Longitudinal hydrodynamic response in heavy-ion collisions

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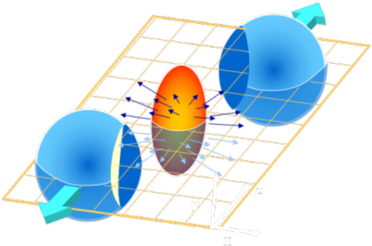
New Development of Hydrodynamics and its applications in Heavy-ion Collisions

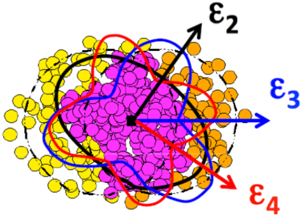
Outline

- Introduction
- Formulation of the longitudinal hydrodynamic response
- Numerical resluts and applications
- Summary

Introduction

- Pressure Gredient (in plane $>$ out of plane)$\rightarrow$ "flow"

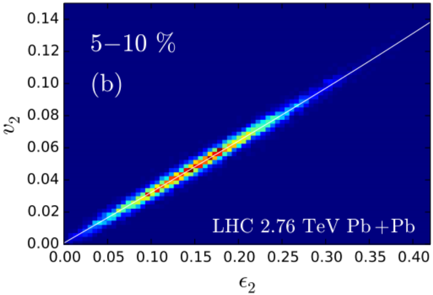




- Define flow harmonics by Fourier expansion of the final observed momentum distribution

$$\frac{dN}{d\phi} = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} V_n exp^{-in\phi}$$

Introduction: Integrated flow V_2



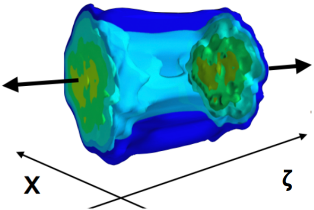
- V_2 is approximately linear to eccentricity $\mathcal{E}_2$: $V_2 = G_0\mathcal{E}_2$

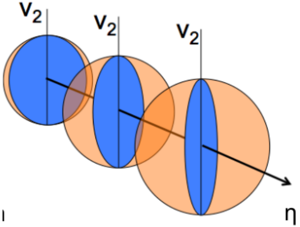
J. Hostler, L. Yan, F. Gardim, J. Ollitrault, Phys. Rev. C93, 014909 (2016)

H.Niemi, K.J.Eskola, R.Paatelainen Phys.Rev. C93, 024907 (2016)

Introduction: Pseudorapidity dependent flow $V_2(\zeta)$

pseudorapidity: $\zeta = \text{atanh}(p_z/|\mathbf{p}|)$





- $V_2 \to V_2(\zeta)$: $V_n(\zeta) = v_n(\zeta)e^{in\Psi_n(\zeta)}$
- Broken of Bjoken symmetry in ζ direction: $V_2(\zeta_1) \neq V_2(\zeta_2) \to$ magnitude and phase(torqued fireball)
- r_2 decorrelation in experiment ...

P. Bozek et al, Phys.Rev. C 83 (2011) 034911

J. Jia et al, Phys. Rev. C 90 (2014) 034905

CMS, Phys.Rev. C92, 034911; ATLAS, Eur. Phys. J. C74, 2982; ALICE, Phys. Lett. B762,

Hydro longitudinal response

- Integrated elliptic flow from hydro:

$$V_2 = G_0\mathcal{E}_2$$

- Generalization to longitudinal hydro response:

$$V_2(\zeta) = \int d\xi G(\zeta - \xi)\mathcal{E}_2(\xi)$$

  - Assuming Bjorken boost-invariant background
  - a non-local response relation
  - hydro response function $G(\zeta - \xi)$ is to be determined

space-time rapidity: $\xi = \mathrm{atanh}(z/t)$

Hydro longitudinal response

- Pseudorapidity dependent harmonic flow $V_n(\zeta)$

$$\frac{dN}{d\phi d\zeta} = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} V_n(\zeta) exp^{-in\phi}$$

here $\phi = \text{atan}(p_y/p_x)$, $V_2 = \int_{-\infty}^{\infty} d\zeta \; V_2(\zeta)$

- Space-time rapidity dependent eccentricity $\mathcal{E}_2(\xi)$

$$\mathcal{E}_2(\xi) = -\frac{\int d^2\vec{x}_\perp \; \rho(\vec{x}_\perp, \xi)(x+iy)^2}{\int d\xi d^2\vec{x}_\perp \; \rho(\vec{x}_\perp, \xi)|x+iy|^2}$$

Here $\mathcal{E}_2 = \int_{-\infty}^{\infty} d\xi \; \mathcal{E}_2(\xi)$

- The definition satisfies $d^m\mathcal{E}_2(\xi)/d\xi^m \to 0$ when $|\xi| \to \infty$

Hydro longitudinal response

- Transform to k-space (wave-number)

$$\begin{cases} \tilde{V}_2(k) = \int_{-\infty}^{\infty} d\zeta \; V_2(\zeta) e^{-ik\zeta}, \\ \tilde{\mathcal{E}}_2(k) = \int_{-\infty}^{\infty} d\xi \; \mathcal{E}_2(\xi) e^{-ik\xi}. \end{cases} \quad (1)$$

$$\tilde{V}_2(k) = \tilde{G}(k) \tilde{\mathcal{E}}_2(k) \quad (2)$$

$\tilde{G}(k)$ can be expanded as: (hydro gradient expansion, $|k| < k^*$)

$$\tilde{G}(k) = G_0 + ikG_1 - k^2 G_2 + O(k^3)$$

Transform back to the ζ-space

$$V_2(\zeta) = G_0 \mathcal{E}_2(\zeta) + G_1 \frac{d\mathcal{E}_2(\zeta)}{d\zeta} + G_2 \frac{d^2 \mathcal{E}_2(\zeta)}{d\zeta^2} + O\left(\frac{d^3}{d\zeta^3}\right)$$

Hydro longitudinal response

- An alternate formulation with new sets of variables:

$$\begin{cases} V_2^{(n)} = \int d\zeta \zeta^n V_2(\zeta)/n!, \\ \mathcal{E}_2^{(n)} = \int d\xi \xi^n \mathcal{E}_2(\xi)/n!. \end{cases} \tag{3}$$

- Pseudorapidity dependent response relation:

$$\overrightarrow{V}_2 = \mathbf{G} \overrightarrow{\mathcal{E}}_2$$

with $\mathbf{G}$ being a matrix where $\overrightarrow{V}_2 = (V_2^{(0)}, V_2^{(1)}, V_2^{(2)} ...)$ and $\overrightarrow{\mathcal{E}}_2 = (\mathcal{E}_2^{(0)}, \mathcal{E}_2^{(1)}, \mathcal{E}_2^{(2)} ...)$

Hydro longitudinal response

- An alternate formulation

$$\overrightarrow{V}_2 = \begin{bmatrix} G_0 & 0 & 0 & \dots & 0 \\ -G_1 & G_0 & 0 & \dots & 0 \\ G_2 & -G_1 & G_0 & \dots & 0 \\ -G_3 & G_2 & -G_1 & G_0 & \dots \\ \dots & & & & \end{bmatrix} \times \overrightarrow{\mathcal{E}}_2, \quad (4)$$

with odd order $G_n = 0$, we have

$$V_2^{(0)} = G_0 \mathcal{E}_2^{(0)}$$
$$V_2^{(1)} = G_0 \mathcal{E}_2^{(1)}$$
$$V_2^{(2)} = G_0 \mathcal{E}_2^{(2)} + G_2 \mathcal{E}_2^{(0)}$$
$$V_2^{(3)} = G_0 \mathcal{E}_2^{(3)} + G_2 \mathcal{E}_2^{(1)}$$
$$\dots$$

Hydro longitudinal response

G_n can be calculated recursively:

$$G_0 = \frac{\langle V_2^{(0)} \mathcal{E}_2^{(0)*} \rangle}{\langle \mathcal{E}_2^{(0)} \mathcal{E}_2^{(0)*} \rangle}$$

$$G_2 = \frac{\langle (V_2^{(0)} - G_0 \mathcal{E}_2^{(2)}) \mathcal{E}_2^{(0)*} \rangle}{\langle \mathcal{E}_2^{(0)} \mathcal{E}_2^{(0)*} \rangle}$$

$$\ldots$$

$$G_n = \frac{\langle (V_2^{(n)} - \sum_{i=0}^{n-1} (-1)^i G_i \mathcal{E}_2^{(n-i)}) \mathcal{E}_2^{(0)*} \rangle}{\langle \mathcal{E}_2^{(0)} \mathcal{E}_2^{(0)*} \rangle}$$

- A series of linear relations realized.
- This alternate form and new variables are accessible in hydro simulations

Model setup: Inition Condition

- AMPT provides non-trival longitudinal distribution with fluctuation
- AMPT construct $T^{\mu\nu}$ by energy depositions of individual partons via a Gaussian smearing:

$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} \frac{K}{\sqrt{(2\pi)^3}\sigma_r^2 \sigma_\xi \tau_0} exp[-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_r^2} - \frac{(\xi-\xi_i)^2}{2\sigma_\xi^2}]$$

- The initial energy density ϵ can be obtained:

$$\epsilon(\tau_0, x_\perp, \xi) \to u_\mu T^{\mu\nu} = \epsilon u^\nu$$

L. Pang, H. Petersen, G, Qin, V. Roy, X. Wang, Eur. Phys. J. A52, 97 (2016)

Model setup: Initial condition + MUSIC

- MUSIC solves 3+1 D relativistic viscous hydrodynamics : $\nabla_\mu T^{\mu\nu} = 0$
  - η/s is constant
  - Eos: Lattice EOS (s95p-v1.2)
  - Cooer-Frye (From hydrodynamics to hadron cascade):

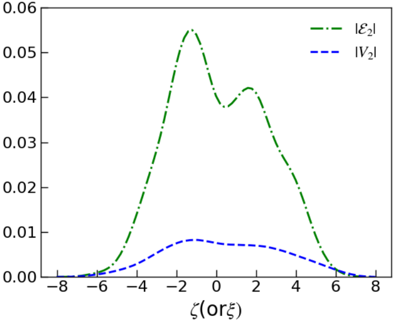
$$E\frac{dN}{d^3p} = \frac{dN}{dyp_T dp_T d\phi_p} = g_i \int_\Sigma f(u^\mu p_\mu) p^\mu d^3\Sigma_\mu$$

  - Calculate the $V_2(\zeta)$ from thermal pions:

B. Schenke, S. Jeon, C. Gale, Phys.Rev.C 82, 014903 (2010); Phys.Rev.Lett. 106, 042301 (2011); Phys. Rev. C 93, 044906 (2016)

Longitudinal distribution of $V_2(\zeta)$ and $\mathcal{E}_2(\xi)$

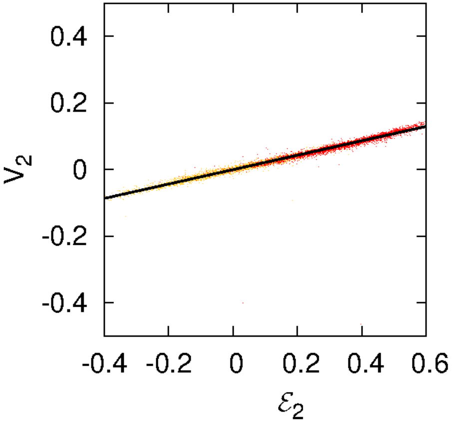
- Longitudinal distribution of $V_2(\zeta)$ and $\mathcal{E}_2(\xi)$ calculated by MUSIC for Pb-Pb collision at $\sqrt{s} = 2.76\,TeV$ for centrality of 30-40% with $\eta/s = 0.08$



Both ξ and ζ are in [-8,8] and $d^m\mathcal{E}_2(\xi)/d\xi^m \to 0$ when $|\xi| \to 8$

Numerical results: G_0

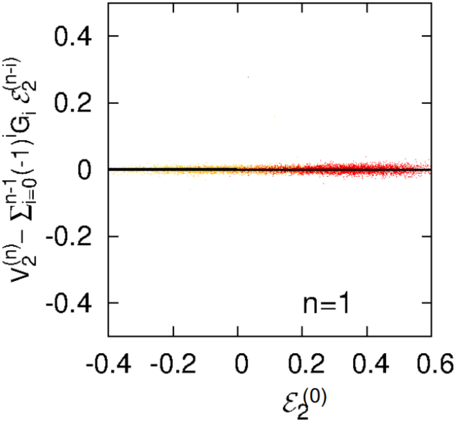
- $n = 0$:



- $G_0 = 0.216 \pm 0.0018$

Numerical results: G_1

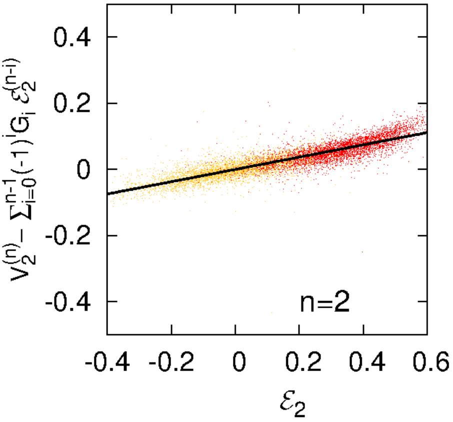
- $n = 1$:



- $G_1 = -0.000398 \pm 0.00198$

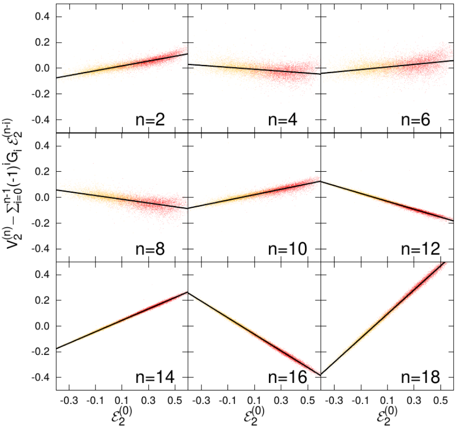
Numerical results: G_2

- $n = 2$:



- $G_2 = 0.185 \pm 0.0029$

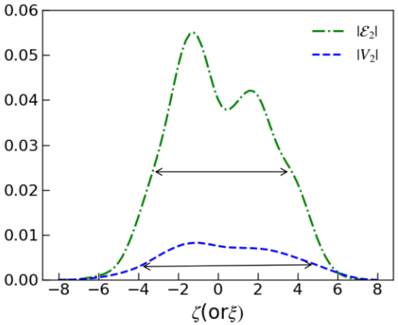
Numerical results: G_n



- Odd oders: $G_n = 0$;
- Even orders: G_n increase with n and flip its sign for every even n.

Application 1: Two-point correlation of V_2

- Width of V_2: $\langle(\Delta\zeta)^2\rangle = \frac{\int d\zeta d\zeta' \langle V_2(\zeta) V_2^*(\zeta')\rangle(\zeta'-\zeta)^2}{\int d\zeta d\zeta' \langle V_2(\zeta) V_2^*(\zeta')\rangle}$



- Width of two-point correlation of $\mathcal{E}_2$ are defined accordingly.

Application 1: Two-point correlation

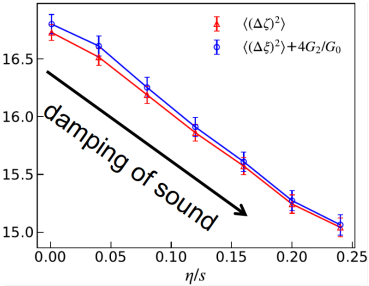
One finds the width of V_2 is enhanced due to fluid longitudinal response:

$$\underbrace{\langle(\Delta\zeta)^2\rangle}_{\text{final state}} = \underbrace{\langle(\Delta\xi)^2\rangle}_{\text{initial state}} + \underbrace{4\frac{G_2}{G_0}}_{\text{fluid response}}$$

- $\langle(\Delta\zeta)^2\rangle$ is entirely determined by the initial model.
- The ratio G_2/G_0 depends on η/s.
- $\langle(\Delta\zeta)^2\rangle$ can be measured in experiment

Application 1: Two-point correlation

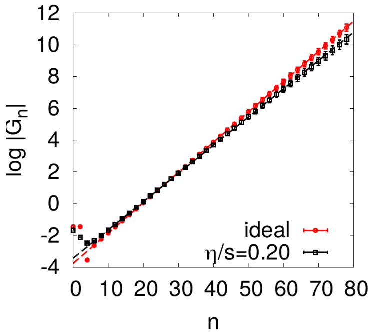
- From numerical simulations : $\langle(\Delta\zeta)^2\rangle = \langle(\Delta\xi)^2 + 4\frac{G_2}{G_0}$ is verified



- $\langle(\Delta\zeta)^2\rangle$ decreases with the increase of η/s due to sound damping.

Application 2: Convergence of gradient expansion

- Magnitudes of G_n increase exponentially, but not a factorial increase.



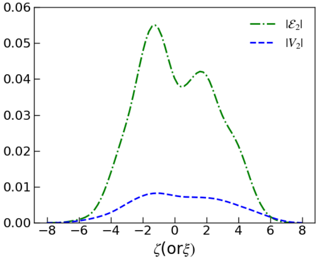
  - Finite radius of convergence, determined by the slopes.
  - For ideal hydro: $k^* = 1.209$; For $\eta/s = 0.2$: $k^* = 1.192$.
  - When the η/s decreases, the k^* increases slightly .

Application 2: $\mathcal{E}_2^{\text{Reg}}$ from finite k^*

- Regularization of the $\mathcal{E}_2$ from finite k^*

$$\mathcal{E}_2^{\text{Reg}} = \int_{-k^*}^{k^*} \frac{dk}{2\pi} e^{ik\xi} \tilde{\mathcal{E}}_2(k) = \int_{-\infty}^{\infty} d\xi^{'} \mathcal{E}_2(\xi^{'}) R(\xi - \xi^{'}; k^*)$$

with $R(\xi - \xi^{'}; k^*) = \frac{\sin(k^*(\xi - \xi^{'}))}{\pi(\xi - \xi')}$.



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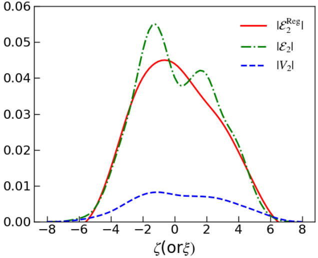

- we consider $k^* = 1.2$ when $\eta/s = 0.08$;
- $k^* \to \infty$: $R(\xi - \xi^{'}; k^*) \to \delta(\xi - \xi^{'})$;
- $k^* \to 0$, $\frac{d}{d\xi} R(\xi - \xi^{'}; k^*) \to 0$ (boost invariant $\mathcal{E}_2^{\text{Reg}}(\xi)$).

Summary

- We have derived the formulation of longitudinal hydro response: $V_2(\zeta) = G_0\mathcal{E}_2(\zeta) + G_1\frac{d\mathcal{E}_2(\zeta)}{d\zeta} + G_2\frac{d^2\mathcal{E}_2(\zeta)}{d\zeta^2} + O\left(\frac{d^3}{d\zeta^3}\right)$. The G_n can be calculated by MUSIC.
- Two-point correlation of flow is understandable in terms of longitudinal fluid response: $\langle(\Delta\zeta)^2\rangle = \langle(\Delta\xi)^2 + 4\frac{G_2}{G_0}$.
- Hydro gradient expansion with finite radius of convergence k^*, results in regularization of the $\mathcal{E}_2$.

Thank you!