Chiral kinetic theory in curved spacetime

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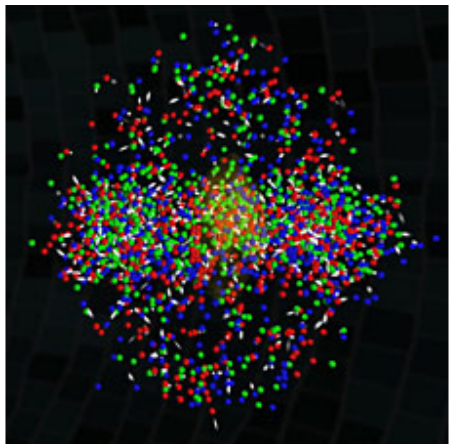
Y.C.Liu, L.L.Gao, KM, X.G.Huang, PRD 99.085014 (2019)
T. Hayata, Y. Hidaka, KM, arXiv : 19xx.xxxxx

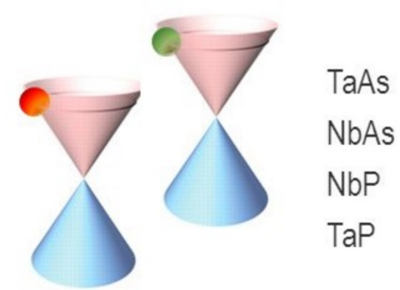
Chiral Transport Phenomena

nuclear physics



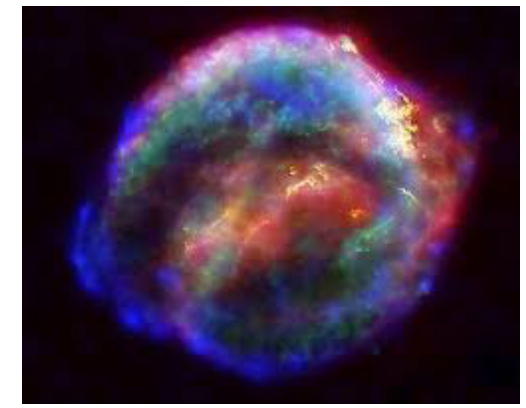
QGP

condensed matter

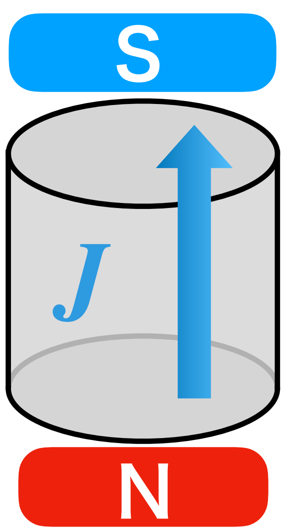


Dirac/Weyl semimetal

astrophysics

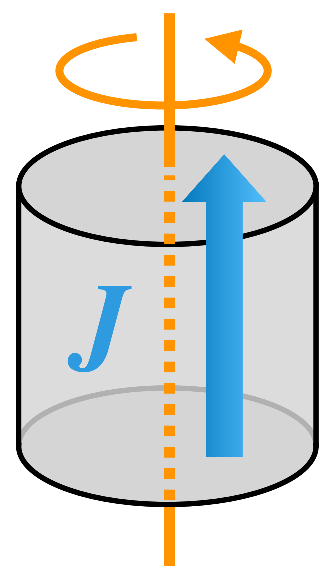


supernovae



chiral magnetic effect

$$\vec{J} = \sigma_{\mathrm{CME}} \vec{B}$$



chiral vortical effect

$$\vec{J} = \sigma_{\mathrm{CVE}} \vec{\omega}$$

Anomaly-induced → universal phenomena in chiral matter

Kinetic Theory for Anomalous Transport

equilibrium

$$\sigma^0_{\mathrm{CME}} = \frac{e^2\mu_5}{2\pi^2}$$

nonequilibrium (with AC mag. field)

$$\sigma^{\mathrm{noneq}}_{\mathrm{CME}}(\omega) = \sigma^0_{\mathrm{CME}}\left(1 + \frac{2}{3}\frac{\omega}{\omega + i\tau^{-1}}\right)$$

Kharzeev, Stephanov, Yee (2017)

Boltzmann kinetic theory + anomaly = Chiral Kinetic Theory

Stephanov, Yin (2012)

$$\left[(1+\hbar\,\#)\partial_t + (\boldsymbol{v} + \hbar\,\#)\cdot\boldsymbol{\nabla} + (\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} + \hbar\,\#)\cdot\boldsymbol{\nabla}_p\right] f = I_{\mathrm{coll}}[f]$$

$$\Rightarrow \quad \partial_\mu J^\mu = \hbar\,\#\boldsymbol{E}\cdot\boldsymbol{B}$$

$$J^0 = \int_{\boldsymbol{p}} (1+\hbar\,\#) f \qquad \boldsymbol{J} = \int_{\boldsymbol{p}} (\boldsymbol{v}+\hbar\,\#) f$$

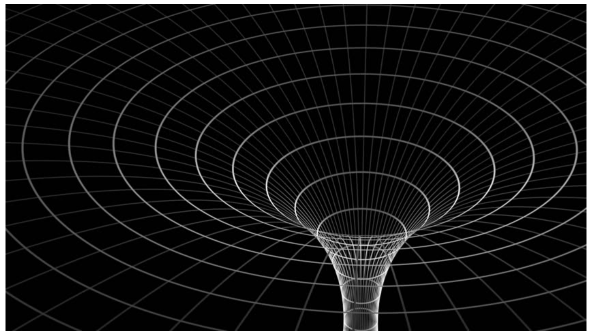
Various Earlier Works

different derivations/Lorentz covariance/collision/mass correction …

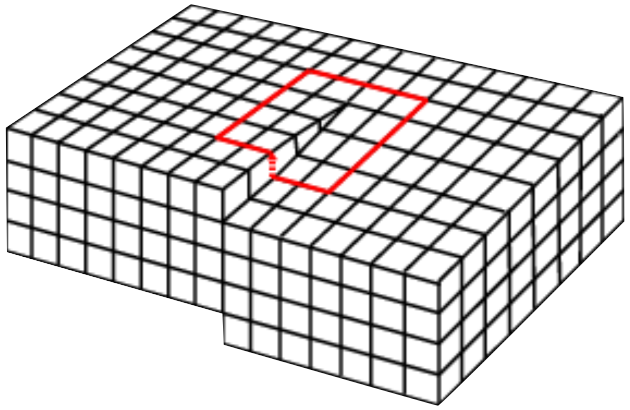
Stephanov, Yin (2012)
Chen, Son, Stephanov, Yee, Yin (2014)
Chen, Son, Stephanov (2015)
Mueller, Venugopalan (2017)
N. Weickgenannt, et al (2019) Gao, Liang (2019)
Son, Yamamoto (2012)
Son, Yamamoto (2013) Lin, Shukla (2019)
Chen, Pu, Wang, Wang (2013)
Hidaka, Pu, Yang (2017) Huang, et al. (2018)
Hattori, Hidaka, Yang, (2019)

vorticity(rotation)

gravity



torsion



Chiral kinetic theory in curved spacetime

From Quantum Field Theory to Chiral Kinetic Theory

Quantum Field Theory $S[\psi, \bar{\psi}]$

1. Construct Wigner function $W(x,p) = \int_y e^{-ip\cdot y/\hbar} \left\langle \bar{\psi}(x + \frac{1}{2}y)\psi(x - \frac{1}{2}y) \right\rangle$

2. Derive dynamic equation of $W(x,p)$ via Dirac eq.

3. Perform the semiclassical expansion e.g. chiral anomaly $\sim O(\hbar)$

4. Extract the right-handed sector $\mathcal{R}^\mu(x,p) = \mathrm{tr}\left[\frac{1+\gamma^5}{2}\gamma^\mu W(x,p)\right]$

Chiral kinetic Theory

Wigner Function

$$W(x,p) = \int_y e^{-ip\cdot y/\hbar} \left\langle \bar{\psi}(x + \tfrac{1}{2}y)\psi(x - \tfrac{1}{2}y) \right\rangle$$

$$\psi(x+y) = \psi(x) + y^\mu \nabla_\mu \psi(x) + \frac{1}{2} y^\mu y^\nu \nabla_\mu \nabla_\nu \psi(x) + \cdots$$

no background $\nabla_\mu = \partial_\mu$

EM field $\nabla_\mu \psi = (\partial_\mu + i A_\mu/\hbar)\psi$

U(1) gauge covariant

curved geometry $\nabla_\mu \psi = (\partial_\mu + i\mathcal{A}_\mu)\psi$ $\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\rho} V^\rho$

local Lorentz covariance

diffeomorphism (general covariance)

1st order CKT in general coordinate

Y.C.Liu, L.L.Gao, KM, X.G.Huang, PRD 99.085014 (2019)

1st order CKT

QFT

$$S = \frac{i}{2}\int \sqrt{-g}\, d^4x\, \bar{\psi}(\gamma^\mu \nabla_\mu \psi + \bar{\psi}\overleftarrow{\nabla}_\mu \gamma^\mu)\psi$$

CKT

kinetic eq.	$\Delta_\mu \mathcal{R}^\mu = 0$	**current cons.** $J^\mu \sim \mathcal{R}^\mu$
constraint	$p_\mu \mathcal{R}^\mu = 0$	**conformal sym.** $T^{\mu\nu} \sim \mathcal{R}^\mu p^\nu$
constraint	$\mathcal{R}^\mu p^\nu - \mathcal{R}^\nu p^\mu - \frac{\hbar}{2}\varepsilon^{\mu\nu\rho\sigma}\Delta_\rho \mathcal{R}_\sigma = 0$	**AM cons.** $S^{\mu\nu\rho} \sim \frac{\hbar}{2}\varepsilon^{\mu\nu\rho\sigma}\mathcal{R}_\sigma$

$$\mathcal{R}^\mu = \mathrm{tr}\left[\gamma^\mu \frac{1+\gamma^5}{2} W\right] \qquad \Delta_\mu = \nabla_\mu + \Gamma^\rho_{\mu\nu} p_\rho \partial^\nu_p$$

Solution

$$\mathcal{R}^\mu = 2\pi\delta(p^2)\left[p^\mu + \Sigma_n^{\mu\nu}(\nabla_\nu + \Gamma^\rho_{\mu\nu} p_\rho \partial_p^\nu)\right] f \qquad \Sigma_n^{\mu\nu} = \frac{\varepsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma}{2p\cdot n}$$

n^μ **Lorentz frame choosing vector**

ex) Minkowski spacetime

$$n^\mu = (1,0,0,0) \qquad \boldsymbol{\Sigma} = (\Sigma^{23}, \Sigma^{31}, \Sigma^{12}) = \frac{\boldsymbol{p}}{2p_0} \sim \frac{\hat{\boldsymbol{p}}}{2}$$

$$J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu + \hbar \Sigma_n^{\mu\nu}$$

decomposed frame-dependently

Chen, Son, Stephanov (2015)

Ex.)Rotating Coordinate

$$g_{\mu\nu} = \delta_{\mu\nu}\,, \quad g_{0i} = -v_i = -(\boldsymbol{\omega} \times \boldsymbol{x})_i$$

$$n^\mu = (1,0,0,0)$$

$$f\Big(|\boldsymbol{p}| - \boldsymbol{\omega}\cdot(\boldsymbol{x}\times\boldsymbol{p}) - \hbar\boldsymbol{\omega}\cdot\hat{\boldsymbol{p}}/2\Big)$$

orbital spin

$$\boldsymbol{J}_{\mathrm{eq}} = \int_p \left(\hat{\boldsymbol{p}} - \hbar\frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|} \times \frac{\partial}{\partial \boldsymbol{x}}\right) f_{\mathrm{eq}} = \hbar\boldsymbol{\omega}\left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)$$

CVE

$$n^\mu = (1, \boldsymbol{x}\times\boldsymbol{\omega})$$

Coriolis force

$$\boldsymbol{J}_{\mathrm{eq}} = \int_p \frac{\hbar}{2|\boldsymbol{p}|^2}\, 2|\boldsymbol{p}|\boldsymbol{\omega}\, f_{\mathrm{eq}} = \hbar\boldsymbol{\omega}\left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)$$

orbital

CVE

cf. chiral magnetic effect $2|\boldsymbol{p}|\boldsymbol{\omega} \leftrightarrow \boldsymbol{B}$ Stephanov, Yin (2012)

$$\boldsymbol{J}_{\mathrm{eq}} = \int_p \frac{\hbar}{2|\boldsymbol{p}|^2} \boldsymbol{B}\, f_{\mathrm{eq}} = \hbar\boldsymbol{B}\frac{\mu}{4\pi^2}$$

Ex.)Rotating Coordinate

$$g_{\mu\nu} = \delta_{\mu\nu}\,, \qquad g_{0i} = -v_i = -(\boldsymbol{\omega} \times \boldsymbol{x})_i$$

$$n^\mu = (1,0,0,0)$$

$$\left[\frac{\partial}{\partial t} + (\hat{\boldsymbol{p}} + \boldsymbol{x} \times \boldsymbol{\omega}) \cdot \frac{\partial}{\partial \boldsymbol{x}} + (\boldsymbol{p} \times \boldsymbol{\omega}) \cdot \frac{\partial}{\partial \boldsymbol{p}}\right] f = 0$$

$$\ddot{\boldsymbol{x}} = 2\dot{\boldsymbol{x}} \times \boldsymbol{\omega} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{x})$$

Coriolis **centrifugal**

$$n^\mu = (1, \boldsymbol{x} \times \boldsymbol{\omega})$$

$$2\,|\boldsymbol{p}|\,\boldsymbol{\omega} \leftrightarrow \boldsymbol{B}$$

$$\left[(1 + \hbar\, 2|\boldsymbol{p}|\boldsymbol{\omega} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})\frac{\partial}{\partial t} + \left\{\boldsymbol{v}_{\boldsymbol{p}} + \hbar(\boldsymbol{v}_{\boldsymbol{p}} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})2|\boldsymbol{p}|\boldsymbol{\omega}\right\} \cdot \frac{\partial}{\partial \boldsymbol{x}} + (\boldsymbol{v}_{\boldsymbol{p}} \times 2|\boldsymbol{p}|\boldsymbol{\omega}) \cdot \frac{\partial}{\partial \boldsymbol{p}}\right] f = 0$$

$$\boldsymbol{v}_{\boldsymbol{p}} = \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \qquad \varepsilon_{\boldsymbol{p}} = |\boldsymbol{p}| - \frac{\hbar}{2}\boldsymbol{\omega} \cdot \hat{\boldsymbol{p}}$$

$$|\boldsymbol{p}|\,\boldsymbol{\omega} \leftrightarrow \boldsymbol{B}$$

2nd order CKT in general coordinate

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T. Hayata, Y. Hidaka, KM, arXiv : 19xx.xxxxx

2nd Order CKT

QFT

$$S = \frac{i}{2}\int \sqrt{-g}\, d^4x\, \bar{\psi}(\gamma^\mu \nabla_\mu \psi + \bar{\psi}\overleftarrow{\nabla}_\mu \gamma^\mu)\psi$$

CKT

kinetic eq.	$\Delta \cdot \mathcal{R} = \frac{\hbar^2}{24}(\nabla_\rho R_{\mu\nu})\partial_p^\rho \partial_p^\mu \mathcal{R}^\nu$	**current cons.**
constraint	$\Pi \cdot \mathcal{R} = \frac{\hbar^2}{8} R_{\mu\nu}\partial_p^\mu \mathcal{R}^\nu$	**conformal sym.**
constraint	$\frac{\hbar}{2}\varepsilon^{\mu\nu\rho\sigma}\Delta_\rho \mathcal{R}_\sigma + \Pi^\mu \mathcal{R}^\nu - \Pi^\nu \mathcal{R}^\mu = \frac{\hbar^2}{16} R^{\mu\nu\rho\sigma}\partial_\rho^p \mathcal{R}_\sigma$	**AM cons.**

$$\Pi_\mu = p_\mu + \frac{\hbar^2}{24} R^\rho{}_{\sigma\mu\nu}\partial_p^\sigma \partial_p^\nu p_\rho + \frac{\hbar^2}{4} R_{\mu\nu}\partial_p^\nu$$

$$\Delta_\mu = D_\mu - \frac{\hbar^2}{12}(\nabla_\rho R_{\mu\nu})\partial_p^\rho \partial_p^\nu - \frac{\hbar^2}{24}(\nabla_\lambda R^\rho{}_{\sigma\mu\nu})\partial_p^\nu \partial_p^\sigma \partial_p^\lambda p_\rho + \frac{\hbar^2}{8} R^\rho{}_{\sigma\mu\nu}\partial_p^\nu \partial_p^\sigma D_\rho$$

$$D_\mu = \nabla_\mu + \Gamma^\rho_{\mu\nu} p_\rho \partial_p^\nu$$

Solution

$$\mathcal{R}_\mu = 2\pi\delta(p^2)\Big[p_\mu(f_{(0)} + \hbar f_{(1)} + \hbar^2 f_{(2)}) + \hbar\underline{\Sigma^n_{\mu\nu}}D^\nu f_{(0)} + \hbar^2\underline{\Sigma^u_{\mu\nu}}D^\nu f_{(1)}\Big]$$

$$+ 2\pi\hbar^2\frac{1}{p^2}\left[-p_\mu X^\lambda p_\lambda + 2p^\nu\Big(Y_{[\mu}p_{\nu]} + Z_{\alpha\mu\nu}p^\alpha\Big)\right]\delta(p^2)f_{(0)}$$

$$+ 2\pi\hbar^2\frac{\delta(p^2)}{p^2}\left[\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}p^\nu D^\rho\underline{\Sigma^{\sigma\lambda}_n}D_\lambda - \underline{\Sigma^u_{\mu\nu}}\left(\frac{1}{2}\tilde{R}^{\alpha\beta\nu\rho}p_\rho p_\alpha\partial^p_\beta + p\cdot D\underline{\Sigma^{\nu\rho}_n}D_\rho\right)\right]f_{(0)}$$

$$D_\mu = \nabla_\mu + \Gamma^\rho_{\mu\nu}p_\rho\partial^\nu_p\,, \quad \tilde{R}_{\alpha\beta\mu\nu} = \frac{1}{2}R_{\alpha\beta}{}^{\rho\sigma}\varepsilon_{\rho\sigma\mu\nu}\,,$$

$$X_\mu = \frac{1}{8}R_{\mu\nu}\partial^\nu_p + \frac{1}{24}R^\rho{}_{\sigma\mu\nu}\partial^\nu_p\partial^\sigma_p p_\rho\,, \quad Y_\mu = \frac{1}{4}R_{\mu\nu}\partial^\nu_p + \frac{1}{24}R^\rho{}_{\sigma\mu\nu}\partial^\nu_p\partial^\sigma_p p_\rho\,,$$

$$Z_{\alpha\mu\nu} = -\frac{1}{16}R_{\lambda\alpha\mu\nu}\partial^\lambda_p$$

two ambiguities (even in flat spacetime)

n^μ — **Lorentz frame**

u^μ — **Lorentz frame?**

Side-Jump term at 2nd Order

ex) Minkowski spacetime

$$n^\mu = u^\mu = (1,0,0,0) \qquad \boldsymbol{\Sigma} = (\Sigma^{23}, \Sigma^{31}, \Sigma^{12}) = \frac{\boldsymbol{p}}{2p_0} \sim \frac{\hat{\boldsymbol{p}}}{2}$$

$$\boldsymbol{\mathcal{R}} \sim \hat{\boldsymbol{p}} f + \frac{1}{|\boldsymbol{p}|}\boldsymbol{\nabla}\times\left(\frac{\hbar}{2}\hat{\boldsymbol{p}} f\right) + \frac{\hbar}{2|\boldsymbol{p}|}\boldsymbol{\nabla}\times\left[\frac{1}{|\boldsymbol{p}|}\boldsymbol{\nabla}\times\left(\frac{\hbar}{2}\hat{\boldsymbol{p}} f\right)\right]$$

$$\boldsymbol{j} = \frac{\hbar}{2i|\boldsymbol{p}|}\left[\psi^\dagger\boldsymbol{\nabla}\psi - (\boldsymbol{\nabla}\psi^\dagger)\psi\right] - \frac{\hbar}{2|\boldsymbol{p}|}\boldsymbol{\sigma}\times\boldsymbol{\nabla}\left(\psi^\dagger\psi\right)$$ **quantum mechanics**

$(\hbar/i)\overset{\leftrightarrow}{\boldsymbol{\nabla}} \to \boldsymbol{p}$

$\boldsymbol{\sigma} \to \boldsymbol{S}$

$$\sim \hat{\boldsymbol{p}} f + \frac{1}{|\boldsymbol{p}|}\boldsymbol{\nabla}\times(\boldsymbol{S} f)$$ **kinetic theory**

Stone (2015)
Fukushima, Pu, Zebin (2018)

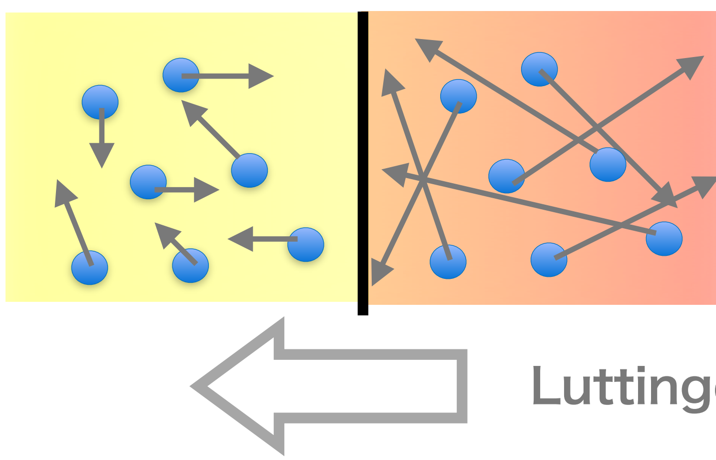
$\boldsymbol{j} \propto \boldsymbol{S} f$

$$\sim \hat{\boldsymbol{p}} f + \frac{1}{|\boldsymbol{p}|}\boldsymbol{\nabla}\times\left(\frac{\hbar}{2}\hat{\boldsymbol{p}} f\right) + \frac{\hbar}{2|\boldsymbol{p}|}\boldsymbol{\nabla}\times\left[\frac{1}{|\boldsymbol{p}|}\boldsymbol{\nabla}\times\left(\frac{\hbar}{2}\hat{\boldsymbol{p}} f\right)\right] + O(\hbar^3)$$

Ex.) Gravity (=T-gradient) and Vorticity

$$g_{00} = 1 - 2\varphi\,, \quad g_{ij} = \delta_{ij}\,, \quad g_{0i} = -v_i(\boldsymbol{x})$$

thermal gradient



Luttinger (1964)

gravity

inhomogeneous vorticity

$$\boldsymbol{E}_g = -\boldsymbol{\nabla}\varphi = -\frac{\boldsymbol{\nabla}T}{T}$$

$$\boldsymbol{B}_g = \frac{1}{2}\boldsymbol{\nabla}\times\boldsymbol{v} = \boldsymbol{\omega}$$

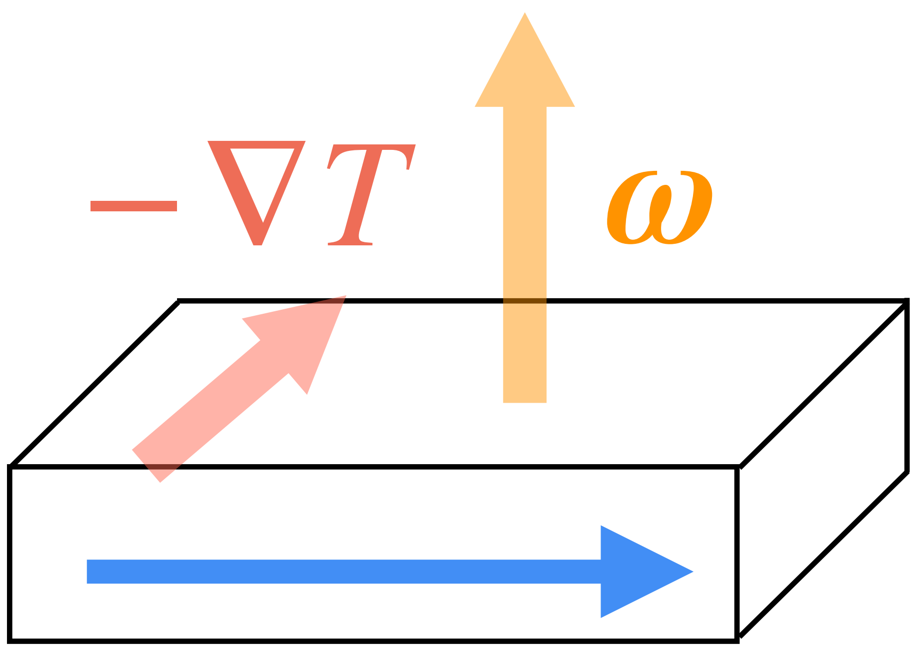
Ricci tensor

$$\boldsymbol{R} = ({R^1}_0, {R^2}_0, {R^3}_0) = 3\boldsymbol{E}_g \times \boldsymbol{B}_g + \boldsymbol{\nabla}\times\boldsymbol{B}_g$$

Current

$$\boldsymbol{J}_R = \int_p \boldsymbol{\mathcal{R}} = \hbar^2 \frac{\mu \boldsymbol{R}}{24\pi^2} = \hbar^2 \frac{\mu}{24\pi^2}\left(3\boldsymbol{E}_g \times \boldsymbol{B}_g + \boldsymbol{\nabla} \times \boldsymbol{B}_g\right)$$

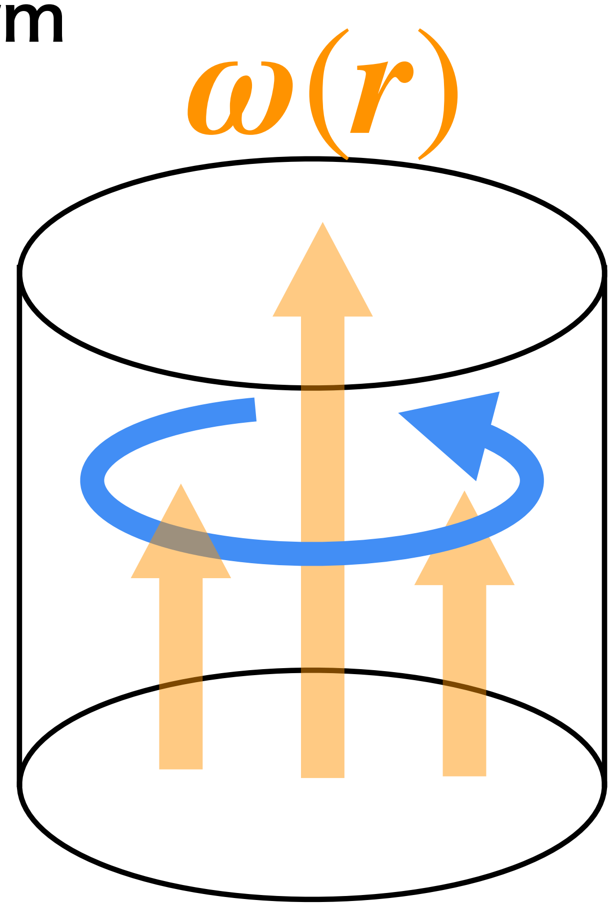
1st term



$$J_{\mathrm{GR}} \sim -\nabla T \times \boldsymbol{\omega}$$

cf. Hall or Nernst Effect

2nd term



$$J_{\mathrm{GR}} \sim \nabla \times \boldsymbol{\omega}$$

Summary & Outlook

[1] The CKT has been extended to general coordinate

The CKT in rot. coord. explains the B-ω duality

The 2nd order CKT in curved spacetime can be solved
(unlike the one in EM backgrounds)

[2] Phenomenological outlook

To describe the dynamics with GR effects
(collisions necessary)

Gravity-induced current can be generated
(practical estimation)

[3] Theoretical outlook

Why two frame vectors emerges

How anomaly is reproduced (Nieh-Yan anomaly?)

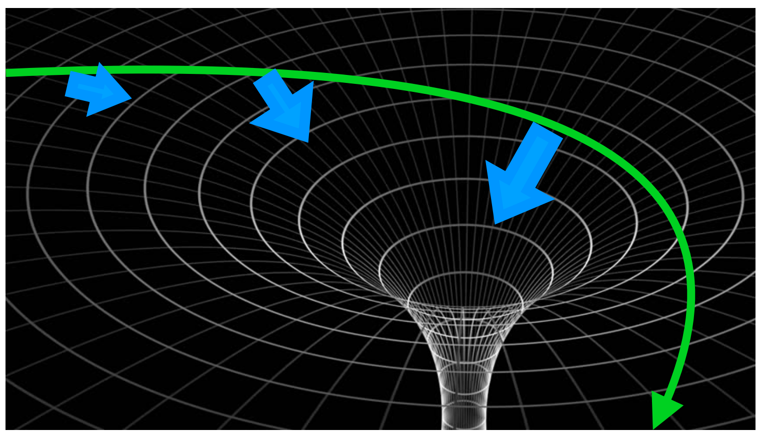
Kinetic Equation

$$\delta(p^2)\left[p\cdot\partial + \Gamma^{\rho}_{\mu\nu}p_\rho p^\mu \partial_p^\nu - \frac{\hbar}{2}\Sigma_n^{\mu\nu}R_{\rho\sigma\mu\nu}p^\rho\partial_p^\sigma + \hbar(D_\mu\Sigma_n^{\mu\nu})D_\nu\right]f = 0$$

motion along geodesic line

spin-curvature force

side-jump term



$$\Rightarrow \quad \dot{p}_\nu = -\left(\Gamma^{\rho}_{\mu\nu}p_\rho + \frac{\hbar}{2}\Sigma_n^{\alpha\beta}R_{\mu\nu\alpha\beta}\right)v^\mu$$

Mathisson (1937)
Papapetrou (1951)
Dixon (1970)

Mathisson-Papapetrou–Dixon equation

Equilibrium Current

$$f_{(0)} = f_{(0)}(-\beta\mu + \beta^\mu p_\mu)$$ **but how about** $f_{(1)}$, $f_{(2)}$ **?**

Lorentz covariance under $n^\mu \to n'^\mu$

$$(\Lambda_n^{-1})_\mu{}^\nu \mathcal{R}'_\nu(x', p') - \mathcal{R}_\mu(x, p) = 0$$

⇨

$$\delta_n f'_{(1)}(x', p') = f'_{(0)} \frac{1}{2} \left(\Sigma_{n'}^{\nu\rho} - \Sigma_n^{\nu\rho} \right) \nabla_\nu \beta_\rho$$

⇨

$$f_{(1)} = f'_{(0)} \frac{1}{2} \Sigma_n^{\mu\nu} \nabla_\mu \beta_\nu$$

$$f_{(0)} + \hbar f_{(1)} = f_{(0)}(g) \qquad g = -\beta\mu + \beta^\mu p_\mu + \frac{\hbar}{2} \Sigma_n^{\mu\nu} \nabla_\mu \beta_\nu$$

Equilibrium Current

Lorentz covariance under $n^\mu \to n'^\mu$ **and** $u^\mu \to u'^\mu$

$$(\Lambda_n^{-1})_\mu{}^\nu \mathcal{R}'_\nu(x', p') - \mathcal{R}_\mu(x, p) = 0$$

⇨

$$\delta_n f_{(2)}(x', p') = \Sigma^u_{\mu\nu} D^\mu \left[\frac{1}{4} f'_{(0)} \varepsilon^{\rho\sigma\nu\lambda} \left(\frac{n'_\lambda}{p \cdot n'} - \frac{n_\lambda}{p \cdot n} \right) \nabla_\rho \beta_\sigma \right]$$

$$\delta_u f_{(2)}(x', p') = \left(\Sigma^{u'}_{\mu\nu} - \Sigma^u_{\mu\nu} \right) D^\mu \left(f'_{(0)} \frac{\varepsilon^{\rho\sigma\nu\lambda} n_\lambda}{4 p \cdot n} \nabla_\rho \beta_\sigma \right)$$

⇨

$$f_{(2)} = \Sigma^u_{\mu\nu} D^\mu \left(f'_{(0)} \frac{\varepsilon^{\nu\rho\sigma\lambda}}{4\, p \cdot n} n_\rho \nabla_\sigma \beta_\lambda \right)$$