Anomalous magneto-hydrodynamics and toward a full solution of complete relativistic Boltzmann equation on GPUs

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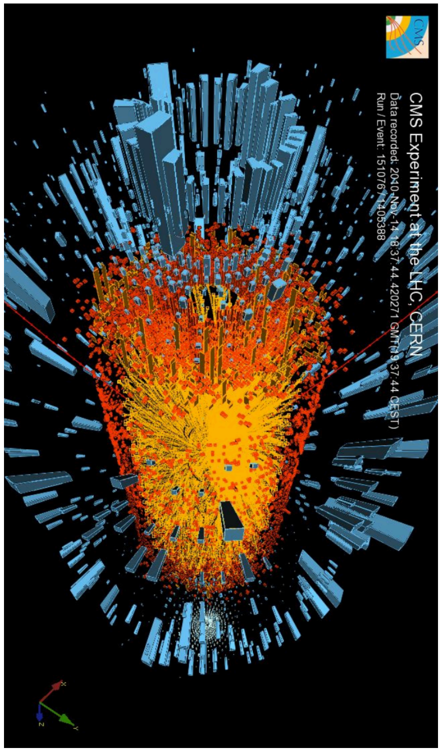
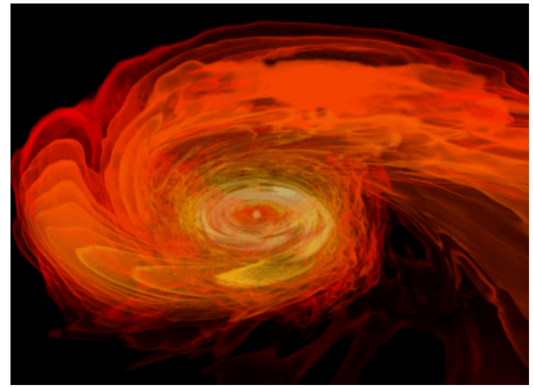
New development of hydrodynamics and its applications in Heavy-Ion Collisions
Oct. 30-Nov. 2, 2019, Fudan University

Outlines



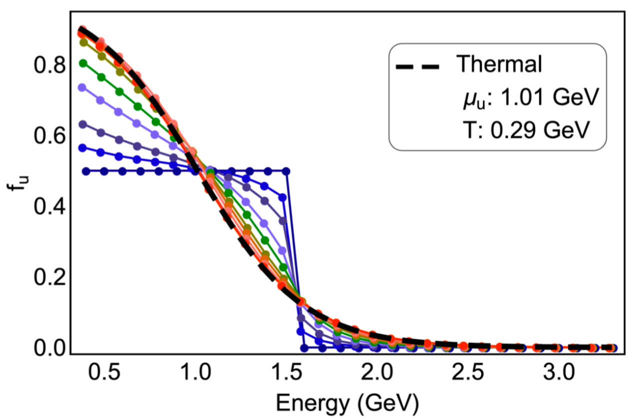
Heavy ion collisions

macroscopic



1. Anomalous magneto-hydrodynamics

microscopic



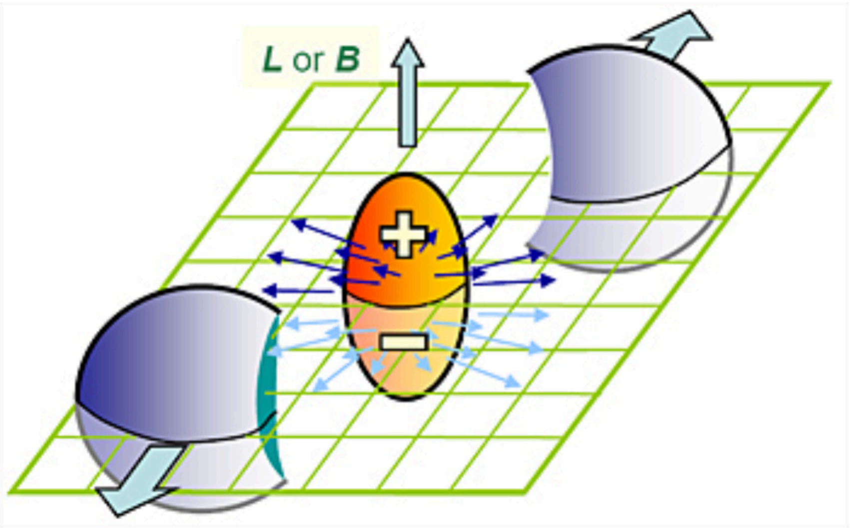
2. Solving Boltzmann equations on GPUs

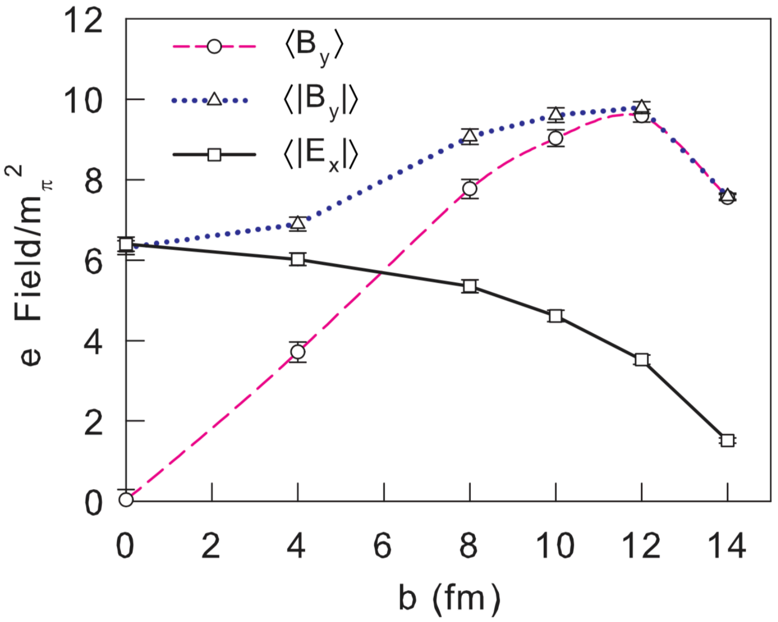
1. Anomalous MagnetoHydroDynamics

Irfan Siddique, Ren-jie Wang, Shi Pu, and Qun Wang, PRD 2019

Strong Electromagnetic fields

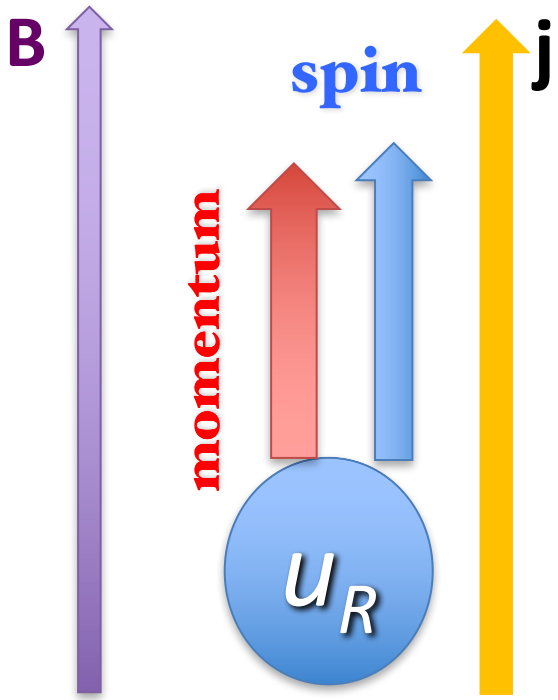
- Theoretical estimation:
  Lienard-Wiechert potential + Event-by-event



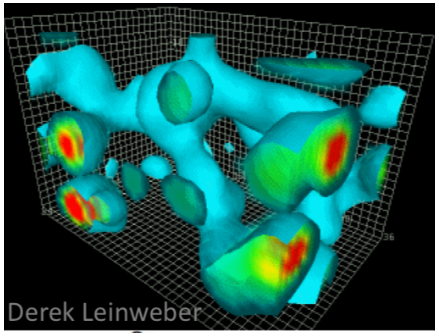


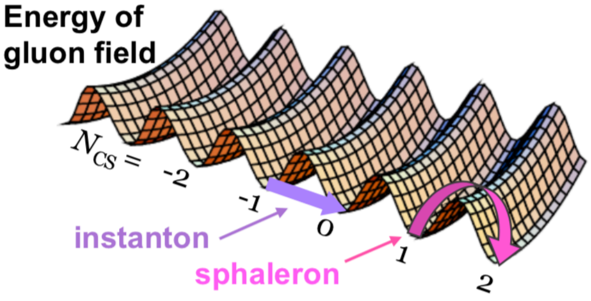
A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015; H. Li, X.l. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013

Chiral Magnetic Effect



- Magnetic fields
- Nonzero axial chemical potential
- Number of Left handed fermions ≠ Number of Right handed fermions





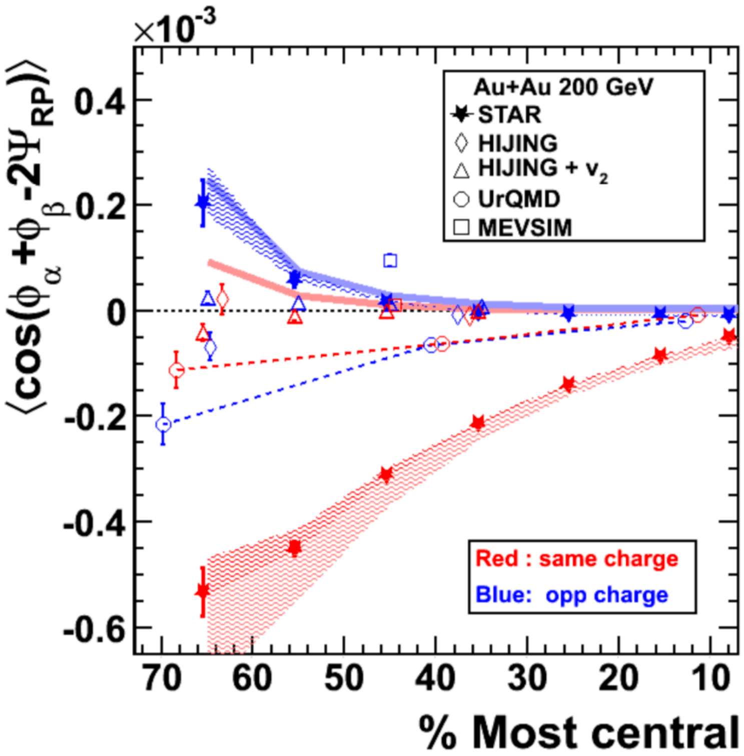
- Charge current: charge separation

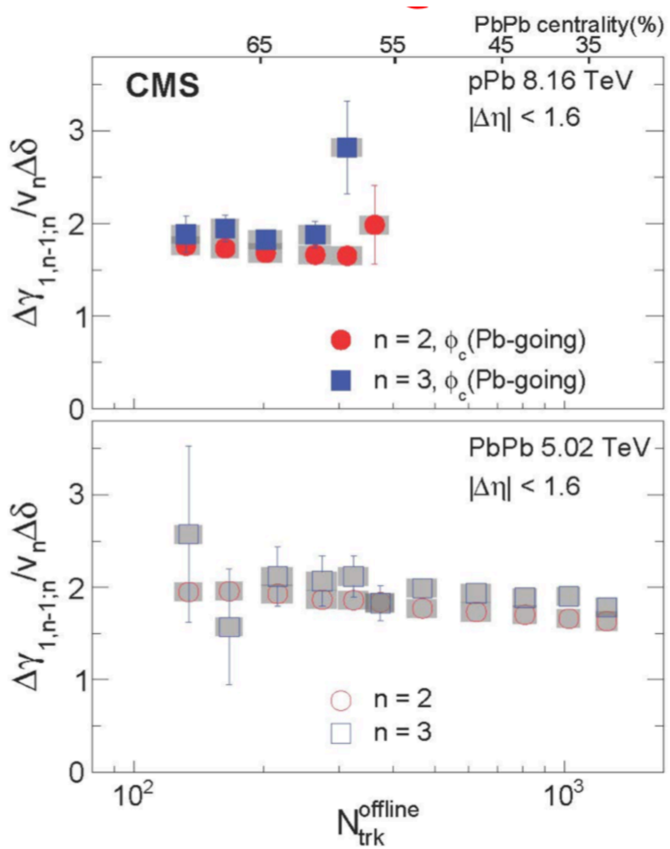
$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B},$$

Kharzeev, Fukushima, Warrigna, (08,09), etc. ...

Experiments: signal VS background

Waiting for the results from Isobar





STAR PRL 103, 251601(2009); PRC 81, 054908

CMS PRL 118, 122301 (2016); PRC 97, 044912

Anomalous MagentoHydroDynamics

- Conservation equations :
  - Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0, \qquad T^{\mu\nu} = T_F^{\mu\nu} + T_{EM}^{\mu\nu}.$$

Fluid part **Electromagnetic part**

  - (anomalous) currents conservation

$$\partial_\mu j_e^\mu = 0, \qquad \partial_\mu j_5^\mu = -e^2 C E \cdot B,$$

Electric Charge current **Chiral current**

- Maxwell' s equation :

$$\partial_\mu F^{\mu\nu} = j_e^\nu, \qquad \partial_\mu(\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0.$$

Previous Studies: ideal MHD without CME

- Preview studies:

➢ 1+1 D ideal MHD Bjorken flow

V.Roy, SP, L. Rezzolla, D. Rischke, Phys.Lett. B750, 45

– With magnetization effects

SP, V. Roy, L. Rezzolla, D. Rischke,Phys.Rev. D93, 074022

➢ 2+1 D ideal MHD Bjorken flow (perturbative)

SP, Di-Lun Yang, Phys.Rev. D93, 054042

➢ Background Magnetic field: contribution to v2

V.Roy, SP, L. Rezzolla, D. Rischke, Phy.Rev. C96, 054909

- Problem:

How to add the CME to Magentohydrodynamics?

Beyond ideal limit of MHD

- Ideal limit of MHD:
  - Electric conductivity is infinite $\sigma \to \infty$

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \Rightarrow \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Maxwell's equation

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = -\nabla \times (\mathbf{v} \times \mathbf{B})$$

- No space for the CME

$$\nabla \times \mathbf{B} = \mathbf{j} + \partial_t \mathbf{E}$$

- **Anomalous MHD needs finite conductivity**

Constitution Eqs. for Anomalous MHD

$$\partial_\mu T^{\mu\nu} = 0,$$

$$T^{\mu\nu} = (\varepsilon + p + E^2 + B^2)u^\mu u^\nu - (p + \frac{1}{2}E^2 + \frac{1}{2}B^2)g^{\mu\nu}$$
$$-E^\mu E^\nu - B^\mu B^\nu - u^\mu \epsilon^{\nu\lambda\alpha\beta}E_\lambda B_\alpha u_\beta - u^\nu \epsilon^{\mu\lambda\alpha\beta}E_\lambda B_\alpha u_\beta,$$

$$\partial_\mu j_e^\mu = 0,$$

Electric Conducting flow

CME

$$j_e^\mu = n_e u^\mu + \sigma E^\mu + \xi B^\mu,$$

$$j_5^\mu = n_5 u^\mu + \sigma_5 E^\mu + \xi_5 B^\mu,$$

CSE

$$\partial_\mu j_5^\mu = -e^2 C E \cdot B,$$

$$\partial_\mu F^{\mu\nu} = j_e^\nu,$$

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta}u_\alpha B_\beta,$$

$$\partial_\mu(\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}) = 0.$$

Equation of States (EoS)

- High (chiral) chemical potential:

Energy density → ε ; pressure → p

$$\varepsilon = c_s^{-2} p,$$

$$n_e = a\mu_e(\mu_e^2 + 3\mu_5^2),$$

$$n_5 = a\mu_5(\mu_5^2 + 3\mu_e^2),$$

Chemical potential → μ_e ; Chiral chemical potentia → μ_5

- High temperature:

$$\varepsilon = c_s^{-2} p,$$

$$n_e = a\mu_e T^2,$$

$$n_5 = a\mu_5 T^2,$$

temperature → T

Bjorken boost invariance

- Profound Bjorken velocity

$$u^\mu = \gamma(1, 0, 0, z/t),$$

- Bjorken invariance: all observed quantity are independent on rapidity.

- Could the Bjorken velocity hold in electromagnetic fields?

Simplification

- Neutral fluid:
  - Electric field will not accelerate the fluid
- Force-free-like magnetic field:
- Configuration of Electromagnetic fields:

$$E^\mu = (0, 0, \chi E(\tau), 0), \; B^\mu = (0, 0, B(\tau), 0),$$

$$\chi = \pm 1$$

τ: proper time

$$E^\mu = F^{\mu\nu} u_\nu, \; B^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta},$$

Not EM fields
In lab frame!

Results: High temperature EoS

- Analytic solutions: (at the order of hbar):

➢ chiral density

$$n_5(\tau) = n_{5,0}\left(\frac{\tau_0}{\tau}\right)\{1 + a_2 e^{\sigma\tau_0}[\mathrm{E}_1(\sigma\tau_0) - \mathrm{E}_1(\sigma\tau)]\},$$

➢ Energy density

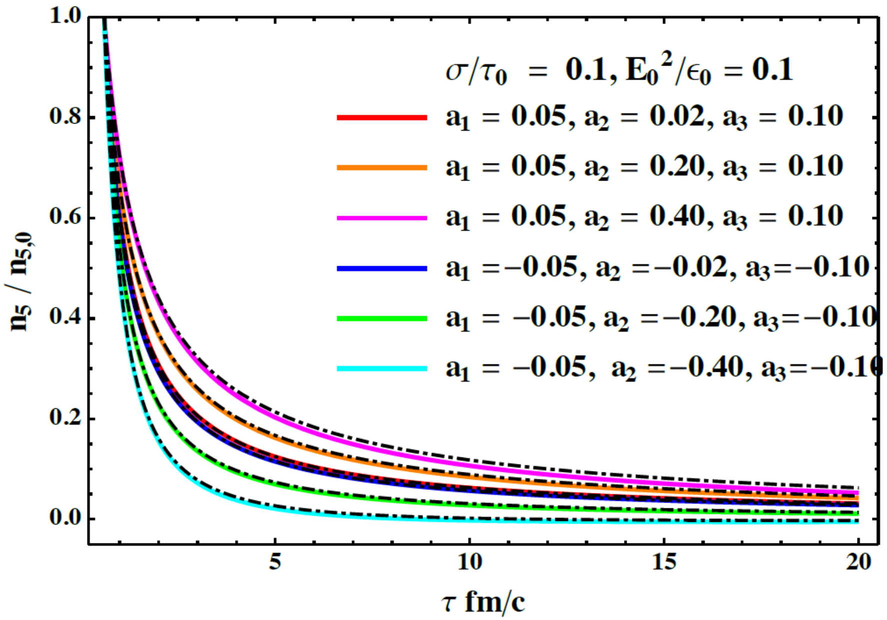
τ: proper time
σ: electric conductivity

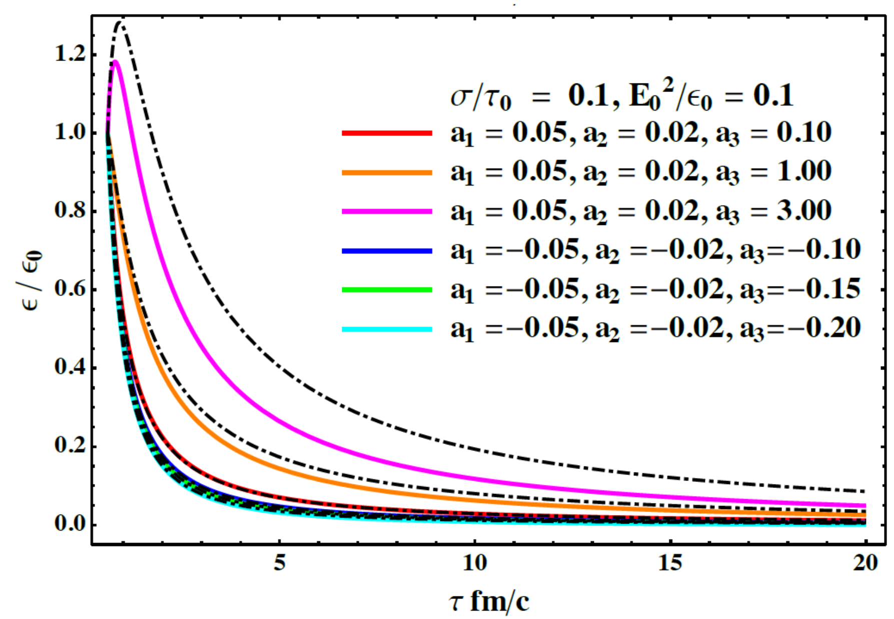
$$\varepsilon(\tau) = \epsilon_0\left(\frac{\tau_0}{\tau}\right)^{1+c_s^2}\left\{1 + \sigma\frac{E_0^2}{\varepsilon_0}e^{2\sigma\tau_0}[\tau_0\mathrm{E}_{1-c_s^2}(2\sigma\tau_0) - \tau\left(\frac{\tau}{\tau_0}\right)^{c_s^2-1}\mathrm{E}_{1-c_s^2}(2\sigma\tau')] + \frac{a_3}{\tau_0}e^{\sigma\tau_0}[\tau_0\mathrm{E}_{2-3c_s^2}(\sigma\tau_0) - \tau\left(\frac{\tau_0}{\tau}\right)^{2-3c_s^2}\mathrm{E}_{2-3c_s^2}(\sigma\tau)]\right\}.$$

a1,a2,a3 are related to the initial EM fields and chirality density

En(x): the generalized exponential integral. $\mathrm{E}_n(z) \equiv \int_1^\infty dt t^{-n} e^{-zt}$

Analytic solution VS numerical results





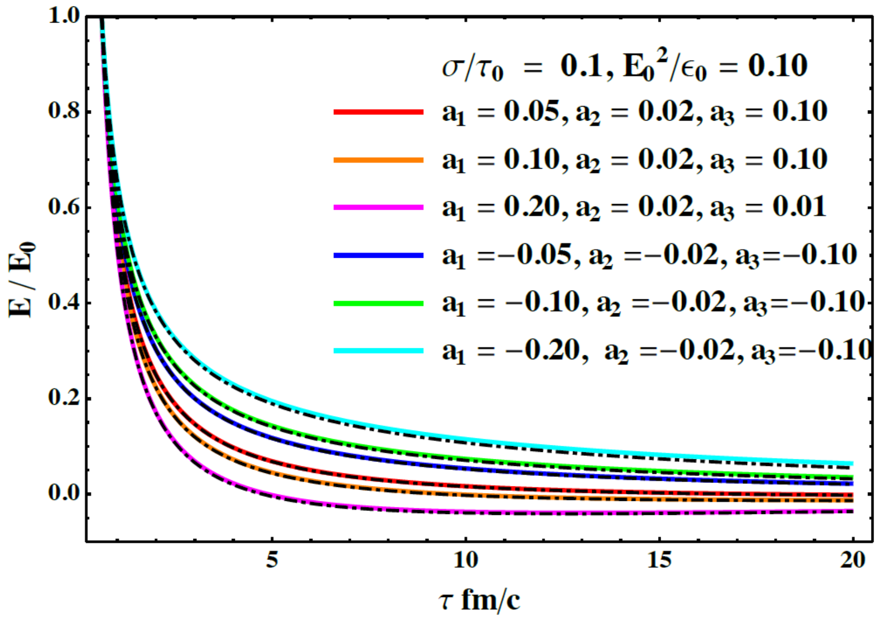
<u>Solid line: numerical results / Dashed line: analytic</u>

$$a_1 = eC\chi\frac{B_0 n_{5,0}}{aT_0^2 E_0}\tau_0, \qquad a_2 = \frac{e^2 C\chi E_0 B_0}{n_{5,0}}\tau_0,$$

$$a_3 = \frac{eC\chi}{a}\frac{n_{5,0}E_0 B_0}{\varepsilon_0 T_0^2}\tau_0.$$

C: chiral anomaly
Coefficients = $\hbar/(2\pi^2)$

Electromagnetic fields in the Lab frame



$$\mathbf{E}_L = (\gamma v^z B(\tau),\ \chi\gamma E(\tau),\ 0),$$

$$\mathbf{B}_L = (-\gamma v^z \chi E(\tau),\ \gamma B(\tau),\ 0),$$

τ: proper time

σ: electric conductivity

$$E(\tau) = E_0\left(\frac{\tau_0}{\tau}\right)\left\{e^{-\sigma(\tau-\tau_0)} - a_1 e^{-\sigma\tau}\left[\mathrm{E}_{1-2c_s^2}(-\sigma\tau_0) - \left(\frac{\tau}{\tau_0}\right)^{2c_s^2}\mathrm{E}_{1-2c_s^2}(-\sigma\tau)\right]\right\},$$

$$B(\tau) = B_0\frac{\tau_0}{\tau},$$

$$a_1 = eC\chi\frac{B_0 n_{5,0}}{aT_0^2 E_0}\tau_0,$$

Decay behavior in the Lab frame:

- By decays ~ 1/τ
- Bx decays ~ exp(- σ τ)/ τ

Much slower than decaying in vacuum

Why Bz and Ez vanish?

- We have checked the Maxwell's eq. in Lab frame.
  Space and time derivatives of Bz, Ez are zero.
- Key point: the currents is different with static case!

$$\nabla \times \mathbf{B}_L = \mathbf{j}_e + \partial_t \mathbf{E}_L.$$

$$\mathbf{j}_{e,||} = \sigma \mathbf{E}_{L,||} + \xi \mathbf{B}_{L,||},$$

$$\mathbf{j}_{e,\perp} = \sigma\gamma(\mathbf{E}_L + \mathbf{v}\times\mathbf{B}_L)_\perp + \xi\gamma(\mathbf{B}_L - \mathbf{v}\times\mathbf{E}_L)_\perp,$$

v: three vector of fluid velocity

- Similar to the force-free EM fields in classical electrodynamics (e.g. Woltjer states)

Qin, Liu, Li, Squire, PRL 109, 235001 (2012);
Xia, Qin, Q. Wang, PRD(2016)

Summary of Anomalous MHD

- Anomalous MHD:

  Hydrodynamic eq. + Maxwell's eq. + Chiral currents

- Analytic solutions of anomalous MHD in Bjorken flow with transverse EM fields

- Decay behavior of EM fields:
  - In lab frame, B field decays much slower than in the vacuum
  - By decays like ~ $1/\tau$,
  - Bx decays like~ $\exp(-\sigma\tau)/\tau$

Microscopic theory: Chiral kinetic theory

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\mathbf{x}} \cdot \nabla_x f + \sqrt{G}\dot{\mathbf{p}} \cdot \nabla_p f = C[f].$$

- **Particle's effective velocity:**

$$\sqrt{G}\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \hbar \left(\frac{\partial \varepsilon}{\partial \mathbf{p}} \cdot \mathbf{\Omega} \right) \mathbf{B} + \hbar \mathbf{E} \times \mathbf{\Omega},$$

- **Effective force:**

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B} + \hbar (\mathbf{E} \cdot \mathbf{B}) \mathbf{\Omega},$$

- **Berry curvature**

$$\sqrt{G} = 1 + \hbar \mathbf{B} \cdot \mathbf{\Omega}, \qquad \mathbf{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3},$$

2. Toward a full solution of complete Relativistic Boltzmann equation on GPUs

Ref: Jun-jie Zhang, Hong-zhong Wu, SP, Guang-you Qin, Qun Wang, in preparation

Relativistic Boltzmann Equations (BE)

- Boltzmann equations: kinetic theory
  - microscopic theory, many body-systems
- Many applications in non-equlibirum systems:
  - Turbulence, self-similar behavior
  - Simulations for heavy ion collisions
  - Gluon condensation

Relativistic BE

$$\frac{\partial}{\partial t} f_p + \frac{\partial \mathbf{x}}{\partial t} \cdot \nabla_x f_p + \frac{\partial \mathbf{p}}{\partial t} \cdot \nabla_p f_p = C[f_p]$$

velocity

force

Collisional term

f: distribution function= function of (t,x,p)

- It is a semi-classical description of many body theory.

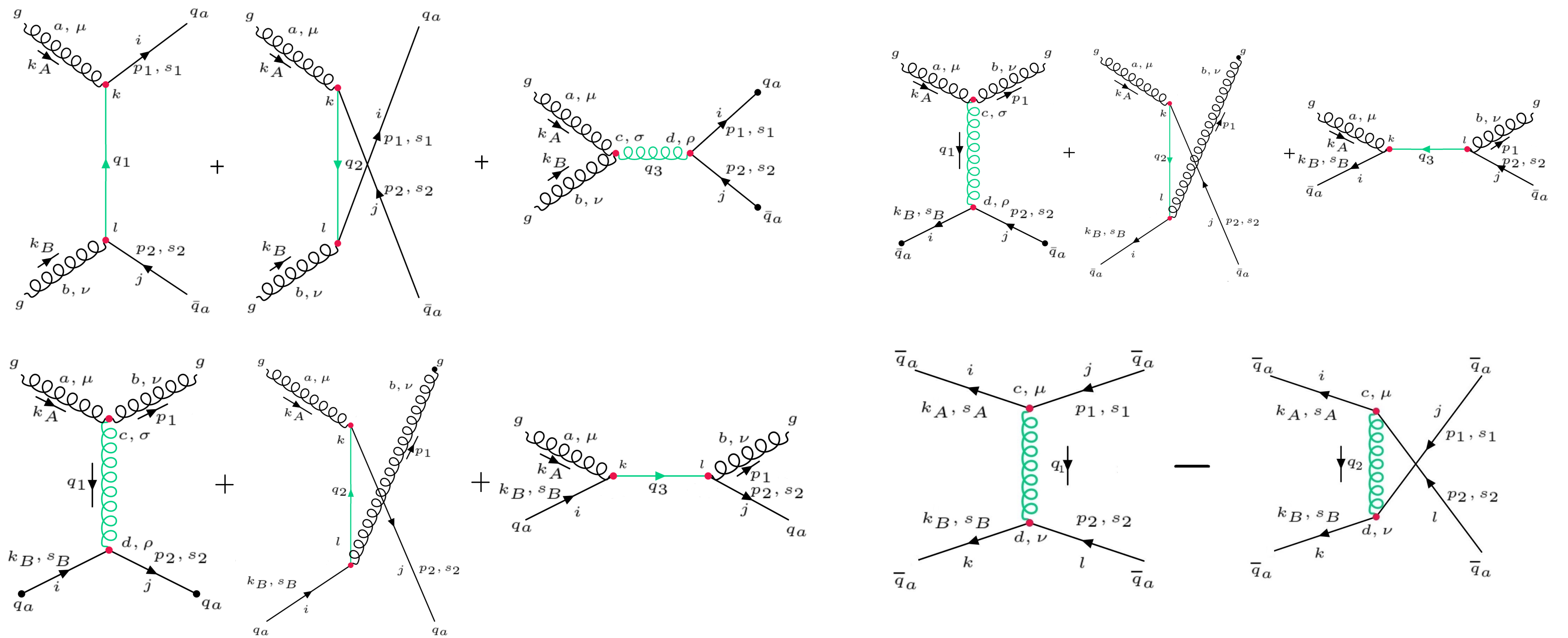
One can derive it symmetrically from quantum field theory with closed-time-path formulism.

(*Blaizot, Iancu, Phys. Rep. 359, 355 (2002).*)

Collisional term

$$C_{ab\to cd} \equiv \int \prod_{i=1}^{3} \frac{d^3k_i}{(2\pi)^3 2E_{k_i}} \frac{|M_{ab\leftrightarrow cd}|^2}{2E_p} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_3 - p)[f^a_{k_1} f^b_{k_2} F^c_{k_3} F^d_p - F^a_{k_1} F^b_{k_2} f^c_{k_3} f^d_p],$$

For 2->2 scatterings including quarks and gluons: e.g.



Main difficulties for relativistic BE

$$\frac{\partial}{\partial t} f_p + \frac{\partial \mathbf{x}}{\partial t} \cdot \nabla_x f_p + \frac{\partial \mathbf{p}}{\partial t} \cdot \nabla_p f_p = C[f_p]$$

- 6+1 dimensional phase space
  - (3 coordinates, 3 momentum, 1 time)
- Extremely complicated collisional term
  - 5 dimensional integrals including many terms

Assuming we only have 10 grids in all space and momentum directions, so totally, we have 10^6 grids. We choose the time step could be 1000. Totally, we need to compute 10^9 times high dimensional integrals. Assuming that it costs 1 sec to compute one 5 dim integral, then totally it will cost 30 years!

Other issues in relativistic BE

- Particle number non-conservation:
  comes from errors of collisional integrals.

- A usual way: Test particle method;
  requires many parameters.

Could we solve a complete BE directly?
Or, with minimal parameters?

What have we done?

- A new numerical framework on GPUs :

➢ A **full solution** of **complete** relativistic BE

➢ High performance

➢ Particle number is strictly conserved

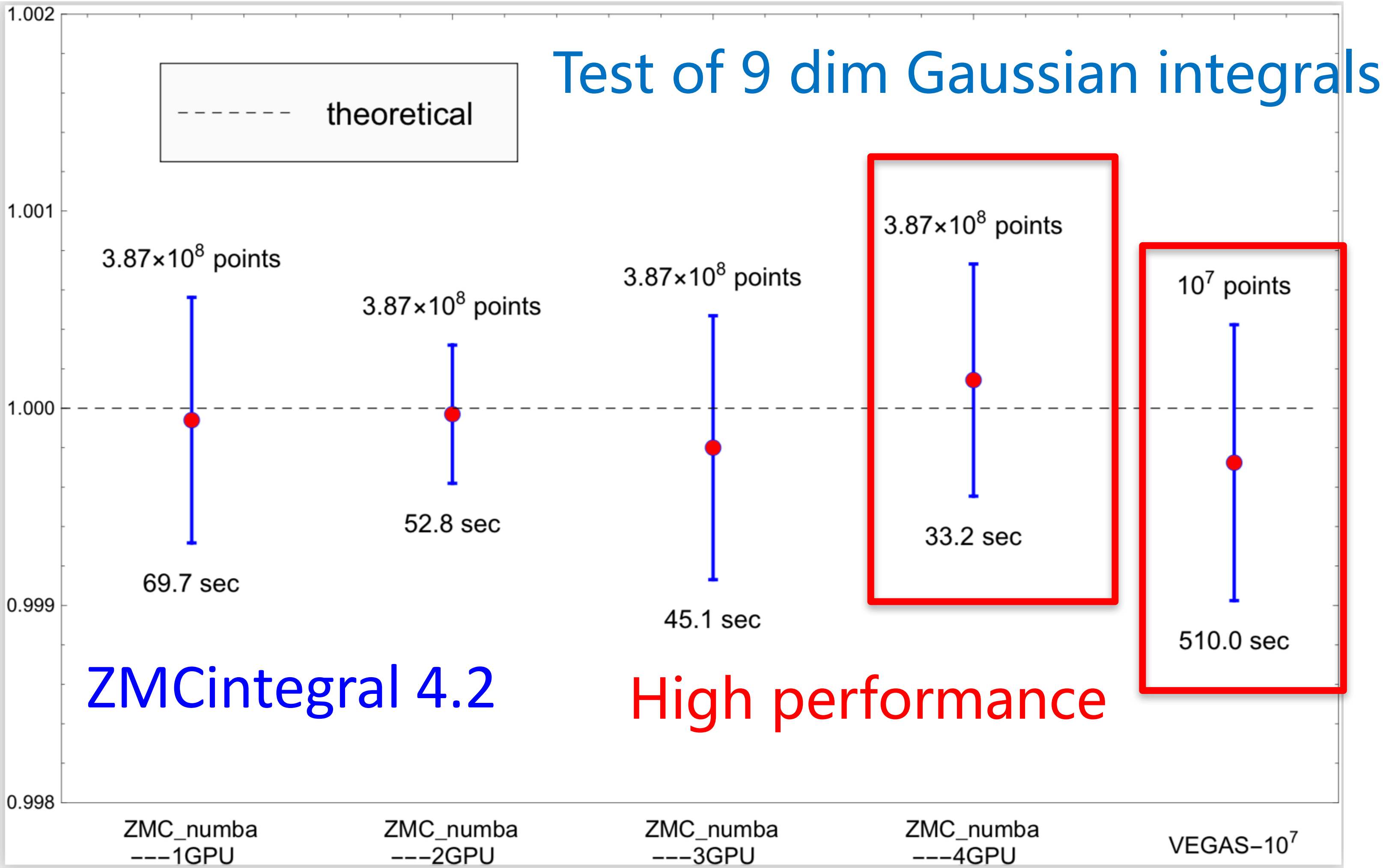
Parameters

- Physical parameters:
  - ➢ coupling constant; initial conditions.
- Parameters for simulations:
  - ➢ Size, number of grids.
- Minimal parameters !

Collisional term via ZMCintegral

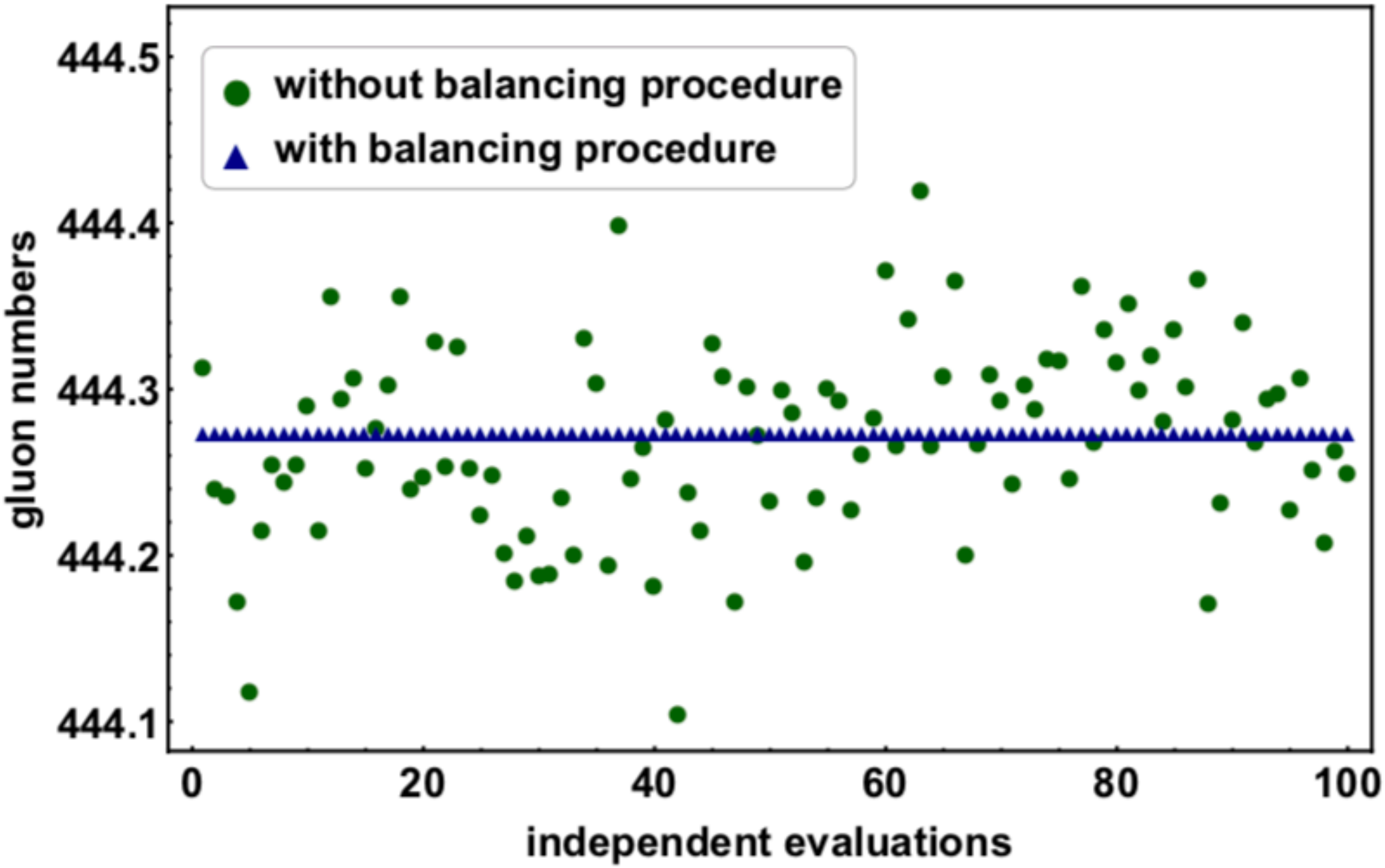
- 5 dimensional integral on each phase space grid:

Wu, Zhang, Pang, Q. Wang, 1902.07916. accepted in Computer Physics Communications



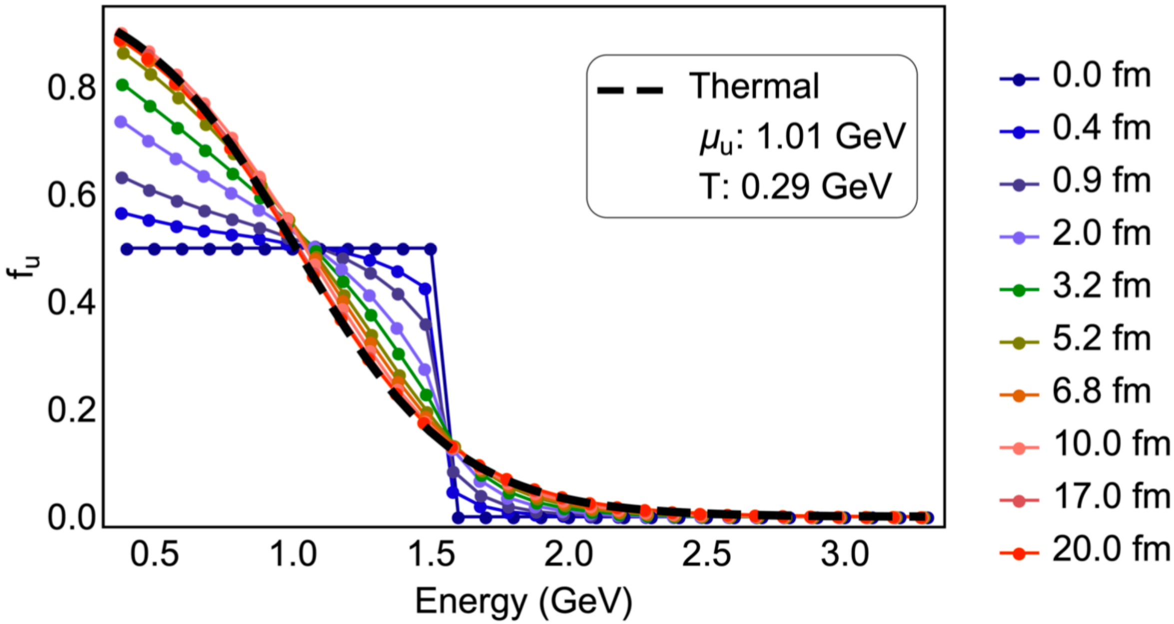
Symmetrical sampling method on GPUs

We introduce a new method to **ensure particle number conservation**.



Time evolution

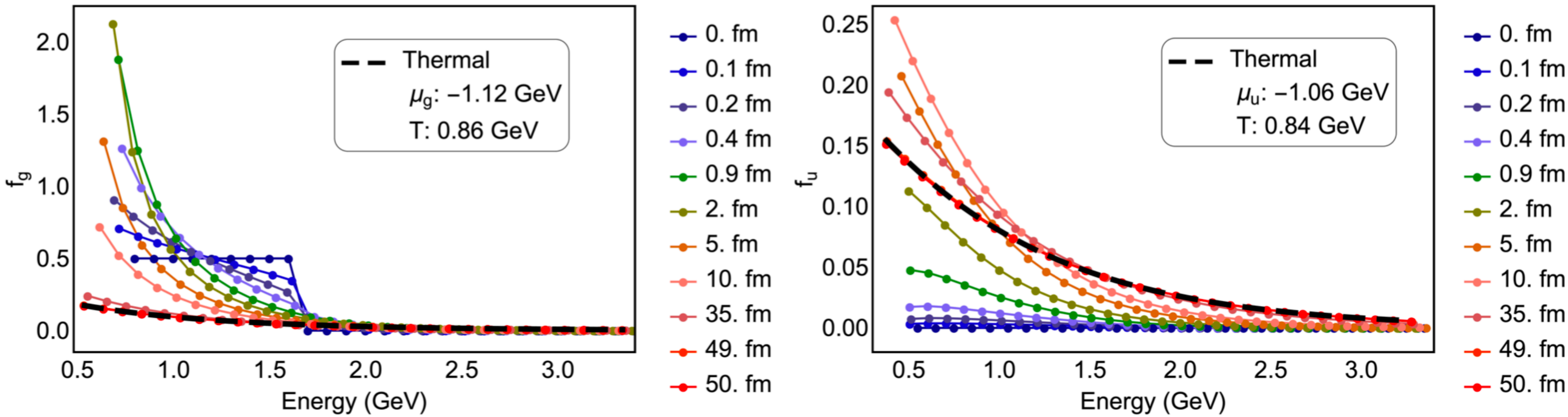
Pure quark case



- space: 1 grid; momentum: 30x30x30=27,000
- Phase space box is of size $[-3\text{fm}, 3\text{fm}]^3 \times [-2\text{GeV}, 2\text{GeV}]^3$.
- Time step dt=0.001fm； 20,000 steps
- on one Nvidia Tesla V100 card: costs around 2 hours

Time evolution

Gluons + quarks



- space: 1 grid; momentum: 30x30x30=27,000
- Phase space box is of size $[-3\text{fm}, 3\text{fm}]^3 \times [-2\text{GeV}, 2\text{GeV}]^3$.
- Time step dt=0.0005fm； 100,000 steps
- on one Nvidia Tesla V100 card: costs around 50 hours

Possible applications

- Gluon condensation
- Effects of inelastic scatterings
- Turbulence, self-similar behavior, non-thermal fixed points
- With strong electromagnetic fields

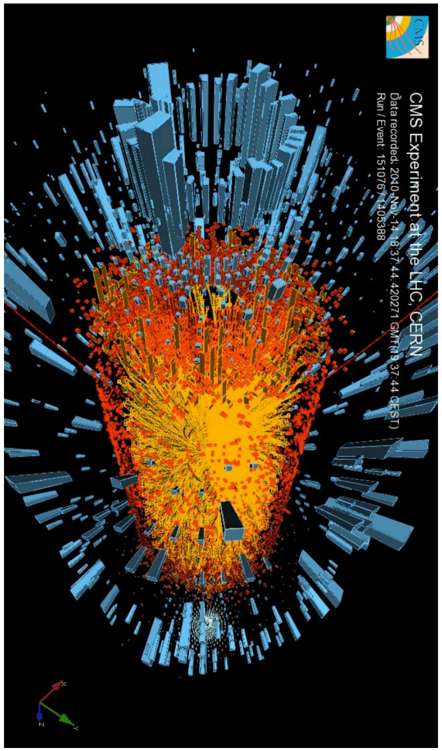
Summary for BE

- We introduce a new **numerical framework** to derive **full solutions** of a **complete relativistic BE** on **GPUs**.

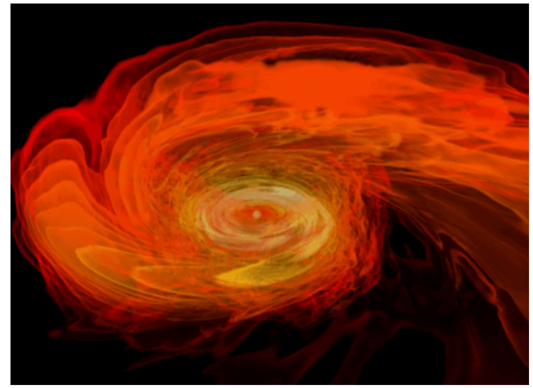
- Full collisional term: high dimensional integrals.
- High performance:
  space 10x10x10, momentum 30x30x30, Time steps: 10^4-10^6, on one Nvidia Tesla V100 card costs a few days!
- Particle number is strictly conserved.

- We did not use any technic like, deep learning, AI, Neural Network. We just program on GPUs.

Summary

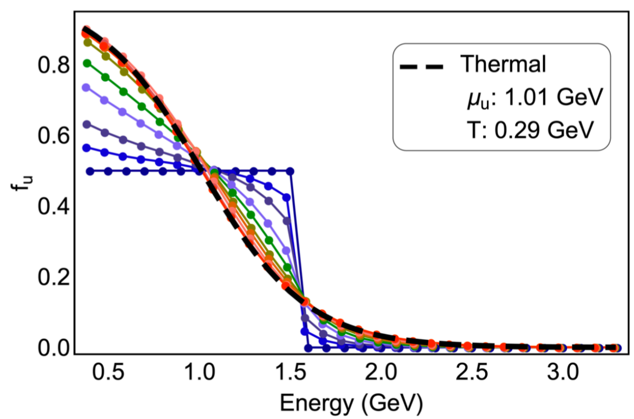


macroscopic



1. Anomalous magneto-hydrodynamics

microscopic

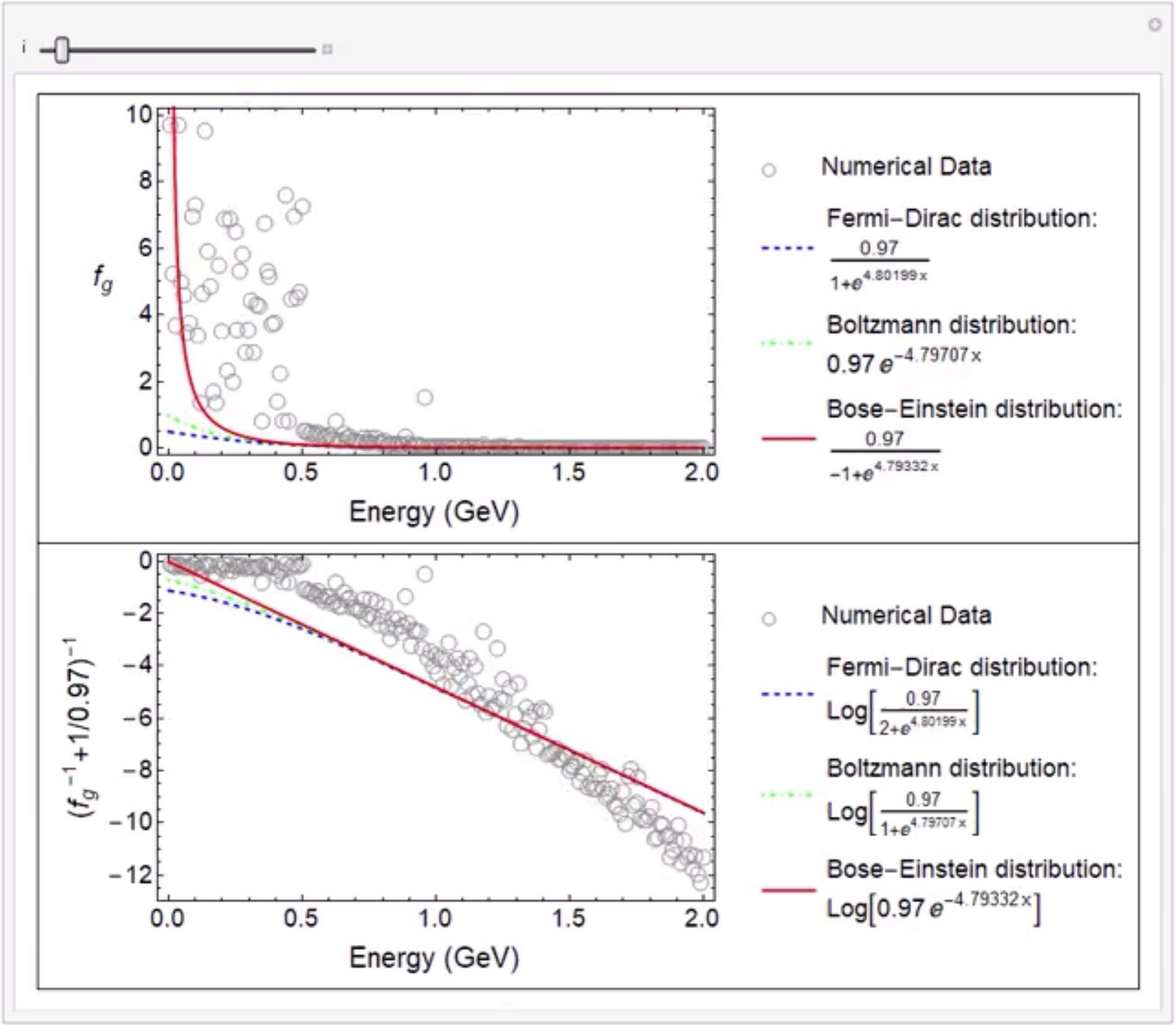


2. Solving Boltzmann equations on GPUs

Heavy ion collisions

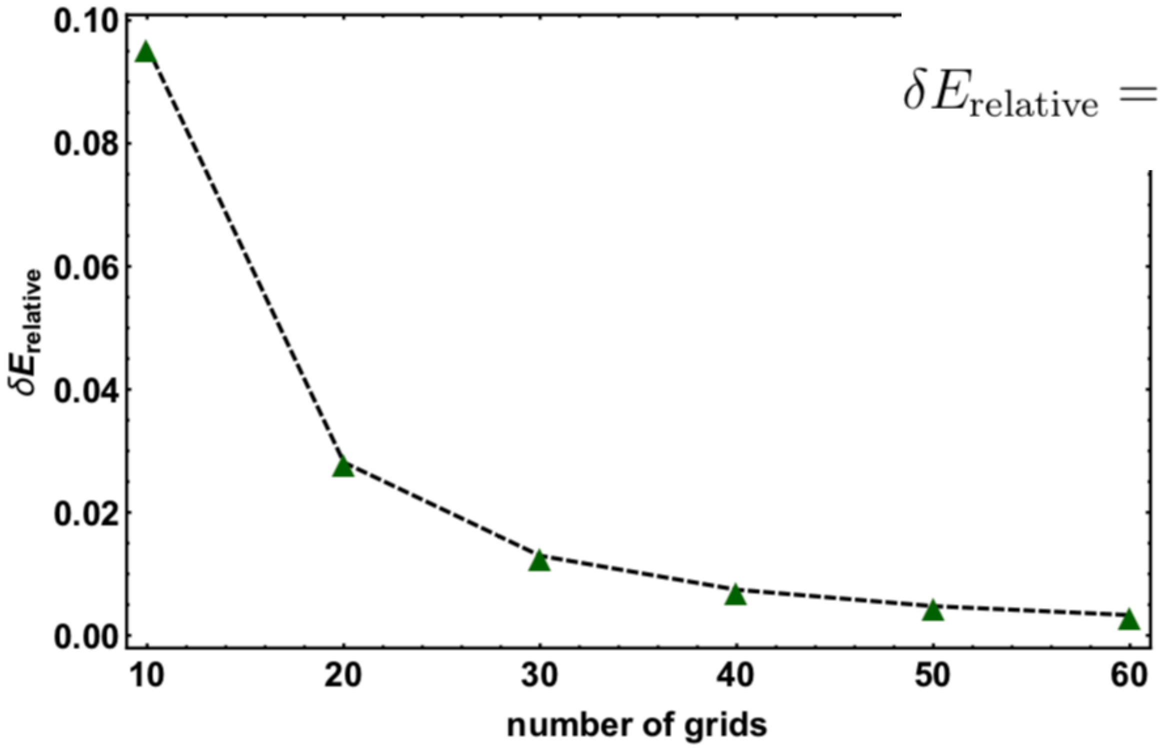
Thank you for your time!

Time evolution



Energy conservation

- Not conserved in HTL (physically)
- Not conserved (from errors): because of discrete grid

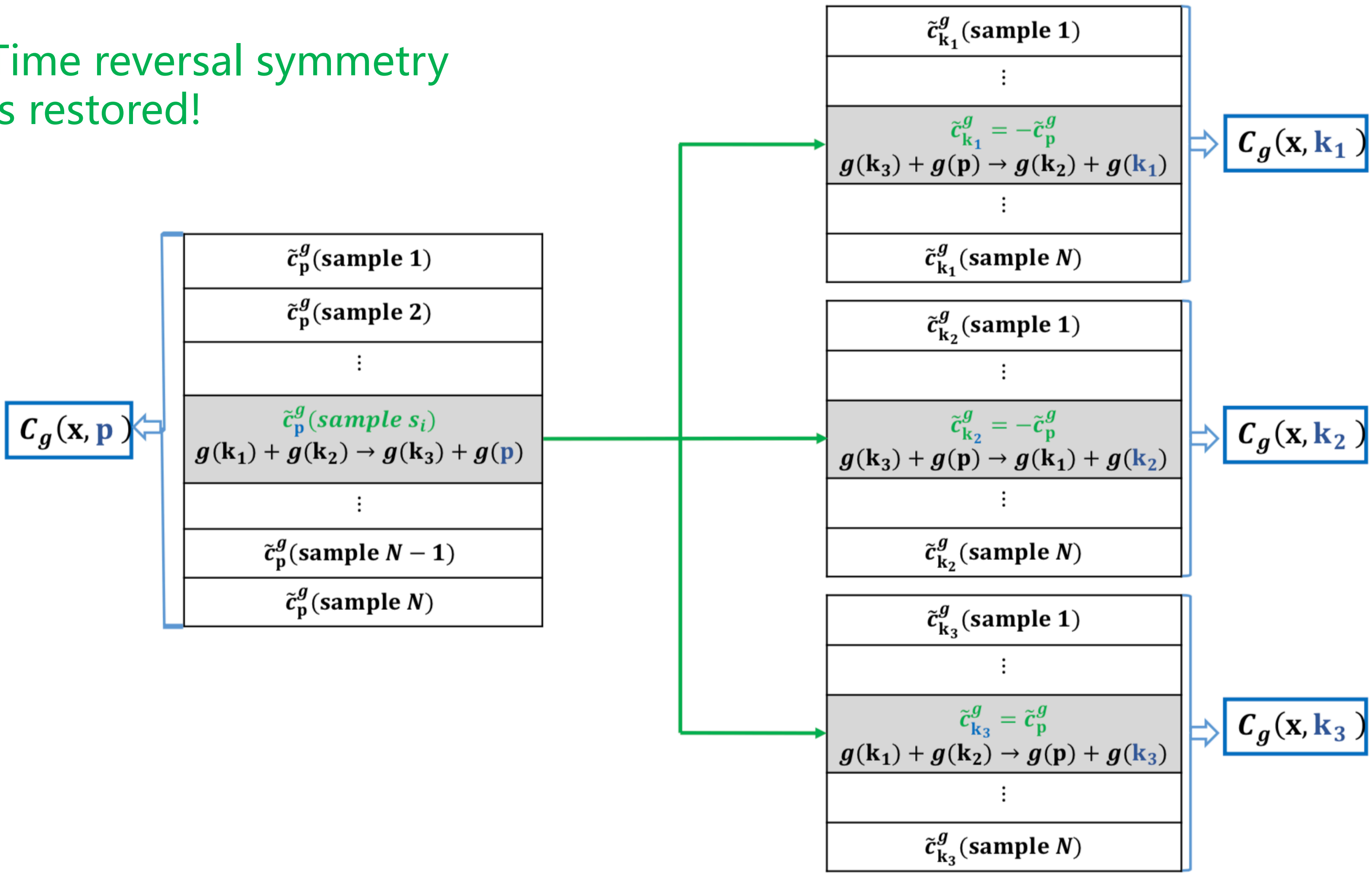


$$\delta E_{\mathrm{relative}} = \frac{|E_{t=0.9999\mathrm{fm}} - E_{t=00.0001\mathrm{fm}}|}{E_{t=0.0001\mathrm{fm}}}$$

- But, if we increases the number of grid, the variation of energy is tiny.

Symmetric Sampling

Time reversal symmetry
is restored!



$$d\Gamma_{ab\to cd} = \frac{1}{2E_p}|M_{ab\to cd}|^2 \prod_{i=1}^{3} \frac{d^3k_i}{(2\pi)^3 2E_{k_i}} \times (2\pi)^4 \delta^{(4)}(k_1+k_2-k_3-p)$$

$$\frac{d\tilde{f}^a_p(x)}{dt} = \mathcal{C}_a$$

5维积分：$k_{1y}, k_{1z}, k_{3x}, k_{3y}, k_{3z}$

6维扫描：p_x, p_y, p_z, x, y, z

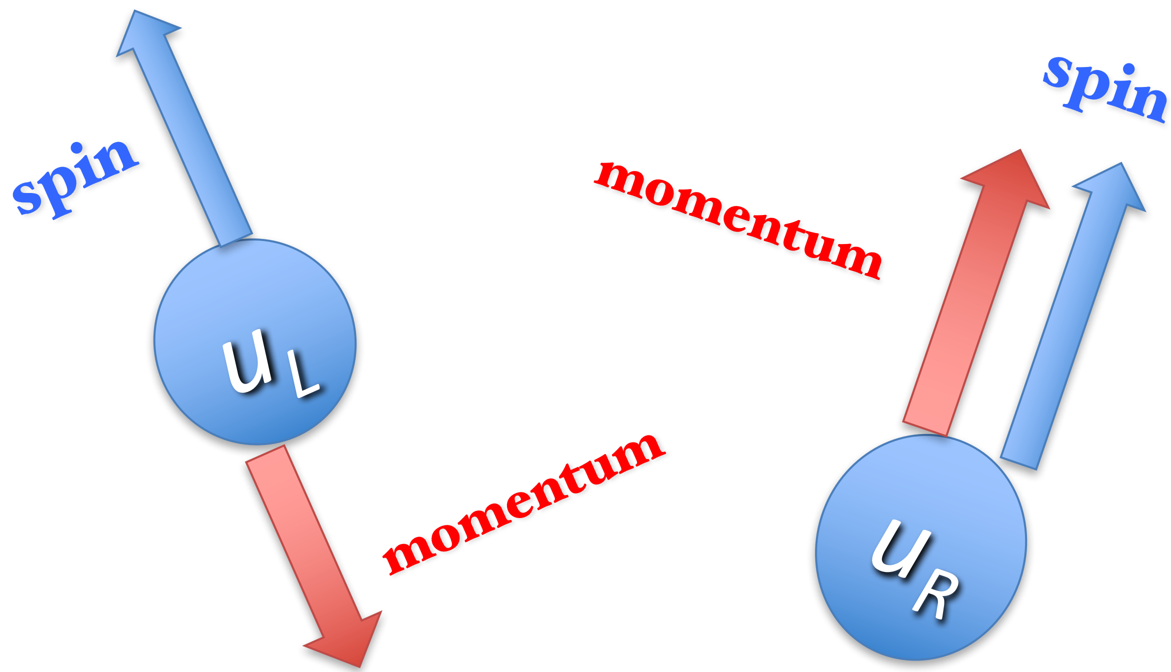
一维时间尺度迭代

即使利用ZMCintegral, 将5维积分压缩到5s，6维（$50\times 25\times 25\times 5\times 5\times 5$）扫描需要~$10^6$s，加上时间维度共需要~$10^9$s~30年！！

结合ZMCintegral编程经验,修改GPU内核程序。将5维积分压缩到~0.01s，6维扫描压缩至~10^2s，有希望将时间控制在~10^5s~1天。Challenging！！

Chirality and massless fermions

Left handed **Right handed**



Polarization by magnetic fields