Chiral and Charged Pion Condensate in Magnetic Field with Rotation

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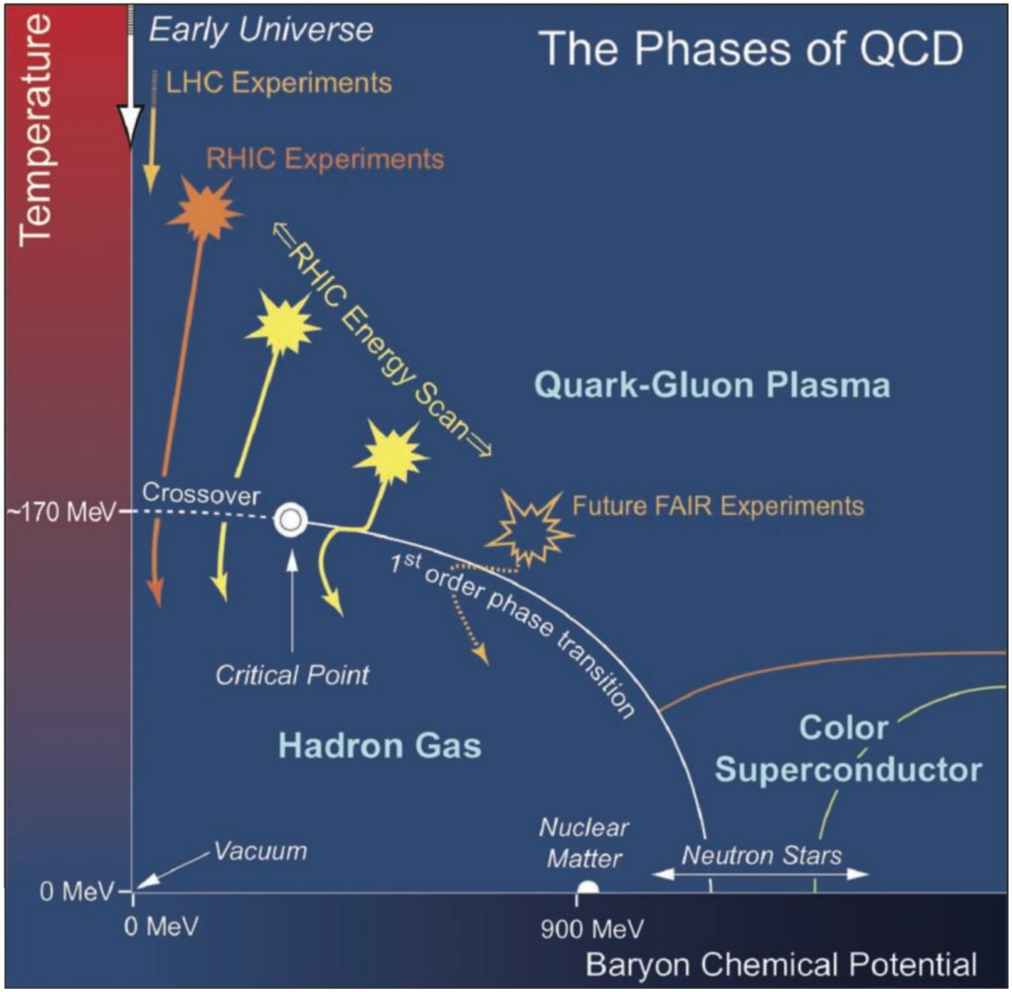
H-L. C, K.Fukushima, X.G.Huang, K. Mameda,
Phys. Rev. D 93, 104052 (2016), arXiv: 1512.08974;
Phys. Rev. D 96, 054032 (2017), arXiv: 1707.09130.
H-L. C, X.G.Huang, K. Mameda, arXiv: 1910.02700.

Outline

- Motivation
- Rotating frame & NJL model
- Rotational magnetic inhibition
- Charged pion condensate in rotating frame

$T\!-\!\mu$ phase diagram of QCD



- Other Backgrounds:

  Isospin chemical potential

  Magnetic Field

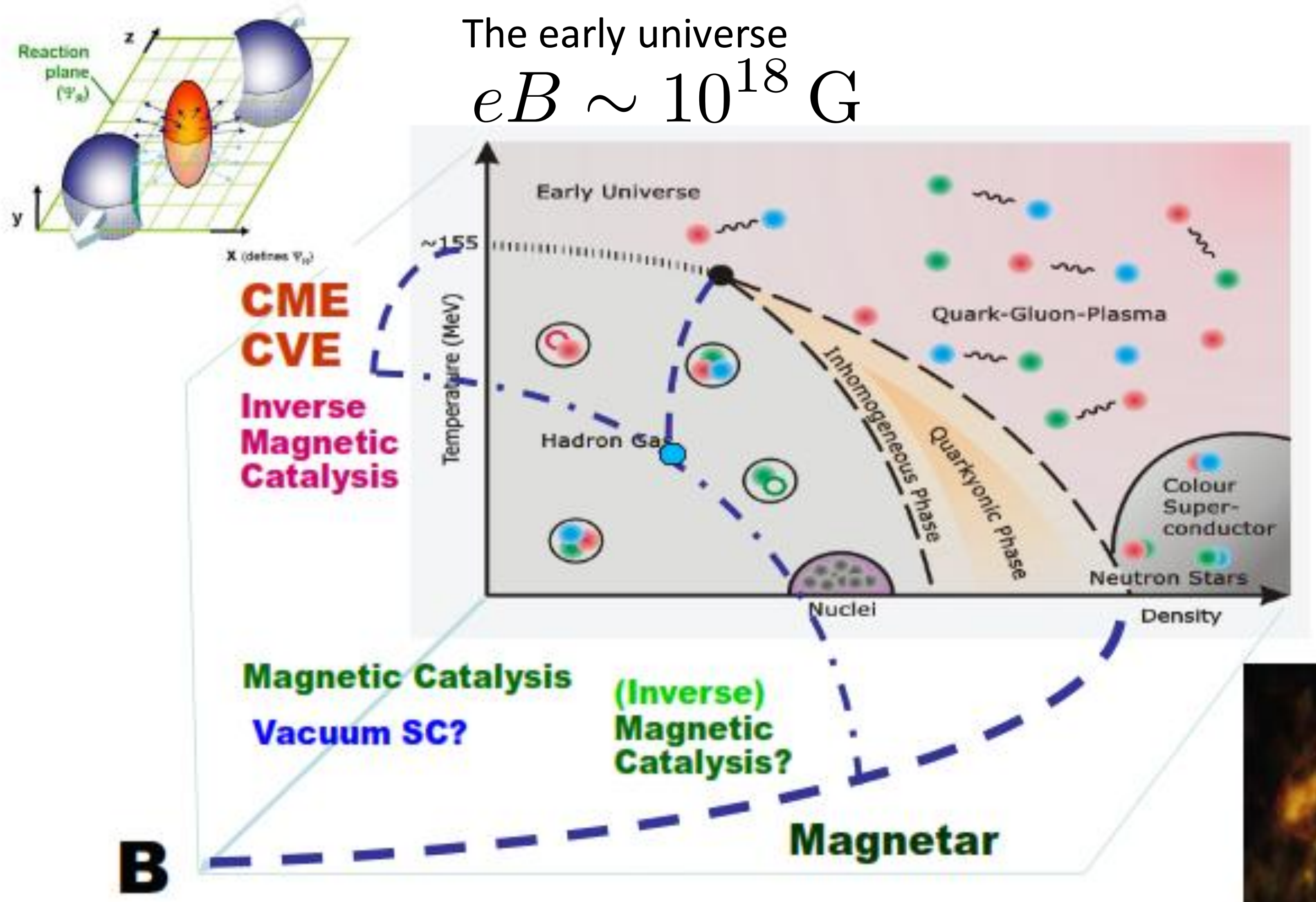
  Rotation

  Gravity

QCD in strong magnetic fields

The early universe

$eB \sim 10^{18}\,\mathrm{G}$

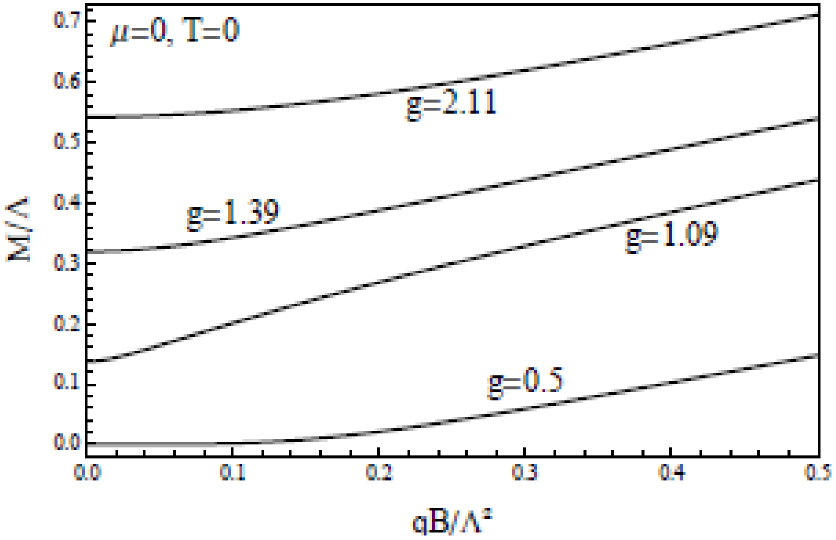


Compact stars
(Magnetar)

$eB \sim 10^{15}\,\mathrm{G}$

Magnetic catalysis

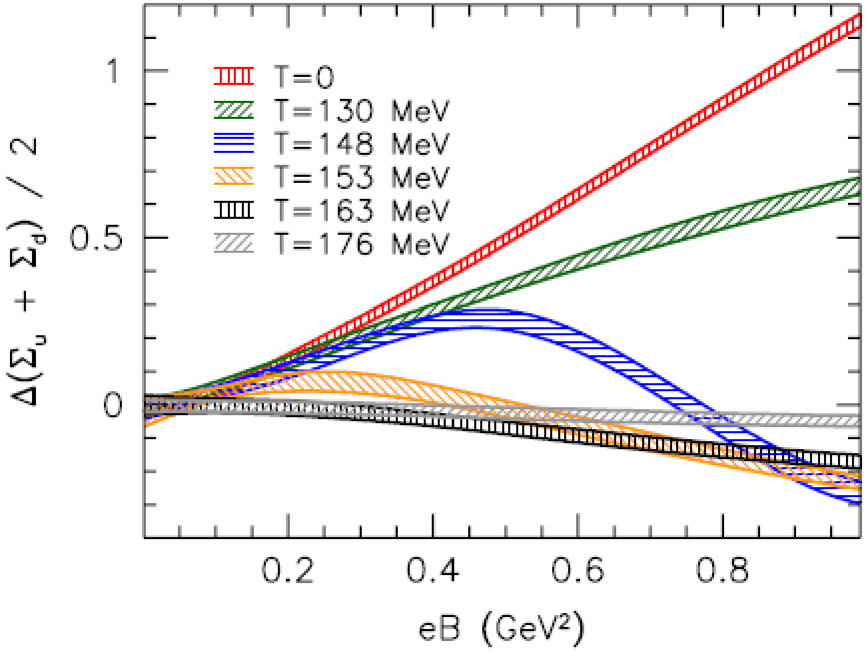
- Condensation m increases with magnetic field eB increasing



F. Preis, et al., Lect. Notes Phys. 871, 51 (2013)

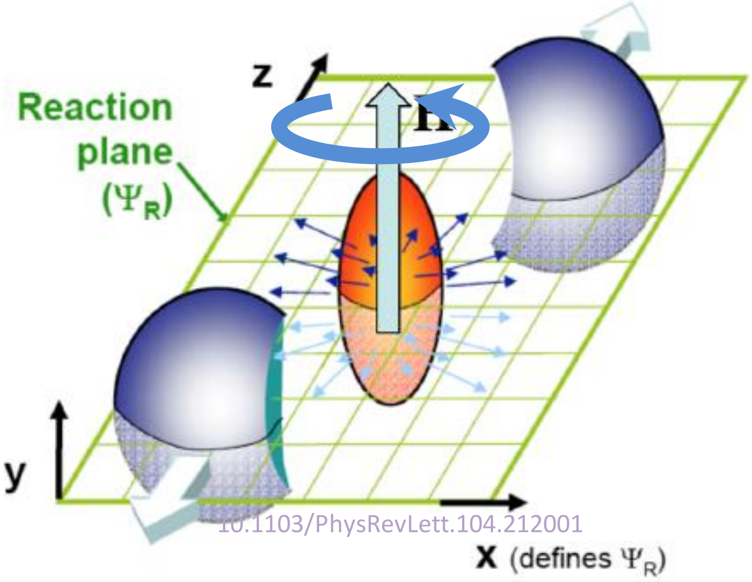
Inverse magnetic catalysis

- Condensation m decreases with magnetic field eB increasing!



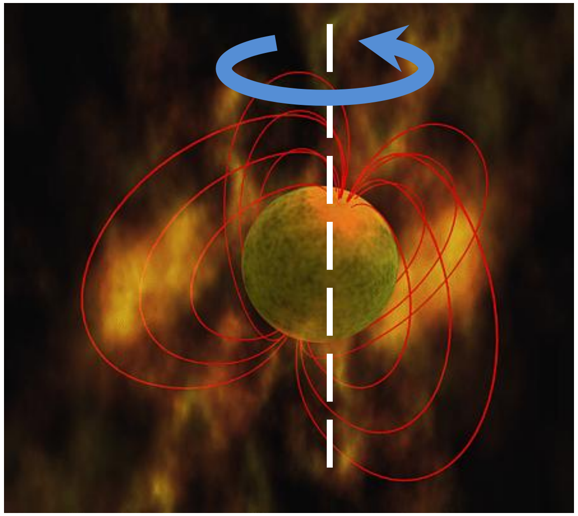
G. Bali, et al., JHEP 1202 (2012) 044

QCD in strong magnetic fields with rotation



10.1103/PhysRevLett.104.212001

$$R\Omega \sim 10^{-1}$$



Rotating Frame

- K-G equation

$$\frac{1}{\sqrt{-g}}D_\mu(\sqrt{-g}g^{\mu\nu}D_\nu\phi)+m^2\phi=0$$

$$g_{\mu\nu}=\begin{pmatrix}1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0\\ y\Omega & -1 & 0 & 0\\ -x\Omega & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$$

- Dirac equation

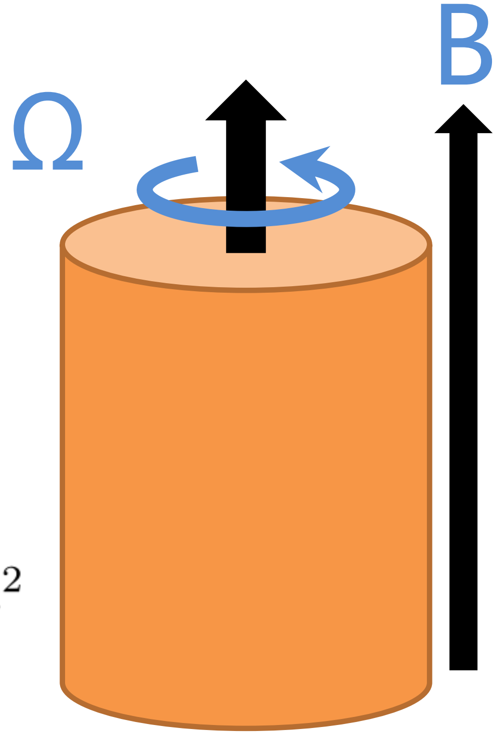
$$[ie_i^\mu\gamma^i(\partial_\mu+iqA_\mu+\Gamma_\mu)-m]\psi(x)=0$$

- e_i^μ is vierbein

  Γ_μ is spin connection

- Dispersion relation

$$[E+\mathrm{sgn}(q)\Omega(l+s_z)]^2=p_z^2+(2\lambda+1-2s_z)|qB|+m^2$$



Schwinger Phase

$$S(x,x') = e^{i\Phi(x,x')} S_{\text{inv}}(x,x')$$

Gauge independent

$$\Gamma^{(2)} = -\int d^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} Tr[(i\gamma^\mu \nabla_\mu - \sigma + \hat{\mu}\gamma^0)^{-1}\gamma^5 \vec{\pi}\cdot\vec{\tau}]^2$$

- For spacetime with R=0

$$\Phi(x,x') = q\int_{x'}^{x} \left[A_\mu(z) + \frac{1}{2}F_{\mu\rho}\nabla_z^\rho \sigma(z,x)\right] dz^\mu$$

- Synge's world function (half the squared geodesic distance between z and x)

$$\sigma(z,x) = \frac{1}{2}\int_0^1 g_{\mu\nu}(y(\tau))\frac{dy^\mu}{d\tau}\frac{dy^\nu}{d\tau} d\tau,$$

- Zero curl (path independent)
- If we choose the integral path as the geodesic

$$\Phi(x,x') = q\int_{x'}^{x} A_\mu(z) dz^\mu,$$

Schwinger Phase

- LLL $\sum_{l=0}^{\infty}\frac{1}{l!}\mathrm{e}^{il(\Delta\theta+\Omega\Delta t)}(\frac{1}{2}qBr_1r_2)^l\mathrm{e}^{-\frac{1}{4}qB(r_2^2+r_1^2)}$

$$= \exp[i\frac{1}{2}qBr_1r_2\sin(\Delta\theta+\Omega\Delta t)-\frac{1}{4}qB(r_2^2-2r_1r_2\cos(\Delta\theta+\Omega\Delta t)+r_1^2)]$$

$$= \exp[\boxed{-iq\int_{x_1}^{x_2}A_\mu \mathrm{d}z^\mu}-\frac{1}{4}qBC_\perp^2].$$

- In curved spacetime: geodesic line $\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\sigma^2}+\Gamma^\mu_{\nu\rho}\frac{\mathrm{d}x^\nu}{\mathrm{d}\sigma}\frac{\mathrm{d}x^\rho}{\mathrm{d}\sigma}=0$

$$-iq\int_{x_1}^{x_2}A_\mu(z)dz^\mu = i\frac{1}{2}qBr_1r_2\sin(\Delta\theta+\Omega\Delta t)$$

- In flat spacetime:straight line

$$-iq\int_{x_1'}^{x_2'}A_\mu'(z)dz^\mu = i\frac{1}{2}qBr_1'r_2'\sin\Delta\theta'$$

$$C_\perp'^2 = (x_2'-x_1')^2+(y_2'-y_1')^2$$

The Nambu-Jona-Lasinio (NJL) model

- The two flavor effective Lagrangian:

$$\mathcal{L}_{NJL} = \bar{\psi} i\gamma^{\mu} \nabla_{\mu} \psi - m_0\bar{\psi}\psi + \mu_B\bar{\psi}\gamma^0\psi + + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma^5\vec{\tau}\psi)^2]$$

$$\nabla_{\mu} = \partial_{\mu} + iQA_{\mu} + \Gamma_{\mu}$$

- Chiral symmetry breaking
- Lack of confinement
- Nonrenormalizable field theory
- The results depend on the regularization scheme and on the UV cut-off that is used

Mean Field Approximation

- Effective action

Hard to diagonalize

$$\Gamma = -\int \mathrm{d}^4x \frac{\sigma^2 + \vec{\pi}^2}{2G} + \frac{1}{i} \ln\det(iD - i\gamma^5 \vec{\pi} \cdot \vec{\tau}),$$

$$iD = i\gamma^\mu \nabla_\mu - \sigma + \mu_B \gamma^0$$

- Magnetic field breaks SU(2) symmetry
- Perturbative expansion in π field

$$\Gamma = \Gamma^{(0)} + \Gamma^{(2)} + \dots,$$

$$\Gamma^{(0)} = -\int \mathrm{d}^4x \frac{\sigma^2}{2G} + \frac{1}{i} \ln\det iD,$$

$$\Gamma^{(2)} = -\int \mathrm{d}^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} \mathrm{Tr}[(iD)^{-1}\gamma^5 \vec{\pi} \cdot \vec{\tau}]^2,$$

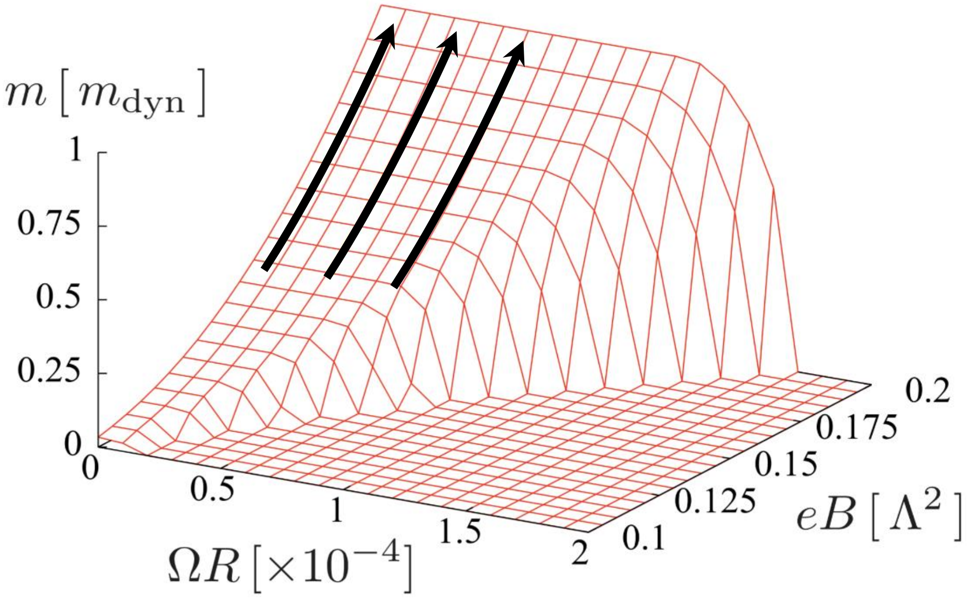
Assumption: $\langle \pi^0 \rangle = 0$

Weak Coupling

$$1/\Omega \geq R \gg 1/\sqrt{qB}$$

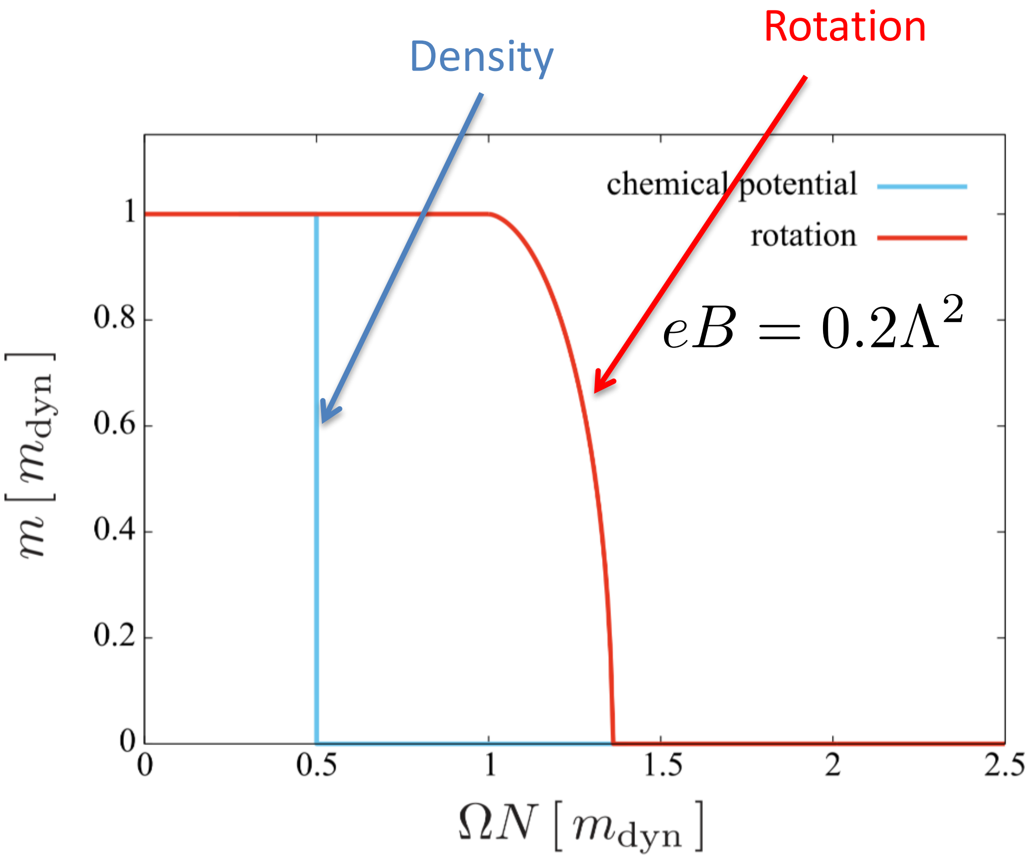
- Magnetic Catalysis

Increasing eB ⟶ Increasing m

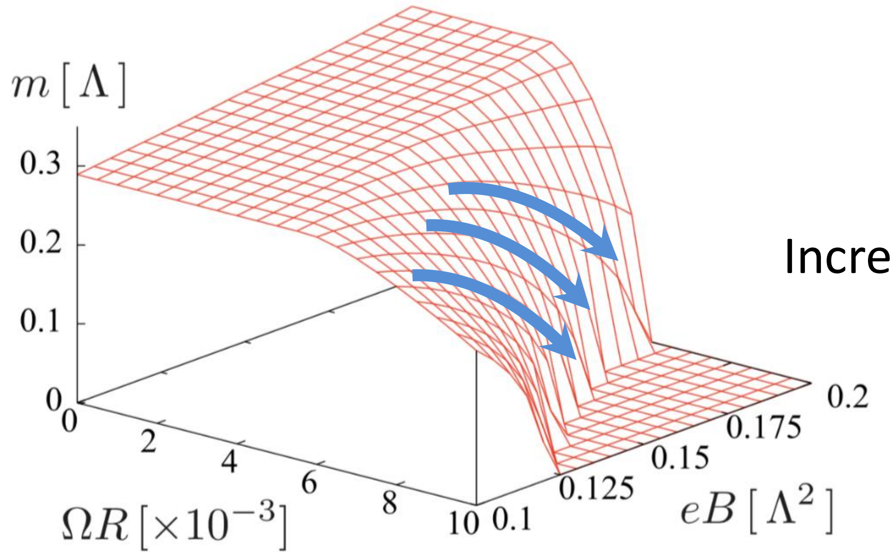


Comparision with density

- Second Order transition



Strong Coupling:
“Rotational Magnetic Inhibition”



$$1/\Omega \geq R \gg 1/\sqrt{qB}$$

Increasing eB $\longrightarrow$ Decreasing m

HLC, et al, Phys. Rev. D 93, 104052 (2016)

- Dropping start around $\Omega N :\ \sqrt{eB}$
- For finite density system the inverse magnetic catalysis start around $\mu :\ \sqrt{eB}$ Preis, Rebhan, Schmitt (2012)

Boundary effect (without rotation)

- Boundary condition

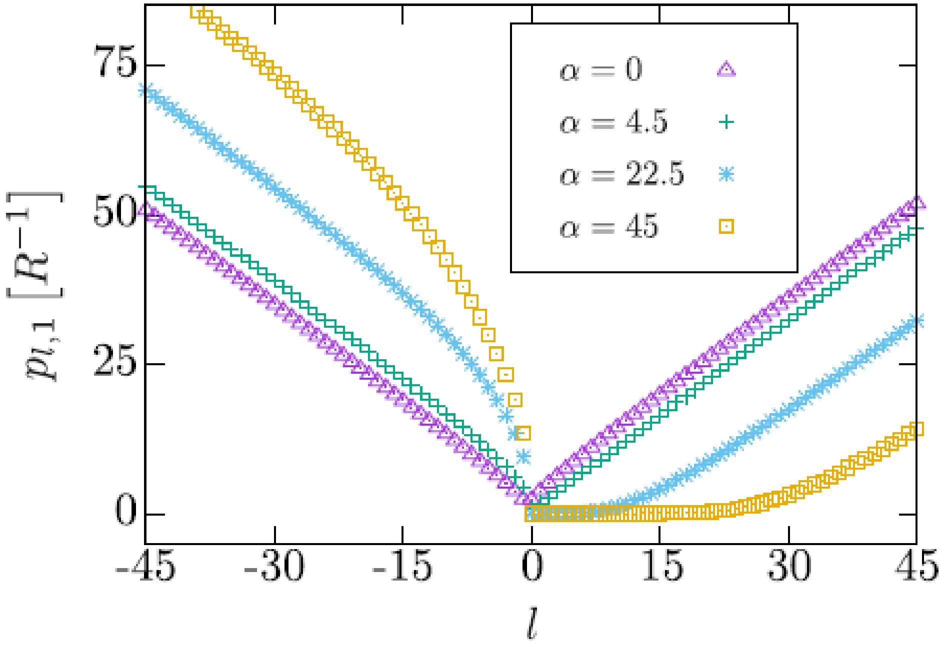
HLC, et al, Phys. Rev. D 96, 054032 (2017)

$$R\int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta\, \bar{\psi}\gamma^r\psi\Big|_{r=R} = 0$$

$$\alpha \equiv \frac{1}{2}eBR^2$$

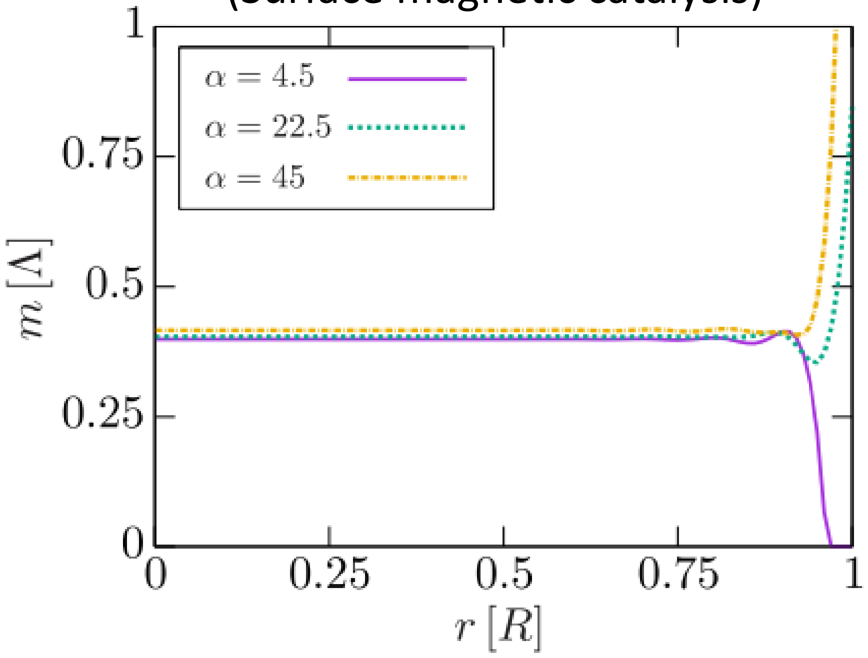
Dispersion relation

$$p_{l,1} = \sqrt{2eB\lambda}$$



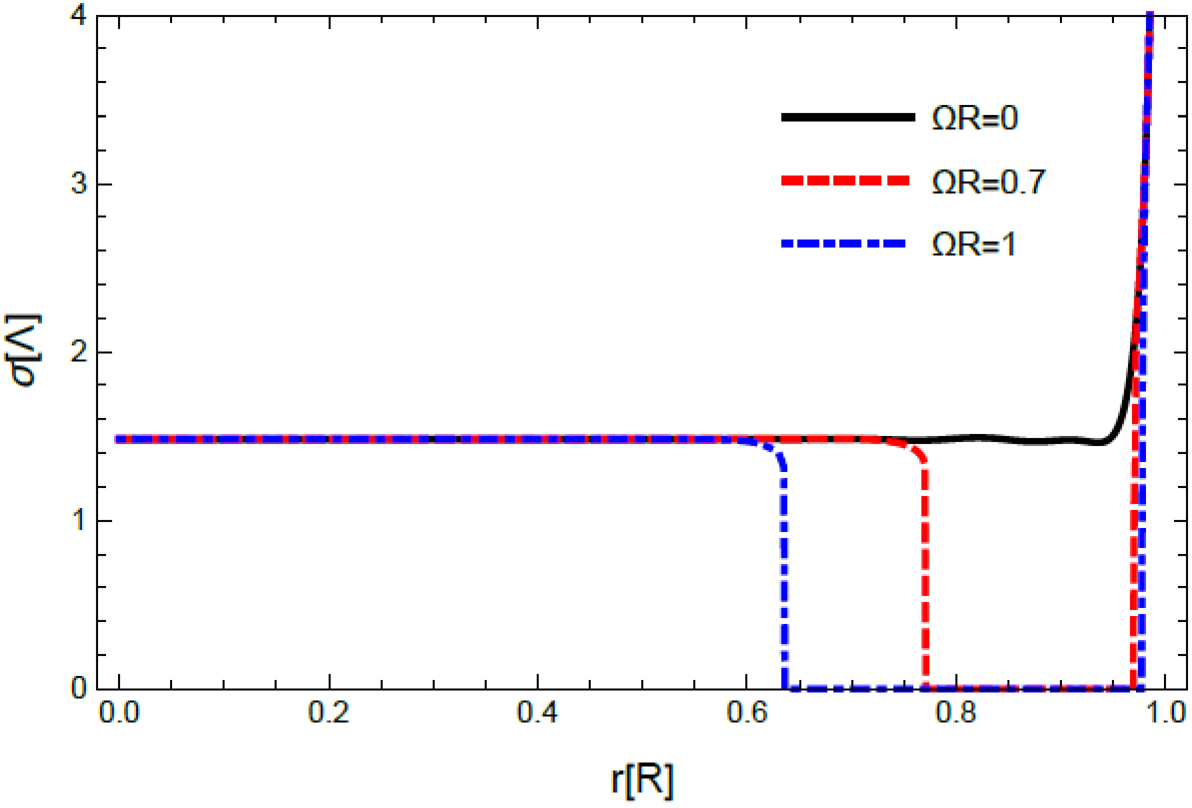
Chiral condensate

(Surface magnetic catalysis)



Rotation & Boundary

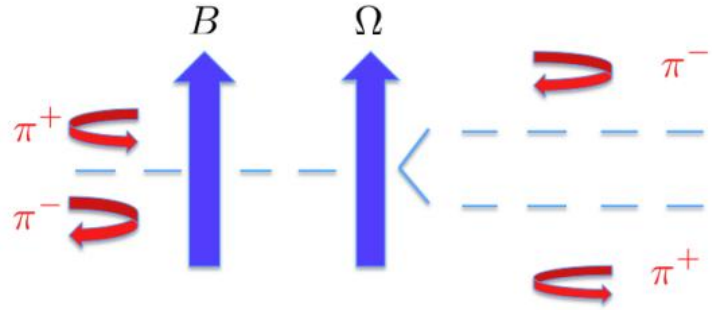
- Rotational Magnetic Inhibition & Surface magnetic catalysis



Pion as the DoF

- Dispersion relation

$$[E + \mathrm{sgn}(q)\Omega l]^2 = p_z^2 + (2\lambda + 1)|qB| + m^2$$



Y. Liu, I. Zahed, Phys.Rev.Lett. 120 (2018)

- $\Omega l \rightarrow \mu \rightarrow$ pion condensate

Charged Pion Condensate (NJL)

Gauge independent

$S(x,x') = e^{i\Phi(x,x')} S_{\text{inv}}(x,x')$

$$\Gamma^{(2)} = -\int d^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} Tr[(i\gamma^\mu \nabla_\mu - \sigma + \hat{\mu}\gamma^0)^{-1}\gamma^5 \vec{\pi}\cdot\vec{\tau}]^2$$

- Ansatz

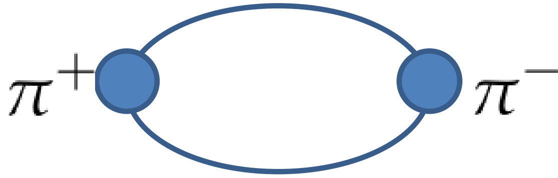
Gauge dependent part:
Wilson line

$$\pi^+(x')\pi^-(x) = \mathrm{e}^{ie\int_{x'}^{x} A_\mu \mathrm{d}z^\mu}\,\tilde{\pi}^+\tilde{\pi}^-,$$

Constant for simplicity

- 2nd order thermodynamic potential

$$V_{eff}^{(2)} = C^{(2)}\tilde{\pi}^+\tilde{\pi}^-$$



- If $C^{(2)} < 0$ $\longrightarrow$ Pion condensate

Choice of the wilson line

$$\pi^+(x')\pi^-(x) = \mathrm{e}^{ie\int_{x'}^{x} A_\mu \mathrm{d}z^\mu}\tilde{\pi}^+\tilde{\pi}^-,$$

- Integrate along geodesic

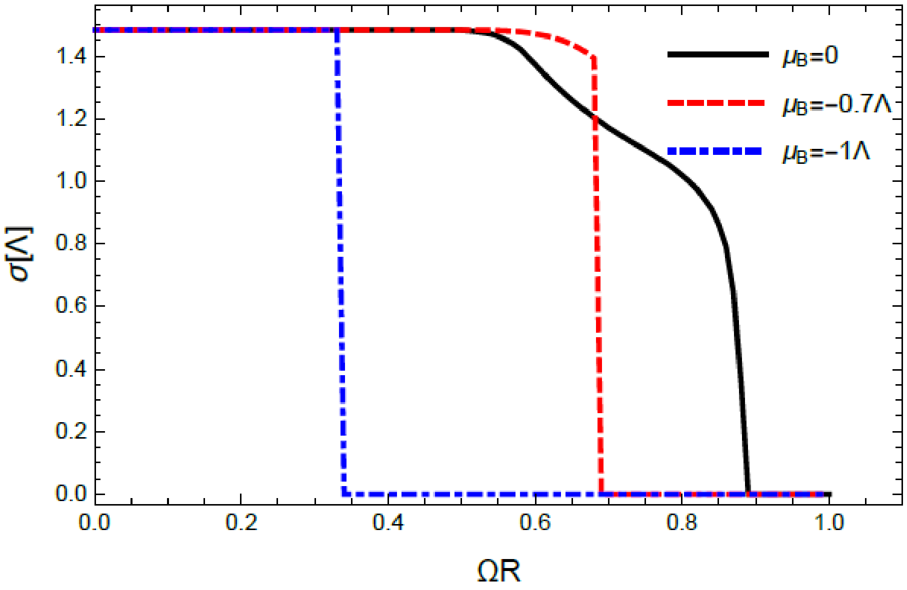
$$ie\int_{x_1}^{x_2} A_\mu dz^\mu = -i\frac{1}{2}eBr_1r_2\sin(\Delta\theta + \Omega\Delta t)$$

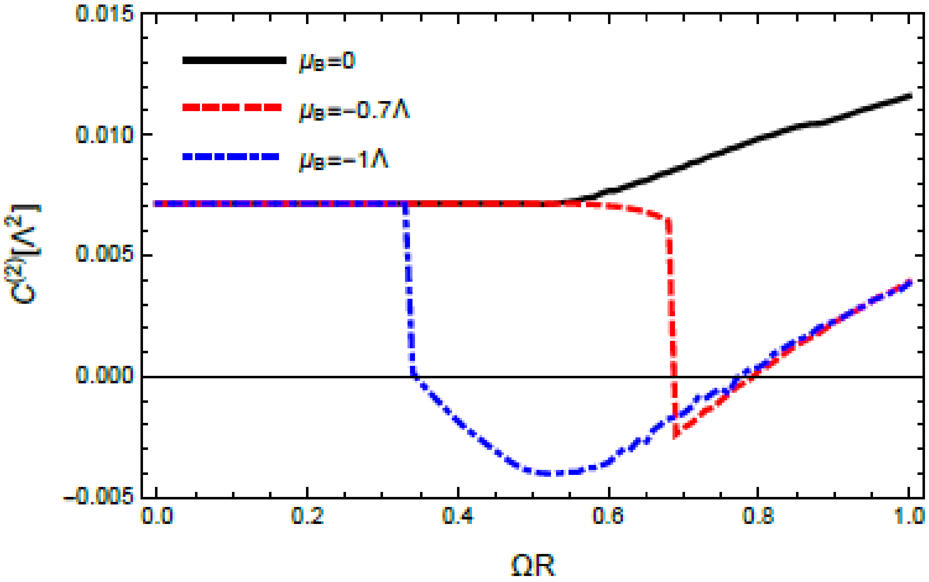
- Exactly cancel the Schwinger phase from the quark propagator
- Unfortunately, no pion condensate($C^{(2)} > 0$)

Charged pion condensate

- Include chemical potential

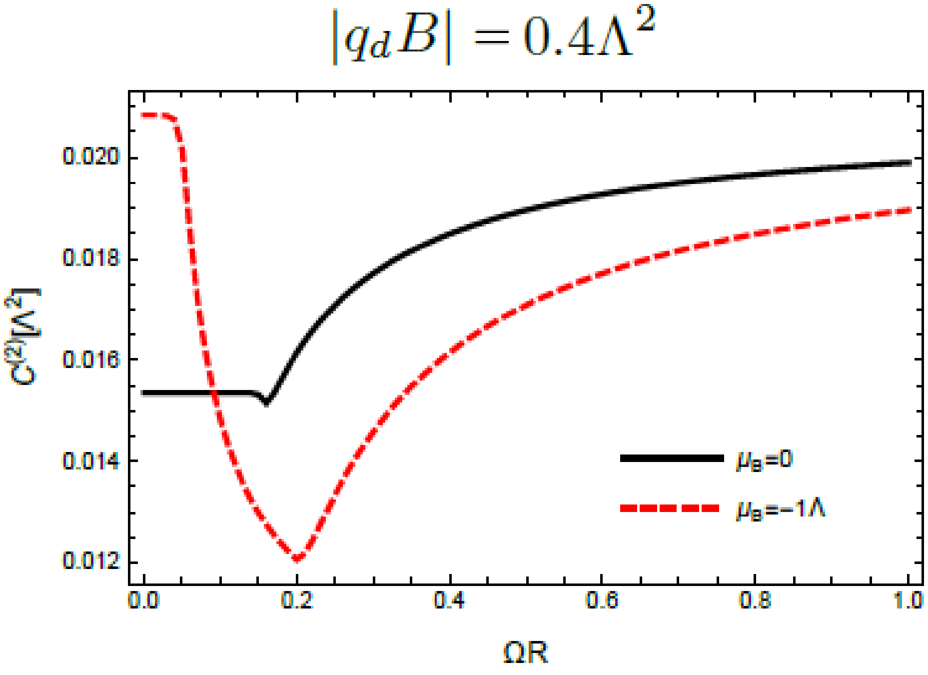
$|q_d B| = 0.1\Lambda^2$





Charged pion condensate

- Condensate never happen if the magnetic field is too strong



$|q_d B| = 0.4\Lambda^2$

Charged pion condensate

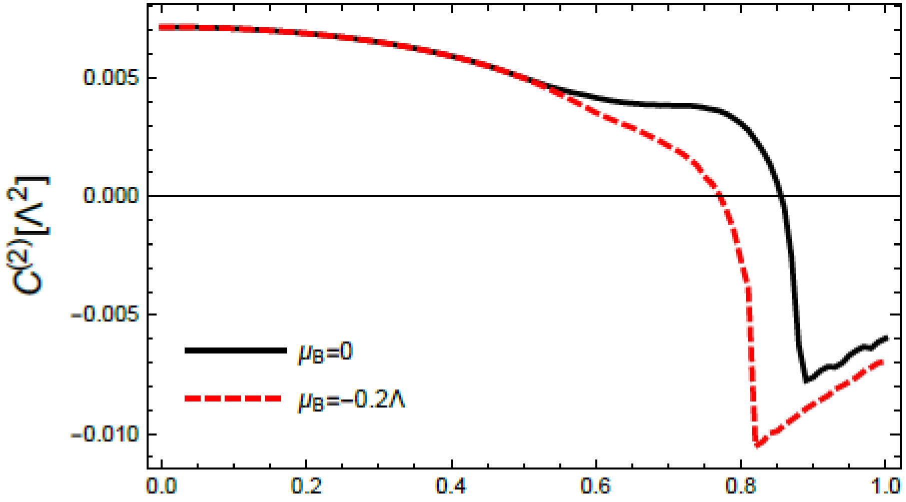
- Both the magnetic field and rotation tend to destroy the charged pion condensate at quark level
- Although the split tend to cause Bose-Einstein condensation at pion level
- Inhibition wins the competition

Another choice

$$(t_1, x_1, y_1, z_1) \to (t_1, 0, 0, z_1) \to (t_2, 0, 0, z_2)$$
$$\to (t_2, x_1, y_1, z_2) \to (t_2, x_2, y_2, z_2),$$

- The same form as the Schwinger phase in flat spacetime $ie \int_{x_1}^{x_2} A_\mu dz^\mu = -i\frac{1}{2}eBr_1r_2 \sin\Delta\theta$

- Physical meaning is not clear



Conclusion & Outlook

- Rotational magnetic inhibition
- Surface magnetic catalysis
- In certain region, rotation can induce pion condensate
- Analogy between rotation and(isospin) chemical potential?
- Superconductivity caused by charged pion condensate
- Rho condensation

Thank you very much!