Hydrodynamic attractors, initial state energy and particle production in relativistic nuclear collisions

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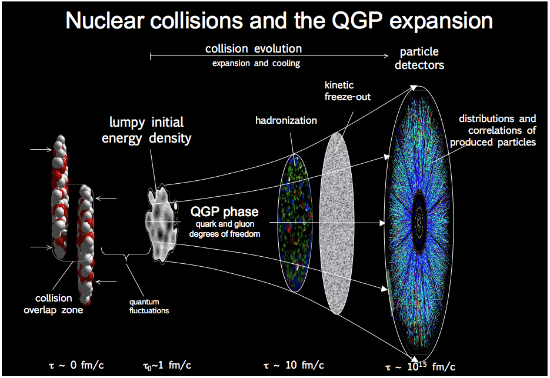
G. Giacalone, AM, S. Schlichting, *Hydrodynamic attractors, initial state energy and particle production in relativistic nuclear collisions* [arXiv:1908.02866]
P. Hanus, AM, K. Reygers, *Entropy production in pp and Pb-Pb collisions at the LHC* [arXiv:1908.02792]

Isolated quantum systems and universality in extreme conditions

Far-from-equilibrium QCD in nucleus-nucleus collisions

Experiments indicate formation and equilibration of Quark-Gluon Plasma

$$\left\langle \frac{dE_\perp}{d\eta} \right\rangle_0 \Rightarrow$$



Sorensen, Quark-gluon plasma 4, 2010

$$\Rightarrow \left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle$$

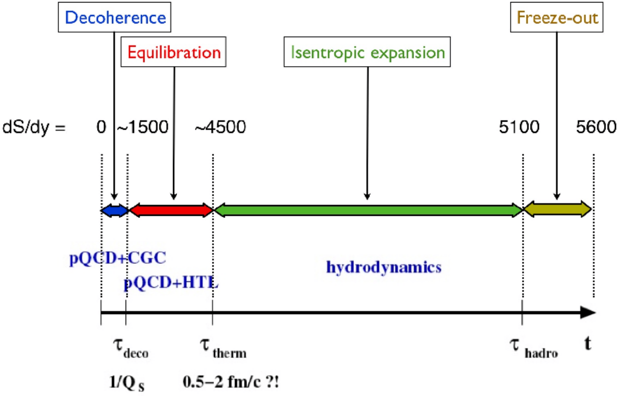
- Non-equilibrium initial-state: tractable in weak coupling QCD (CGC)
- Final-state observables: produced particle spectra and correlations

Most basic question: how many particles will be produced in a collision?

Estimates of entropy production in central Au-Au collisions at RHIC

Particle multiplicity is directly proportional to entropy at thermalization

$$\left\langle \frac{dS}{dy} \right\rangle_{\tau_{\text{therm}}} = \langle s\tau A_\perp \rangle_{\tau_{\text{therm}}} \approx \frac{S}{N_{\text{ch}}} \left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle .$$



Muller and Schafer (2011)

Most of entropy production occurs at early times during equilibration.

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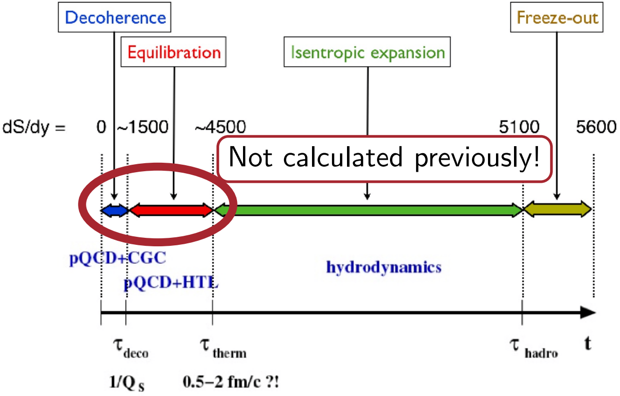

Muller and Schafer (2011)

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Measuring entropy in heavy ion collisions

Entropy formula for dilute gas of hadrons

$$S = \int \frac{d^3r d^3p}{(2\pi)^3}\left[-f \ln f + (1 \pm f) \ln (1 \pm f)\right]$$

$f(r,p)$—one-body particle phase-space distribution.

Measured particle spectra $\Longrightarrow$ coordinate-integral of $f(r,p)$

$$\frac{dN}{d^3p} = \int d^3r f(r,p).$$

Measured HBT radii $\Longrightarrow$ spatial spread of distribution

$$f(\vec{p},\vec{r}) = \mathcal{F}(\vec{p}) \exp\left(-\frac{x_{\rm out}^2}{2R_{\rm out}^2} - \frac{x_{\rm side}^2}{2R_{\rm side}^2} - \frac{x_{\rm long}^2}{2R_{\rm long}^2}\right)$$

$$\mathcal{F}(\vec{p}) = \frac{(2\pi)^{3/2}}{R_{\sf out} R_{\sf side} R_{\sf long}} \frac{dN}{d^3p}.$$

Data driven calculation of entropy in the collision. Pal and Pratt (2003)

Entropy budget for 0–10% Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

Individual contributions of measured and inferred hadron species.

Hanus, AM and Reygers (2019)

particle	$(dS/dy)^{\text{one state}}_{y=0}$	factor	$(dS/dy)^{\text{total}}_{y=0}$
π	2182	3	6546
K	605	4	2420
η	399	1	399
η'	66	1	66
p	266	2	532
n	266	2	532
Λ	160	2	320
Σ	58	6	348
Ξ	39	4	156
Ω	8	2	16
total			11335

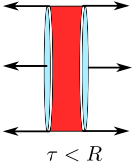
Entropy per measured charged hadron $S/N_{ch} = 6.7 \pm 0.8$.

Smaller than 7.7-8.5 in Hadron Resonance Gas models at $T_{ch} = 156$ MeV

Boost-invariant equations of motion of 1D expansion at early times

Energy-momentum conservation $T^{\mu}{}_{\nu} = \text{diag}\,(e, P_T, P_T, P_L)$

$$\partial_\tau e = -\frac{e + P_L}{\tau},$$



Need microscopic input: constitutive relation $P_L = P_L(e, \tau)$.

- Equilibrium: equation of state

$$\frac{P_L}{e} \approx \frac{1}{3} \quad \Longrightarrow \quad e \propto \tau^{-\frac{4}{3}}.$$

- Near-equilibrium: viscous constitutive equations

$$\frac{P_L}{e} = \frac{1}{3} - \frac{16}{9}\frac{\eta/s}{\tau T} + \dots.$$

η/s —specific shear-viscosity.

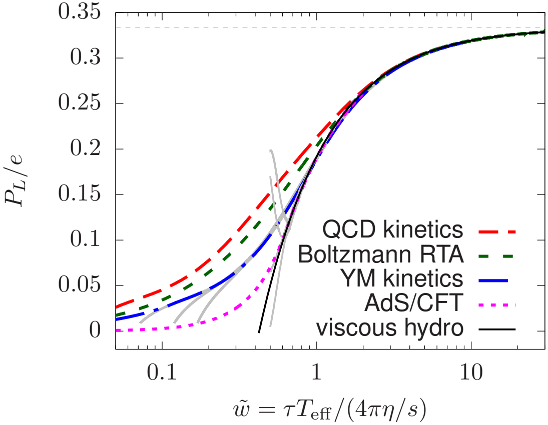
Macroscopic evolution far from equilibrium?

Macroscopic theory of equilibration: hydrodynamic attractors

Apparent emergence of constitutive relations far-from-equilibrium

Heller and Spaliǹksi (2015)

$$\frac{P_L}{e} = f\left[\tilde{w} = \frac{\tau T_{\text{eff}}}{4\pi\eta/s}\right], \quad \text{where} \qquad T_{\text{eff}} \propto e^{1/4}.$$



see reviews by Florkowski, Heller and Spalinksi (2017), Romatschke and Romatschke (2017) [1, 2]

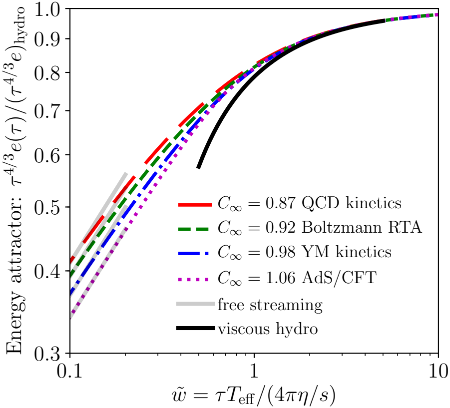
Similarities of energy evolution in different theories

Integrating equations of motion:

Giacalone, AM, Schlichting (2019)

$$e(\tau_{\text{therm}}) = e_0 \exp\left(- \int_{\tilde{w}_0}^{\tilde{w}_{\text{therm}}} \frac{d\tilde{w}}{\tilde{w}} \frac{1 + f(\tilde{w})}{\frac{3}{4} - \frac{1}{4} f(\tilde{w})} \right) .$$

Final-state entropy: $\frac{dS}{dy} = A_\perp (s\tau)_{\tau_{\text{therm}}} \propto \left(e\tau^{\frac{4}{3}} \right)^{\frac{3}{4}}_{\tau_{\text{therm}}} \equiv \left(e\tau^{\frac{4}{3}} / \mathcal{E}(\tilde{w}) \right)^{\frac{3}{4}}$



Universal early/late asymptotics

Viscous hydro:

$\mathcal{E}(\tilde{w} \gg 1) = 1 - \frac{2}{3\pi\tilde{w}}$

Free-streaming ($e \sim \tau^{-1}$):

$\mathcal{E}(\tilde{w} \ll 1) = C_\infty^{-1} \tilde{w}^{4/9}$

Entropy-production from hydrodynamic attractor

Substitute the early time asymptotics

$$(s\tau)_{\tau_{\rm therm}} = \frac{4}{3}\left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{1/4}\left(\frac{e\tau^{4/3}}{C_\infty^{-1}\left(\frac{T\tau}{4\pi\eta/s}\right)^{4/9}}\right)^{3/4}.$$

Final state entropy density:

$$(s\tau)_{\tau_{\rm therm}} = \frac{4}{3}C_\infty^{3/4}\left(4\pi\frac{\eta}{s}\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{1/3}(e\tau)_0^{2/3}.$$

Consider nucleus transverse overlap area $A_\perp$

$$\underbrace{\left\langle\frac{dN_{\rm ch}}{d\eta}\right\rangle}_{\text{final-state}} \approx A_\perp\ \frac{N_{\rm ch}}{S}\underbrace{\frac{4}{3}C_\infty^{3/4}\left(4\pi\frac{\eta}{s}\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{\rm eff}\right)^{1/3}}_{\text{medium properties}}\underbrace{\left(\frac{1}{A_\perp}\left\langle\frac{dE_\perp}{d\eta}\right\rangle_0\right)^{2/3}}_{\text{initial-state}}$$

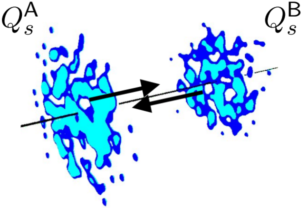
All relevant-prefactors and powers included!

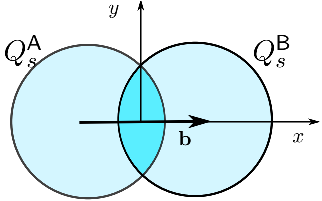
Important to model initial-state energy density $(e\tau)_0$

$\Longrightarrow$ tractable with first principle theory of high-energy QCD.

Energy deposition in high energy nucleus-nucleus collisions

Collisions of glasma sheets in color-glass condensate effective theory





Local saturation scale is proportional to nuclear thickness

$$Q_s^2(\mathbf{x}_\perp) \propto T(\mathbf{x}_\perp).$$

Gluon liberation (up to $\log$–corrections)

$$\text{gluon number} \quad (n\tau)_0(\mathbf{x}_\perp) \propto \; T^<(\mathbf{x}_\perp),$$

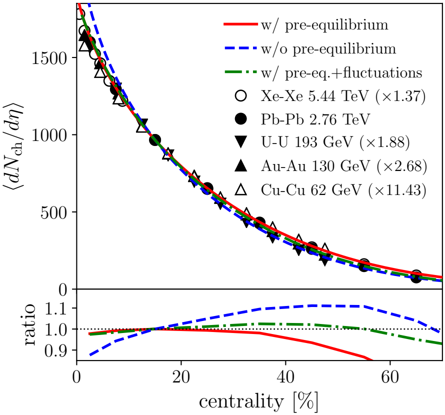
$$\text{gluon energy} \quad (e\tau)_0(\mathbf{x}_\perp) \propto \; T^<(\mathbf{x}_\perp)\sqrt{T^>(\mathbf{x}_\perp)}.$$

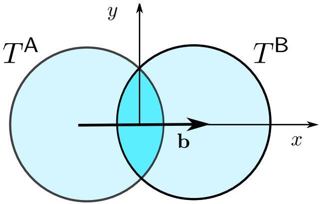
Can now determine centrality dependence of $dN_{ch}/d\eta$

Universal centrality dependence of particle multiplicity

Collapse of rescaled multiplicity $\Longrightarrow$ compare with theory models

$$\left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle \propto \underbrace{\frac{dS_{\text{therm}}}{d\eta}}_{\text{equilibration}}, \quad \underbrace{\frac{dN_{\text{gluons}}}{d\eta}}_{\text{no equilibration}}, \quad \underbrace{\left\langle \frac{dS_{\text{therm}}}{d\eta} \right\rangle}_{\text{e-by-e fluctuations}}.$$





$$centrality = \pi b^2/\sigma_{\text{AA}}$$

Entropy production and e-by-e fluctuations improve agreement with data.

Initial state energy density

Bjorken formula for initial state energy density

$$e_0^{\text{Bjorken}} \approx \frac{1}{\tau_0 A_\perp} \frac{dE_\perp^{\text{final}}}{dy}.$$

Does not include work done during expansion!

$$\frac{dE_\perp^{\text{initial}}}{dy} = A_\perp (\tau e)_0 > \frac{dE_\perp^{\text{final}}}{dy}.$$

Including the longitudinal work during expansions in central Pb-Pb get

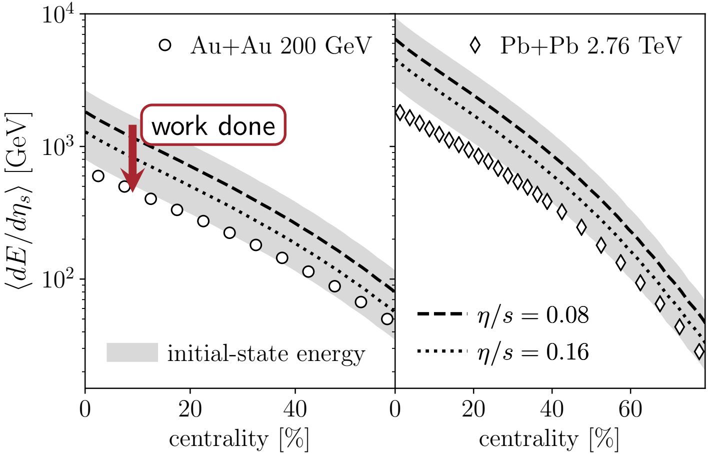
$$e_0 \approx 270 \text{ GeV/fm}^3 \left(\frac{\tau_0}{0.1\text{fm}/c}\right)^{-1} \left(\frac{C_\infty}{0.87}\right)^{-9/8} \left(\frac{\eta/s}{2/4\pi}\right)^{-1/2}$$
$$\left(\frac{A_\perp}{138\text{fm}^2}\right)^{-3/2} \left(\frac{dN_{\text{ch}}/d\eta}{1600}\right)^{3/2} \left(\frac{\nu_{\text{eff}}}{40}\right)^{-1/2} \left(\frac{S/N_{\text{ch}}}{7.5}\right)^{3/2},$$

c.f. $e \approx 0.3 \text{ GeV/fm}^3$ near QCD cross-over.

Centrality dependence of initial state energy

Matching multiplicity allows to infer the initial-state energy per rapidity

Bands are variations of $C_\infty = [0.8\text{-}1.15]$, $\eta/s = [0.08\text{-}0.24]$



Initial state energy ⇔ non-equilibrium properties of QGP.

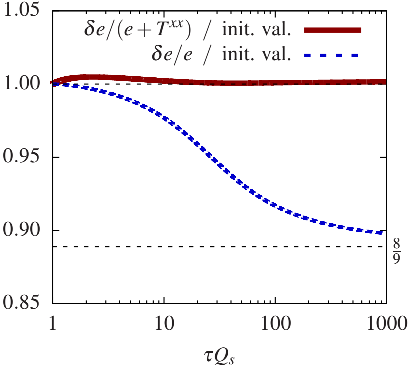
Equilibration of perturbations

Non-linearities change the perturbation spectra

$$s_{\text{therm}} \propto e_0^{\frac{2}{3}} \Longrightarrow \frac{\delta s_{\text{therm}}}{s_{\text{therm}}} = \frac{2}{3}\frac{\delta e}{e_0}.$$

$k = 0$ perturbation evolution in kinetic theory: $\delta e/(e + T^{xx}) = \text{const.}$

Keegan, Kurkela, AM and Teaney (2016) [3]



$$\frac{\delta e_{\text{therm}}}{e_{\text{therm}}} = \frac{8}{9}\frac{\delta e_0}{e_0}$$

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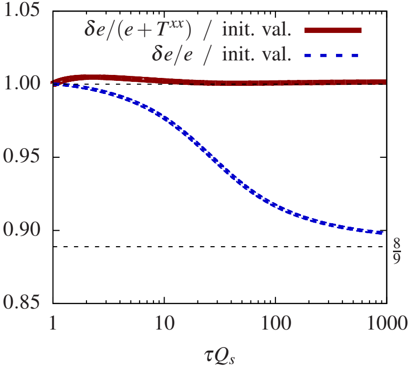

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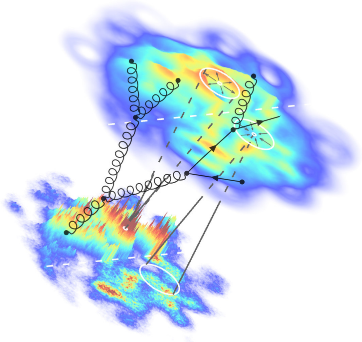
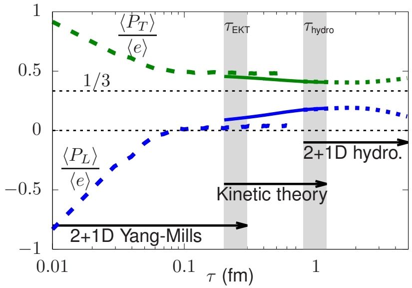
Transverse pre-equilibrium evolution

KøMPøST— event-by-event kinetic pre-equilibrium for heavy ion collisions.

$$\underbrace{\delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\text{hydro}}, \mathbf{x}')}_{\text{goes into hydro}} = \int d^2\mathbf{x}' \; \underbrace{G^{\mu\nu}_{\alpha\beta}\left(\mathbf{x}-\mathbf{x}', \tau_{\text{hydro}}, \tau_{\text{EKT}}\right)}_{\text{linear response function}} \underbrace{\delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\text{EKT}}, \mathbf{x}')}_{\text{initial}}.$$



https://github.com/KMPST/KoMPoST [6]



Kurkela, AM, Paquet, Schlichting and Teaney (2018)[4, 5]

Linearized 2+1D pre-equilibrium propagator based on scaled response functions of kinetic theory.

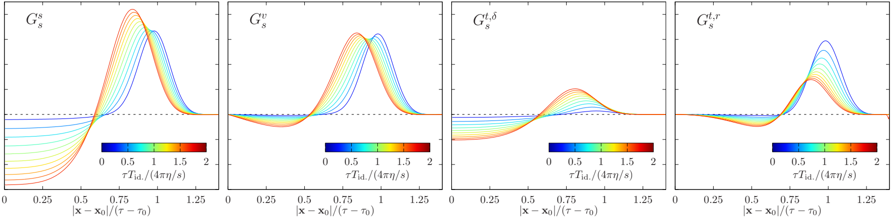
Kinetic theory response functions

Invariant form of non-equilibrium response functions

$$G^{\mu\nu}\Big(\tau, \tau_0, |\mathbf{x}-\mathbf{x}_0|, e(\tau_0), \lambda\Big) \Rightarrow G^{\mu\nu,\mathrm{univ}}\left(\frac{\tau T_{\mathrm{Id.}}}{\eta/s}, \frac{|\mathbf{x}-\mathbf{x}_0|}{(\tau-\tau_0)}\right)$$

All components of energy-momentum tensor generated by kinetic response

energy→energy | energy→velocity | energy→shear-stress



Kinetic response functions evolve from free-str.-like to hydrodynamic-like.

for details see Kurkela, AM, Paquet, Schlichting and Teaney (2018) [4]

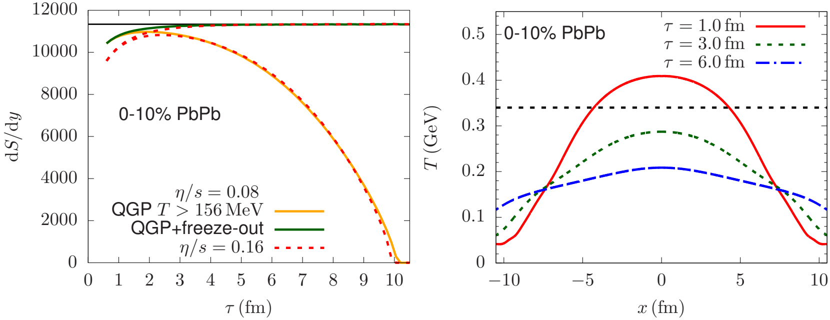
Space-time evolution of entropy in an averaged Pb-Pb collision

"Back-propagation" of entropy density in an averaged event.

- KøMPøST+FluiduM hydrodynamic simulation

for FluiduM, see Floerchinger, Grossi and Lion (2018) [7]

- Final entropy fixed at $\frac{dS}{dy} \approx 11300$
- Initial conditions given by averaged MC-Glauber

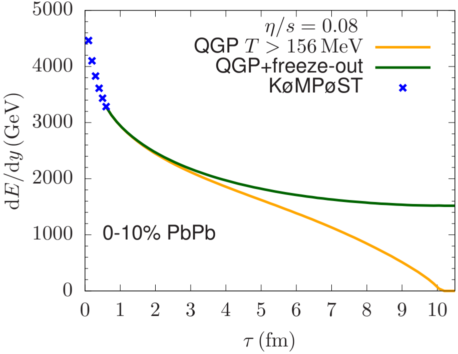


Hanus, AM and Reygers (2019)

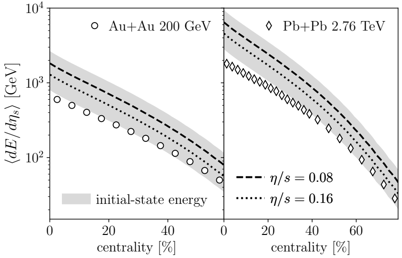
Central $T \sim 400$ MeV at $\tau = 1$ fm, but depends on the value of η/s.

Initial state energy in central Pb-Pb collisions

KøMPøST evolution of energy from $0.1\,\text{fm}$ to $0.6\,\text{fm}$.



Hanus, AM and Reygers (2019)



Good agreement between KøMPøST simulation and attractor formula.

Summary and Outlook

- Significant progress in understanding and modelling pre-equilibrium.
  $\Longrightarrow$ *That's where entropy is generated.*
- Direct connection between initial and final states.
  $\Longrightarrow$ *Simple formula for final state entropy.*
- Universal centrality dependence of particle multiplicity.
  $\Longrightarrow$ *Naturally born out with pre-equilibrium entropy production.*
- Quantitative estimation of initial state energy.
  $\Longrightarrow$ *New constraints on non-equilibrium QGP properties.*
- Fluctuations are also affected by equilibration
  $\Longrightarrow$ *Can be propagated with KøMPøST*

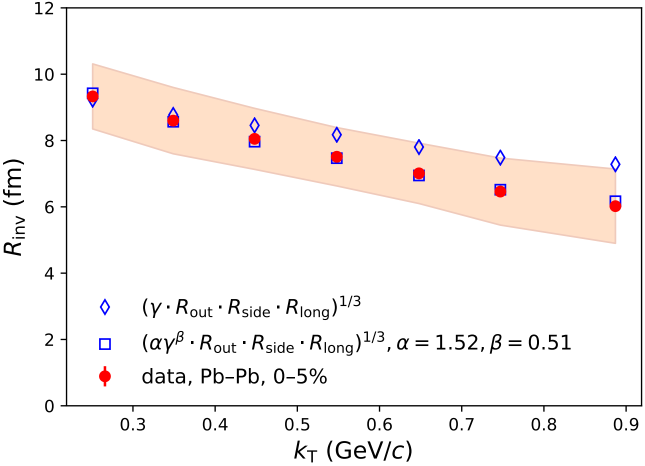
Outlook:

- Corrections due to transverse expansion and incomplete equilibration.
- Equilibration of event-by-event fluctuation spectra.

Extraction of $R_{\text{out}} R_{\text{side}} R_{\text{long}}$ from data

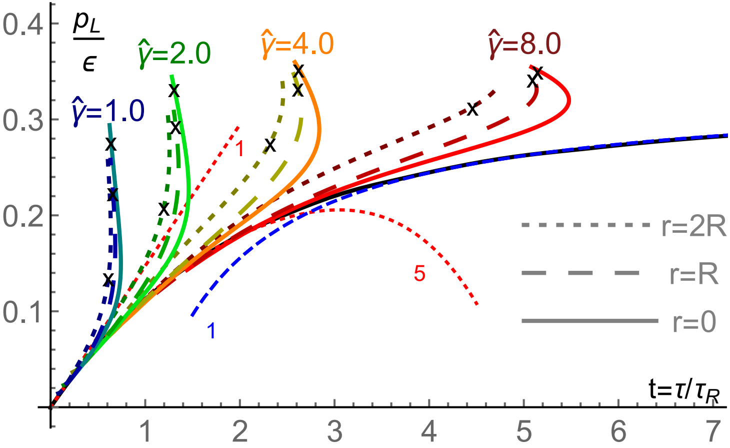
Experimentally easier to measure one-dimensional Gaussian radius R_{inv}

We use phenomenological parametrizations to infer $R_{\text{out}} R_{\text{side}} R_{\text{long}}$



Entropy integral only logarithmically sensitive to radius.

Radial expansion in isotropization time approximation.



Kurkela, Wiedemann and Wu (2019)[8]

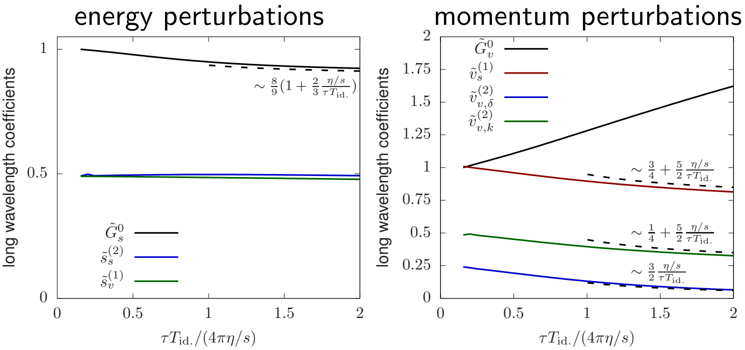
Opaque ($\hat{\gamma} \gg 1$) systems follow 1D attractor for $\tau \ll 2R$.

Low k expansion of kinetic reponse functions

$$\tilde{G}^{\text{energy}}_{\text{energy}}(\tau,\tau_0,|\mathbf{k}|) = \tilde{G}^0_s(\tau,\tau_0)\ \left(1 - \frac{1}{2}|\mathbf{k}|^2(\tau-\tau_0)^2\ \tilde{s}^{(2)}_s + ...\right) ,$$

$$\tilde{G}^{\text{mom.}}_{\text{energy}}(\tau,\tau_0,|\mathbf{k}|) = \tilde{G}^0_s(\tau,\tau_0)\ \left(|\mathbf{k}|(\tau-\tau_0)\ \tilde{s}^{(1)}_v + ...\right) ,$$

Taylor expansion coefficients of response functions to (initial):



Smooth evolution of low-wavelength response in kinetic pre-equilibrium.

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