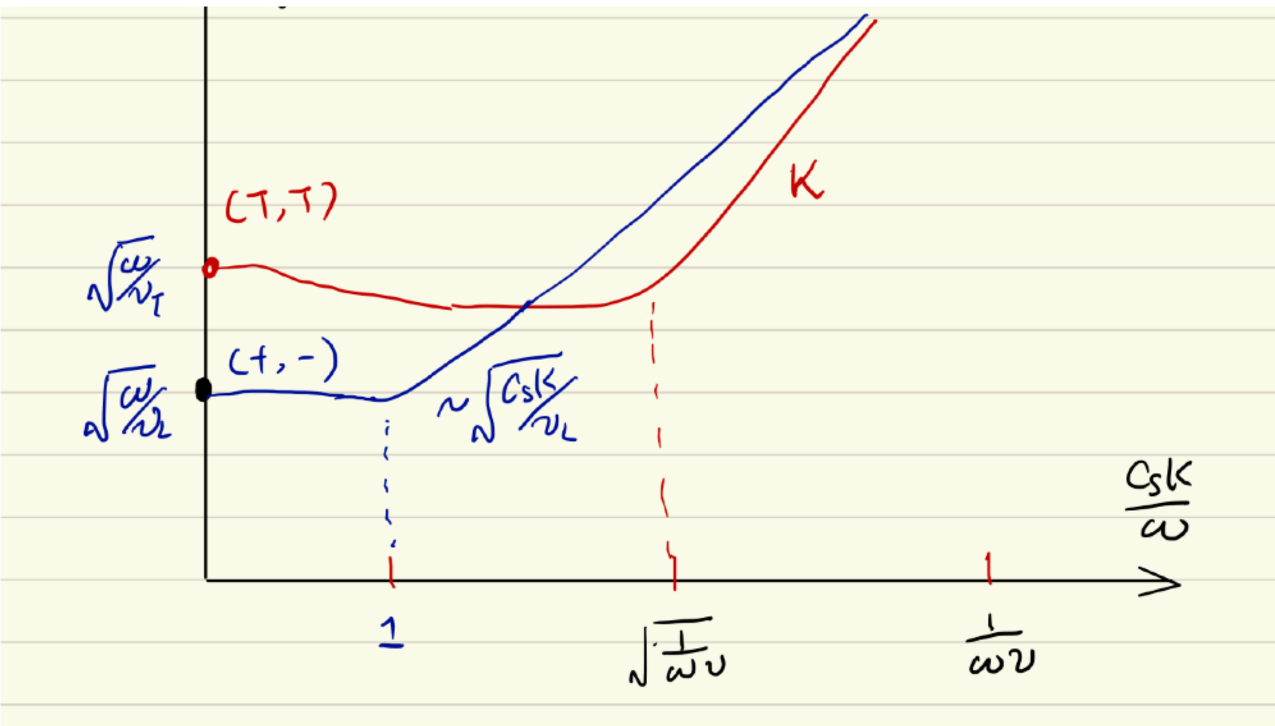
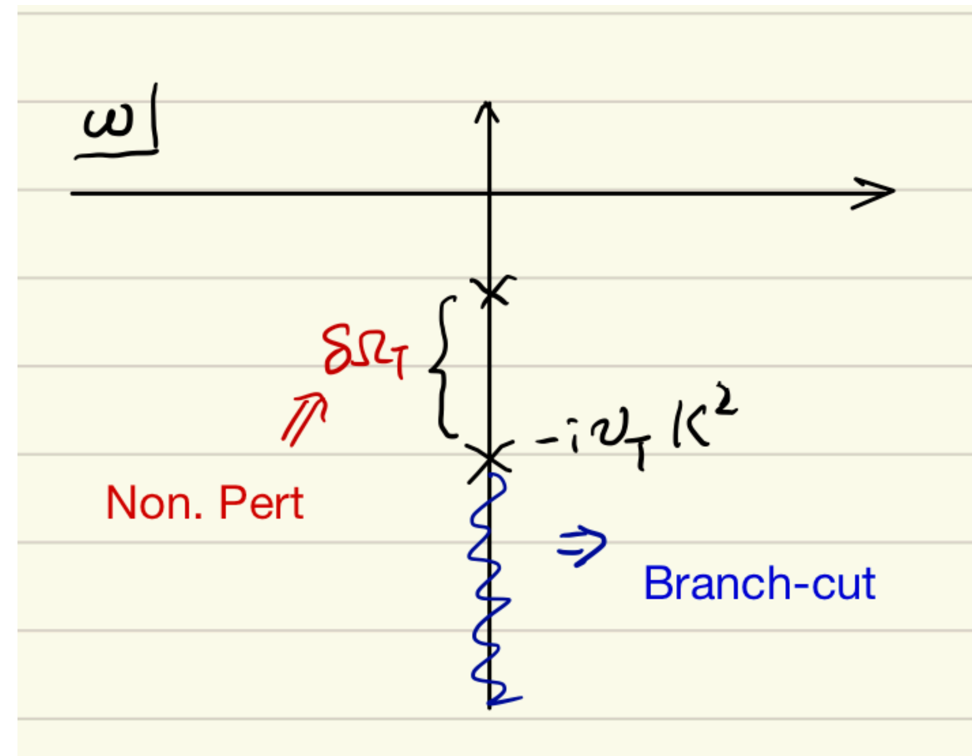
Sound and diffusion, signifying the multi-scale dynamics of hydrodynamic fluctuations



Finite time and size correction to shear viscosity



Non-perturbative modification of shear dispersion relation

Chris Lau, Hong Liu, YY, in preparation

Institute of Modern Physics (Lanzhou), Chinese academy of sciences

Yi Yin

Fudan, Oct.31, 2019

Introduction

There are three approaches for studying hydro. with fluctuations in general.

I. Stochastic hydro. approach: (adding noise to hydro. equations).

Landau-Lifshitz, Statistical Mechanics; Kapusta-Mueller-Stephanov, PRC '11;…

II. Treating off-equilibrium fluctuations as slow modes in additional to "hydro" modes.

⇒ Coupled deterministic equation.

Kawasaki, Ann. Phys. '70; Andreev, JTEP, '1971; …"hydro-kinetic", Akamatsu-Mazeliauskas-Teaney, PRC 16, PRC '18

III: "Effective field theory" (EFT) approach: formulating hydro on the Schwinger-Keldysh contour.

Kovtun-Moore-Romatschke, JHEP 14'; Glorioso-Crossley-Liu, JHEP 17'; Haehl-Loganayagam-Rangamani, 1803.11155, …

Warm-up exercise: Brownian motion

$$\frac{dp}{dt} = -\,\eta_D\, p + \xi\,, \qquad \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

The physics of Brownian is characterized by three parameters: η_D , κ and temperature of the medium. They are related by fluctuation-dissipation theorem:

$$\eta_D = 2MT\kappa$$

For fluctuating hydrodynamics, transport coefficients and diffusive constant are analogous to η_D , κ . For example,

Approach 1: Stochastic hydro. approach

Adding noise term to hydro. , e.g.

$$\partial_\mu \left[T^{\mu\nu}_{\rm ave}(\epsilon, n, u^\mu) + S^{\mu\nu}_{\textbf{noise}} \right] = 0 \qquad \langle S_T S_T \rangle \propto \eta$$

Landau-Lifshitz, Statistical Mechanics; Kapusta-Mueller-Stephanov, PRC '11;…

Output: the probability distribution of hydro. variables (as a function of space&time).

Advantages:

a) Equations are known b) contains full Information.

Challenging:

a) numerically demanding. b) Subtleties on extracting physical results (cut-off dependence). c) multiplicative noise.

Approach 2: deterministic "extended hydro".

$$\partial_\mu \left[T^{\mu\nu}_{\rm ave}(\epsilon, n, u^\mu) + \Delta T^{\mu\nu}(\text{2pt,3pt}, \dots)\right] = 0$$

E.o.Ms for 2pt, 3pt,…

Advantages:

Kawasaki, Ann. Phys. '70; Andreev, JTEP, '1971; …

a) deterministic equations.

b) Treating fluct. at an equal-footing as hydro. variables

c) convenient to gain qualitative insights.

d) Less d.o.f. after truncation.

Challenging:

a) Requiring non-trivial formulations. b) Relies on assumption about the truncation and decoupling.

The study of hydro-kinetic equation: rapid progress

	Flow	Baryon density	Conformal E.o.S
Akamatsu-Mazeliauskas-Teaney, PRC 16	Bjorken	No	Yes
Akamatsu-Mazeliauskas-Teaney, PRC 18	Bjorken	No	No
Schaffer-Martinez, 1812.05279	Bjorken	Yes	Yes
Xin An-Basar-Stephanov -H.-U. Yee, PRD 19'	General	No	No

In general, those calculations reproduce hydro. tail obtained from diagrammatic computation at one loop for a static background, and obtain the manifestation of hydro. tail for Bjorken expansion.

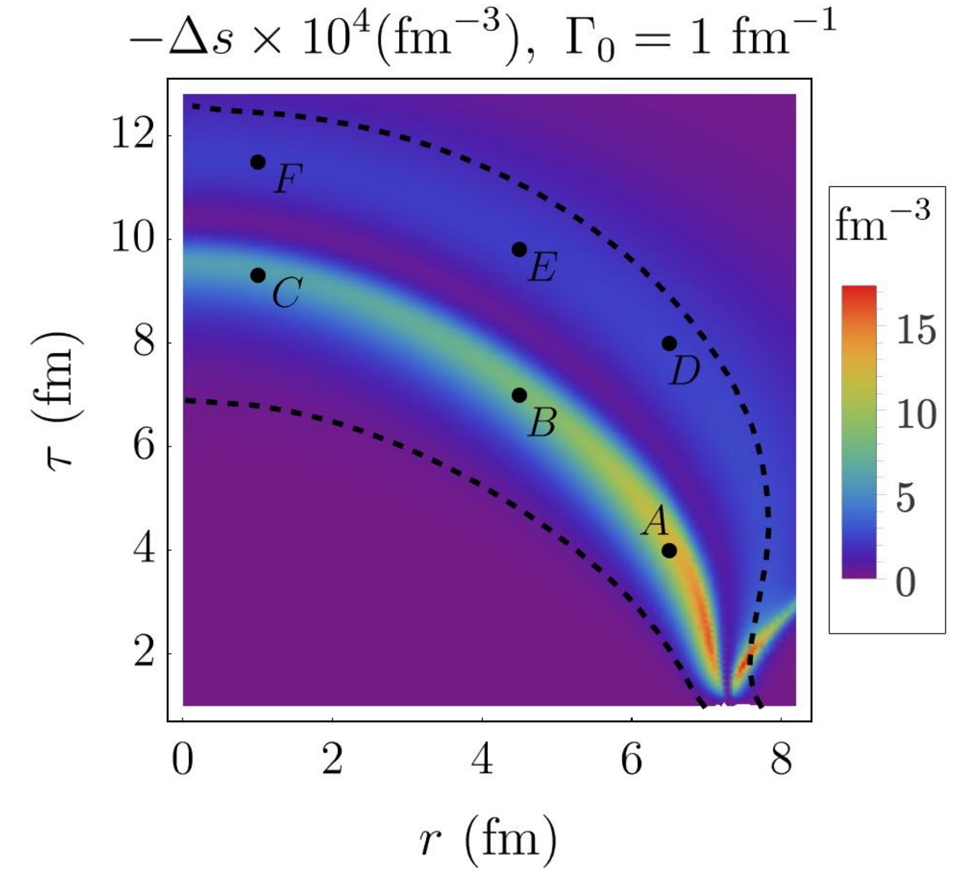
Akamatsu-Mazeliauskas-Teaney, PRC 16'

$$\frac{\langle T^{zz}\rangle}{e+p} = \Big[\underbrace{\frac{p}{e+p}}_{\sim 1} - \underbrace{\frac{4}{3}\frac{\gamma_\eta}{\tau}}_{\text{1st order}} + \underbrace{\frac{1.08318}{s\,(4\pi\gamma_\eta\tau)^{3/2}}}_{\text{3/2 order!}} + \underbrace{\frac{(\lambda_1 - \eta\tau_\pi)}{e+p}\frac{8}{9\tau^2}}_{\text{2nd order}} + \dots\Big]$$

The application of deterministic approach near the QCD critical point and its simulation

"Hydro+" : couples hydro. d.o.f with long wavelength critical fluctuations using deterministic equations.

Stephanov-YY, 1712.10305, PRD '18;



First simulation: Rajagopal-Ridgway-Weller-YY, 1908.08539

Equations for the evolution of long wavelength fluctuations.

+

Gradient of $p_{(+)}$+dissipation ≃ acceleration of flow

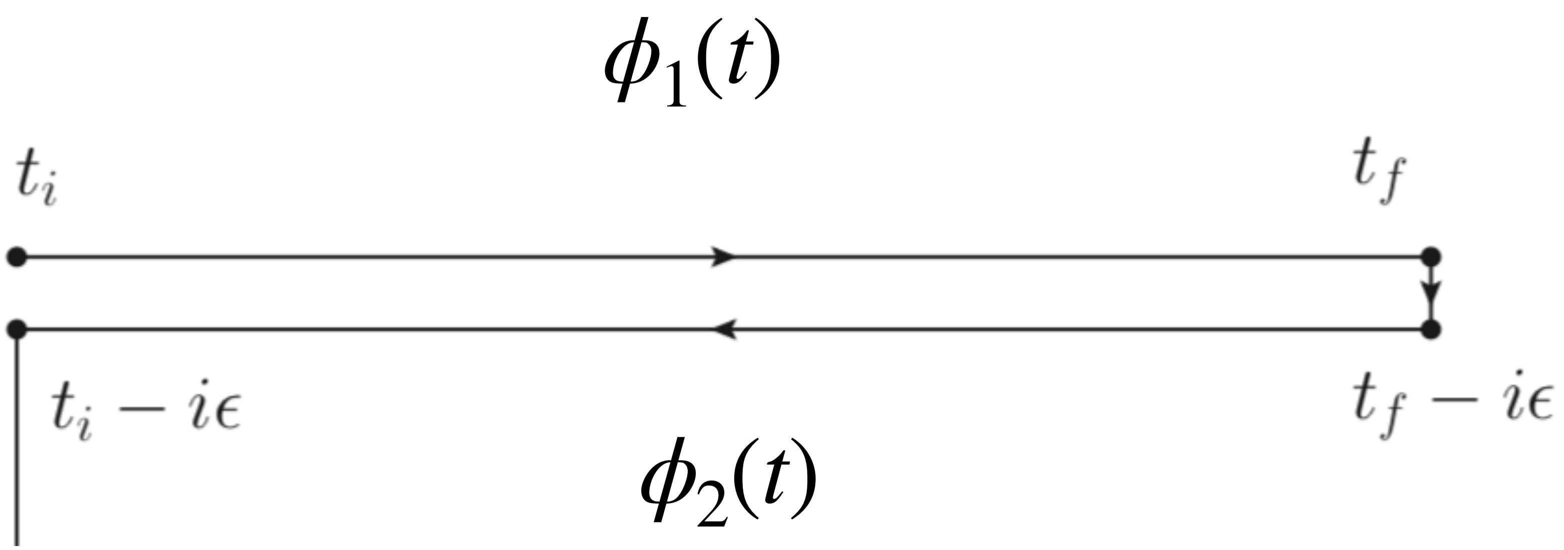
In addition, there is third approach ("Effective field theory" approach): formulating hydro on the Schwinger-Keldysh contour. (difficult for numerical simulation, but useful to gain analytic insights)

Kovtun-Moore-Romatschke, JHEP 14'; Glorioso-Crossley-Liu, JHEP 17'; Haehl-Loganayagam-Rangamani, 1803.11155, …

This talk: evaluating the non-linear effect due to hydro. fluctuations use EFT approach.

Set-up

Fluctuation, dissipation and Schwinger-Keldysh formalism



$$\phi_r \equiv \frac{1}{2}\left(\phi_1 + \phi_2\right) \ , \qquad \phi_a \equiv \phi_1 - \phi_2$$

The action for fluctuating hydrodynamic in Schiwinger-Keldysh formalism

$$\mathscr{L}_{\rm Hydro} = T^{\mu\nu}_{\rm Hydro} \nabla_\nu X_\mu + i\beta^{-1} H^{\mu\nu\alpha\beta} \nabla_\mu X_\alpha \nabla_\nu X_\beta$$

with:

$$T^{\mu\nu}_{\rm hydro} = \epsilon\, u^\mu u^\nu + p(\epsilon)\left(g^{\mu\nu} + u^\mu u^\nu\right) + T^{\mu\nu}_{\rm vis}$$

$$H^{\mu\nu\alpha\beta} = \eta \left[\Delta^{\mu\alpha}\Delta^{\nu\beta} + (\textbf{permutation})\right] + (\propto \zeta)$$

The form of this action is not difficult to obtain with bottom-up approach:

e.g., Kovtun-Moore-Romatschke, JHEP 14';

The variation of the first term w.r.t. a-field X gives the deterministic hydro. eqn.

The variation of the second term w.r.t X introduces the noise to hydro. Eqns. and hence the form of H is completely determined by fluct-dissipation theorem.

However, "top-down" construction identifies the "inverse temperature current" β^μ as the natural hydro. fields (r-field).

Glorioso-Crossley-Liu, JHEP 17';

The properties of β^μ :

In equilibrium, $\beta^\mu \to \beta_0 u_0^\mu$

Thermodynamic function (such as pressure p) and transport coefficients are viewed as a function of $\beta \equiv \sqrt{-\beta^\mu \beta_\mu}$

Fluid velocity is defined as: $u^\mu \equiv \dfrac{\beta^\mu}{\beta}$.

Note: β^μ is also argued to be the natural hydro. variables from a different perspective, see Becattini's talk.

We shall see this later that the identification of β^μ as dynamical variable for hydro. action is important for the description of non-linear systematically.

Retarded and symmetrized Green function/propagators

Consider the fluctuation around a homogenous and static background with <X>=0.

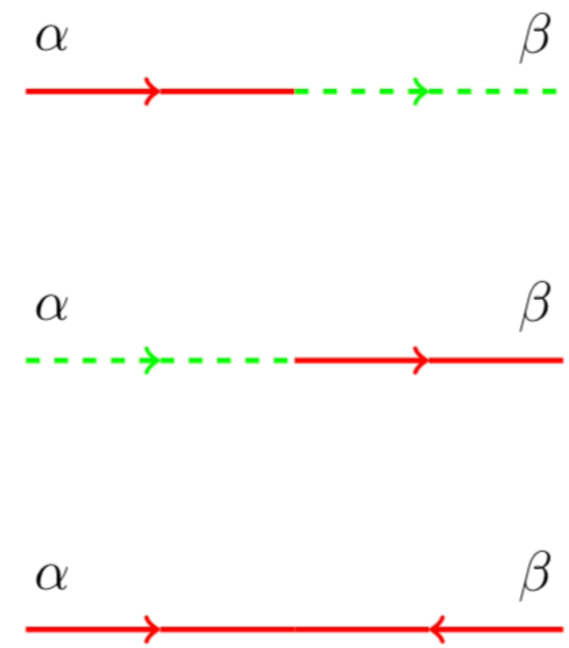
$$\beta^\mu = \beta_0 u^\mu + \beta_0 \lambda^\mu$$

Then define retarded and symmetrized Green function.

$G^R_{\alpha,\beta}(\omega,k)$:

$G^A_{\alpha,\beta}(\omega,k)$:

$G^S_{\alpha,\beta}(\omega,k)$:



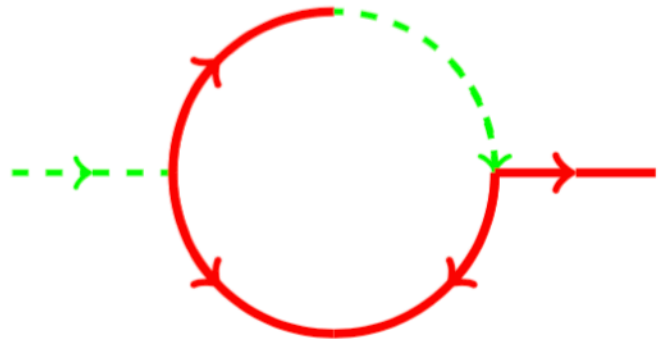
From hydro. action, we obtain:

$$G^R_{\mu\nu}(\omega,k) = \frac{1}{\omega - \Omega_T(k)} P^T_{\mu\nu} + \frac{\omega}{\left(\omega - \Omega_+(k)\right)\left(\omega - \Omega_-(k)\right)} P^L_{\mu\nu} + \ldots$$

$$\Omega_T(k) = -i\nu_T k^2\,, \qquad \Omega_\pm(k) = \pm c_s k - i\nu_T k^2\,,$$

Self-energy at one-loop c.f. Hydro-kinetic approach:

$\Sigma^{ar}(\omega, k)$:



$\propto V_{arr} \times V_{arr}$

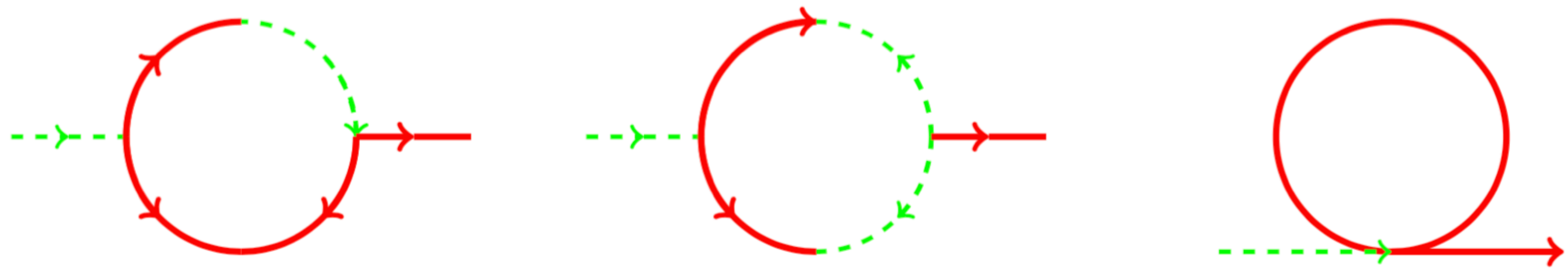
$$\delta_2(T^{\mu\nu}_{\text{hydro}})\,\nabla_\mu X_\nu \to V_{arr}$$

The contribution from this diagram is captured in hydro-kinetic theory follows:

$$\delta_2(T^{\mu\nu}_{\text{hydro}}) \to W^{\mu\nu\alpha\beta}\lambda_\alpha\lambda_\beta \to W^{\mu\nu\alpha\beta}G^S_{\alpha\beta}$$

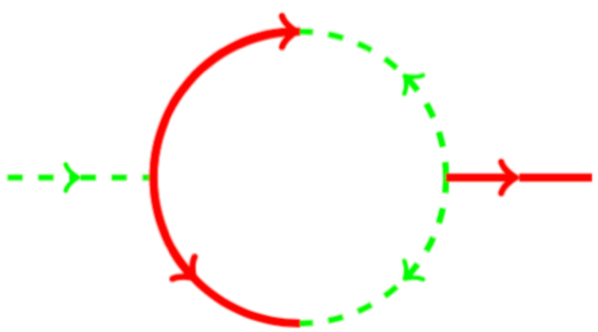
However, they are more diagrams!

$\Sigma^{ar}(\omega, k)$:



$\propto V_{arr} \times V_{arr}$ $\propto (V_{aar} \times V_{arr})$ $\propto V_{arrr}$

The effects of the last two diagrams are not captured in hydro-kinetic approach in its current form.



$\propto (V_{aar} \times V_{arr})$

Representing effects of hydro. - noise coupling.

$$\delta_1(\beta^{-1}\eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\ldots)\nabla_\mu X_\alpha \nabla_\nu X_\beta \to V_{aar}$$

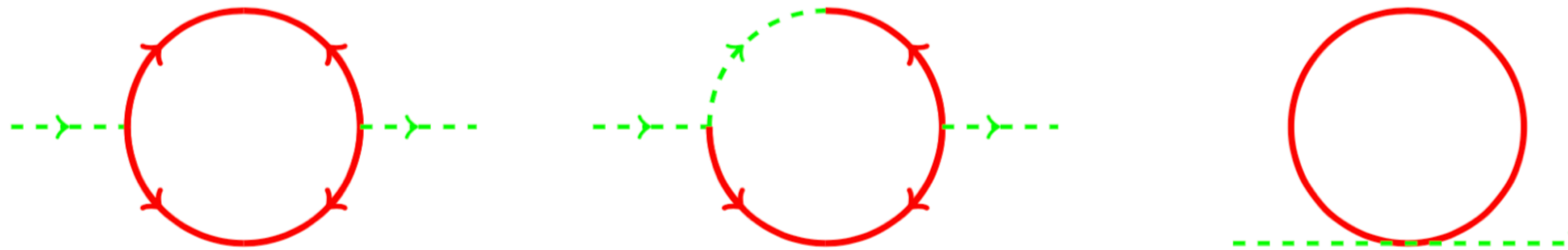
Since $V_{aar} \propto$ thermal derivatives of transport coefficients, it might be enhanced near the critical point.

Those overlooked diagrams could potentially lead to qualitative new features, e.g. .

Lin-Delacrétaz and Hartnoll, PRL 19'

Similarly, the contribution to Σ^{ar} is also rich in pattern

$\Sigma_{aa}(\omega, k)$:



$\propto (V_{arr} \times V_{arr})$ $\propto (V_{aar} \times V_{arr})$ $\propto V_{aarr}$

The contribution of V_{aarr} manifests multiplicative noise effects:

$$\delta_2(\beta^{-1}\eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\ldots)\nabla_\mu X_\alpha \nabla_\nu X_\beta \to V_{aar}$$

Results

Self-energy in shear channel

$$g_T^{S,R}(\omega,k) \equiv P_T^{\mu\nu}(\hat{k}) G_{\mu\nu}^{S,R}(\omega,k) \quad \sigma^{ar,aa}(\omega,k) = -\left[P_T^{\mu\nu}(\hat{k})\Sigma_{\mu\nu}^{ar,aa}(\omega,k)\right]/k^2$$

$\sigma^{ar}(\omega,k)$ and $\sigma^{aa}(\omega,k)$ describe the finite time and size correction for ν_T and η respectively.

$$g_T^S(\omega,k) \equiv \frac{2\left[\beta_0^{-1}\eta - \sigma_{aa}(\omega,k)\right]k^2}{\left[\omega + i\left(\nu_T - \sigma_{ar}^T(\omega,k)\right)k^2\right]\left[\omega + i\left(\nu_T - \sigma_{ar}^T(\omega,k)\right)k^2\right]^*}$$

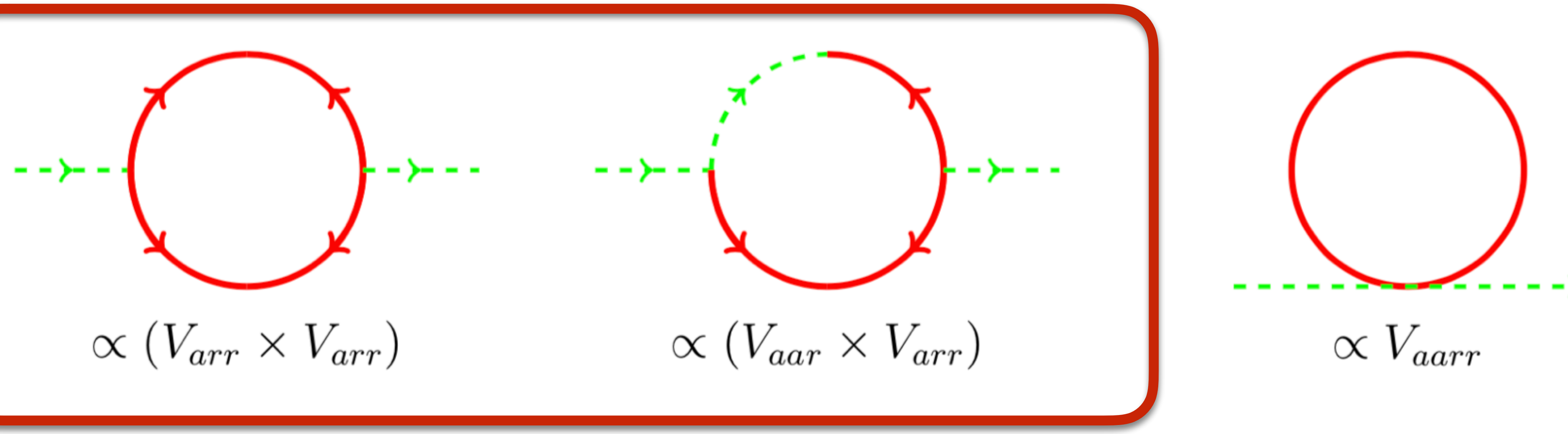
The previous calculations is limited to small k limit.

$$\lim_{k\to 0}\sigma^{aa}(\omega,k) \sim \frac{1}{s\nu}\sqrt{\frac{\omega}{\nu}}$$

The behavior for general ω, k. ?

The contribution from different pairs of hydrodynamic modes

$\Sigma_{aa}(\omega, k)$:



The bubble diagram represents the contribution from different pairs of hydro. modes .

$$\left(\frac{k^2}{s}\right) \int_{k/2}^{\Lambda} dp p^2 \int_{\cos\theta} \sum_{A,B=\pm,T} F_{A,B}(p_+, p_-; k) \int \frac{dp_0}{2\pi} \times \frac{1}{p_0 - \Omega_A(p_+)} \frac{1}{p_0 - \omega - \Omega_B(p_-)}$$

$$\Omega_T(k) = -i\nu_T k^2 , \qquad \Omega_\pm(k) = \pm c_s k - i\nu_L k^2 ,$$

Cut-off dependence

Inspecting on:

$$(\frac{1}{s})\int_{k/2}^{\Lambda} dpp^2 \frac{1}{\omega - \Omega_A(p) - \Omega_B(p)}$$

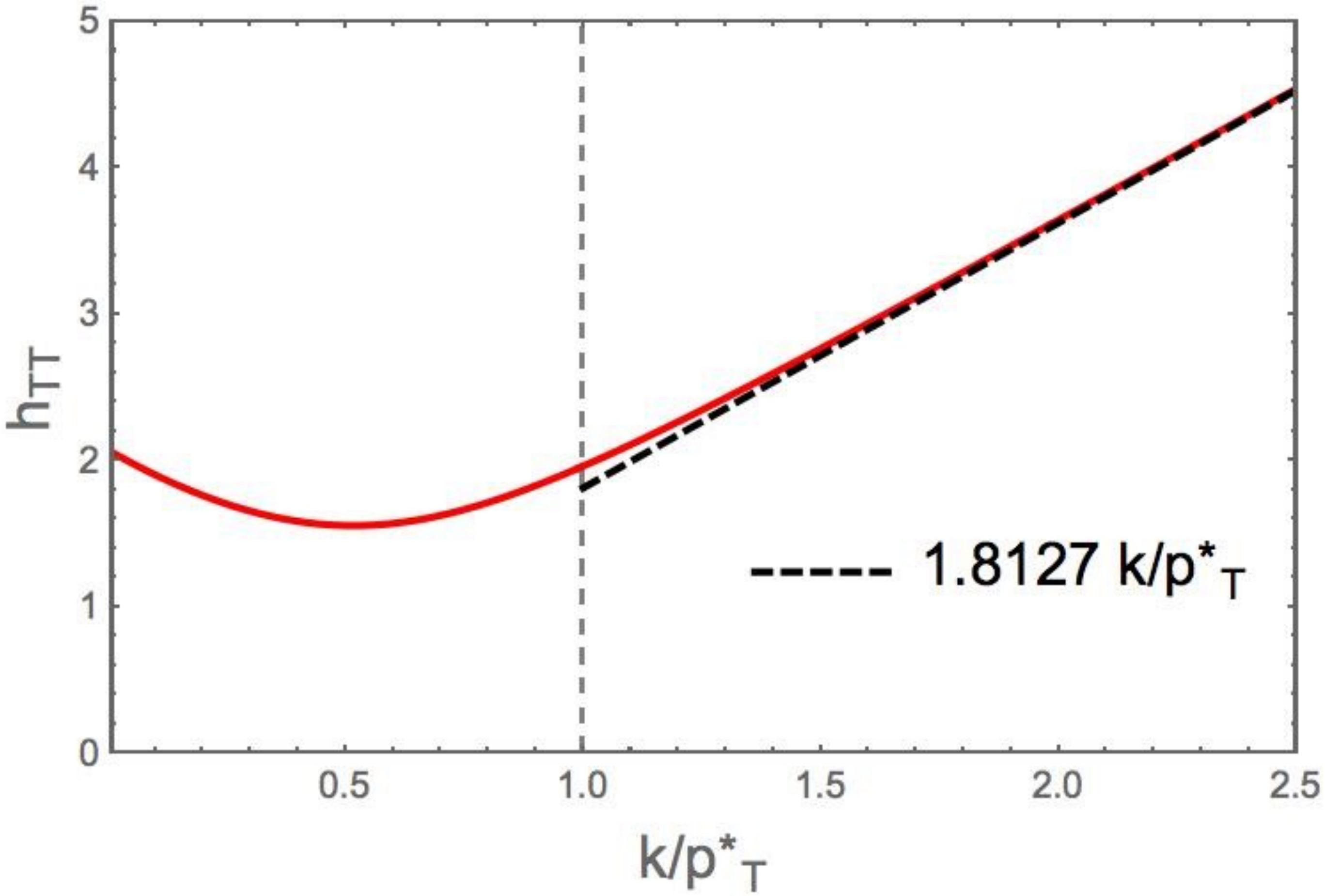
➡

(T,T)	(+, -)	(T,T)	(+,T)
$(\frac{1}{s\nu_T})\,\Lambda$	$(\frac{1}{s\nu_L})\,\Lambda$	$(\frac{\Lambda^2}{sc_s^2})$	$(\frac{\Lambda^2}{sc_s^2})$

$$\Delta\nu \sim \sigma^{aa,ar} \sim (\frac{\Lambda}{s\nu})\ \left[1 + (\nu\Lambda/c_s) + (\nu\Lambda/c_s)^2 + \ldots\right]\ ,$$

NB: $(\nu\Lambda/c_s) \ll 1$ in hydro. regime , but can this condition be satisfied in numerical simulation?

The universal scaling function for (T,T) channels



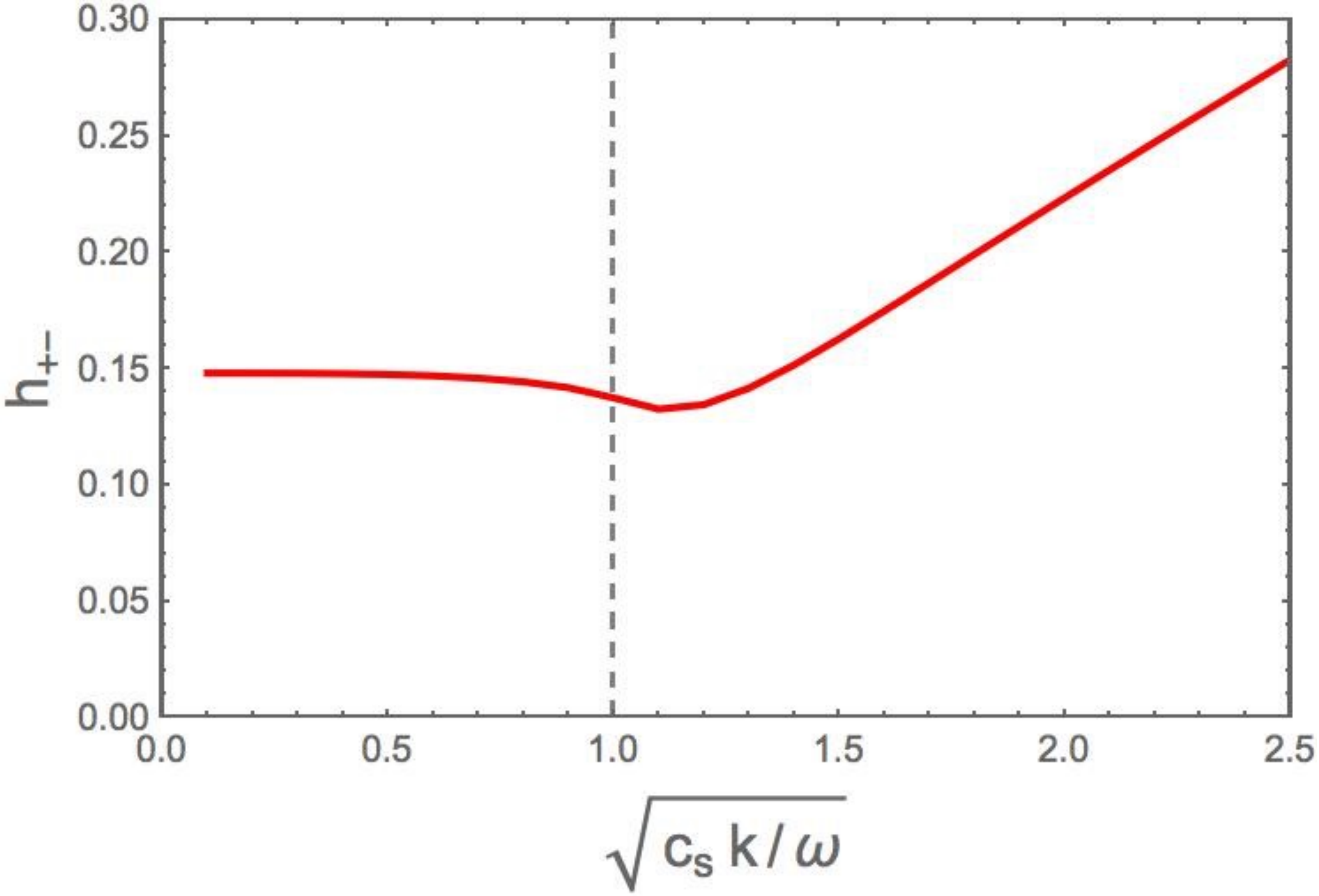
$$\sigma^{aa}_{T,T}(\omega, k) \sim \sqrt{\frac{\omega}{\nu_T}} h_{TT}(k/\sqrt{\omega/\nu_T})$$

The k-dependence is captured by the universal scaling function:

$$h_{TT}(0) = 2.0734, \qquad k < \sqrt{\omega/\nu_T}$$

$$h_{TT} = 1.8127 k/\sqrt{\omega/\nu_T}, \qquad k > \sqrt{\omega/\nu_T} \to \sigma^{aa} \sim k$$

The universal scaling function for (+,-) channels



$$\sigma^{aa}_{T,T}(\omega, k) \sim \sqrt{\frac{\omega}{\nu_L}} h_{+-}(z)$$

$$z = \frac{\sqrt{c_s k/\nu}}{\sqrt{\omega/\nu_L}} = \sqrt{c_s k/\omega}$$

The k-dependence is qualitatively different between (T,T) channel and (+,-) channel, in particular

$$h_{+-}(\sqrt{c_s k/\omega}) \sim \sqrt{\frac{c_s k}{\omega}} \to \sigma^{aa}_{+-} \sim \sqrt{\frac{c_s k}{\nu_L}} \gg k$$

The characteristic momentum p^*

The behavior of σ^{aa}/w is characterized by the typical internal momentum p^*:

$$\frac{\sigma^{aa}(\omega,\nu)}{w} \sim \frac{p^*(\omega,k)}{s\nu}$$

p^* can be estimated by solving

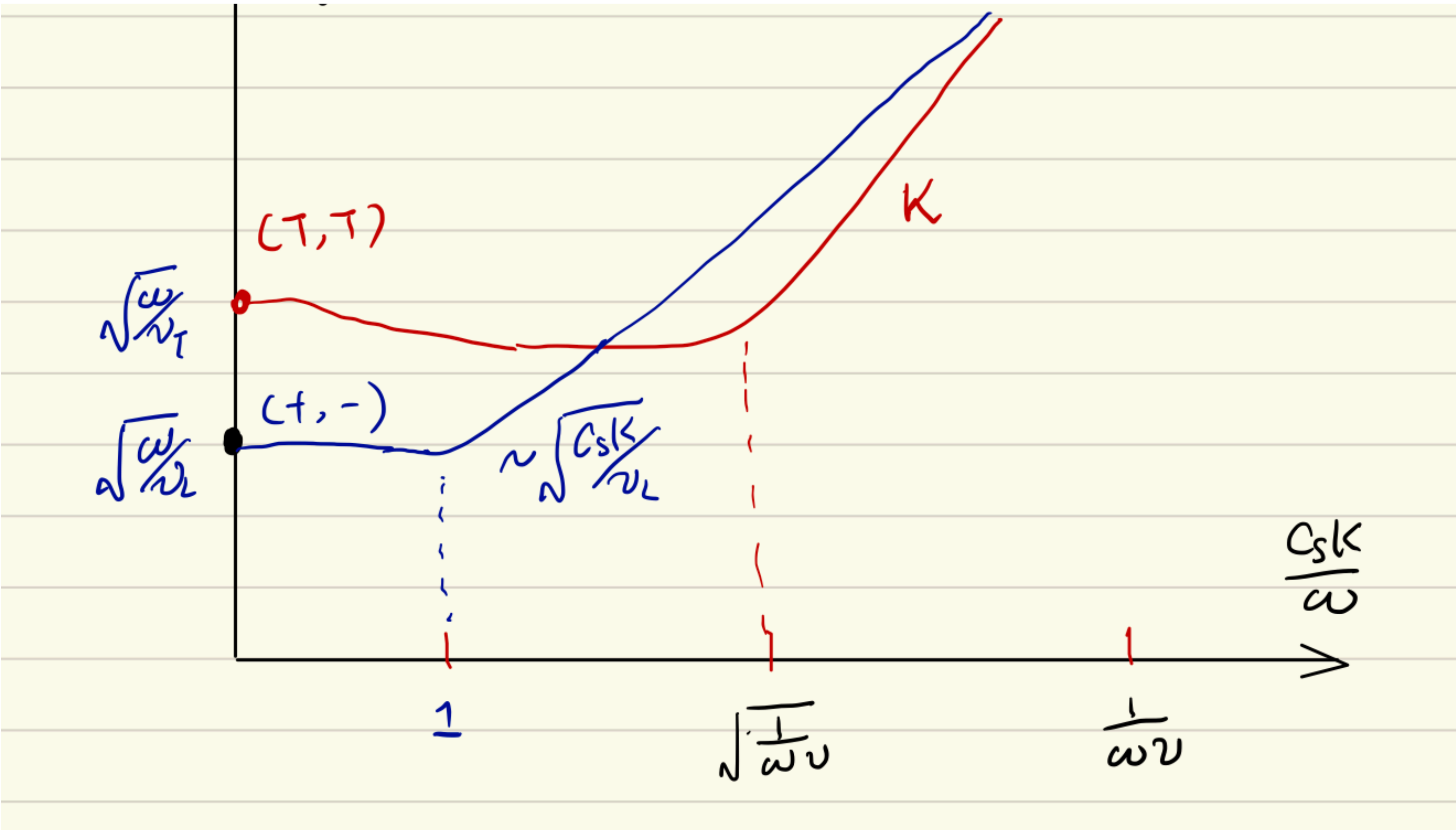
$$\omega - \Omega_A(|\vec{p} - \vec{k}/2|) - \Omega_B(|\vec{p} + \vec{k}/2|) = 0$$

and therefore depends on a) the "flavor" of hydro. modes in the loop and b) the hierarchy between frequency and momentum.

For (T,T): $P^{*,<}_{TT} = \sqrt{\frac{\omega}{\nu_T T}}, \quad \nu k^2 < \omega; \qquad P^{*,>}_{TT} = k, \quad \nu k^2 < \omega;$

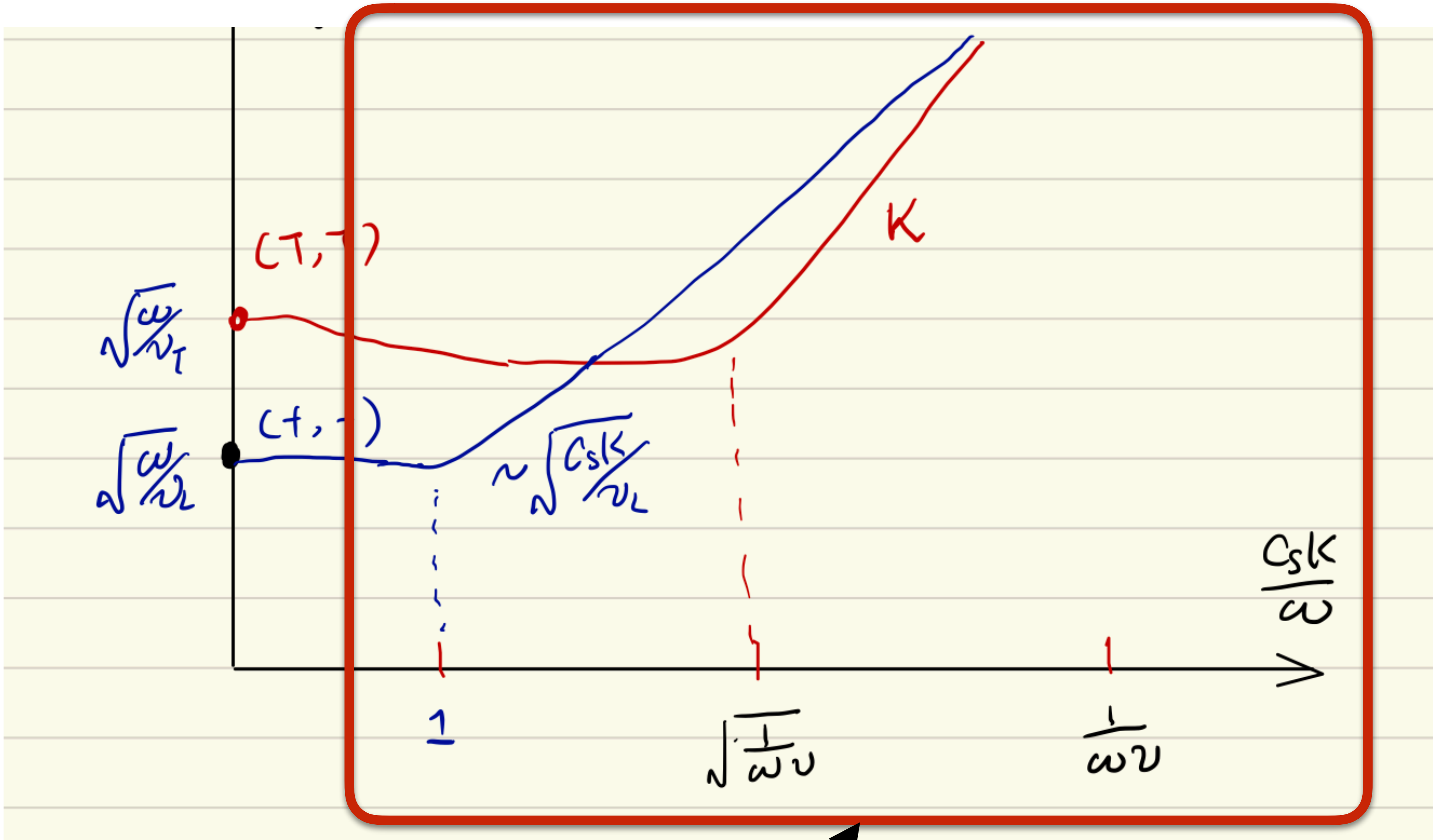
For (+, -): $P^{*,<}_{+-} = \sqrt{\frac{\omega}{\nu_L T}}, \quad c_s k < \omega; \qquad P^{*,>}_{+-} = k, \quad c_s k > \omega$

A summary on the computation of self-energy



Finite time and size correction to shear viscosity due to non-linearity

Going beyond hydro-kinetic approach



The regime which can not be matched with hydro-kinetic approach

Σ^{ar} : straightforward evaluation, surprising outcome.

(This talk) Focus on the (T, T) contribution in those bubble diagrams from now. Previous calculation obtains a simple relation:

$$(2w\beta^{-1})\mathrm{Im}\Sigma^{ar}\left(\omega,k\right) = \Sigma^{rr}\left(\omega,k\right) \qquad \text{or } \Delta\eta = w\Delta\nu_T$$

Our result is different:

$$(2w\beta^{-1})\mathrm{Im}\Sigma^{ar}\left(\omega,k\right) \neq \Sigma^{rr}\left(\omega,k\right) \qquad \text{or } \Delta\eta \neq w\Delta\nu_T$$

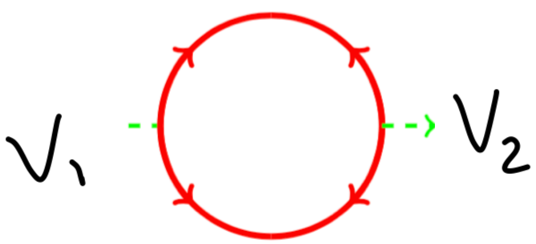
NB: since η, ν and w=ε+p will all be renormalized due to the non-linear effect. The fluctuation-dissipation relation η=w ν does not necessarily mean $\Delta\eta = w\,\Delta\nu_T$.

The origin of the difference

The relevant vortex (K denotes the momentum of the a-field):

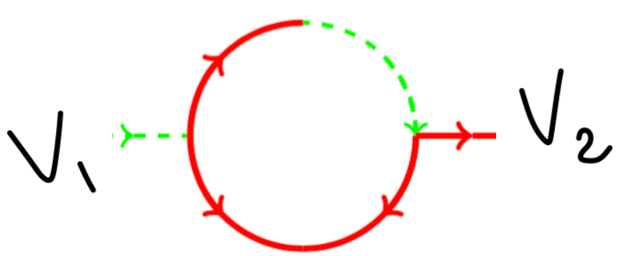
$$\delta_2(T^{\mu\nu}_{\rm hydro})\,\nabla_\mu X_\nu \to V_1^{\alpha\beta\nu} = \Delta^{\nu\alpha}(iK_\perp^\beta)\,, \qquad V_2^{\alpha\beta\nu} = c_s^2\Delta^{\alpha\beta}(iK_\perp^\nu)$$

For Σ^{aa} , the mixing between such two vortex vanishes.



$$P^T_{\mu\nu}(\hat{\boldsymbol{k}})\,V^{\alpha\beta\mu}_{\rm ideal}(\boldsymbol{k},\omega)\,V^{\rho\sigma\nu}_{\rm ideal}(\boldsymbol{k},\omega) = 0$$

For Σ^{ar} , the momentum of this a-field is internal and will survive from the result of the contraction.



$$P^T_{\mu\rho}(\hat{\boldsymbol{k}})\,V^{\alpha\beta\mu}_{\rm ideal}(k,\omega)\,V^{\rho\sigma\nu}_{\rm ideal}(p_+,\omega-p_0) \neq 0$$

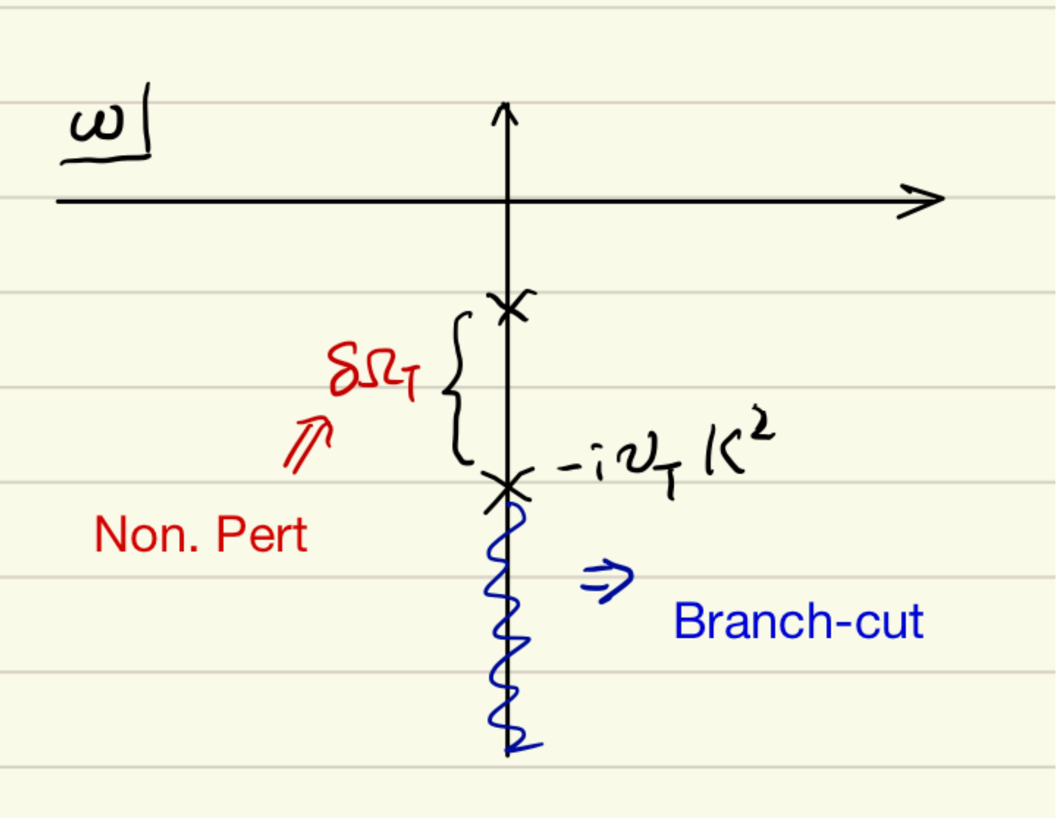
V_2 is due to the dependence of the pressure on the fluctuating "(inverse) temperature" current:

$$\delta_2 p(\beta = \sqrt{-\beta^\mu\cdot\beta_\mu})/w = \frac{1}{2}\lambda_\perp^2 + \dots \qquad (\beta^\mu = \beta_0 u^\mu + \beta_0\lambda^\mu)$$

Σ^{ar} *in complex frequency plane.*

$\Sigma^{ar}(\omega,k)$ and consequently $G^R(\omega,k)$ will feature a branch point at $\omega=-i\,\nu_T\,k^2$.

$$\Sigma^{ar}(\omega, k) \sim \frac{k^2}{2s\nu_T}\int_{k/2}^{\Lambda} dp \frac{p^2}{\omega + 2i\nu_T(p^2 + k^2/2)}$$



We examine modification of the shear dispersion relation

$$\delta_T(k) \equiv 1 - \left(\frac{-\omega}{i\nu k^2}\right)$$

Although δ_T is small for small $(k/s\nu^2)$, its magnitude is enhanced *logarithmically*.

$$\delta_T + \left(\frac{c_0 k}{s\nu_T^2}\right)\log(\delta_T) = 0$$

The non-linear effects on shear dispersion relation is non-perturbative!

Conclusion

Conclusion and discussion

We studied the finite frequency and finite size effects on the hydrodynamic Green function (shear channel) based on EFT approach.

The physics of hydro. fluctuations are rich in scales because of the nonlinear coupling between sound and shear modes.

Some of the qualitative features can not be described in the current hydro-kinetic approach.

Capturing those non-linear effects in the studies of expanding QGP calls for *new developments.*

Back-up

The expansion of ideal part of constitutive relation

$$\delta_2(\beta^{-1}\eta\Delta^{\mu\alpha}\Delta^{\nu\beta}\ldots)\,\nabla_\mu X_\alpha \nabla_\nu X_\beta \to V_{aar}$$

Vortex from the viscous part of the action

Three approaches for studying hydro. with thermal fluctuations in general.

I. Stochastic hydro. approach: (adding noise to hydro. equations).

Landau-Lifshitz, Statistical Mechanics; Kapusta-Mueller-Stephanov, PRC ’11;…

Nahrgang-Bluhm-Schaefer-Bass, 1804.05728 (with a critical E.o.S);Sakai’s talk

II. “Effective field theory” (EFT) approach: formulating hydro on the Schwinger-Keldysh contour.

Kovtun-Moore-Romatschke, JHEP 14’; Glorioso-Crossley-Liu, JHEP 17’; Haehl-Loganayagam-Rangamani, 1803.11155, …

III: Treating off-equilibrium fluctuations as slow modes in additional to “hydro” modes.

⇒ Coupled deterministic equation.

Kawasaki, Ann. Phys. ’70; Andreev, JTEP, ‘1971; …

“hydro-kinetic”, Akamatsu-Mazeliauskas-Teaney, PRC 16, PRC ’18 (*Bjorken-flow*);

“hydro+”, Stephanov-YY, 1712.10305 (*near a critical point*)

Theoretical perspective: fruitful cross-fertilization among different approaches

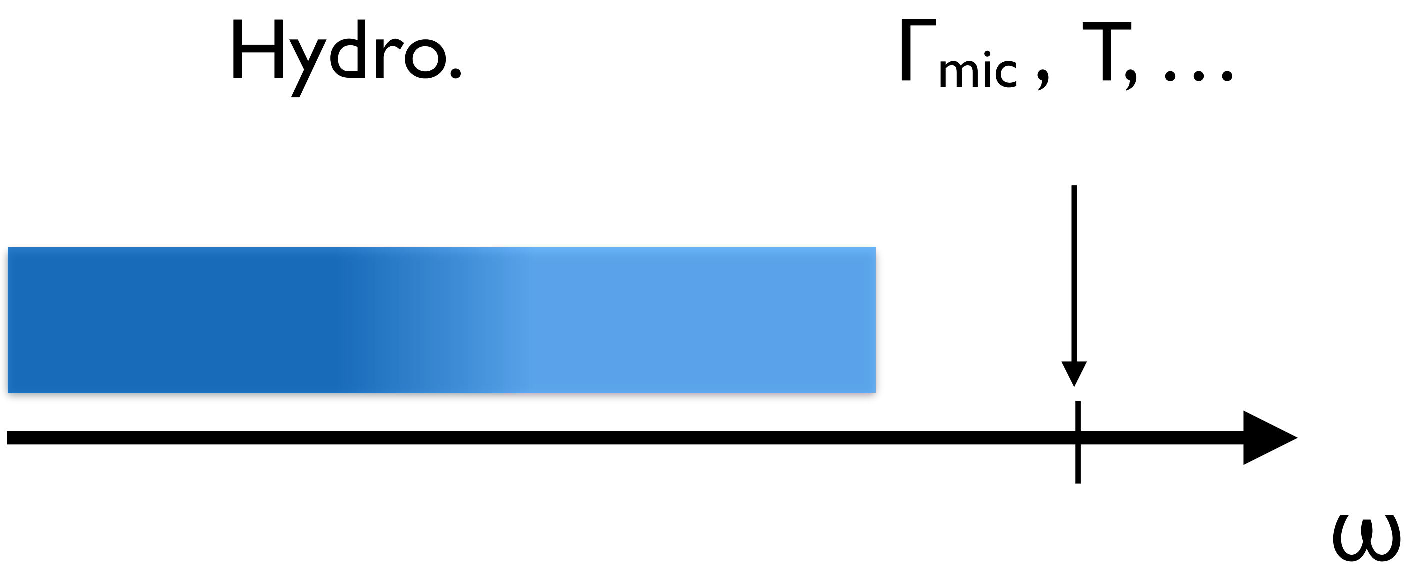
The identification of non-equilibrium legnth $l_{n.e}$ from "approach II" is instrumental for the recent progress on 3d stochastic hydro simulations ("approach I", by McGill group)

Applying Wilsonian method to EFT approach, equations in "approach II" emerge.

Liu Hong-Lau-YY, in progress

Near future: direct comparison between "approach I" and "approach II" numerically.

Story gets interesting as we attack the same problem from complementary ways.


Hydro.
Γ_{mic} , T, …
ω