Adiabatic hydrodynamization in the rapidly-expanding quark-gluon plasma

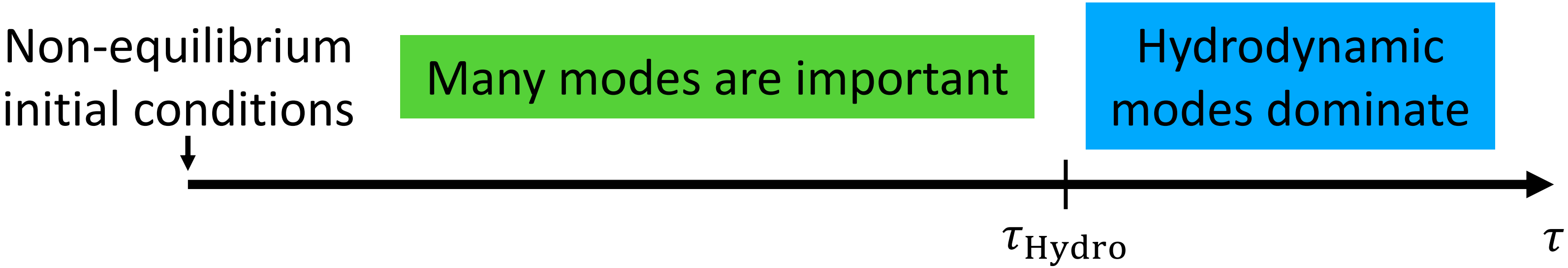
Jasmine Brewer

Based on:

JB, Li Yan, and Yi Yin [arXiv:1910.00021]

and ongoing work

Expected applicability of hydrodynamics

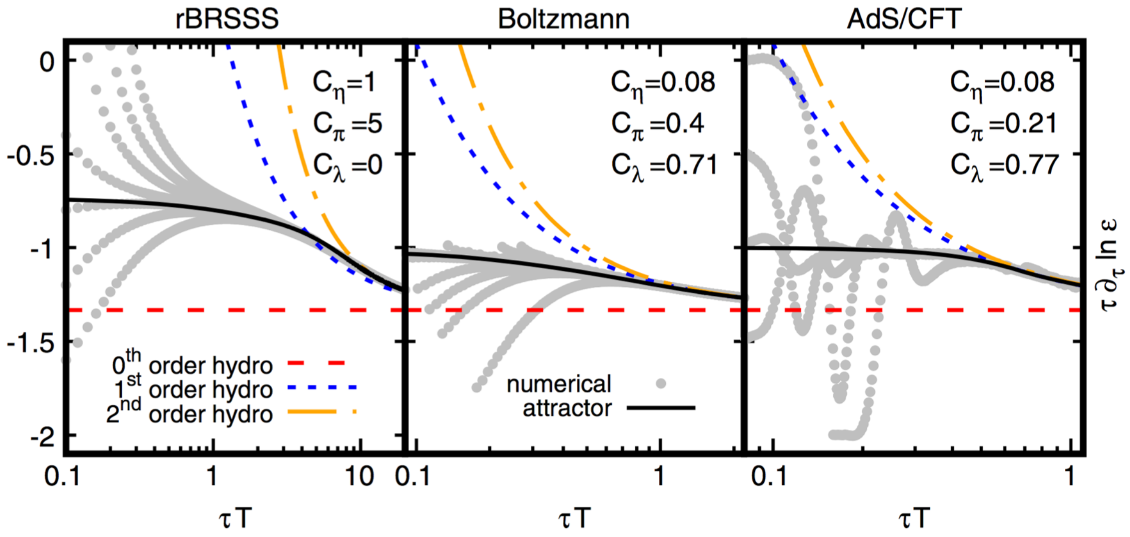


"Gradients are large so hydro doesn't work"

"Gradients are small so hydro works"

Observation of attractor behavior

Heller and Spalinski [1503.07514], many follow-ups



Suggestive of simplified bulk description before τ_{Hydro}

Fig: Romatschke [1704.08699]

If the evolution of a system follows an attractor, does this imply it is hydrodynamized?

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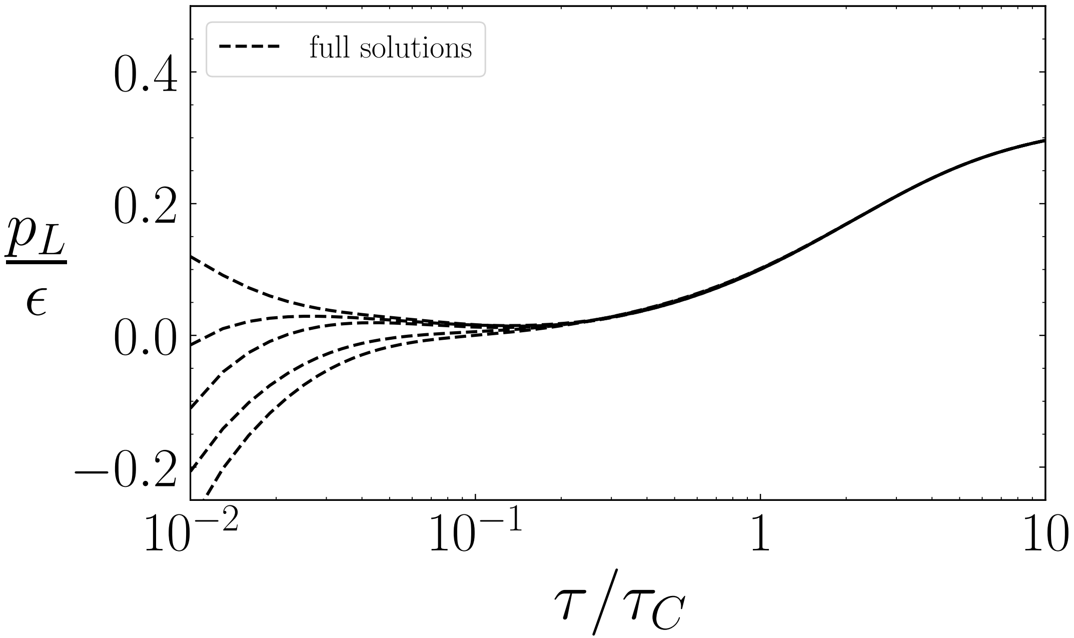
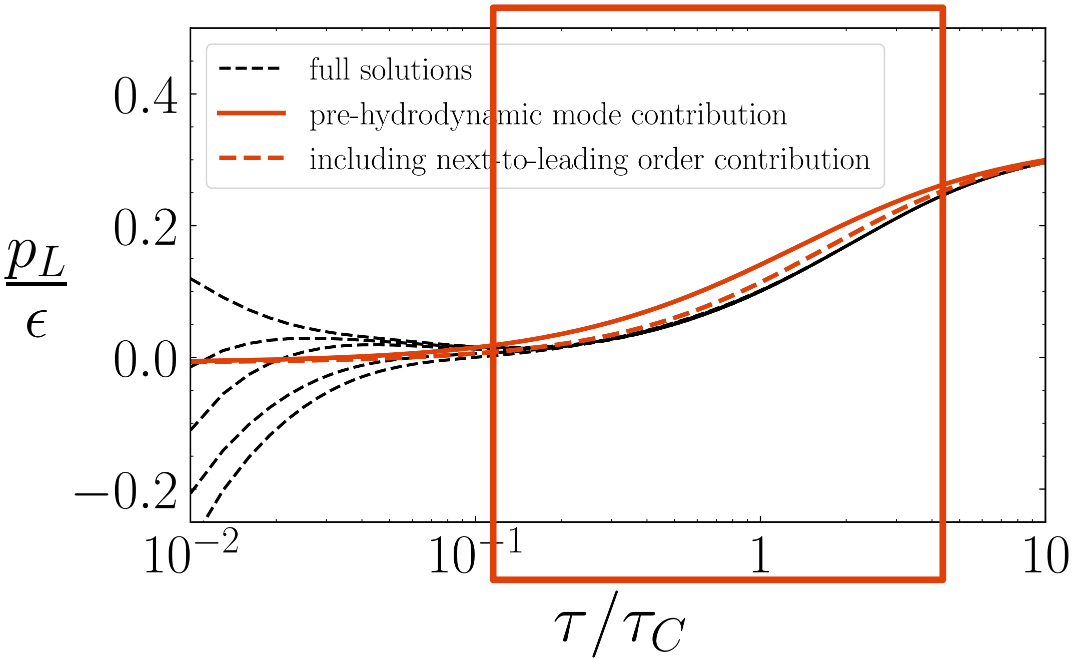
If by hydrodynamization you mean that hydrodynamic modes dominate the evolution, then no

A new interpretation of far-from-equilibrium behavior



Bjorken-expanding kinetic theory in the relaxation time approximation

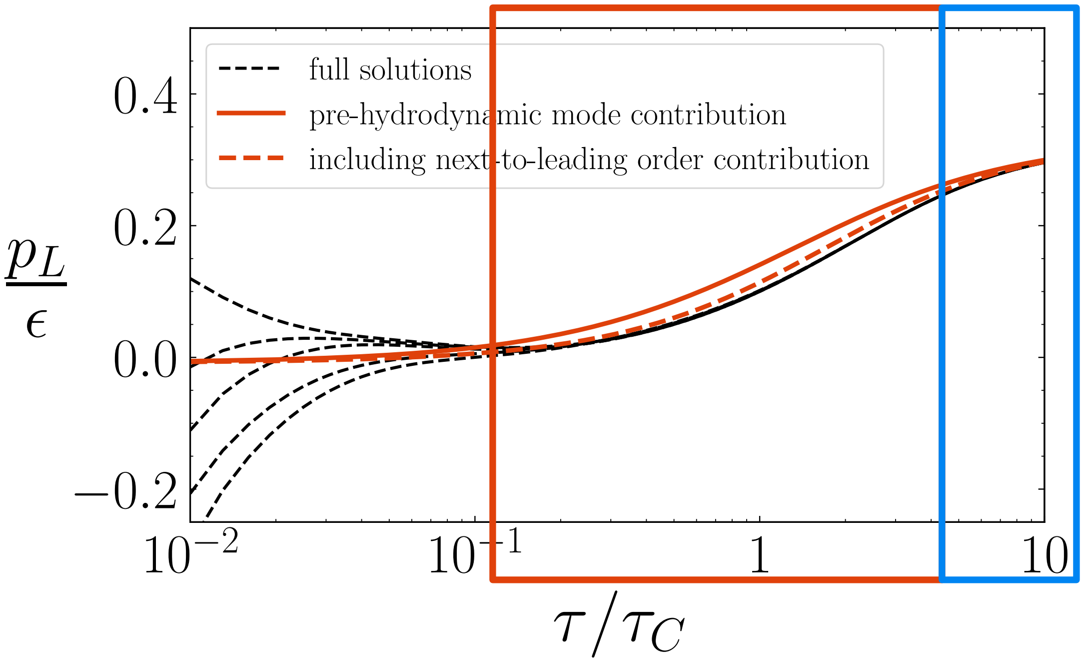
A new interpretation of far-from-equilibrium behavior



A single mode dominates the evolution, but it is not the hydrodynamic mode

"Pre-hydrodynamic" mode

A new interpretation of far-from-equilibrium behavior

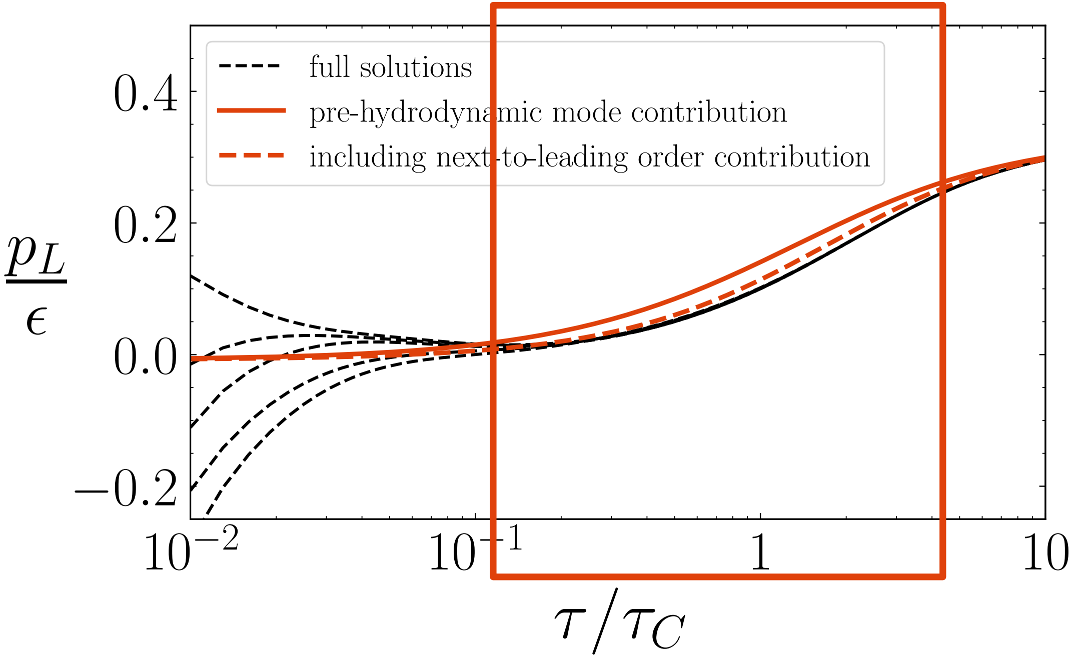


Pre-hydrodynamic mode evolves continuously into hydrodynamic mode

Adiabatic theorem:

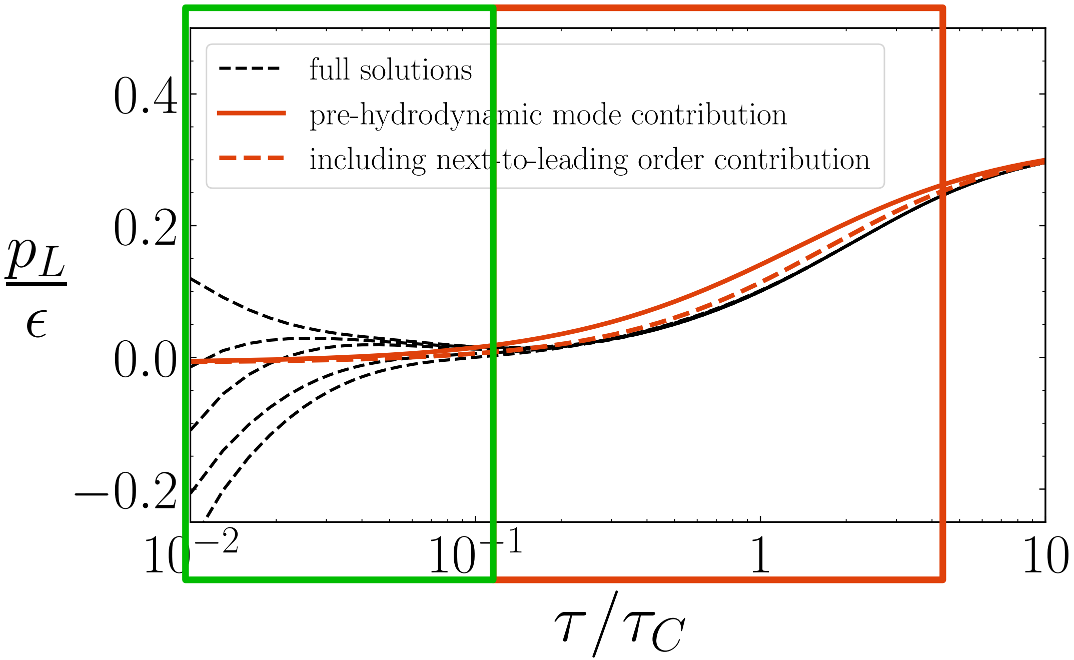
"A system prepared in its (instantaneous) ground state will remain in its (instantaneous) ground state under adiabatic evolution of the Hamiltonian"

Adiabatic interpretation of far-from-equilibrium behavior



Pre-hydrodynamic mode is instantaneous ground state of an effective Hamiltonian

Adiabatic interpretation of far-from-equilibrium behavior



Dominance of ground state at early times driven by rapid longitudinal expansion

Hamiltonian formulation from kinetic theory

Bjorken-expanding kinetic theory

$$\frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau) = -\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau) - \hat{C}[f]$$

longitudinal expansion ⟷ collisions

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Moments of kinetic equation give evolution of more macroscopic quantities

$$\int_{\mathbf{p}} p\ f(p_z, p_\perp; \tau) = \epsilon(\tau)$$

Energy density

$$\int_{|\mathbf{p}|} p\ f(p_z, p_\perp; \tau) = F_\epsilon(\cos\theta; \tau)$$

Angular distribution contributing to energy density

Hamiltonian formulation from kinetic theory

$$\int_{|\mathbf{p}|} p \left(\underbrace{\frac{\partial}{\partial \tau} f(p_z, p_\perp; \tau)}_{\frac{\partial}{\partial \tau} F_\epsilon} = \underbrace{-\frac{p_z}{\tau} \frac{\partial}{\partial p_z} f(p_z, p_\perp; \tau)}_{-\frac{1}{\tau}(\ldots) F_\epsilon} - \hat{C}[f] \right)$$

For some $\hat{C}[f]$ $\quad \tau \frac{\partial}{\partial \tau} F_\epsilon = (\ldots) F_\epsilon \longleftrightarrow \partial_y \psi = -\mathcal{H}(y) \psi \qquad y = \log\left(\frac{\tau}{\tau_I}\right)$

Implies Hamiltonian formulation

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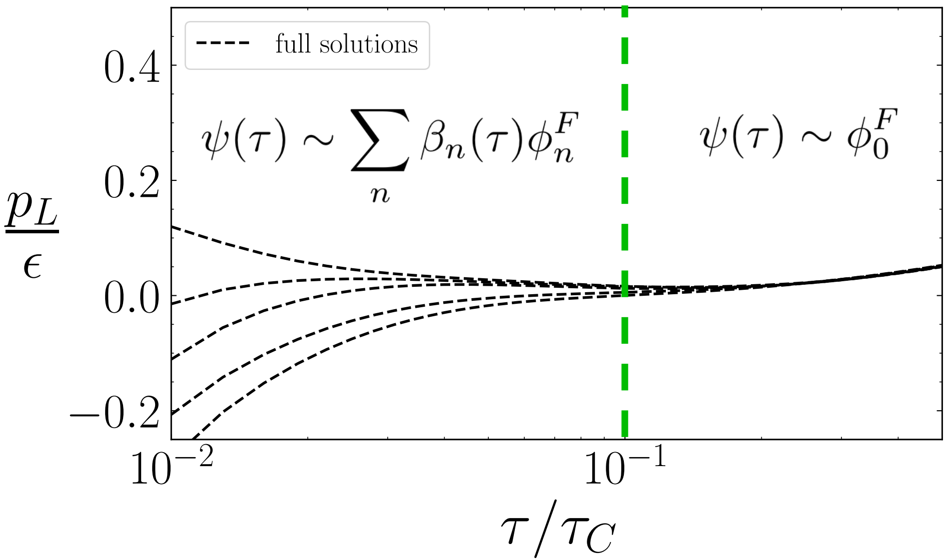
Implies Hamiltonian formulation

In relaxation time approximation, has simple form $\quad \mathcal{H} = \mathcal{H}_F + \frac{\tau}{\tau_C} \mathcal{H}_R$

free streaming $\longrightarrow$ hydrodynamics

Collision-less expansion at early times: $\hat{C}[f] = 0$ $\partial_y \psi = -\mathcal{H}_F \psi$



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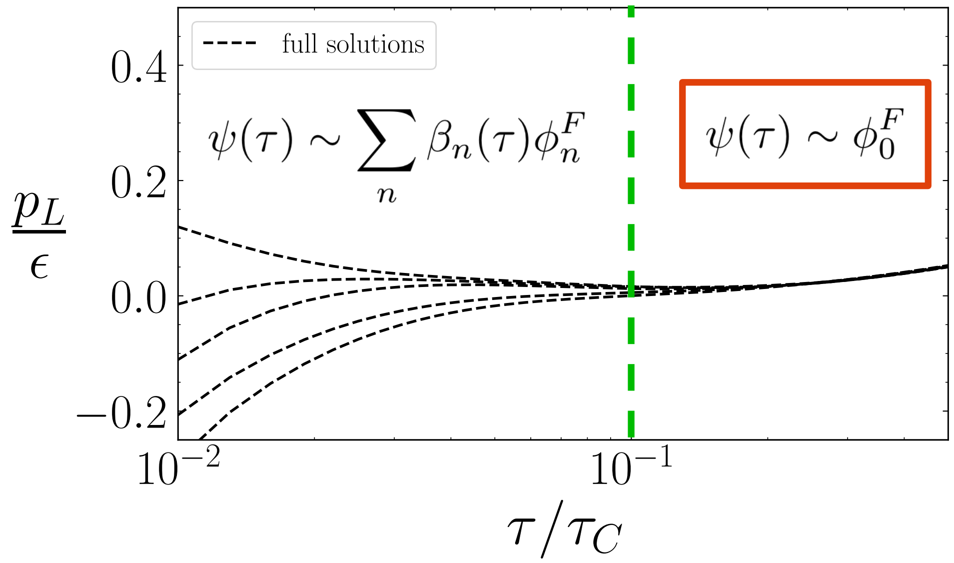

Since $\mathcal{H}_F$ is gapped, ψ decays toward the ground state

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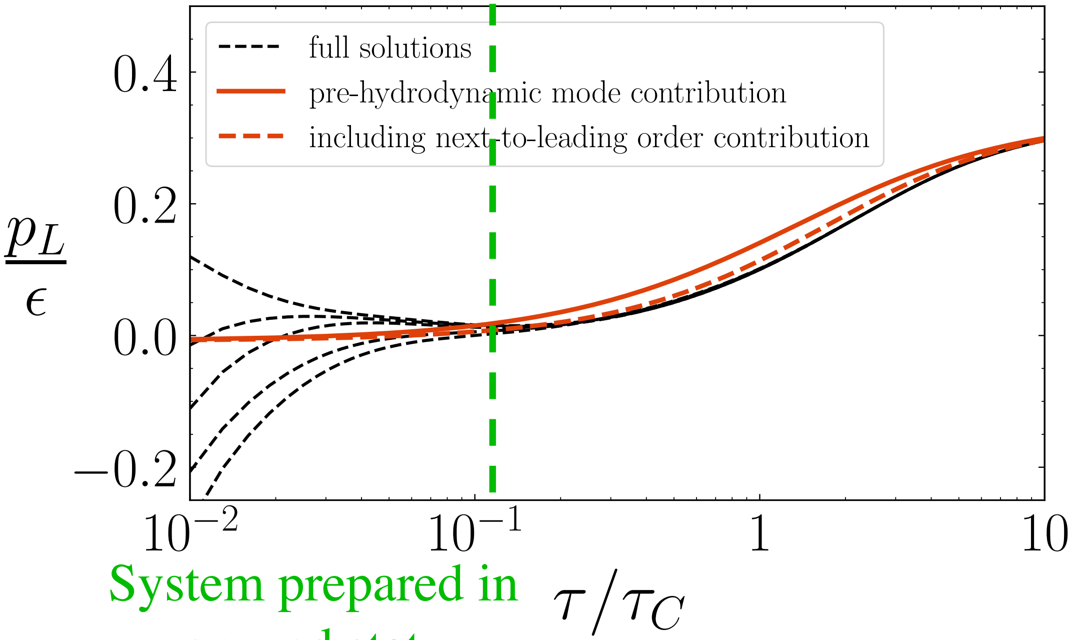

System prepared in ground state

Is the subsequent evolution adiabatic?

Adiabatic hydrodynamization

Definition of adiabatic hydrodynamization:

System evolution determined by the instantaneous ground state

$$\psi(y) \sim \phi_0(y)$$

$$\partial_y \psi = -\mathcal{H}(y)\psi \longrightarrow \partial_y \phi_0(y) = -\mathcal{E}_0(y)\phi_0(y)$$

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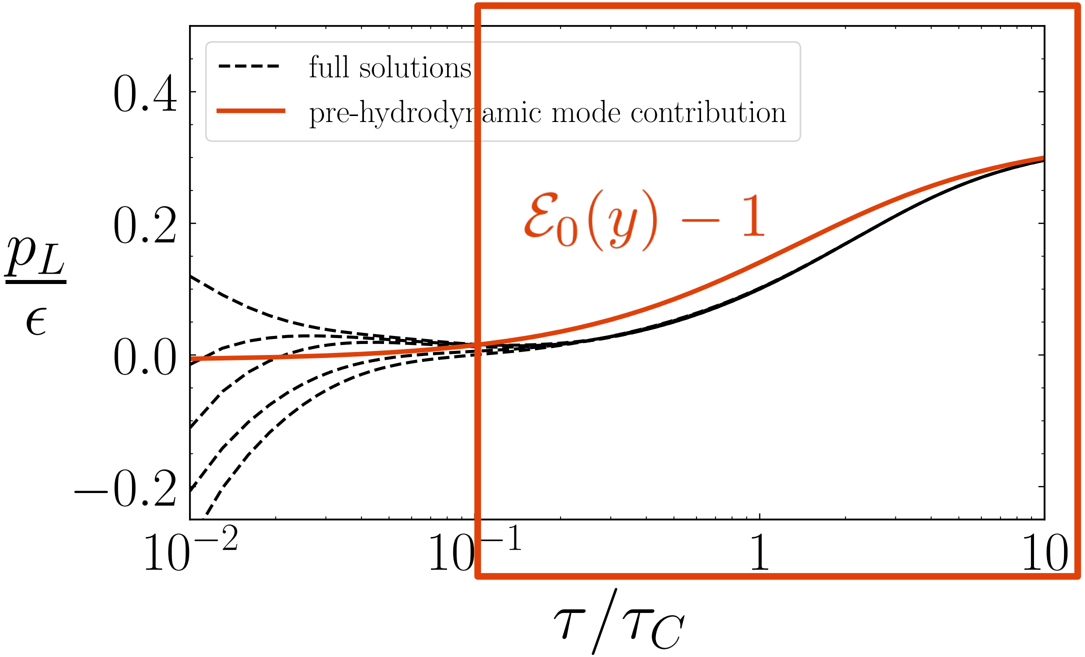
Predicts non-trivial relations between components of ψ, e.g.

$$g(y) \equiv 1 + p_L/\epsilon = \mathcal{E}_0(y)$$

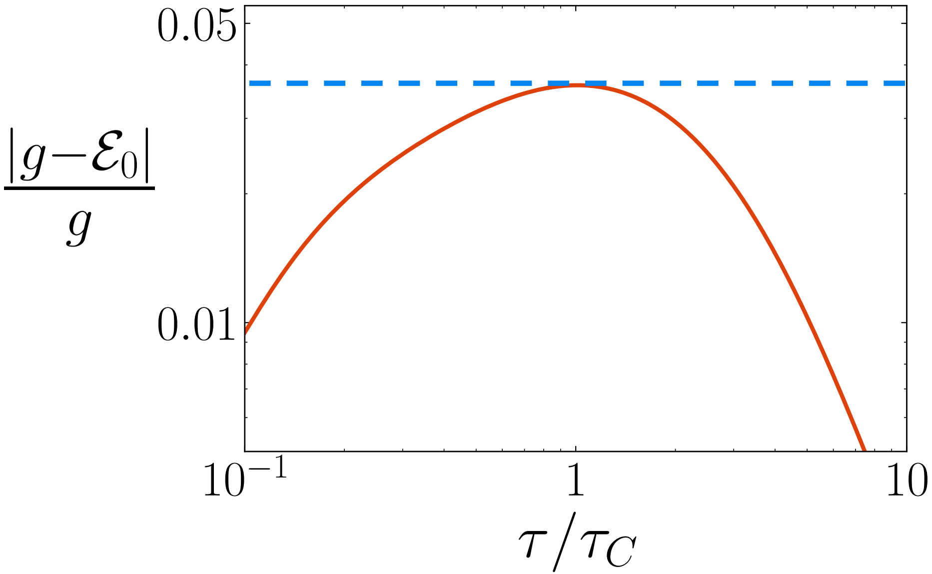
Test extent to which these relations are satisfied!

Adiabatic hydrodynamization in RTA kinetic theory



Far-from-equilibrium evolution dominated just by evolution of instantaneous ground state (“pre-hydrodynamic”) mode!

Implies presence of a small "adiabatic" parameter that suppresses contributions from other modes



$>$ 95% of g described by instantaneous ground state mode

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- Hamiltonian evolution slow compared to energy gap
- Small close to hydro limit

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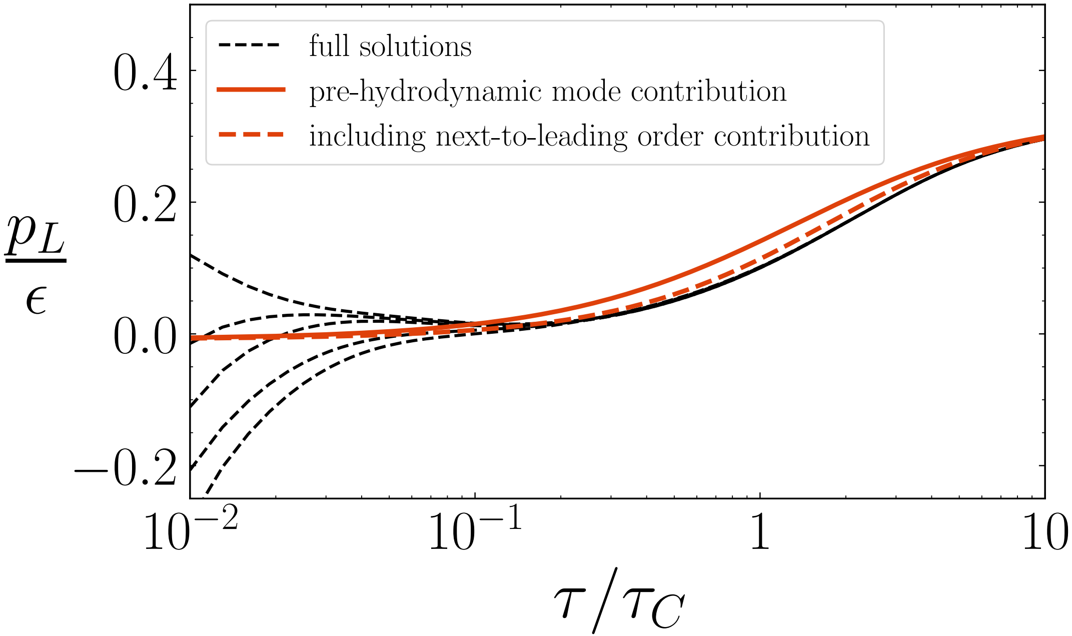
"Fast quench" adiabaticity

- Matrix element for transitions between ground and excited states suppressed
- Small at early times because H suppressed by λ

A new perturbative expansion



Including contributions from the excited states to the evolution at $\mathcal{O}(\delta_A)$ shows explicitly that they are a small correction

How generic is adiabatic hydrodynamization?

- Independence of initial conditions at early times driven by gap from rapid longitudinal expansion

- Adiabatic parameter generically small at early and late times. In bottom-up thermalization there appears to be a parametrically narrow window when it can be large

- How to formulate this idea in more general theories, e.g. strong coupling?