Zilch currents and chiral kinetic theory for vector particles

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Work in progress

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New development of hydrodynamics and its applications in Heavy-Ion Collisions

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Outline

- Zilch currents: gauge-invariant measure of helicity separation
- Zilch Vortical Effect
  - Chiral kinetic theory
  - Wigner function for spin-1 particles
- Conclusions

Chiral Vortical Effect for fermions

$$J^{\mu}_{R/L} = \pm\left(\frac{T^2}{12} + \frac{\mu^2_{R/L}}{4\pi^2}\right)\omega^{\mu}$$

Kharzeev, Liao, Voloshin, Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016)

- Vilenkin PRD 21, 2260 (1980)
- Chiral kinetic theory: Berry phase $\Rightarrow$ Chiral anomaly and CVE
  Son, Yamamoto, PRL **109**, 181602 (2012): Stephanov, Yin, PRL **109**, 162001 (2012)
- Wigner function
  e.g. Chen, Pu, Q. Wang and X. N. Wang, PRL **110**, no. 26, 262301 (2013)
  Hidaka, Pu, Yang, PRD **95**, no. 9, 091901 (2017);
- Connection to gravitational anomaly
  Landsteiner, Megias, Melgar, Pena-Benitez, JHEP **1109**, 121 (2011)
  Flachi, Fukushima, PRD **98**, no. 9, 096011 (2018)
- Anomalous hydrodynamics
  Son, Surowka, PRL **103**, 191601 (2009)

Chiral Vortical Effect for photons

$$J^{\mu}_{R/L} = \pm \frac{T^2}{6} \omega^{\mu}$$

Avkhadiev, Sadofyev, PRD **96**, no. 4, 045015 (2017); N. Yamamoto, PRD **96**, no. 5, 051902 (2017)

- $K^{\mu} = \epsilon^{\mu\nu\alpha\beta} A_{\nu} \partial_{\alpha} A_{\beta}$ Avkhadiev, Sadofyev, (2017);
- Connection to Berry curvature in same way as fermions Yamamoto (2017)
- Generalization to higher spins Huang, Sadofyev, JHEP **1903**, 084 (2019)

Is there a local gauge-invariant measure of photon helicity to study CVE?

Zilch currents

- Extra conservation law for Maxwell's equation **independent** of energy and momentum conservation Lipkin, J. Math. Phys. **5**, 696 (1964)

$$\partial_\mu Z^\mu = 0$$

Zilch charge and current:

$$Z^0 = \mathbf{E}\cdot(\nabla\times\mathbf{E}) + \mathbf{B}\cdot(\nabla\times\mathbf{B}), \qquad \mathbf{Z} = \mathbf{E}\times\dot{\mathbf{E}} + \mathbf{B}\times\dot{\mathbf{B}}$$

- Zilch is actually a component of a tensor Kibble, J. Math. Phys. 6, 1022 (1965)

$$\partial^\rho Z_{\mu\nu\rho} = 0$$

$$\begin{aligned} Z_{\mu\nu\rho} &= \tilde{F}^{\lambda}{}_{\mu}\partial_\rho F_{\nu\lambda} - F^{\lambda}{}_{\mu}\partial_\rho \tilde{F}_{\nu\lambda} \\ &= \tilde{F}^{\lambda}{}_{\mu} \overleftrightarrow{\partial}_{\rho} F_{\nu\lambda} + \tilde{F}^{\lambda}{}_{\nu} \overleftrightarrow{\partial}_{\rho} F_{\mu\lambda} \end{aligned}$$

with $Z_{\mu 00} = Z_\mu$, $\overleftrightarrow{\partial} = \frac{1}{2}(\overrightarrow{\partial} - \overleftarrow{\partial})$

- Infinite set of conserved quantities $Z^{(s)}_{\alpha_1\cdots\alpha_s}$ Kibble, J. Math. Phys. 6, 1022 (1965)
- It is convenient to define a **symmetrized Zilch**

$$\bar{Z}^{(s)}_{\alpha_1\cdots\alpha_s} = \tilde{F}^{\lambda}{}_{\{\alpha_1} \overleftrightarrow{\partial}_{\alpha_2} \cdots \overleftrightarrow{\partial}_{\alpha_{s-1}} F_{\alpha_s\}\lambda}$$

$\bar{Z}^{(s)}_{\alpha_1\cdots\alpha_s}$ and $Z^{(s)}_{\alpha_1\cdots\alpha_s}$ have the same charge and both are conserved

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Physical meaning of Zilch

- After quantization:

$$\int d^3x :Z_0 := \sum_J \omega_J^2 (a_J^{(+)\dagger} a_J^{(+)} - a_J^{(-)\dagger} a_J^{(-)})$$

Chernodub, Cortijo, Landsteiner, PRD **98**, no. 6, 065016 (2018)

$\Rightarrow$ Zilch measures helicity separation weighted with frequency squared

- Zilch was used to describe interaction of circularly polarized light with chiral molecules
  Tang, Cohen, PRL **104**, 163901 (2010)

- K^μ - **Pro**: 'right' dimension for a current. **Con**: local non gauge-invariant
  Z^μ - **Pro**: local gauge-invariant. **Con**: non-canonical dimensionality

Chiral kinetic theory - I

- **Symmetrized** Zilch

$$\bar{Z}^{(s)}_{\alpha_1 \cdots \alpha_s} = \tilde{F}^{\lambda}{}_{\{\alpha_1} \overleftrightarrow{\partial}_{\alpha_2} \cdots \overleftrightarrow{\partial}_{\alpha_{s-1}} F_{\alpha_s\}\lambda}$$

- Construction of current in CKT
  - (i) Vectors at our disposal: p^μ and $j^\mu = p^\mu f + S^{\mu\nu}\partial_\nu f$
  - (ii) Dimensionality
- Case $s = 3$, $[\bar{Z}] = [E]^5$, thus

$$\bar{Z}_{i00} = \int \frac{d^4 p}{(2\pi)^3}\, \delta(p^2)\, p_{\{0}\, p_0\, j_{i\}}$$

- Global equilibrium, first order in vorticity ω:

Zilch Vortical Effect

$$\bar{Z}_{i00} = \frac{2\pi^2 T^4}{27}\omega_i$$

- Relation to the **Berry curvature** through $S^{\mu\nu}$!

Chiral kinetic theory - II

- Case $s > 3$

$$\bar{Z}_{i0\dots0} = \int \frac{d^4p}{(2\pi)^3}\,\delta(p^2)\,p_{\{0}\cdots p_0\,j_{i\}}$$

- Global equilibrium, first order in vorticity ω:

$$\bar{Z}_{i0\dots0} = \omega_i \frac{1}{2\pi^2}\,\frac{(s+1)(s+2)}{3s}\int_0^{+\infty} d|\mathbf{p}||\mathbf{p}|^s f^{(0)}$$

$$f^{(0)} = (e^{|\mathbf{p}|/T} - 1)^{-1}$$

- Infinite set of Zilch Vortical Effects related to Berry curvature!

Gauge-dependent Wigner function

$$W^{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \langle : A^\mu\left(x+\frac{y}{2}\right) A^\nu\left(x-\frac{y}{2}\right) : \rangle$$

- Equations of motion (we choose $\partial_\mu A^\mu = 0$)

$$\left(p_\alpha - i\frac{\hbar}{2}\partial_\alpha\right) W^{\alpha\mu}(x,p) = 0,$$
$$\left(p_\alpha + i\frac{\hbar}{2}\partial_\alpha\right) W^{\mu\alpha}(x,p) = 0,$$
$$\left(p^2 - m^2 - \frac{\hbar^2}{4}\partial_\alpha\partial^\alpha\right) W^{\mu\nu}(x,p) = 0,$$
$$\hbar\, p\cdot\partial W^{\mu\nu}(x,p) = 0,$$

- **Semiclassical expansion**

$$W^{\mu\nu} = W^{(0)\mu\nu} + \hbar W^{(1)\mu\nu} + \dots .$$

Gauge-dependent Wigner function - Zeroth order

- Equations of motion at zeroth order in $\hbar$

$$p_\alpha W^{(0)\alpha\mu}(x,p) = 0,$$
$$p_\alpha W^{(0)\mu\alpha}(x,p) = 0,$$
$$(p^2 - m^2) W^{(0)\mu\nu}(x,p) = 0,$$
$$p \cdot \partial W^{(0)\mu\nu}(x,p) = 0$$

- Explicit calculation of Wigner function by plugging the free field

$$A^\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E}} \sum_\lambda \left(\epsilon^\mu_\lambda a_{\mathbf{p},\lambda} e^{-ip\cdot x} + (\epsilon^\mu_\lambda)^* a^\dagger_{\mathbf{p},\lambda} e^{+ip\cdot x} \right)$$

We obtain

$$W^{(0)\mu\nu}(x,p) = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) f^{(0)}(x,p) \delta(p^2 - m^2)$$

which is a solution of EOMs

Gauge-dependent Wigner function - First order

- Equations of motion at first order in $\hbar$

$$
\begin{aligned}
p_\alpha W^{(1)\alpha\mu}(x,p) - \frac{i}{2}\partial_\alpha W^{(0)\alpha\mu}(x,p) &= 0 \\
p_\alpha W^{(1)\mu\alpha}(x,p) + \frac{i}{2}\partial_\alpha W^{(0)\mu\alpha}(x,p) &= 0 \\
(p^2 - m^2) W^{(1)\mu\nu}(x,p) &= 0 \\
p\cdot\partial W^{(1)\mu\nu}(x,p) &= 0
\end{aligned}
$$

- $W^{\mu\nu} = W_S^{\mu\nu} + W_A^{\mu\nu}$ ($W_S^{\mu\nu} = W_S^{\nu\mu}$ and $W_A^{\mu\nu} = -W_A^{\nu\mu}$)

$$
\begin{aligned}
p_\alpha W_S^{(1)\alpha\mu} - \frac{i}{2}\partial_\alpha W_A^{(0)\alpha\mu} &= 0 \\
p_\alpha W_A^{(1)\alpha\mu} - \frac{i}{2}\partial_\alpha W_S^{(0)\alpha\mu} &= 0
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
p_\alpha W_S^{(1)\alpha\mu} &= 0 \\
p_\alpha W_A^{(1)\alpha\mu} &= \frac{i}{2}\partial^\mu f^{(0)}\delta(p^2 - m^2)
\end{aligned}
$$

- Global equilibrium: $f^{(0)} = (1/(2\pi)^3)[\exp(\beta\cdot p) - 1]^{-1}$

$$
W^{\mu\nu} = \left[\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)(f^{(0)} + \hbar f^{(1)}) + i\hbar\,\omega^{\mu\nu} f^{(0)\prime}\right]\delta(p^2 - m^2)
$$

$\omega^{\mu\nu} = \frac{1}{2}(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu)$, $f^{(0)\prime} = \frac{\partial f^{(0)}}{\partial(\beta\cdot p)}$

Gauge-invariant Wigner function

$$Y_{\sigma\delta\nu\alpha} = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \langle : F_{\sigma\delta}\left(x+\frac{y}{2}\right) F_{\nu\alpha}\left(x-\frac{y}{2}\right) : \rangle$$

- It can be computed from Gauge-dependent Wigner function

$$Y_{\sigma\delta\nu\alpha} = k^*_\sigma k_\nu W_{\delta\alpha} - k^*_\sigma k_\alpha W_{\delta\nu} - k^*_\delta k_\nu W_{\sigma\alpha} + k^*_\delta k_\alpha W_{\sigma\nu}$$

with $k^\mu = p^\mu + i\frac{\hbar}{2}\partial^\mu$

- Global equilibrium at first order in $\hbar$

$$\begin{aligned} Y_{\sigma\delta\nu\alpha} = \\ = & \left[p_\sigma p_\nu g_{\delta\alpha}(f^{(0)} + \hbar f^{(1)}) + i\hbar p_\sigma p_\nu \omega_{\delta\alpha} f^{(0)\prime} + i\frac{\hbar}{2} g_{\delta\alpha}(p_\sigma \partial_\nu f^{(0)} - p_\nu \partial_\sigma f^{(0)})\right]\delta(p^2 - m^2) \\ - & \left[p_\sigma p_\alpha g_{\delta\nu}(f^{(0)} + \hbar f^{(1)}) + i\hbar p_\sigma p_\alpha \omega_{\delta\nu} f^{(0)\prime} + i\frac{\hbar}{2} g_{\delta\nu}(p_\sigma \partial_\alpha f^{(0)} - p_\alpha \partial_\sigma f^{(0)})\right]\delta(p^2 - m^2) \\ - & \left[p_\delta p_\nu g_{\sigma\alpha}(f^{(0)} + \hbar f^{(1)}) + i\hbar p_\delta p_\nu \omega_{\sigma\alpha} f^{(0)\prime} + i\frac{\hbar}{2} g_{\sigma\alpha}(p_\delta \partial_\nu f^{(0)} - p_\nu \partial_\delta f^{(0)})\right]\delta(p^2 - m^2) \\ + & \left[p_\delta p_\alpha g_{\sigma\nu}(f^{(0)} + \hbar f^{(1)}) + i\hbar p_\delta p_\alpha \omega_{\sigma\nu} f^{(0)\prime} + i\frac{\hbar}{2} g_{\sigma\nu}(p_\delta \partial_\alpha f^{(0)} - p_\alpha \partial_\delta f^{(0)})\right]\delta(p^2 - m^2) \end{aligned}$$

- Terms with $1/p^2$ cancel $\Longrightarrow$ $m \to 0$ safely

CVE and ZVE from Wigner function

- **Chiral Vortical Effect**

$$K^\mu = -i\epsilon^{\mu\nu\alpha\beta}\int d^4p\, k_\alpha W_{\nu\beta} = \frac{T^2}{6}\omega^\mu \qquad O(\hbar)$$

with $k^\mu = p^\mu + i\frac{\hbar}{2}\partial^\mu$

- **Zilch Vortical Effect**
  - Typical terms are of the form

$$\langle: \tilde{F}^\lambda{}_\mu \partial_\rho F_{\nu\lambda} :\rangle = \frac{1}{2}\int d^4p\, k_\rho \epsilon^\lambda{}_\mu{}^{\sigma\delta} Y_{\sigma\delta\nu\lambda}$$

$$\bar{Z}^{\mu 00} = \frac{2\pi^2 T^4}{27}\omega^\mu \qquad O(\hbar)$$

in agreement with CKT!

- $\bar{Z}^{(s)}_{\alpha_1 \dots \alpha_s}$ from Wigner function also agrees with CKT

Conclusions

Summary

- Zilch currents from chiral kinetic theory
- Zilch currents from Wigner function
- Zilch currents from equilibrium QFT (not shown in this talk)
- Results of CKT, Wigner function and equilibrium QFT agree
- Connection between Zilch Vortical Effects and **Berry curvature**

Outlook

- Origin of Chiral Vortical Effects
- Transport theory for vector particles
- Polarization effects in heavy-ion collisions