Primordial non-Gaussianity in heavy-ion collisions

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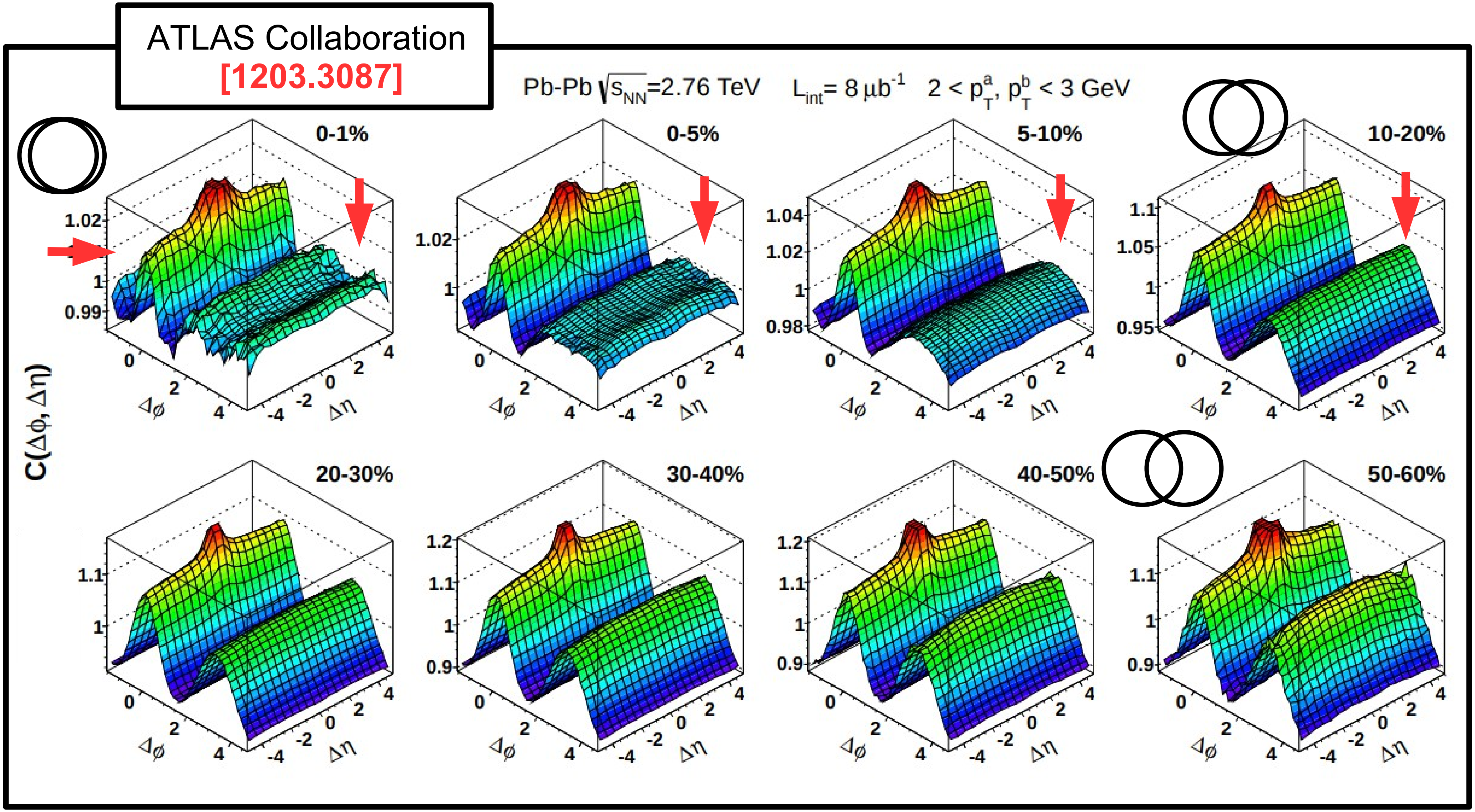
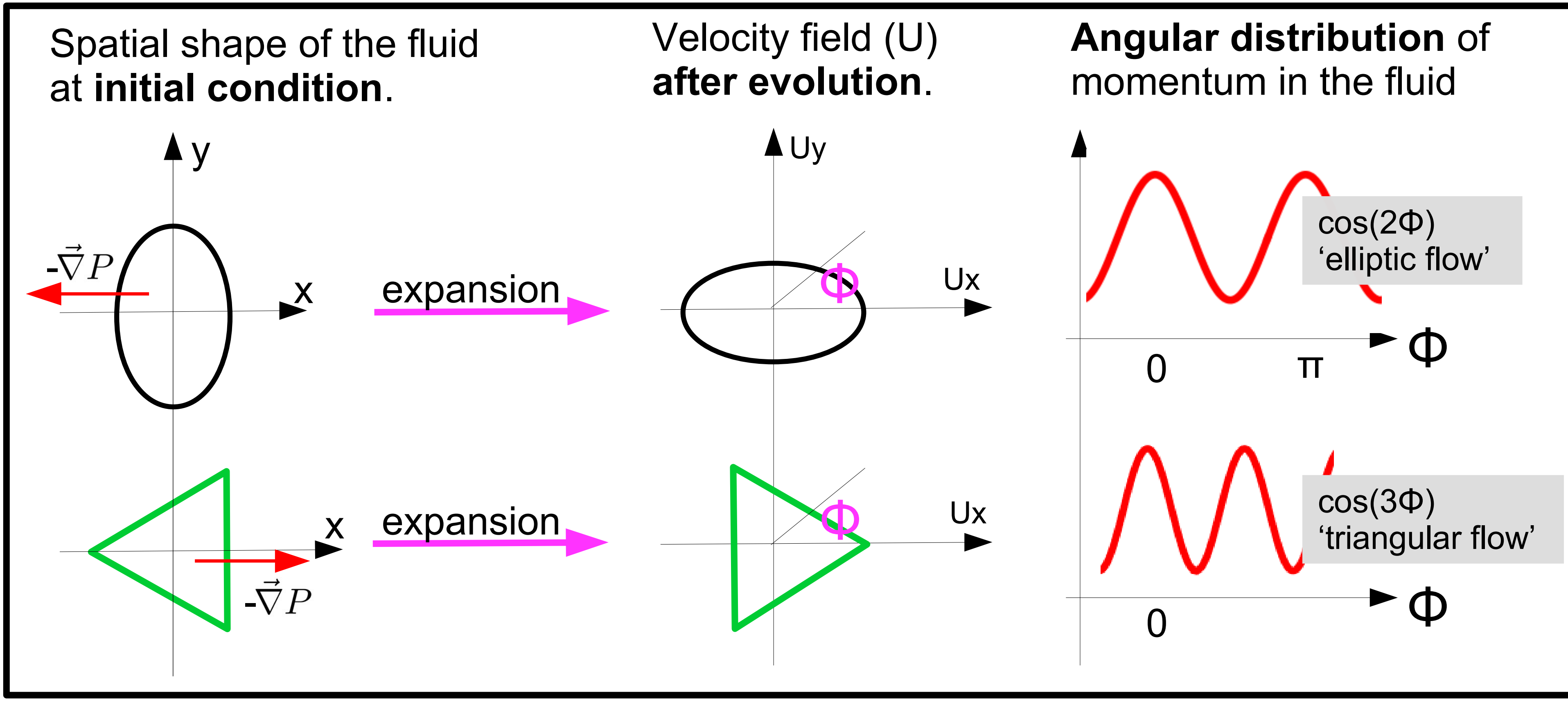
(slides borrowed from Giuliano Giacalone)

Heavy-ion collisions. What do we want to explain?



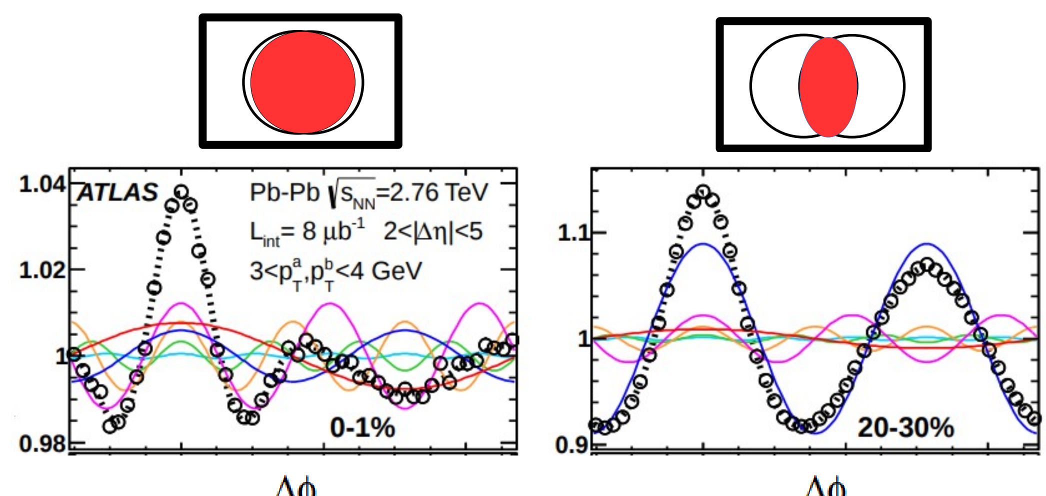
Long-range azimuthal correlations: A new feature of high-energy physics.

Natural explanation in a hydrodynamic paradigm.
The system created in a collision is a fluid. $F = -\nabla P$

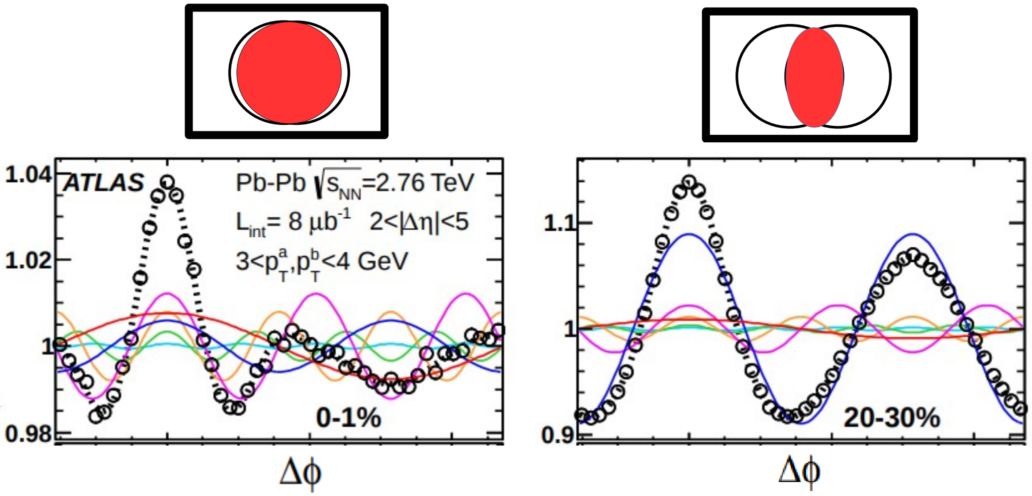


ANISOTROPIC FLOW

It predicts the structures observed in the experimental data



Structures conveniently characterized by Fourier spectrum



$$P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \boxed{V_n} e^{-in\phi}$$

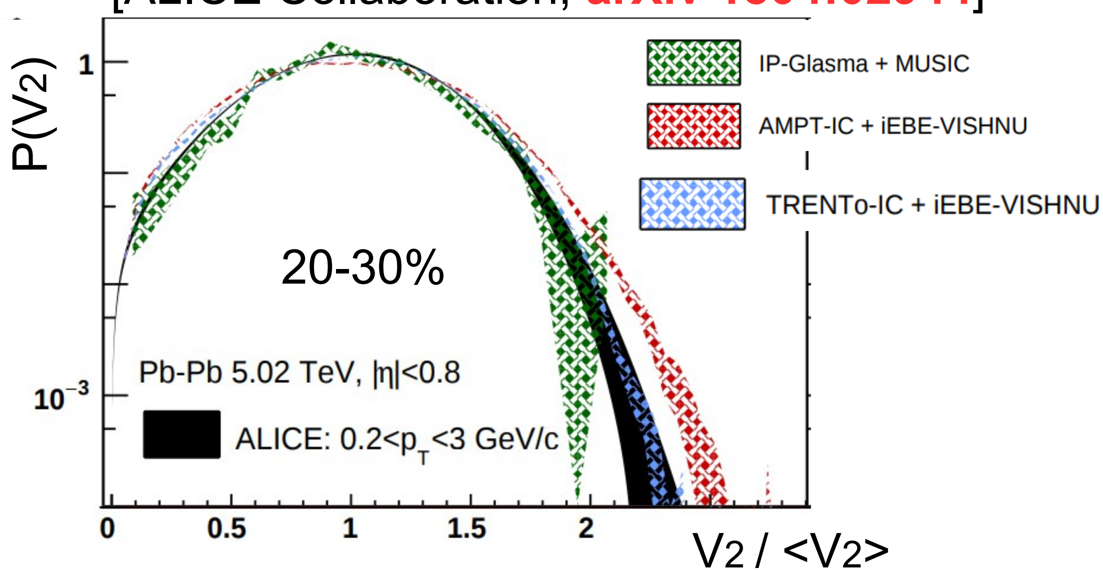
$$V_{-n} = V_n^*$$

Fourier coefficients are complex quantities that fluctuate event-by-event:

$P(v_x, v_y)$ [x=real, y=imaginary]

We only have access to the distribution of the magnitude: $v_n \equiv |V_n|$

[ALICE Collaboration, arXiv 1804:02944]



Experimentally one can measure **moments of the vn distribution** by means of **multi-particle** azimuthal correlations.

$$\langle v_n^2 \rangle = \langle e^{in(\phi_1-\phi_2)} \rangle$$
$$\langle v_n^4 \rangle = \langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle$$
$$\langle v_n^6 \rangle = \langle e^{in(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \rangle$$
$$\langle v_m^2 v_n^2 \rangle = \langle e^{im(\phi_1-\phi_2)} e^{in(\phi_3-\phi_4)} \rangle$$

Average over events in a class of multiplicity

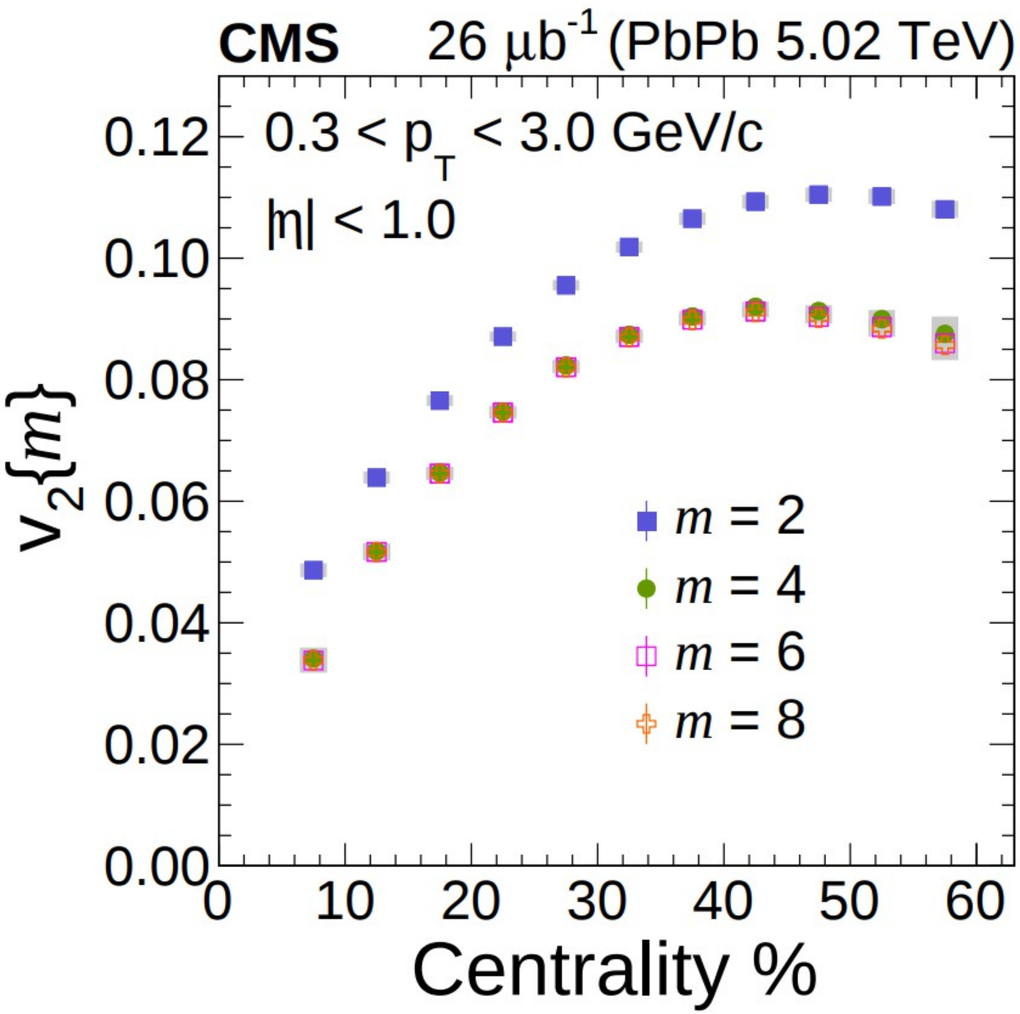
et cetera....

One usually constructs **cumulants**:

[Borghini, Dinh, Ollitrault, nucl-th/0105040]

$$v_n\{2\}^2 = \langle v_n^2 \rangle$$
$$v_n\{4\}^4 = 2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle$$
$$v_n\{6\}^6 = \frac{1}{4}\left[\langle v_n^6 \rangle - 9\langle v_n^2 \rangle\langle v_n^4 \rangle + 12\langle v_n^2 \rangle^3\right]$$
$$v_n\{8\}^8 = \frac{1}{33}\left[144\langle v_n^2 \rangle^4 - 144\langle v_n^2 \rangle^2\langle v_n^4 \rangle + 18\langle v_n^4 \rangle^2 + 16\langle v_n^2 \rangle\langle v_n^6 \rangle - \langle v_n^8 \rangle\right]$$

How do cumulants look like?



[CMS Collaboration, **arXiv 1711:05594**]

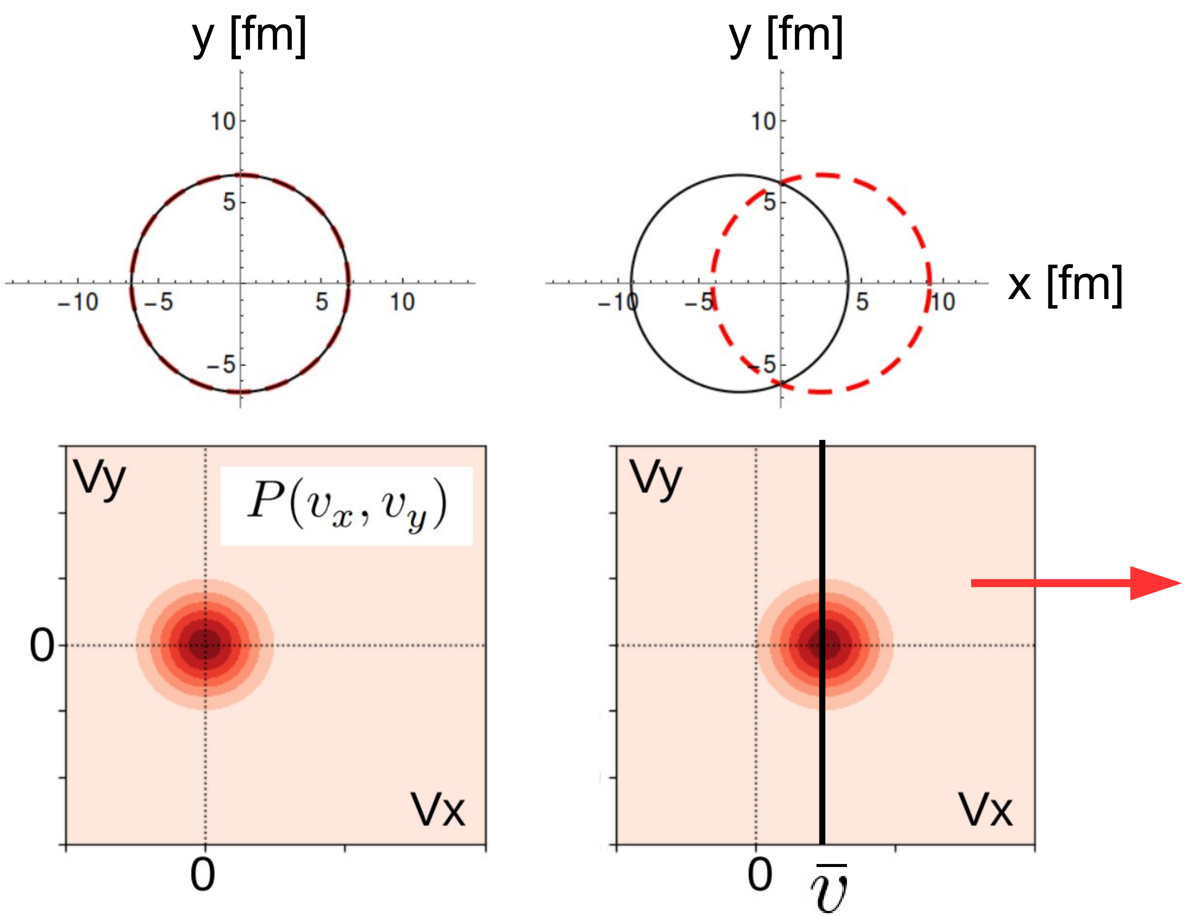
Good approximation: **v2{2} > v2{4} = v2{6} = v2{8}**

Do we understand that???

Immediately explained by a Gaussian Ansatz:

$$P(v_x, v_y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(v_x - \bar{v})^2 + v_y^2}{2\sigma^2}\right)$$

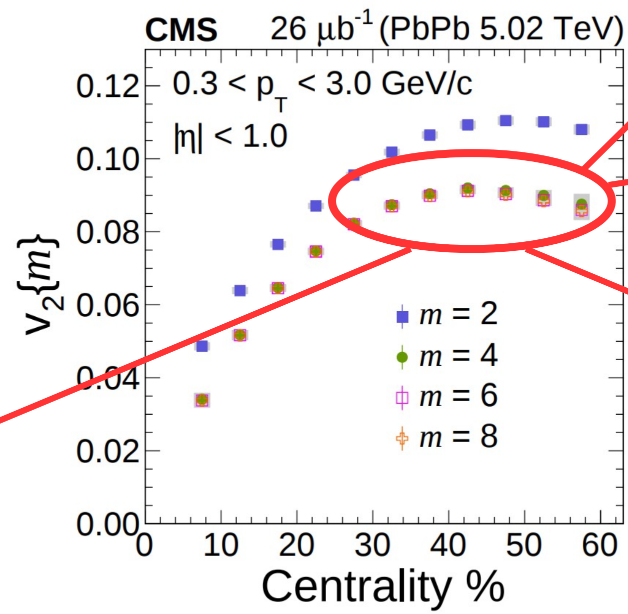
[Voloshin, Poskanzer, Tang, Wang, **arXiv 0708:0800**]



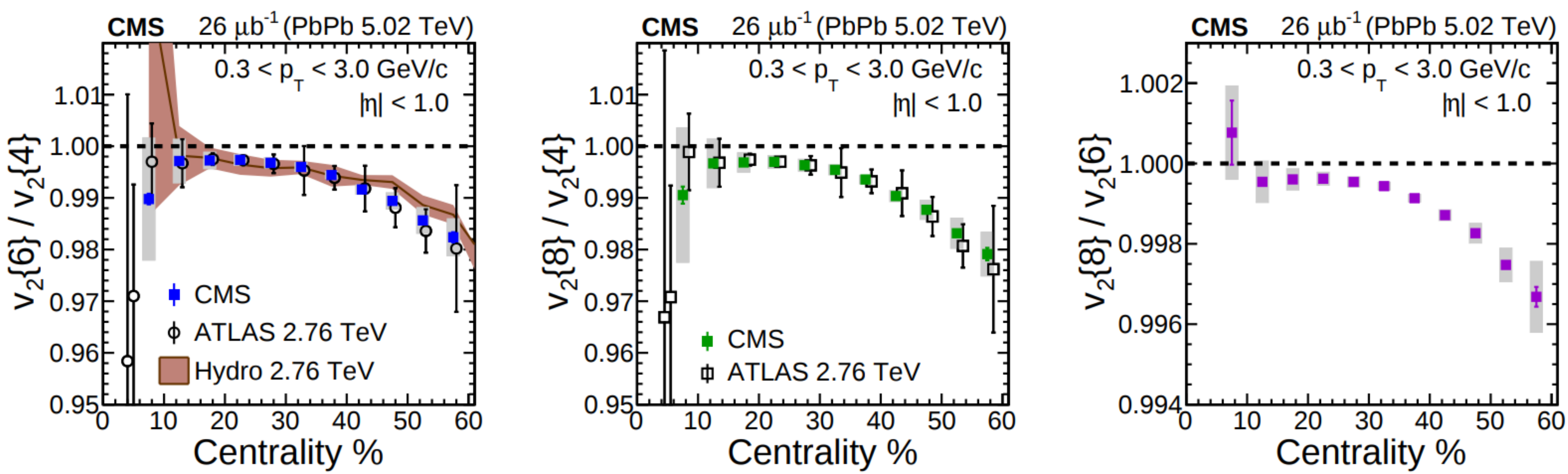
$$v_2\{2\} = \sqrt{\bar{v}^2 + 2\sigma^2}$$
$$v_2\{4\} = \bar{v}$$
$$v_2\{6\} = \bar{v}$$
$$v_2\{8\} = \bar{v}$$
$$v_2\{\ldots\} = \bar{v}$$

Predicts degeneracy of cumulants!

The end of the story ?

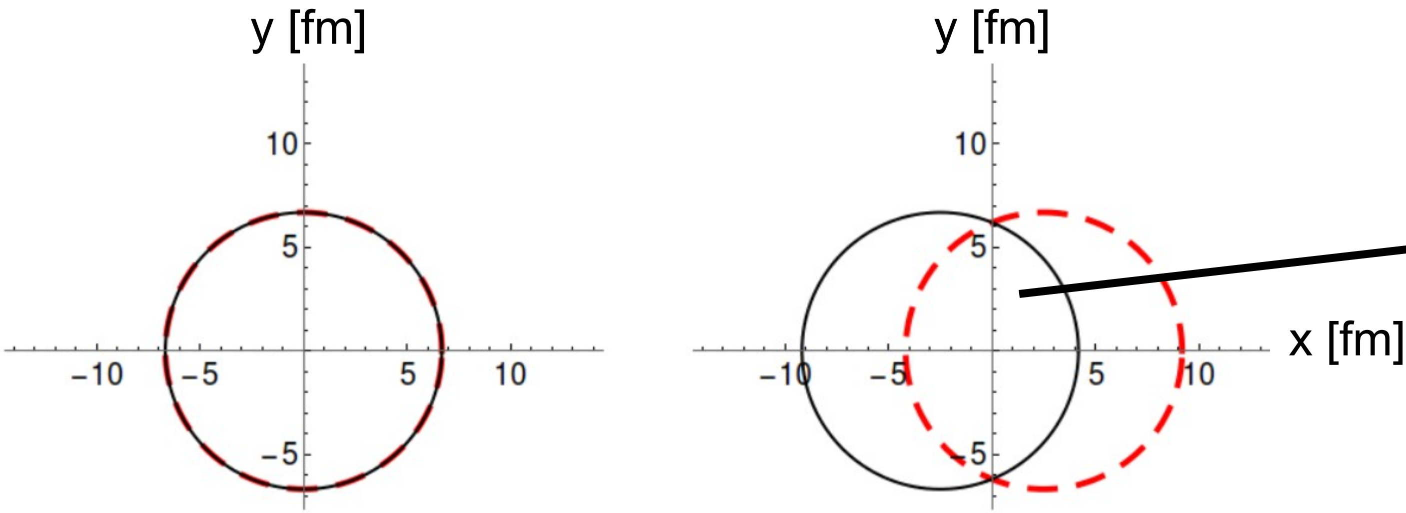


[CMS Collaboration, **arXiv 1711:05594**]



They are not degenerate. Elliptic flow fluctuations are non-Gaussian!

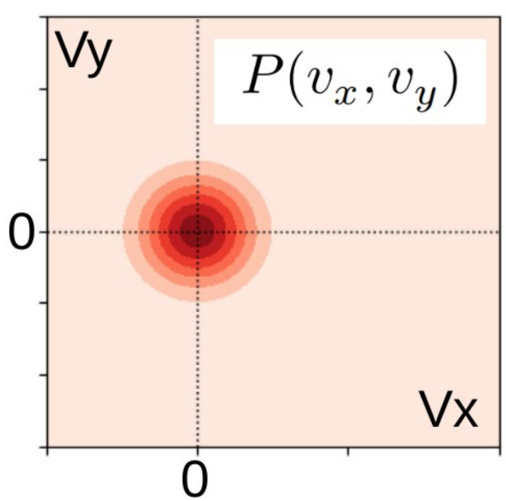
What about triangular flow fluctuations (**V₃**) ?



NO intrinsic triangular anisotropy

Gaussian Ansatz:

$$P(v_x, v_y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{v_x^2 + v_y^2}{2\sigma^2}\right)$$



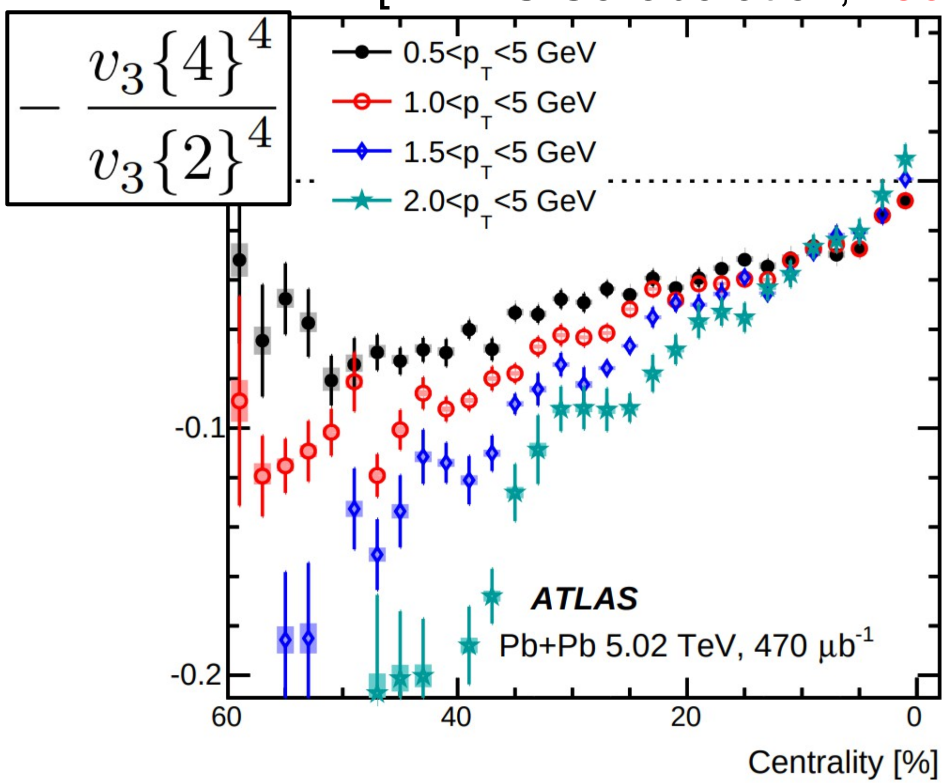
$$v_3\{2\} = \sqrt{2}\sigma$$
$$v_3\{4\} = 0$$
$$v_3\{6\} = 0$$
$$v_3\{\ldots\} = 0$$

The (standardized) kurtosis of triangular flow fluctuations: $-\dfrac{v_3\{4\}^4}{v_3\{2\}^4}$

[Abbasi, Allahbakhshi, Davody, Taghavi, **1704.06295**]

Recent measurement :

[ATLAS Collaboration, **1904.04808**]



$V_3\{4\} \neq 0$. **Fluctuations of triangular flow are non-Gaussian.**

In summary, the fluctuations of the Fourier harmonics of the azimuthal particle distributions are non-Gaussian.

In a hydrodynamic framework,
what is the origin of non-Gaussianity?

Anisotropy is of geometric origin.
Need to identify the spatial anisotropy of the initial state. **How?**

[Teaney, Yan **1010.1876**]

$$\varepsilon_n \equiv \frac{\int_{\mathbf{s}} \mathbf{s}^n \rho(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \rho(\mathbf{s})} \xrightarrow{\text{hydro}} P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} V_n e^{-in\phi}$$

ε_n: eccentricity

$\mathbf{s} = x + iy$

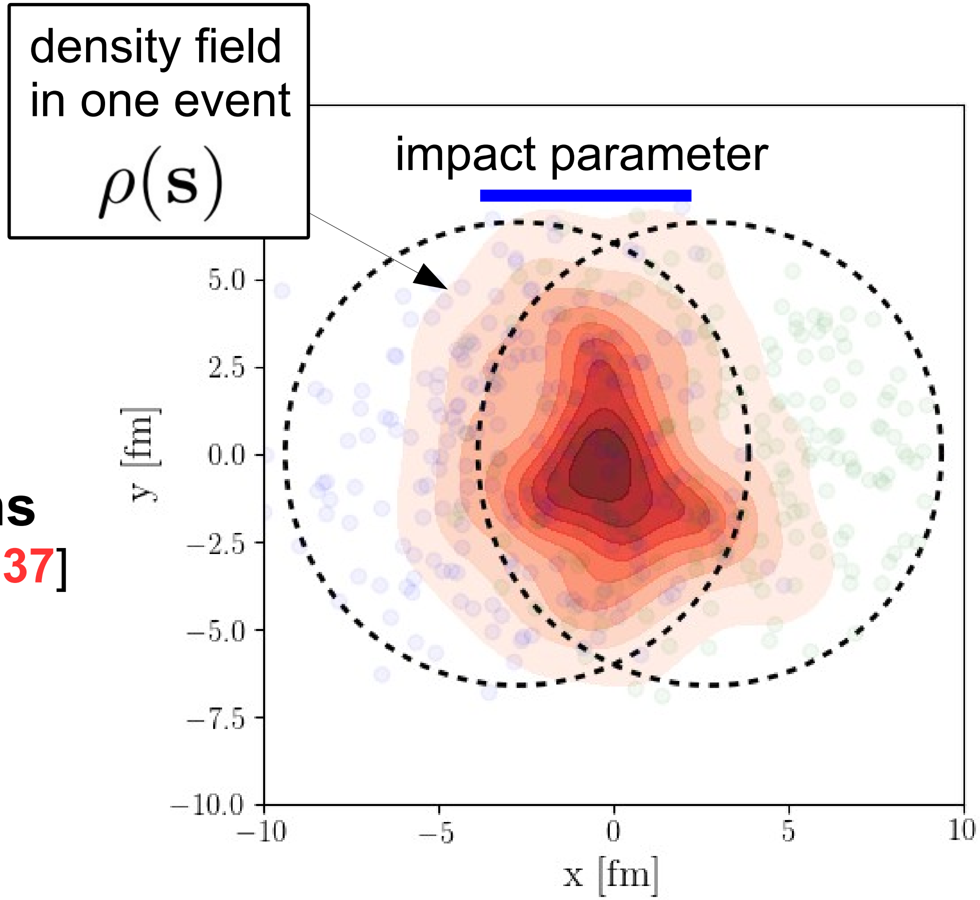
V_n: flow coefficient

The largest anisotropies are
n=2 (elliptical) and n=3 (triangular).

Their origin:

Elliptic flow → **geometry + fluctuations**
[PHOBOS Collaboration **nucl-ex/0610037**]

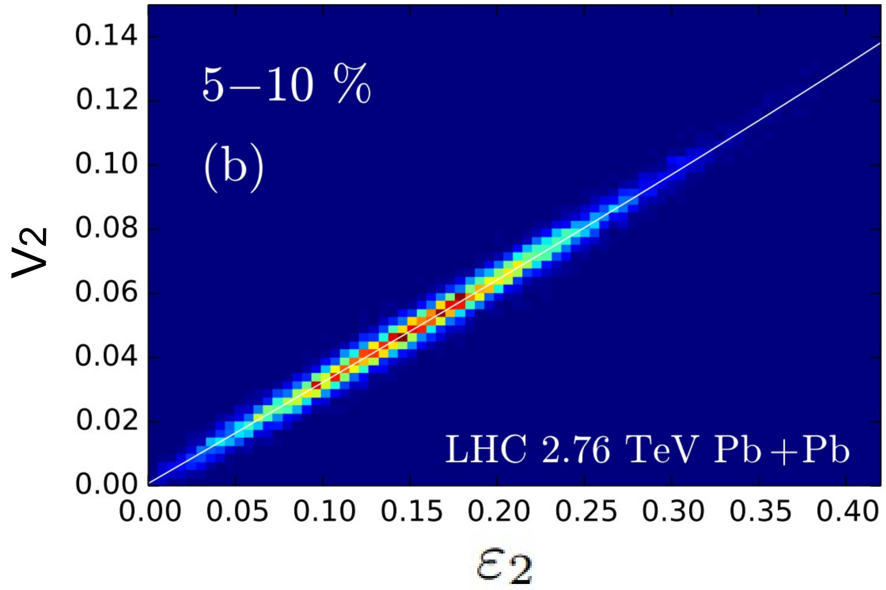
Triangular flow → **fluctuations only**
[Alver, Roland **1003.0194**]



Hydrodynamic simulations indicate that the relation is linear:

[Niemi, Eskola, Patelaainen, **arXiv 1505:02677**]

$$v_n = k_n \varepsilon_n$$



If true in every event,
then it applies as well to averaged
quantities, relevant for experiments.

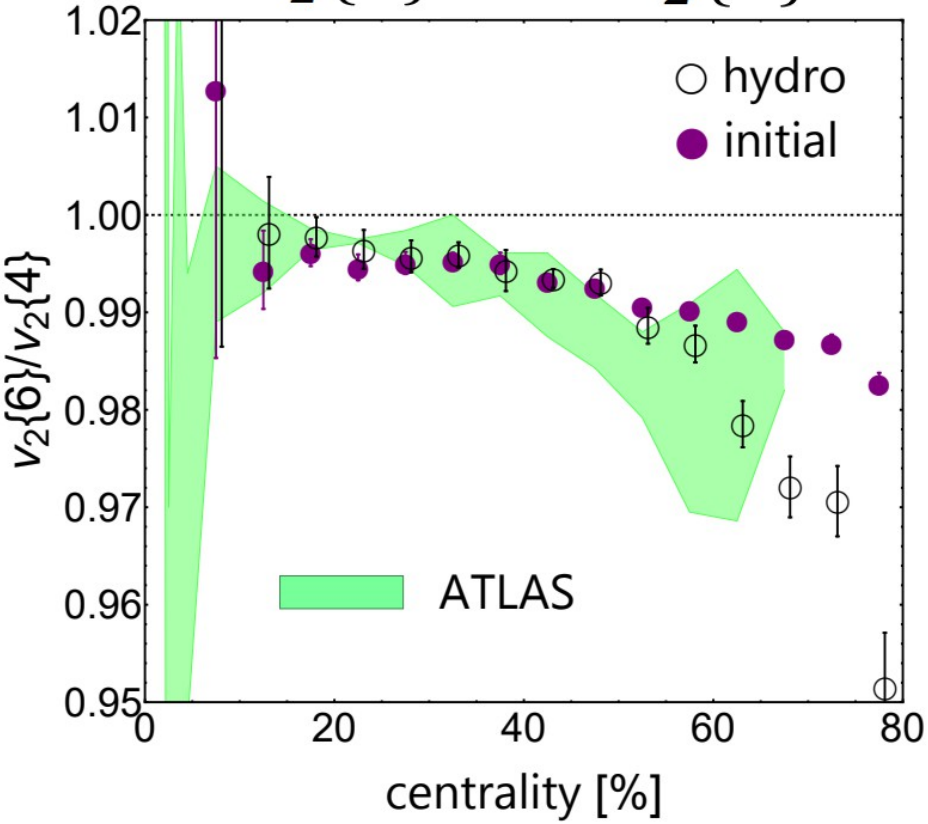
$$v_n\{2\} = \kappa_n \varepsilon_n\{2\} = \kappa_n \sqrt{\langle \varepsilon_2^2 \rangle}$$

$$v_n\{4\} = \kappa_n \varepsilon_n\{4\} = \kappa_n \sqrt[4]{2\langle \varepsilon_n^2 \rangle^2 - \langle \varepsilon_n^4 \rangle}$$

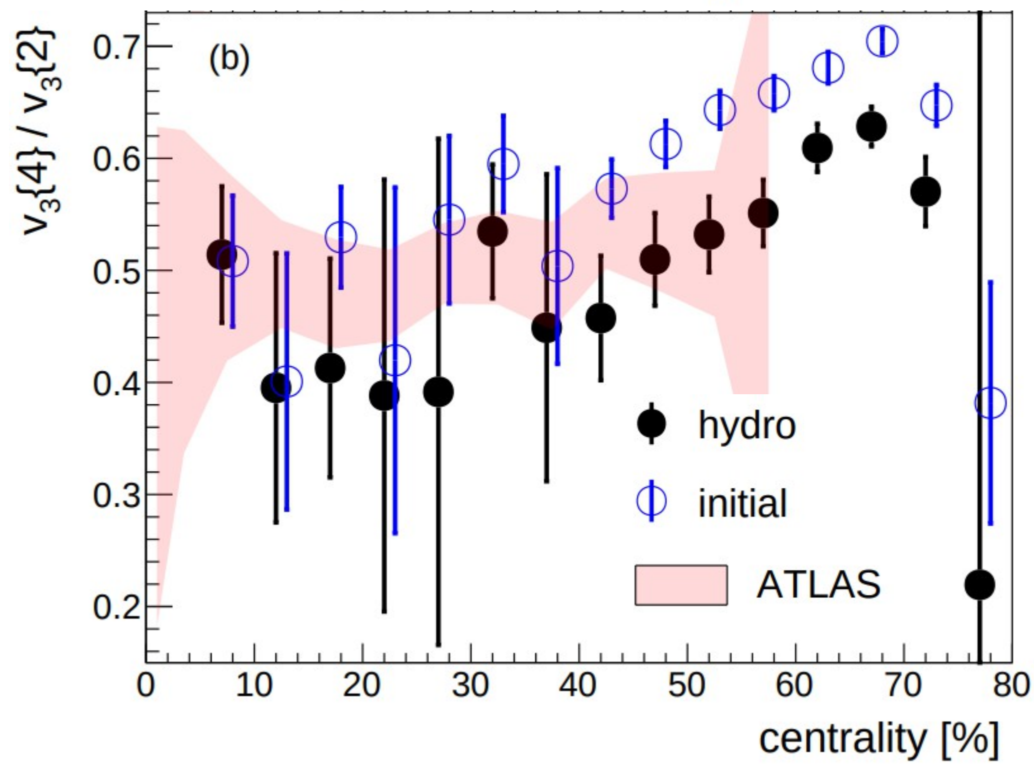
$$v_n\{6\} = \kappa_n \varepsilon_n\{6\} = \kappa_n \sqrt[6]{\frac{1}{4}\left(\langle \varepsilon_n^6 \rangle - 9\langle \varepsilon_n^2 \rangle \langle \varepsilon_n^4 \rangle + 12\langle \varepsilon_n^2 \rangle^3\right)}$$

[Giacalone, Noronha-Hostler, Yan, Ollitrault **arXiv 1608:01823**]
[Giacalone, Noronha-Hostler, Ollitrault **arXiv 1702:01730**]

initial $\frac{\varepsilon_2\{6\}}{\varepsilon_2\{4\}}$ → hydro $\frac{v_2\{6\}}{v_2\{4\}}$



initial $\frac{\varepsilon_3\{4\}}{\varepsilon_3\{2\}}$ → hydro $\frac{v_3\{4\}}{v_3\{2\}}$



In hydrodynamics non-Gaussian flow fluctuations
are simply due to non-Gaussian eccentricity fluctuations.
What do we learn from that?

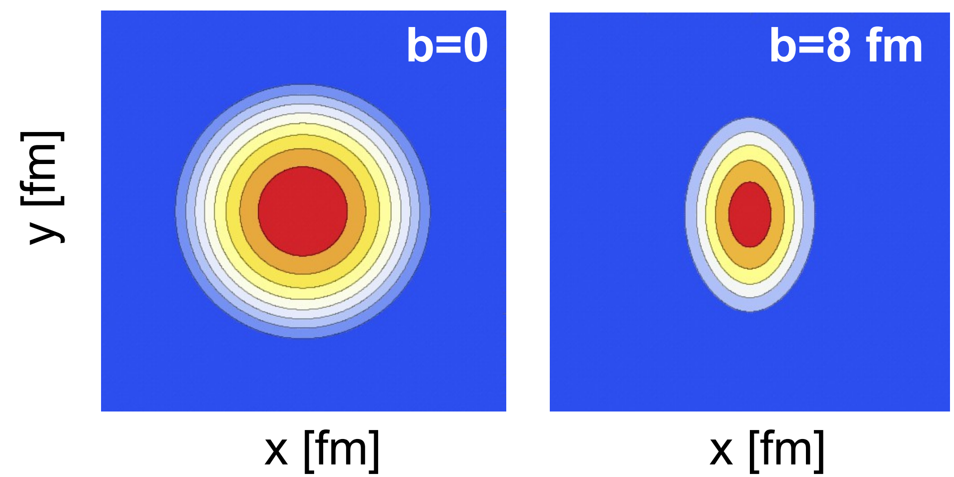
PERTURBATIVE APPROACH TO ANISOTROPY

Energy density created in heavy-ion collisions as average + fluctuation:

$$\rho(\mathbf{s}) = \langle \rho(\mathbf{s}) \rangle + \delta\rho(\mathbf{s})$$

The average density looks something like:

$$\langle \rho(\mathbf{s}) \rangle \longrightarrow$$



On long wavelengths we shall assume:

$$\langle \rho(\mathbf{s}) \rangle \gg \delta\rho(\mathbf{s})$$

The anisotropy of the average density:

$$\bar{\varepsilon}_2 \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} = \frac{\langle \mathbf{s}^2 \rangle}{\langle |\mathbf{s}|^2 \rangle}$$

With shorthand notations:

$$\langle f \rangle \equiv \frac{1}{\langle E \rangle} \int_{\mathbf{s}} f(\mathbf{s}) \langle \rho(\mathbf{s}) \rangle$$

$$\langle E \rangle = \int_{\mathbf{s}} \langle \rho(\mathbf{s}) \rangle$$

[Blaizot, Broniowski, Ollitrault,

The eccentricity of Teaney and Yan is:

$$\varepsilon_n = \frac{\int_{\mathbf{s}} (\mathbf{s}-\mathbf{s}_0)^n \rho(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}-\mathbf{s}_0|^n \rho(\mathbf{s})} \quad \text{with} \quad \mathbf{s}_0 \equiv \delta \mathbf{s}$$

recenterin
g
correction

Decompose ρ = <ρ> + δρ and expand to first nontrivial order:

$$\varepsilon_2 = \bar{\varepsilon}_2 + \frac{\delta \mathbf{s}^2}{\langle |\mathbf{s}|^2 \rangle} - \bar{\varepsilon}_2 \frac{\delta |\mathbf{s}|^2}{\langle |\mathbf{s}|^2 \rangle} - \frac{(\delta |\mathbf{s}|^2)(\delta \mathbf{s}^2)}{\langle |\mathbf{s}|^2 \rangle^2} + \bar{\varepsilon}_2 \frac{(\delta |\mathbf{s}|^2)^2}{\langle |\mathbf{s}|^2 \rangle^2} - \frac{(\delta \mathbf{s})^2}{\langle |\mathbf{s}|^2 \rangle} + \bar{\varepsilon}_2 \frac{(\delta \mathbf{s})(\delta \mathbf{s}^*)}{\langle |\mathbf{s}|^2 \rangle}$$

background

δ

δδ

$$\varepsilon_3 = \frac{\delta \mathbf{s}^3}{\langle |\mathbf{s}|^3 \rangle} - \frac{(\delta |\mathbf{s}|^3)(\delta \mathbf{s}^3)}{\langle |\mathbf{s}|^3 \rangle^2} - 3 \frac{(\delta \mathbf{s}^2)(\delta \mathbf{s})}{\langle |\mathbf{s}|^3 \rangle}$$

δ

δδ

With shorthand notation:

$$\delta f \equiv \frac{1}{\langle E \rangle} \int_{\mathbf{s}} f(\mathbf{s}) \delta \rho(\mathbf{s})$$

Eccentricity is equal to background in absence of fluctuations.
The triangularity originates solely from fluctuations.

[Bhalerao, Giacalone, Guerrero-Rodriguez, Luzum, Marquet, Ollitrault, **1903.06366**]

2-PARTICLE CORRELATIONS

The simplest cumulant we can build is a variance

$$c_n\{2\} \equiv \langle \varepsilon_n \varepsilon_n^* \rangle$$

Neglect for a second the recentering correction. One finds:

$$\langle \varepsilon_2 \varepsilon_2^* \rangle = \varepsilon_2\{2\}^2 = \bar{\varepsilon}_2^2 + \frac{\int_{\mathbf{s}_1,\mathbf{s}_2} (\mathbf{s}_1)^2 (\mathbf{s}_2^*)^2 S(\mathbf{s}_1, \mathbf{s}_2)}{\left(\int_{\mathbf{s}} |\mathbf{s}|^2 \langle \rho(\mathbf{s}) \rangle \right)^2}$$

background

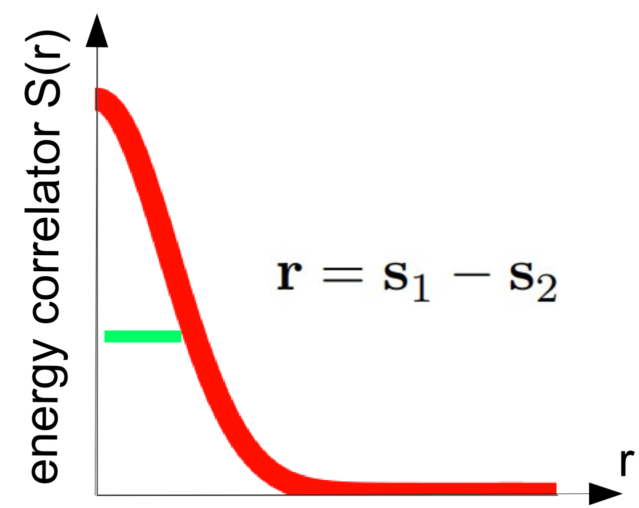
fluctuations

$$\langle \varepsilon_3 \varepsilon_3^* \rangle = \varepsilon_3\{2\}^2 = \frac{\int_{\mathbf{s}_1,\mathbf{s}_2} (\mathbf{s}_1)^3 (\mathbf{s}_2^*)^3 S(\mathbf{s}_1, \mathbf{s}_2)}{\left(\int_{\mathbf{s}} |\mathbf{s}|^3 \langle \rho(\mathbf{s}) \rangle \right)^2}$$

fluctuations only

We introduce the **connected 2-point function**:

$$S(\mathbf{s}_1, \mathbf{s}_2) = \langle \rho(\mathbf{s}_1) \rho(\mathbf{s}_2) \rangle - \langle \rho(\mathbf{s}_1) \rangle \langle \rho(\mathbf{s}_2) \rangle$$



Explicit check on a model that describes heavy-ion collision data.
CGC-motivated analytic model for **average and fluctuations**:

- $\langle \rho(\mathbf{s}) \rangle = \frac{4}{3g^2} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s})$

[Giacalone, Guerrero-Rodriguez, Luzum, Marquet, Ollitrault, 1902.07168]

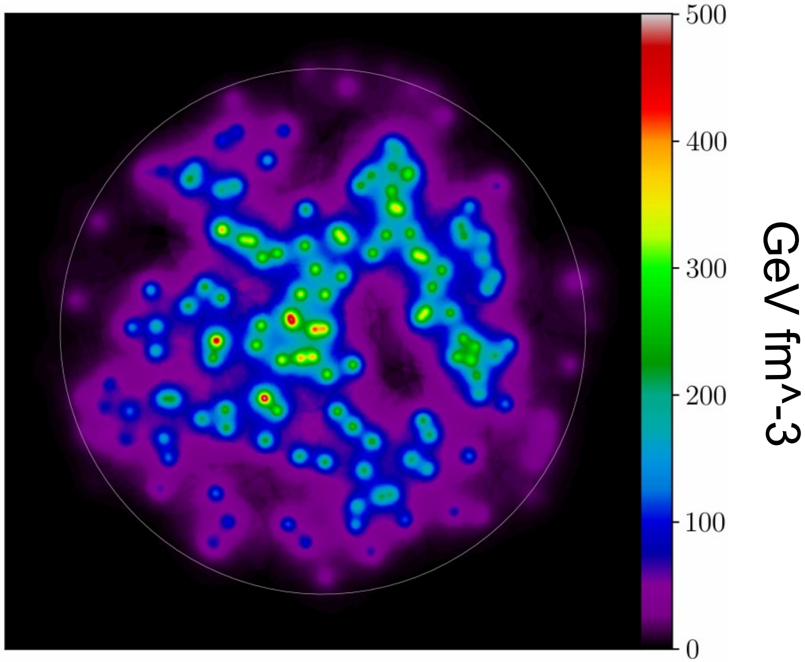
- $S(\mathbf{s}_1, \mathbf{s}_2) = \xi(\mathbf{s}) \delta(\mathbf{r}) \qquad \mathbf{s} = \frac{\mathbf{s}_1 + \mathbf{s}_2}{2}, \ \mathbf{r} = \mathbf{s}_1 - \mathbf{s}_2$

$$\xi(\mathbf{s}) = \frac{16\pi}{9g^4} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s}) \left[Q_A^2(\mathbf{s}) \ln\left(\frac{Q_B^2(\mathbf{s})}{m^2} \right) + Q_B^2(\mathbf{s}) \ln\left(\frac{Q_A^2(\mathbf{s})}{m^2} \right) \right]$$

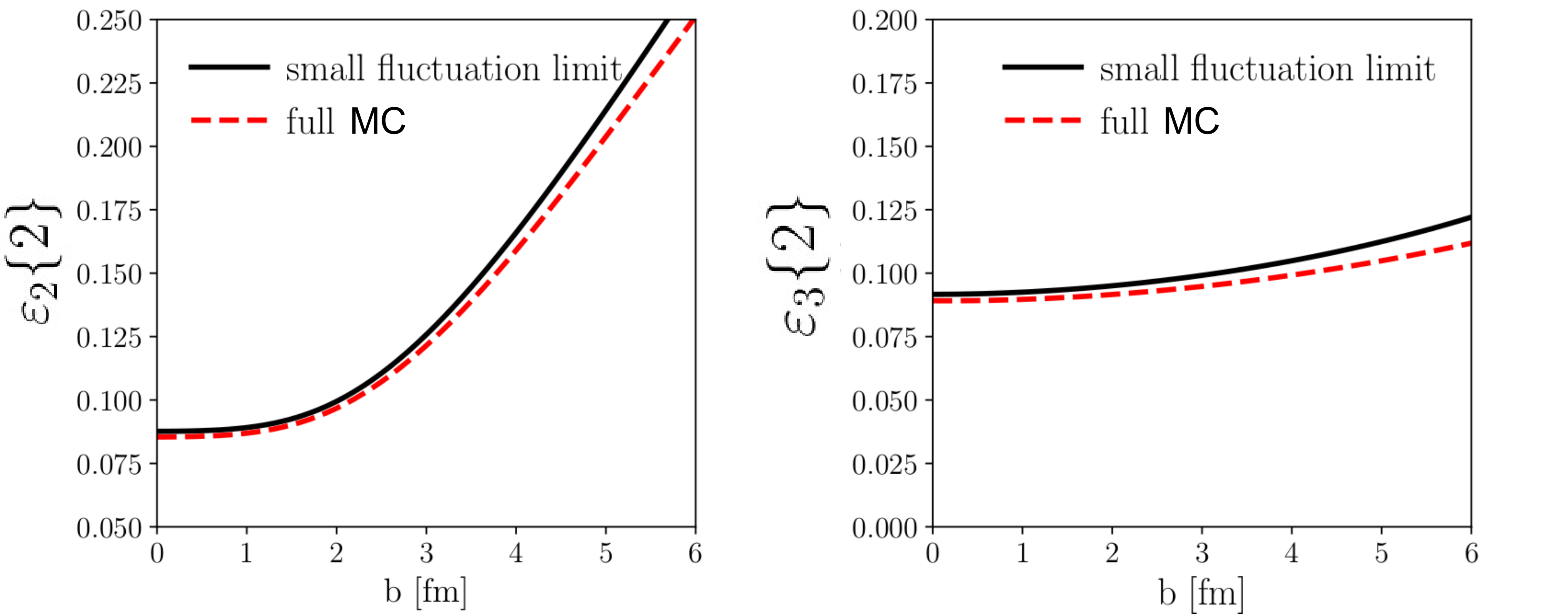
IR cutoff

Monte Carlo realization
for event-by-event calculations

[Gelis, Giacalone, Guerrero-Rodriguez, Marquet, Ollitrault, 1907.10498]



Put the 1- and 2-point functions in the formulas for eccentricities:



Recentering correction yields negligible
contributions up to b~7fm

[Bhalerao, Giacalone, Guerrero-Rodriguez, Luzum, Marquet, Ollitrault, **1903.06366**]

The small fluctuation formula is correct within few % accuracy

4-PARTICLE CORRELATIONS

Non-Gaussianity appears in higher-order correlations.
Kurtosis and mixed kurtosis:

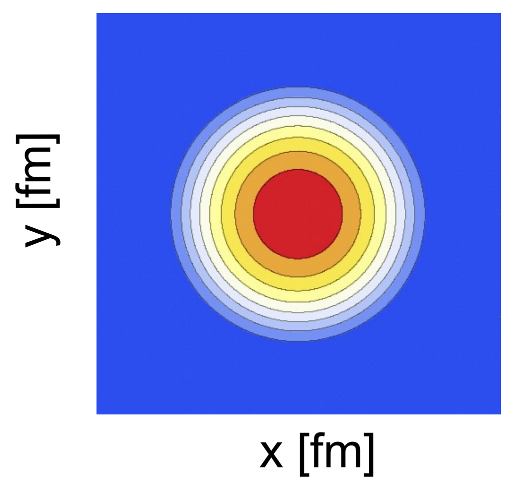
[Blaizot, Grönqvist, Ollitrault, **1604.07230**]
[Bhalerao, Giacalone, Ollitrault, **1904.10350**]

$$c_n\{4\} \equiv \langle \varepsilon_n \varepsilon_n \varepsilon_n^* \varepsilon_n^* \rangle - 2\langle \varepsilon_n \varepsilon_n^* \rangle \langle \varepsilon_n \varepsilon_n^* \rangle$$

$$SC(3,2) \equiv \langle \varepsilon_2 \varepsilon_3 \varepsilon_2^* \varepsilon_3^* \rangle - \langle \varepsilon_2 \varepsilon_2^* \rangle \langle \varepsilon_3 \varepsilon_3^* \rangle$$

We shall work with an azimuthally symmetric background (central collisions):

$\langle \rho(\mathbf{s}) \rangle \longrightarrow$



$\longrightarrow \quad \bar{\varepsilon}_2 = 0$

Now have fun with **algebra + orders of magnitude**.

INGREDIENTS

$$\langle \delta f \delta g \rangle = \frac{1}{\langle E \rangle^2} \int_{\mathbf{s}_1,\mathbf{s}_2} f(\mathbf{s}_1) g(\mathbf{s}_2) C_2(\mathbf{s}_1,\mathbf{s}_2)$$

$$C_2(\mathbf{s}_1,\mathbf{s}_2) \rightarrow \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2) \rangle$$

$$\langle \delta f \delta g \delta h \rangle = \frac{1}{\langle E \rangle^3} \int_{\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3} f(\mathbf{s}_1) g(\mathbf{s}_2) h(\mathbf{s}_3) C_3(\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3)$$

$$C_3(\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3) \rightarrow \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\delta\rho(\mathbf{s}_3) \rangle$$

$$\langle \delta f \delta g \delta h \delta k \rangle_c = \frac{1}{\langle E \rangle^4} \int_{\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4} f(\mathbf{s}_1) g(\mathbf{s}_2) h(\mathbf{s}_3) k(\mathbf{s}_4) C_4(\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4)$$

$$C_4(\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4) \rightarrow \begin{aligned} &\langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\delta\rho(\mathbf{s}_3)\delta\rho(\mathbf{s}_4) \rangle \\ &- \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2) \rangle \langle \delta\rho(\mathbf{s}_3)\delta\rho(\mathbf{s}_4) \rangle \\ &- \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_3) \rangle \langle \delta\rho(\mathbf{s}_2)\delta\rho(\mathbf{s}_4) \rangle \end{aligned}$$

- C_3 and C_4 are zero for a Gaussian density field.
- for C_3 there is a positive contribution from $\rho(s)>0$.
- **In a standard scenario (e.g. Poisson) C_2, C_3, C_4 are positive and short-range. This implies that <δδδ....> are positive.**

RESULTS

$$c_n\{4\} = c_n\{4\}_K + c_n\{4\}_S + c_n\{4\}_V$$

δδδδ δδδδδ δδδδδδ

$$c_n\{4\}_V = 8\frac{\langle \delta \mathbf{s}^n \delta \mathbf{s}^{*n} \rangle^2 \langle \delta |\mathbf{s}|^n \delta |\mathbf{s}|^n \rangle}{\langle |\mathbf{s}|^n \rangle^6}$$

$$c_n\{4\}_S = -8\frac{\langle \delta \mathbf{s}^n \delta \mathbf{s}^{*n} \rangle \langle \delta |\mathbf{s}|^n \delta \mathbf{s}^n \delta \mathbf{s}^{*n} \rangle}{\langle |\mathbf{s}|^n \rangle^5}$$

$$c_n\{4\}_K = \frac{\langle \delta \mathbf{s}^n \delta \mathbf{s}^n \delta \mathbf{s}^{*n} \delta \mathbf{s}^{*n} \rangle_c}{\langle |\mathbf{s}|^n \rangle^4}$$

For short-range correlations, this is the only negative contribution

$$SC(3,2) = SC(3,2)_K + SC(3,2)_S + SC(3,2)_V$$

δδδδ δδδδδ δδδδδδ

$$SC(3,2)_V = 4\frac{\left\langle \delta \mathbf{s}^2 \delta \mathbf{s}^{*2} \right\rangle \left\langle \delta \mathbf{s}^3 \delta \mathbf{s}^{*3} \right\rangle \left\langle \delta |\mathbf{s}|^2 \delta |\mathbf{s}|^3 \right\rangle}{\langle |\mathbf{s}|^2 \rangle^3 \langle |\mathbf{s}|^3 \rangle^3} + 9\frac{\left\langle \delta \mathbf{s}^2 \delta \mathbf{s}^{*2} \right\rangle^2 \langle \delta \mathbf{s} \delta \mathbf{s}^* \rangle}{\langle |\mathbf{s}|^2 \rangle^2 \langle |\mathbf{s}|^3 \rangle^2}$$

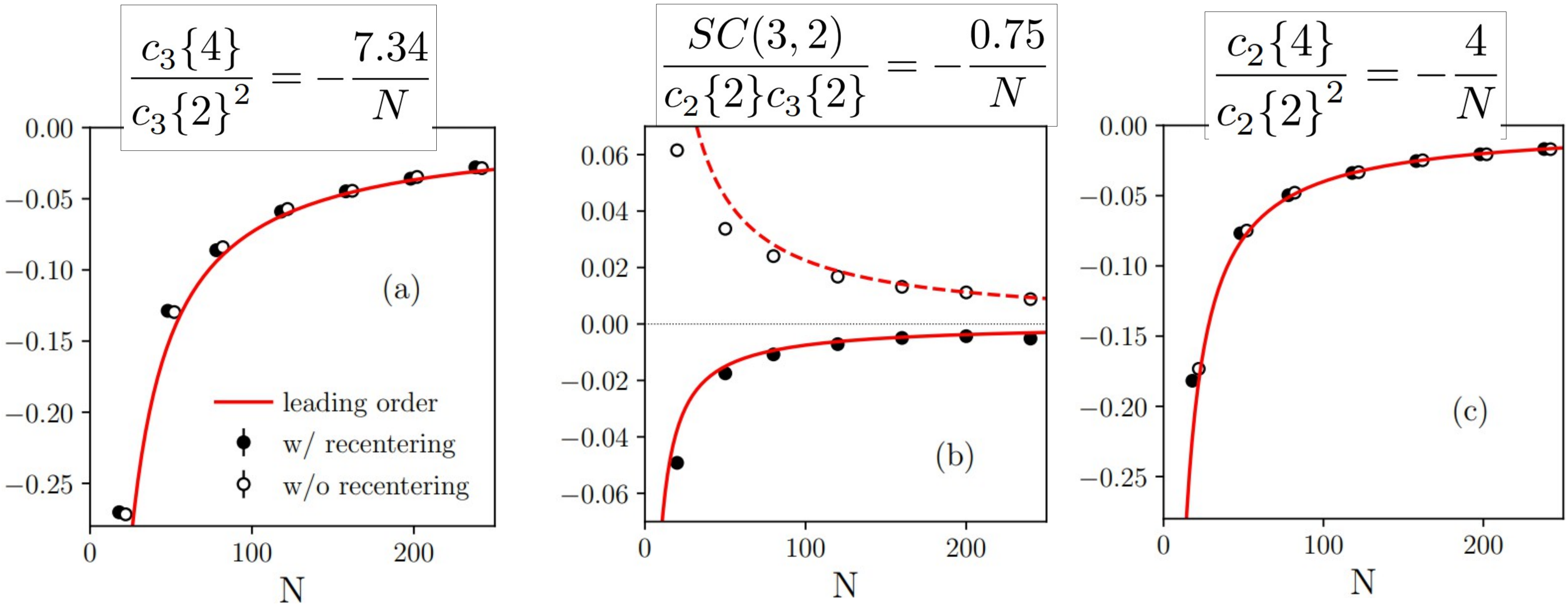
$$\begin{aligned} SC(3,2)_S = &- 2\frac{\left\langle \delta \mathbf{s}^3 \delta \mathbf{s}^{*3} \right\rangle \left\langle \delta |\mathbf{s}|^3 \delta \mathbf{s}^2 \delta \mathbf{s}^{*2} \right\rangle}{\langle |\mathbf{s}|^2 \rangle^2 \langle |\mathbf{s}|^3 \rangle^3} \\ &- 2\frac{\left\langle \delta \mathbf{s}^2 \delta \mathbf{s}^{*2} \right\rangle \left\langle \delta |\mathbf{s}|^2 \delta \mathbf{s}^3 \delta \mathbf{s}^{*3} \right\rangle}{\langle |\mathbf{s}|^2 \rangle^3 \langle |\mathbf{s}|^3 \rangle^2} \\ &- 6\frac{\left\langle \delta \mathbf{s}^2 \delta \mathbf{s}^{*2} \right\rangle \left\langle \delta \mathbf{s} \delta \mathbf{s}^2 \delta \mathbf{s}^{*3} \right\rangle}{\langle |\mathbf{s}|^2 \rangle^2 \langle |\mathbf{s}|^3 \rangle^2} \end{aligned}$$

For short-range correlations, this is the only negative contribution

$$SC(3,2)_K = \frac{\left\langle \delta \mathbf{s}^2 \delta \mathbf{s}^3 \delta \mathbf{s}^{*2} \delta \mathbf{s}^{*3} \right\rangle_c}{\langle |\mathbf{s}|^2 \rangle^2 \langle |\mathbf{s}|^3 \rangle^2}$$

Check in a Gaussian model of N identical independent sources where 3- and 4- point correlators can be evaluated analytically:

Points: full numerics (standardized quantities)
Lines: small fluctuation limit

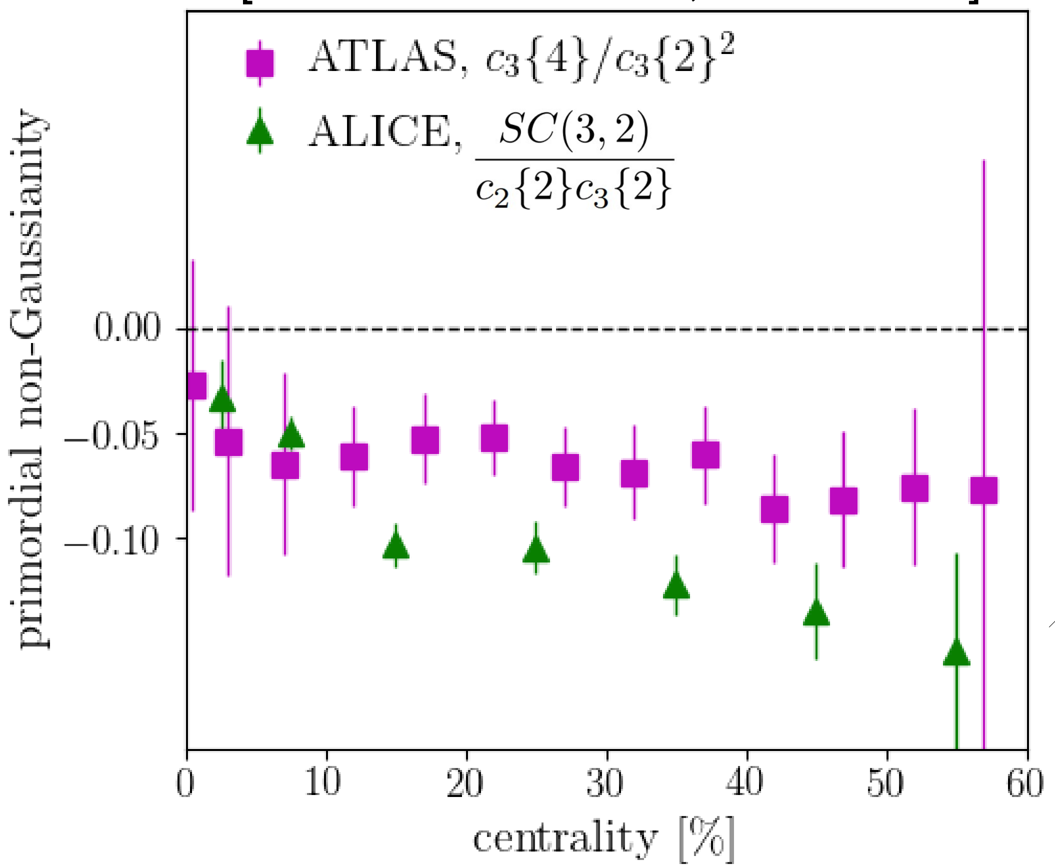


[Bhalerao, Giacalone, Ollitrault, 1904.10350]

They are all negative:
short-range correlations + positive skew of the field

Now look at these quantities in the data.

[ATLAS Collaboration, 1408.4342]
[ALICE Collaboration, 1604.07663]

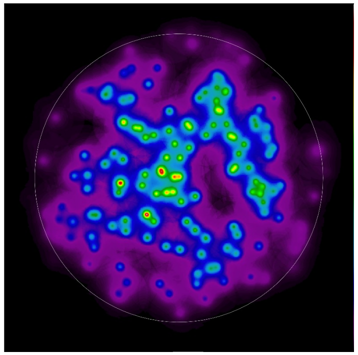


sc(3,2) is larger in magnitude. Interesting.

Negative and **mildly dependent on centrality**. Consistent with:

short-range correlations + positive skew of the energy-density field

HEP-PHENOMENOLOGY FOR LARGE SYSTEMS?



PRIMORDIAL FLUCTUATIONS AND ANISOTROPY

Multi-point correlators of energy-density field

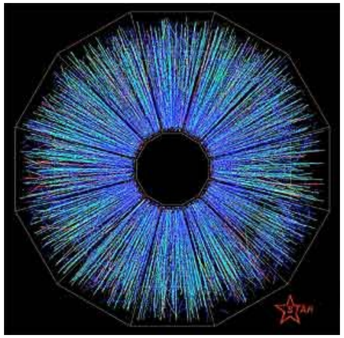
$$\langle \rho(\mathbf{s}) \rangle, \ \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2) \rangle, \ \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2)\rho(\mathbf{s}_3) \rangle \ \ldots$$

Possibly from first principles

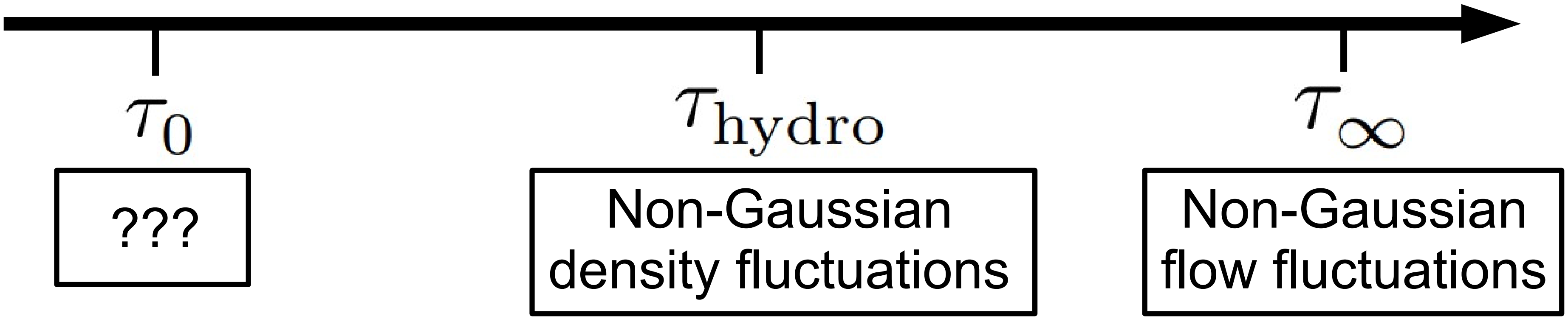
FINAL-STATE FLUCTUATIONS AND ANISOTROPY

Multi-particle azimuthal correlations in the detectors

$$\langle e^{in(\phi_1-\phi_2)} \rangle, \ \langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle, \ \ldots$$

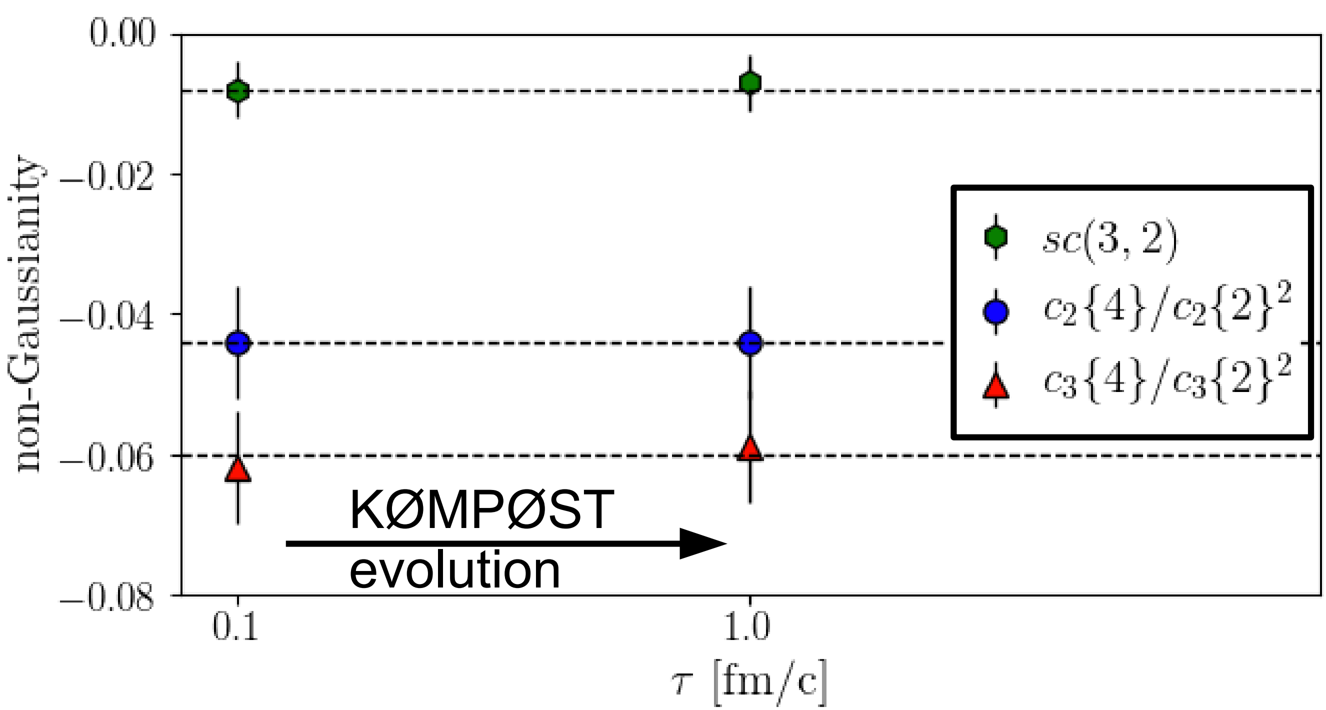


When is non-Gaussianity created? So far we know that:



Is it present at the very initial time? Check using the KØMPØST code and the Gaussian independent source model.

[Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney, 1805.00961]

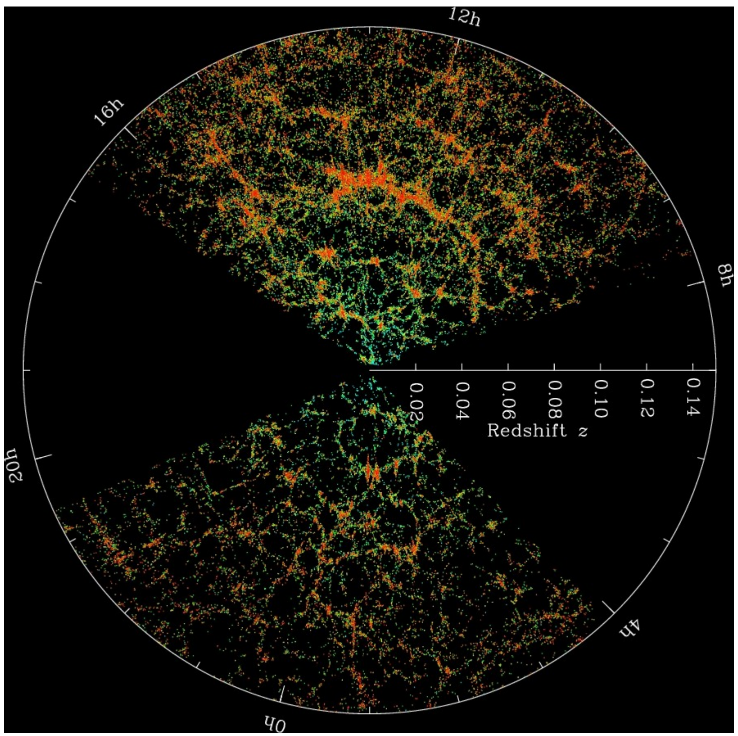


Results indicate that non-Gaussianity is of <u>primordial</u> origin.

primordial non-Gaussianity = 0

expansion

[Sloan Digital Sky Survey, sdss.org]

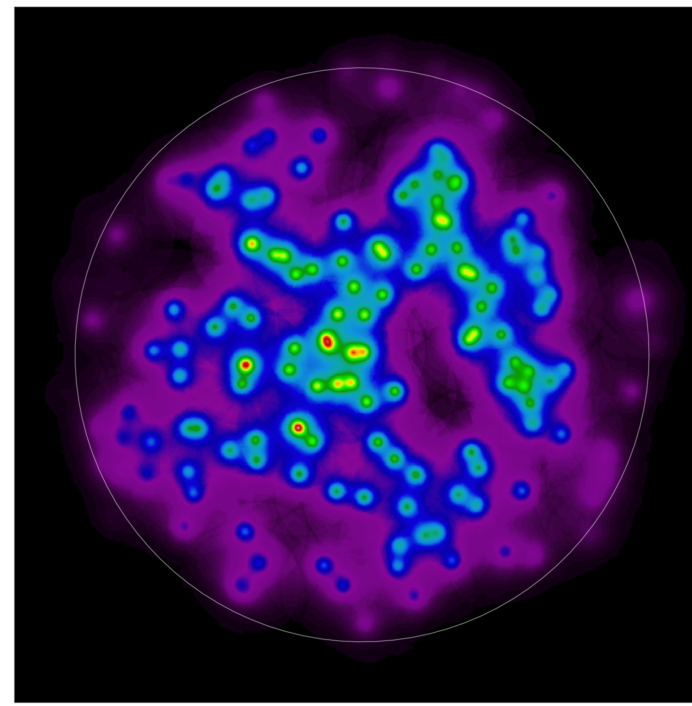


10^{26} m

In cosmology, **structures** are formed during the expansion.

In heavy-ion collisions **structures** are there at the beginning.

primordial non-Gaussianity ≠ 0



10^{-14} m

expansion