\ &= \frac{1}{T}\omega_{\mu\nu}^{(K)} + \omega_{\mu\nu}^{(\mathrm{th})}(T), \end{aligned}$$

Becattini, Inghirami, Rolando, et al., Eur. Phys. J. C75,406 (2015) [1501.04468]

Wu, Pang, Huang, QW,
PRR 1, 033058(2019) [1906.09385]

A test of different vorticities in (3+1)D hydro

- **Polarization at freezeout**

Becattini, et al., Ann.Phys. 338,32(2013)

$$S^{\mu}(p)=-\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\int d\Sigma_{\lambda}p^{\lambda}\Omega_{\rho\sigma}f_{FD}(1-f_{FD})}{\int d\Sigma_{\lambda}p^{\lambda}f_{FD}}$$

- **where we choose different vorticities**

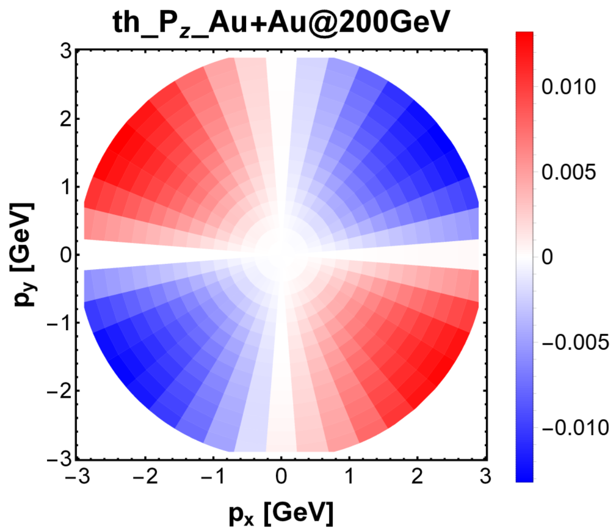
$$\Omega_{\rho\sigma}=\frac{1}{T}\omega_{\rho\sigma}^{(K)},\frac{1}{T^{2}}\omega_{\rho\sigma}^{(T)},\omega_{\rho\sigma}^{(\mathrm{th})},\frac{1}{T}\omega_{\rho\sigma}^{(\mathrm{NR})}$$

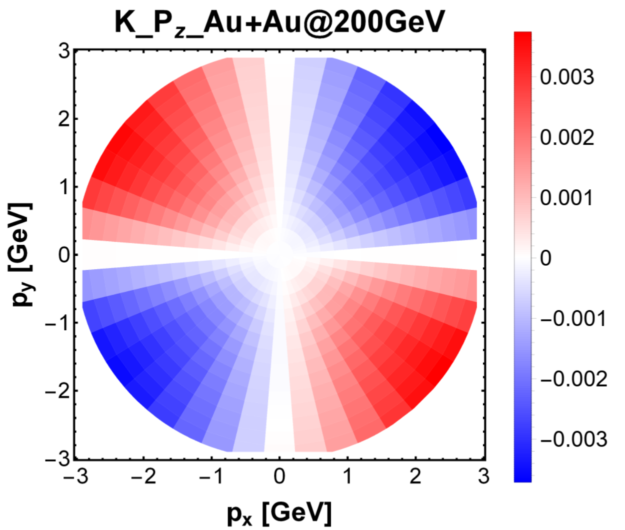
Wu, Pang, Huang, QW,
PRR 1, 033058(2019)
[1906.09385]

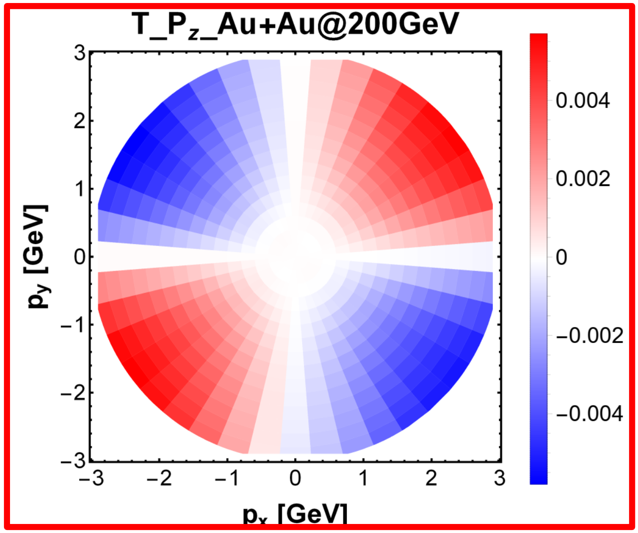
- **(3+1)D Hydro model CLVisc: with AMPT initial condition (OAM encoded)**

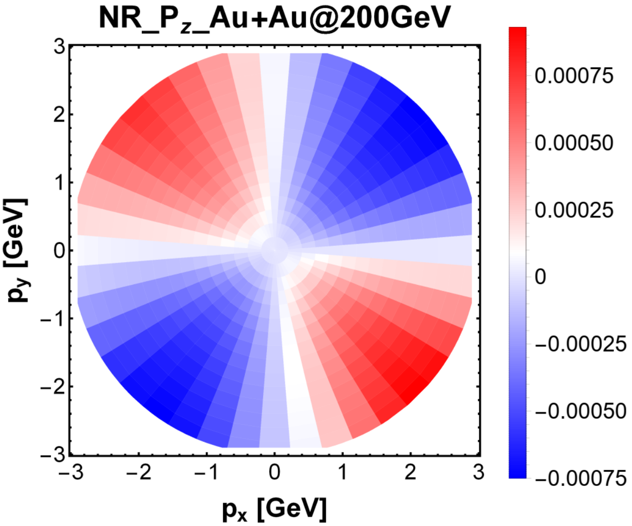
  [Pang, QW, Wang (2012); Pang, Petersen, Wang (2018)]

Longitudinal polarization


th_P_z_Au+Au@200GeV

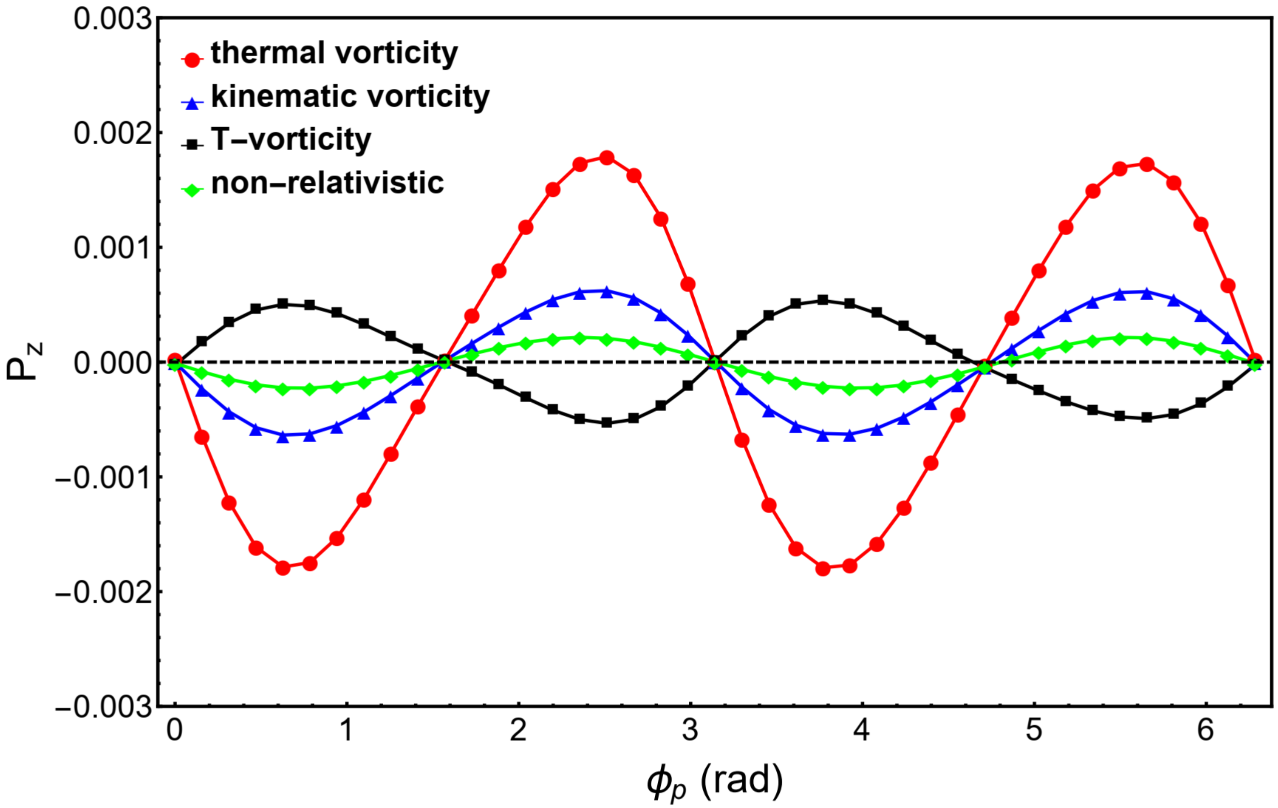

K_P_z_Au+Au@200GeV


T_P_z_Au+Au@200GeV


NR_P_z_Au+Au@200GeV

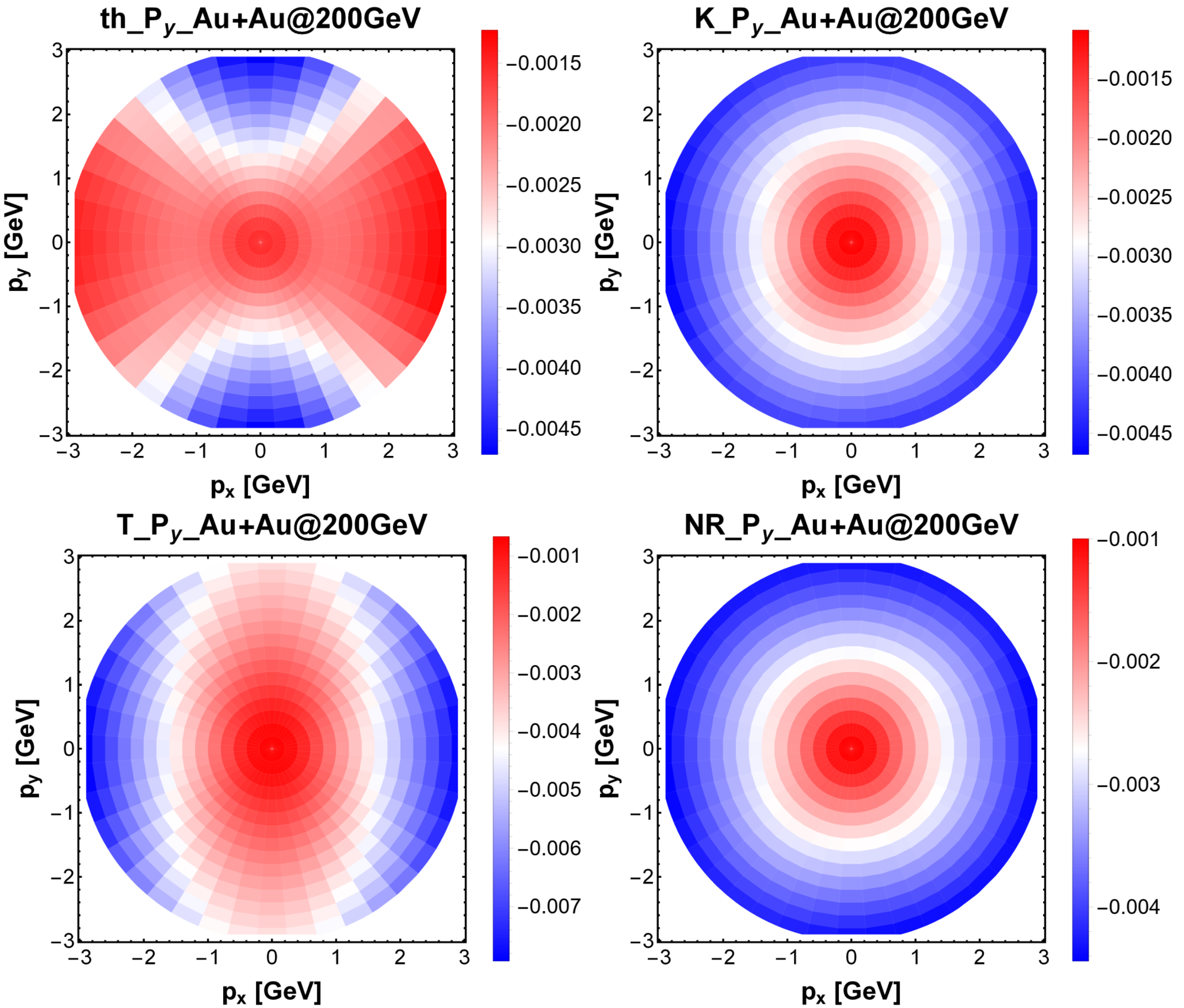
The longitudinal polarization in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

Longitudinal polarization



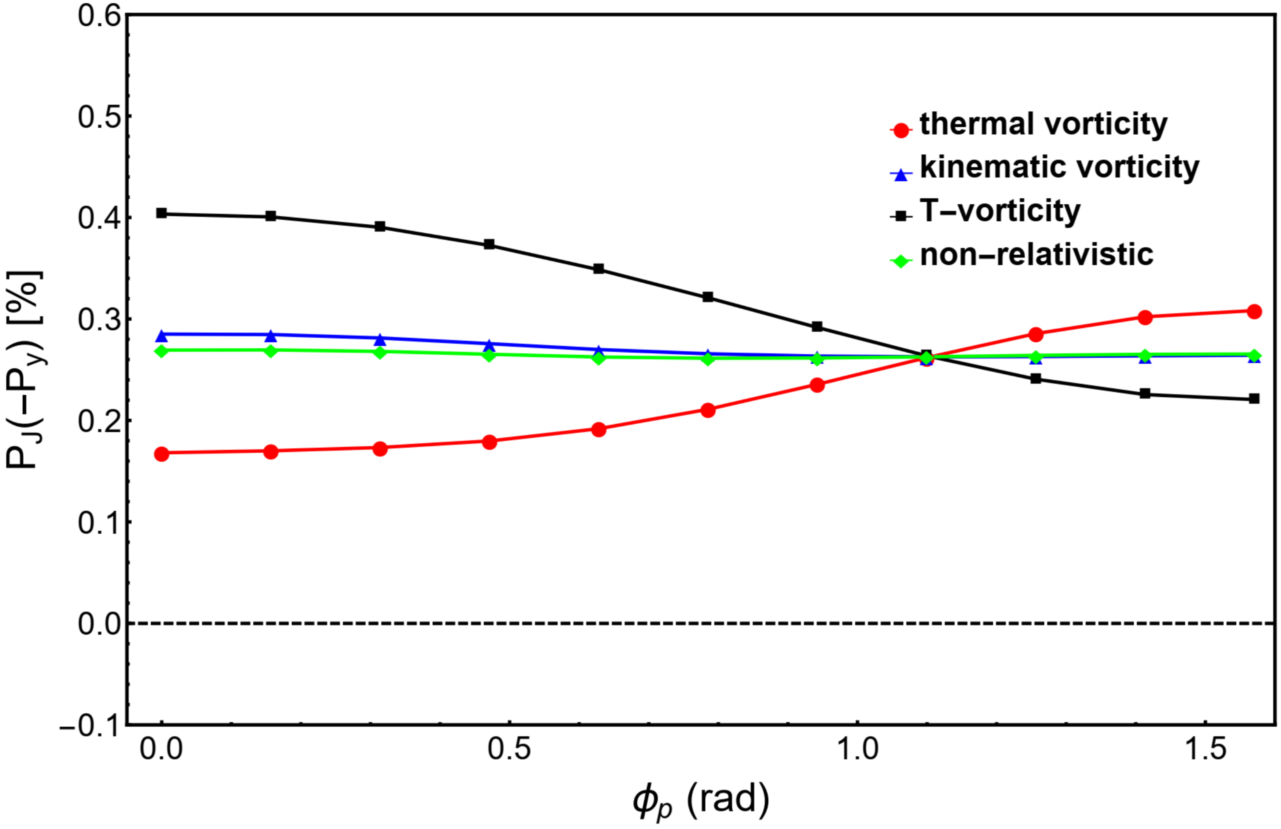
The longitudinal polarization in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

Polarization in direction of OAM



The polarization along –y direction in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

Polarization in direction of OAM



The polarization along –y direction in Au+Au collisions at 200 GeV in the rapidity range Y=[-1; 1] with the AMPT initial condition as functions of (p_x; p_y).

Discussions and Messages

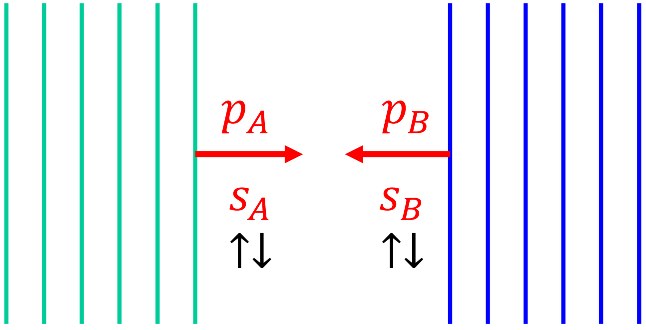
The implication of the T-vorticity by the data may possibly indicate:

1. **The time behavior of the temperature at the freezeout is essential for the T-vorticity to reproduce the correct sign of P_z**
2. **The T-vorticity might be coupled with the spin similar to the way that a magnetic moment is coupled to a magnetic field.**
3. **It is also possible that it is a coincidence from the main assumption that the spin vector is given by the T-vorticity in the same way as the thermal vorticity.**

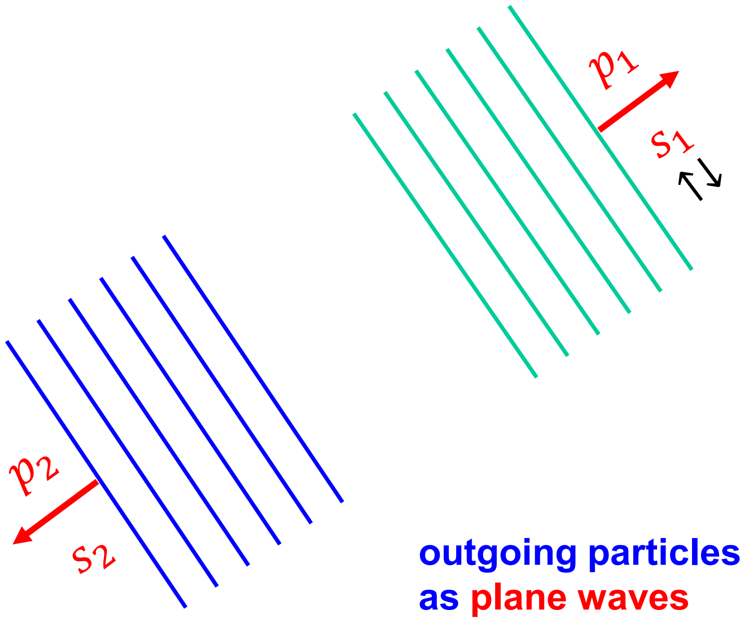
A microscopic model for global polarization through spin-orbit couplings in particle scatterings

[J.-J. Zhang, R.-H. Fang, QW, X.-N. Wang, 1904.09152]

Collisions of particles as plane waves



incident particles
as plane waves



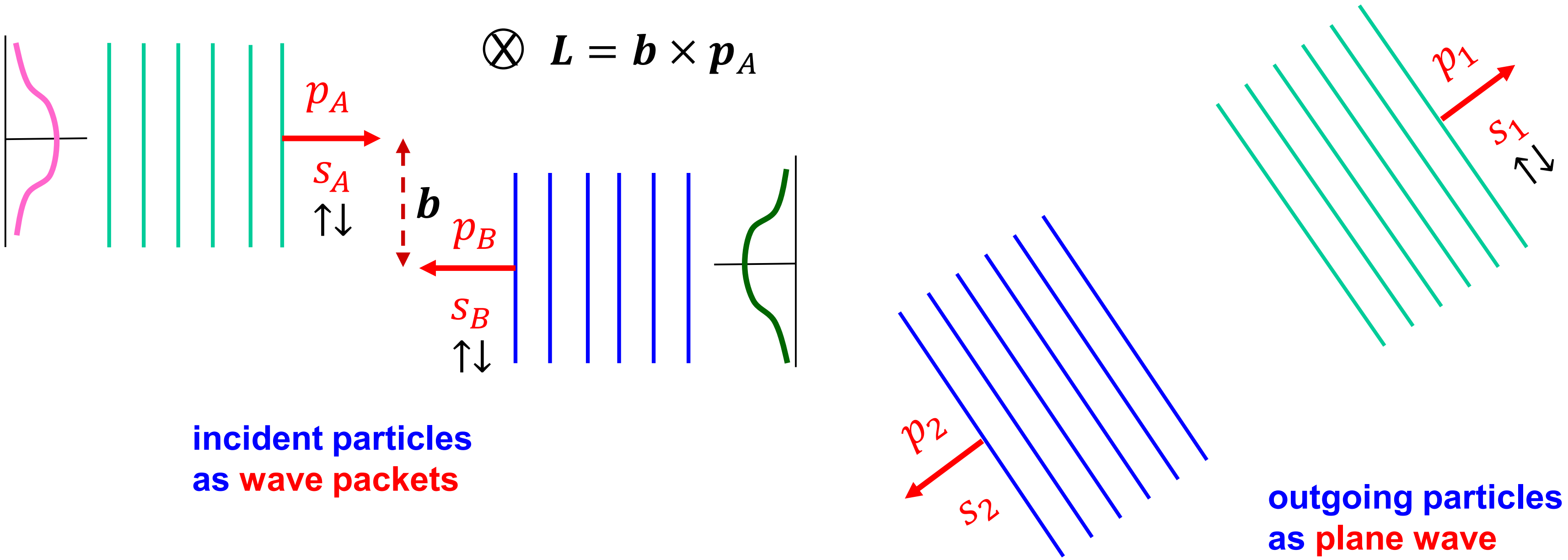
outgoing particles
as plane waves

s_2 is specified

Particle collisions as plane waves:
since there is no preferable position for particles, so there is no OAM and polarization

$$\langle \hat{\boldsymbol{x}} \times \hat{\boldsymbol{p}} \rangle = 0 \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow} = \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

Collisions of particles as wave packets



incident particles as wave packets

outgoing particles as plane wave

s_2 is specified

Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L}=\boldsymbol{b}\times\boldsymbol{p}_A \Longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow}\neq\left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

Incident particles as wave packets

- **Wave packets for incident particles $\boldsymbol{i = A, B}$ located in phase space $(\boldsymbol{x}, \boldsymbol{p})$**

$$\underbrace{|\phi_i(x_i,p_i)\rangle_{\rm in}}_{\text{WP as Wigner function}} = \int \frac{d^3k_i}{(2\pi)^3}\frac{1}{\sqrt{2E_{i,k}}}\underbrace{\phi_i(\mathbf{k}_i-\mathbf{p}_i)}_{\text{WP amplitude}}\underbrace{e^{-i\mathbf{k}_i\cdot\mathbf{x}_i}}_{\text{phase factor}}\underbrace{|\mathbf{k}_i\rangle_{\rm in}}_{\text{plane wave}}$$

- **Gaussian form of the wave packet amplitude in p-space**

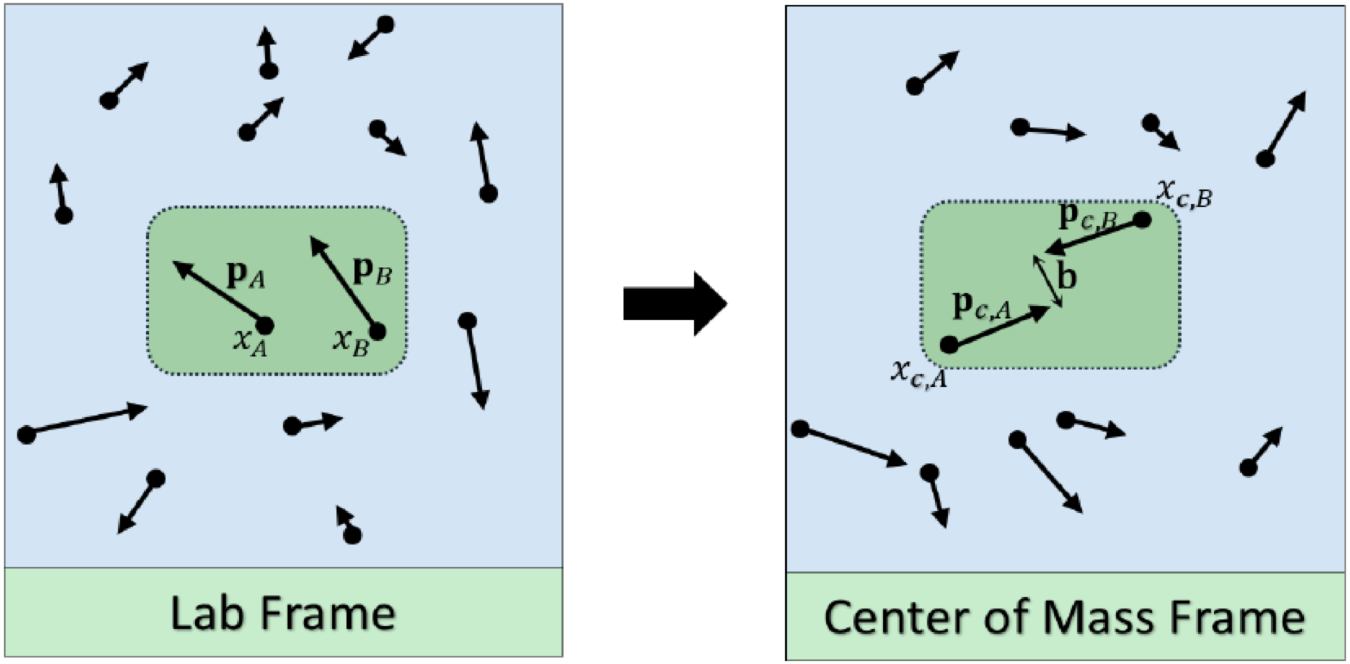
$$\phi_i(\mathbf{k}_i - \underbrace{\mathbf{p}_i}_{\text{central momentum}}) = \frac{(8\pi)^{3/4}}{\alpha_i^{3/2}}\exp\left[-\frac{(\mathbf{k}_i-\mathbf{p}_i)^2}{\underbrace{\alpha_i^2}_{\text{Gaussian width}}}\right]$$

- **Outgoing particles are momentum states in plane waves**

$$|p_1\rangle\,,\,|p_2\rangle$$

Peskin, Schroeder (1995)

Collisions of particles at different space-time points



(1) Momentum distributions depend on $u^\alpha(x)$ in Lab frame

(2) Collisions of momentum states at one space-time point does not contain information about gradient of $u^\alpha(x)$

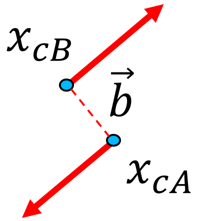
(3) The gradient of $u^\alpha(x)$ can only be probed by collisions of particles at different space-time points

Collisions of particles at different space-time points

- Two incident particles at $x_A = (t_A, \mathbf{x}_A)$ and $x_B = (t_B, \mathbf{x}_B)$
- We have

$$t_A = t_B \xrightarrow{\boxed{\mathbf{x}_A \neq \mathbf{x}_B}} t_{c,A} \neq t_{c,B}$$

$$t_A \neq t_B \xrightarrow[\boxed{\mathbf{x}_{c,A} \neq \mathbf{x}_{c,B}}]{} t_{c,A} = t_{c,B}$$



$\left|\vec{b}\right| < b_0$

CM frame

- We impose the causality condition in CM frame for scattering of particles at two different space-time points (the time interval and longitudinal distance of two space-time points should be small enough for scattering to take place)

$$\Delta t_c = t_{c,A} - t_{c,B} = 0$$

$$\Delta x_{c,L} = \hat{\mathbf{p}}_{c,A} \cdot (\mathbf{x}_{c,A} - \mathbf{x}_{c,B}) = 0$$

$$\left|\vec{b}\right| < b_0$$

Collisions of particles at different space-time points

- **Collision rate of two particles at two space-time points in CMS**

$$
\begin{aligned}
R_{AB\to 12} &= \int \frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2} \\
&\quad \times \frac{1}{C_{AB}} \int d^4x_A d^4x_B \underline{\delta(\Delta t)\delta(\Delta x_L)} \\
&\quad \times \underline{f_A(x_A,p_A) f_B(x_B,p_B)} G_1 G_2 |v_A - v_B| \\
&\quad \times (2E_A)(2E_B) \underline{|_{\mathrm{out}}\langle p_1 p_2 | \phi_A(x_A,p_A)\phi_B(x_B,p_B)\rangle_{\mathrm{in}}|^2}
\end{aligned}
$$

$$\boxed{C_{AB} \equiv \int d^4X = t_X \Omega_{\mathrm{int}}}$$

equal time and L-position

distributions for
incident particles
At two points

scattering amplitude

- **We carry out integral over x_A and x_B**

$$
\begin{aligned}
I &= \int \underline{d^4x_A d^4x_B}\, \delta(\Delta t)\delta(\Delta x_L) f_A(x_A,p_A) f_B(x_B,p_B) \\
&\quad \times \exp\left(-i\mathbf{k}_A\cdot\mathbf{x}_A - i\mathbf{k}_B\cdot\mathbf{x}_B + i\mathbf{k}'_A\cdot\mathbf{x}_A + i\mathbf{k}'_B\cdot\mathbf{x}_B\right) \\
&\approx \int d^4X d^2\mathbf{b} f_A\left(X + \frac{y_T}{2}, p_A\right) f_B\left(X - \frac{y_T}{2}, p_B\right) \\
&\quad \times \exp\left[\underline{i(\mathbf{k}'_A - \mathbf{k}_A)\cdot\mathbf{b}}\right]
\end{aligned}
$$

$$
\begin{aligned}
X &= \frac{1}{2}(x_A + x_B) \\
y &= x_A - x_B.
\end{aligned}
$$

$$\boxed{\vec{b} = \vec{x}_A - \vec{x}_B}$$

phase from impact parameter

all variables are defined in CMS but we suppress index ‘c’ for simplicity

Polarization of spin-1/2 particles from scatterings (general formula)

- **Polarization rate from particle scatterings $A+B\rightarrow 1+2$ at different space-time points**

all variables are defined in CMS index ‘c’

$$\begin{array}{ccc} (s_A,p_A)+(s_B,p_B) & \rightarrow & (s_1,p_1)+(s_2,p_2) \\ \textit{wave packets} & & \textit{plane waves} \\ \textit{sum over } (s_A,s_B,s_1) & & s_2 \textit{ is open} \end{array}$$

$$\frac{d^4\mathrm{P}_{AB\rightarrow 12}(X)}{dX^4} = \frac{1}{(2\pi)^4}\int\frac{d^3p_{c,A}}{(2\pi)^3 2E_{c,A}}\frac{d^3p_{c,B}}{(2\pi)^3 2E_{c,B}}\frac{d^3p_{c,1}}{(2\pi)^3 2E_{c,1}}\frac{d^3p_{c,2}}{(2\pi)^3 2E_{c,2}}$$

$$\times|v_{c,A}-v_{c,B}|G_1 G_2\int \underline{d^3k_{c,A}d^3k_{c,B}d^3k'_{c,A}d^3k'_{c,B}}$$

wave packet momenta

wave packet

$$\underline{\times\phi_A(\mathbf{k}_{c,A}-\mathbf{p}_{c,A})\phi_B(\mathbf{k}_{c,B}-\mathbf{p}_{c,B})\phi^*_A(\mathbf{k}'_{c,A}-\mathbf{p}_{c,A})\phi^*_B(\mathbf{k}'_{c,B}-\mathbf{p}_{c,B})}$$

$$\times\delta^{(4)}(k'_{c,A}+k'_{c,B}-p_{c,1}-p_{c,2})\delta^{(4)}(k_{c,A}+k_{c,B}-p_{c,1}-p_{c,2})$$

distributions for incident particles displaced by b<b$_0$

$$\underline{\times\int d^2\mathbf{b}_c f_A\left(X_c+\frac{y_{c,T}}{2},p_A\right)f_B\left(X_c-\frac{y_{c,T}}{2},p_B\right)}\ \underline{\exp\left[i(\mathbf{k}'_{c,A}-\mathbf{k}_{c,A})\cdot\mathbf{b}_c\right]}$$

phase factor

$$\times\sum_{s_A,s_B,s_1,s_2}\boxed{2s_2\mathbf{n}_c}\underline{\mathcal{M}\left(\{s_A,k_{c,A};s_B,k_{c,B}\}\rightarrow\{s_1,p_{c,1};s_2,p_{c,2}\}\right)}$$

scattering amplitude

polarization direction

$\vec{n}_c=\vec{b}\times\vec{p}_{cA}$

$$\times\underline{\mathcal{M}^*\left(\{s_A,k'_{c,A};s_B,k'_{c,B}\}\rightarrow\{s_1,p_{c,1};s_2,p_{c,2}\}\right)}$$

scattering amplitude

Application: quark polarization in 22 parton scatterings in QGP (locally thermalized in p)

- **Asumptions:**

(1) local equilibrium in momentum but not in spin

(2) $f(x,p)$ depends on x^μ through $f(x,p) = f[\beta(x)p\cdot u(x)]$

(3) All 22 scatterings with at least one quark in the final state

- **Expansion of $f_A(x_{cA},p_{cA})f_B(x_{cB},p_{cB})$ in small $y_{c,T} = (0,\vec{b})$**

$$
\begin{aligned}
& f_A\left(X_c+\frac{y_{c,T}}{2},p_{c,A}\right)f_B\left(X_c-\frac{y_{c,T}}{2},p_{c,B}\right)\\
= \;& f_A\left(X_c,p_{c,A}\right)f_B\left(X_c,p_{c,B}\right)+\boxed{\frac{1}{2}y^\mu_{c,T}\frac{\partial(\beta u_{c,\rho})}{\partial X^\nu_c}}\\
& \times\left[\boxed{p^\rho_{c,A}}f_B\left(X_c,p_{c,B}\right)\frac{df_A\left(X_c,p_{c,A}\right)}{d(\beta u_c\cdot p_{c,A})}-\boxed{p^\rho_{c,B}}f_A\left(X_c,p_{c,A}\right)\frac{df_B\left(X_c,p_{c,B}\right)}{d(\beta u_c\cdot p_{c,B})}\right]
\end{aligned}
$$

$$
\begin{aligned}
& y^\mu_{c,T}p^\rho_{c,A}\frac{\partial(\beta u_\rho)}{\partial X^\mu_c}\\
= \;& -\frac{1}{2}\boxed{y^{[\mu}_{c,T}p^{\rho]}_{c,A}}\omega^{(c)}_{\mu\rho}\\
& +\frac{1}{4}y^{\{\mu}_{c,T}p^{\rho\}}_{c,A}\underline{\left[\frac{\partial(\beta u_{c,\rho})}{\partial X^\mu_c}+\frac{\partial(\beta u_{c,\mu})}{\partial X^\rho_c}\right]}
\end{aligned}
$$

local OAM
L-ω coupling

generally
non-zero

Quark polarization rate

- **Quark polarization rate per unit volume: 10D + 6D integration**

$$\frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} = \frac{\pi}{(2\pi)^4}\frac{\partial(\beta u_\rho)}{\partial X^\nu}\int\boxed{\frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B}}$$

6D integral

$$\times\left|v_{c,A}-v_{c,B}\right|\boxed{\left[\Lambda^{-1}\right]^{\nu}_{\ j}}\mathbf{e}_{c,i}\epsilon_{ikh}\hat{\mathbf{p}}^{h}_{c,A}$$

Lorentz boost from cms to Lab frame

$$\times f_A\left(X,p_A\right)f_B\left(X,p_B\right)\left(p^{\rho}_A-p^{\rho}_B\right)\boxed{\Theta_{jk}(\mathbf{p}_{c,A})}$$

10D integral

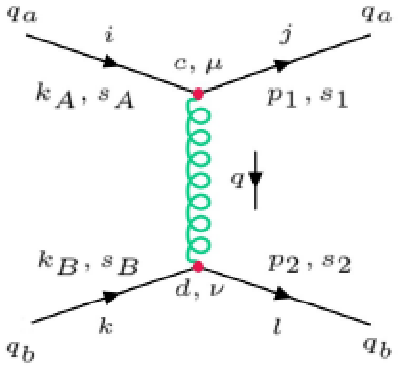
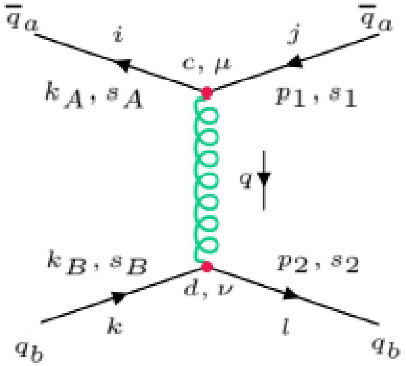
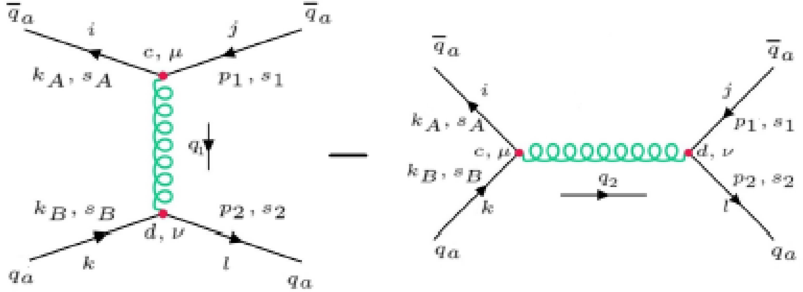
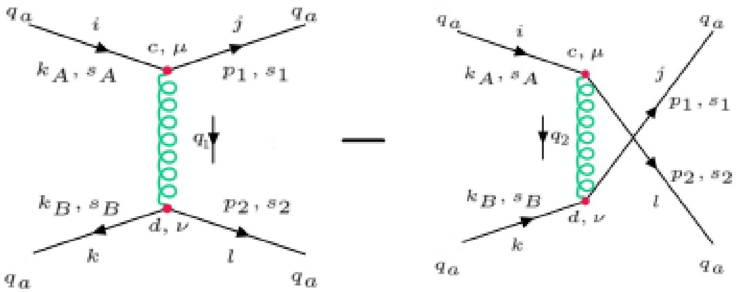
$$\equiv\frac{\partial(\beta u_\rho)}{\partial X^\nu}\boxed{\mathbf{W}^{\rho\nu}}$$

16D integral !!

- **Numerical challenge !!!** **We use newly developed ZMCintegral-3.0, a Monte Carlo integration package that runs on multi-GPUs** **[Hong-zhong Wu, Junjie Zhang, Long-gang Pang, QW, Comp.Phys.Comm. (2019), 1902.07916.]**
- **Another challenge:** **there are more than 5000 terms in polarized amplitude squared**

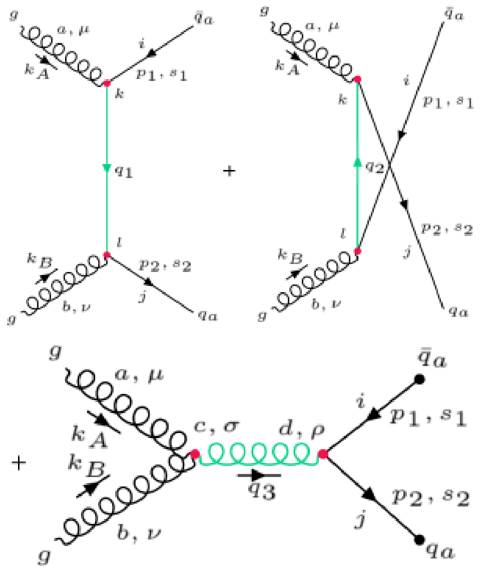
$$I_M^{q_aq_b\to q_aq_b}(s_2) = \sum_{s_A,s_B,s_1}\sum_{i,j,k,l}\mathcal{M}\left(\{s_A,k_A;s_B,k_B\}\to\{s_1,p_1;s_2,p_2\}\right)\mathcal{M}^*\left(\{s_A,k'_A;s_B,k'_B\}\to\{s_1,p_1;s_2,p_2\}\right)$$

All 22 parton scaterings for quark polarization

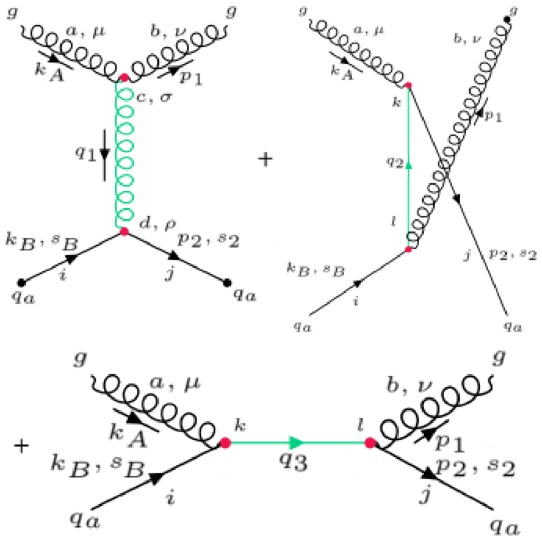
$q_aq_b \to q_aq_b$  	$\bar{q}_aq_b \to \bar{q}_aq_b$ 
$\bar{q}_aq_a \to \bar{q}_aq_a$  	$q_aq_a \to q_aq_a$ 

All 22 parton scaterings for quark polarization

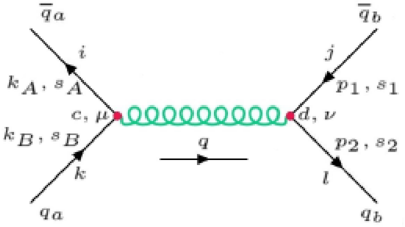
$gg \to \bar{q}_a q_a$



$g q_a \to g q_a$



$\bar{q}_a q_a \to \bar{q}_b q_b$



Numerical results for quark polarization

- **Numerical results show $W^{\rho\nu}$ has anti-symmetric structure**

$$\mathbf{W}^{\rho\nu} = W\epsilon^{0\rho\nu j}\mathbf{e}_j \quad \Longrightarrow \quad \mathbf{W}^{\rho\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & W\mathbf{e}_z & -W\mathbf{e}_y \\ 0 & -W\mathbf{e}_z & 0 & W\mathbf{e}_x \\ 0 & W\mathbf{e}_y & -W\mathbf{e}_x & 0 \end{pmatrix}$$

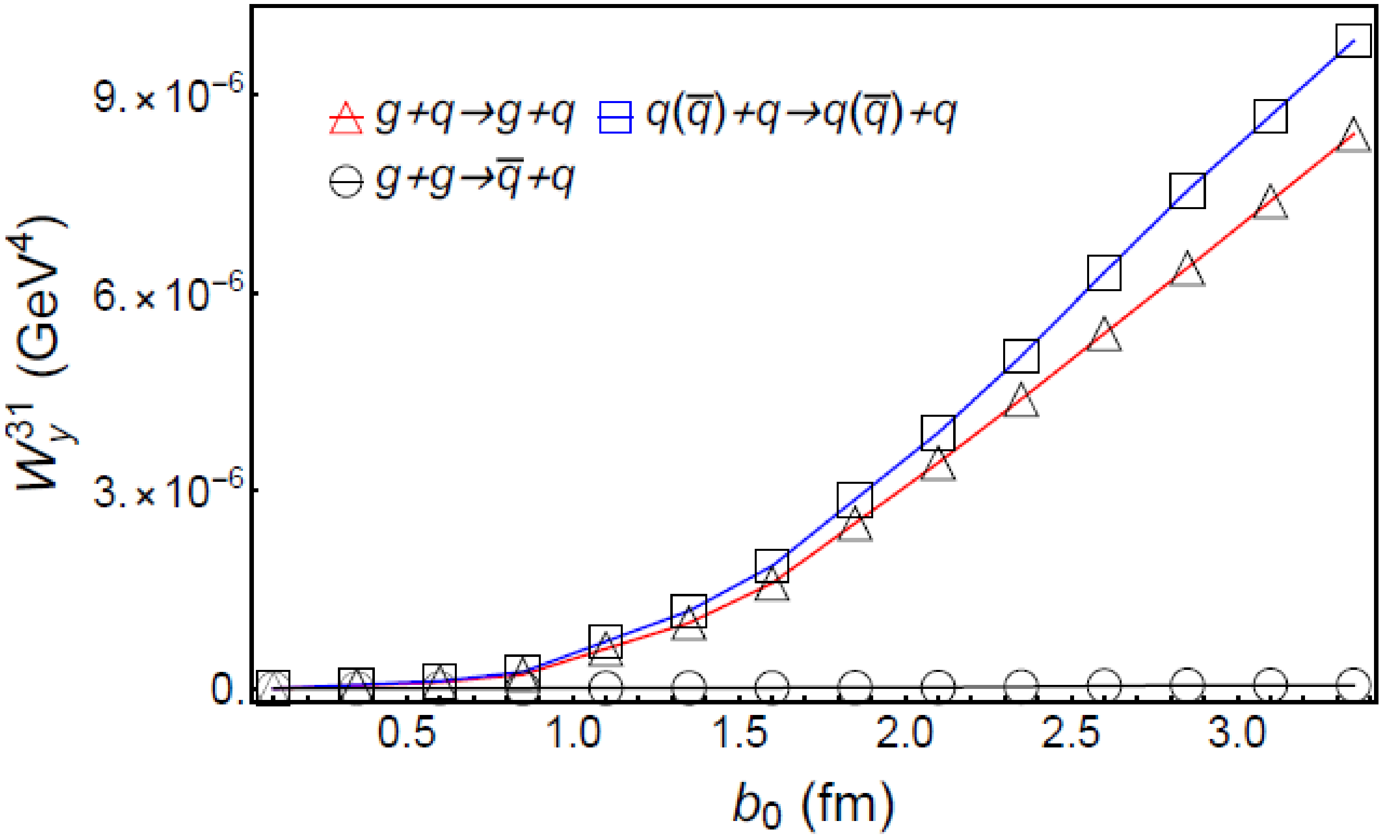
$$\begin{aligned} \frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} &= \epsilon^{0j\rho\nu}\frac{\partial(\beta u_\rho)}{\partial X^\nu}W\mathbf{e}_j = 2\epsilon_{jkl}\underline{\omega_{kl}}W\mathbf{e}_j \\ &= \boxed{2W\nabla\times(\beta\mathbf{u})} \end{aligned}$$

$$\omega_{\rho\nu} = -(1/2)[\partial_\rho^X(\beta u_\nu) - \partial_\nu^X(\beta u_\rho)]$$

$$\omega_{kl} = (1/2)[\nabla_k(\beta\mathbf{u}_l) - \nabla_l(\beta\mathbf{u}_k)]$$

Polarization is given by the vorticity
up to a coefficient W
W can be calculated numerically

Numerical results for quark polarization



The cutoff b_0 is of the order of hydro length scale $1/\partial u(x)$ and larger than interaction scale $1/m_D$: $b_0 \sim \frac{1}{\partial u(x)} > \frac{1}{m_D}$

$$\frac{d^4 \mathrm{P}_{AB\to 12}(X)}{dX^4} = 2W\nabla_X \times (\beta \mathbf{u})$$

Summary

- A microscopic model for the polarization through the spin-orbit coupling in particle collisions is constructed.
- It is based on scatterings of particles as wave packets, an effective method to deal with particle scatterings at specified impact parameters.
- The spin-vorticity coupling naturally emerges from the LS coupling encoded in polarized scattering amplitudes of collisional integrals.
- The polarization is then the consequence of particle collisions in a non-equilibrium state of spins.
- Applications: high energy HIC (parton collisions), low energy HIC (NN collisions)