Inelastic meson-meson scattering in hadronic matter

Xiao-Ming Xu

Department of Physics

Shanghai University

Collaborators: Y.-Q. Li, Z.-Y. Shen, K. Yang, T.-T. Wang, W.-X. Li (Shanghai University)

H. J. Weber (University of Virginia)

Hadronic matter consists of π, ρ, K, K^*, p, n and other hadrons.
All possible meson-meson scattering takes place in hadronic matter.

Types of meson-meson scattering:

1. Involve quark interchange between mesons.
2. Involve quark-antiquark annihilation.
3. Involve quark-antiquark creation.
4. Involve quark-antiquark annihilation and creation.
5. Involve quark interchange as well as quark-antiquark annihilation and creation.
6. Involve resonances as well as quark-antiquark annihilation and creation.

Pions, kaons and rho mesons are dominant meson species in hadronic matter.

Study scattering among π, ρ, K and K^*.

Reactions governed by quark interchange

Y.-Q. Li, X.-M. Xu, Nucl. Phys. A794 (2007) 210

Z.-Y. Shen, X.-M. Xu, J. Korean Phys. Soc. 66 (2015) 754

I=2 $\pi\pi \to \rho\rho$

I=1 $KK \to K^*K^*$

I=1 $KK^* \to K^*K^*$

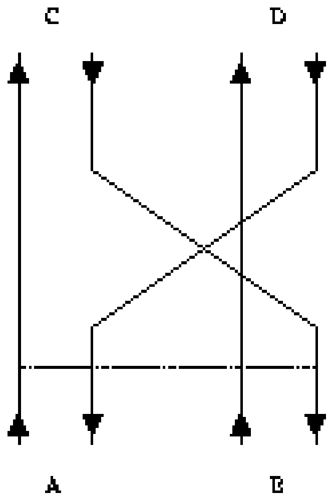
I=3/2 $\pi K \to \rho K^*$

I=3/2 $\pi K^* \to \rho K^*$

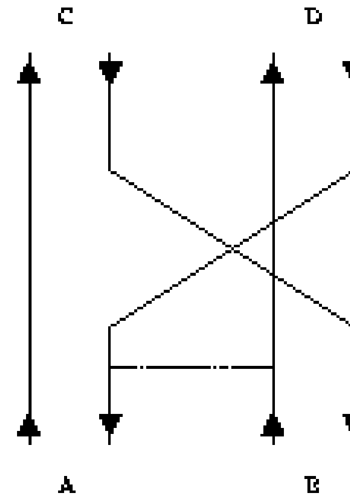
I=3/2 $\rho K \to \rho K^*$

I=3/2 $\pi K^* \to \rho K$

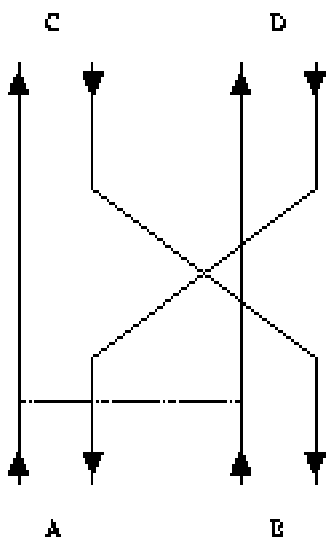
Scattering in the prior form: gluon exchange occurs before quark interchange



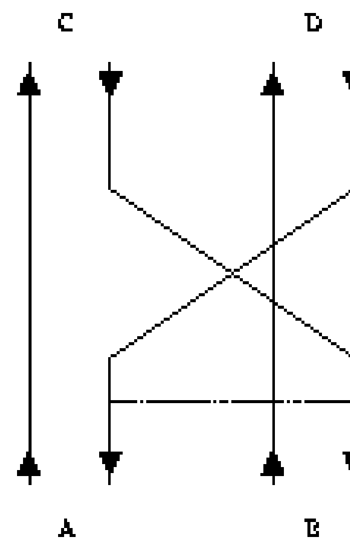
C1 prior



C2 prior

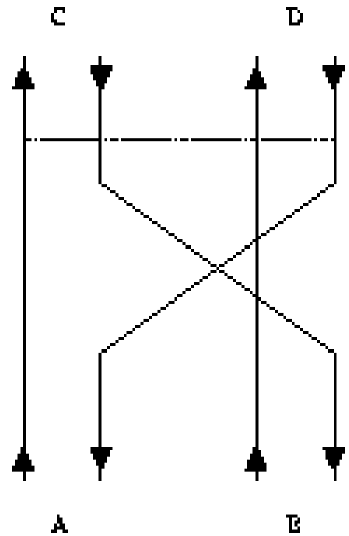


T1 prior

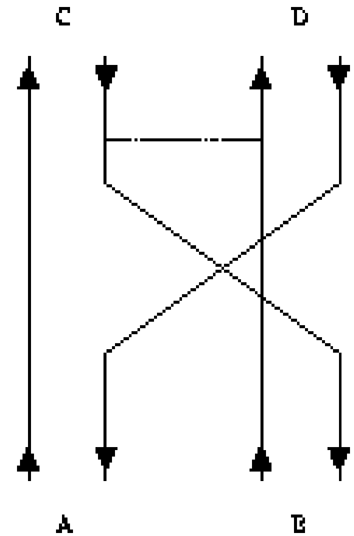


T2 prior

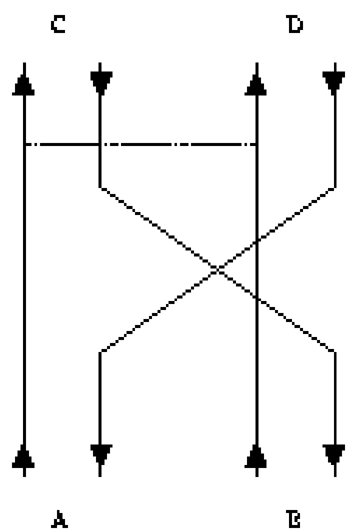
Scattering in the post form: gluon exchange occurs after quark interchange



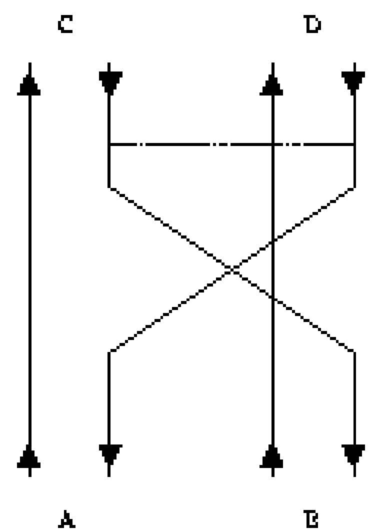
C1 post



C2 post



T1 post



T2 post

S-matrix element

for $A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \rightarrow C(q_1\bar{q}_2) + D(q_2\bar{q}_1)$

$$S_{fi} = \delta_{fi} - 2\pi i\delta(E_f - E_i) < q_1\bar{q}_2, q_2\bar{q}_1|H_I|q_1\bar{q}_1, q_2\bar{q}_2 >$$

$H_I = V_{q_1\bar{q}_2} + V_{\bar{q}_1 q_2} + V_{q_1 q_2} + V_{\bar{q}_1\bar{q}_2}$ in the prior form

$H_I = V_{q_1\bar{q}_1} + V_{\bar{q}_2 q_2} + V_{q_1 q_2} + V_{\bar{q}_1\bar{q}_2}$ in the post form

quark interaction

$$V_{ab} = V_{si} + V_{ss}$$

Spin-independent potential

$$V_{si}(\vec{r}) = \frac{\vec{\lambda}_a}{2} \cdot \frac{\vec{\lambda}_b}{2} \{-\frac{3}{4}\mathrm{D}[1.3 - (\frac{T}{T_c})^4]\tanh(\mathrm{Ar}) + \frac{6\pi}{25}\frac{v(\lambda r)}{r} e^{-Er}\}$$

$T_c = 0.175$ GeV, D=0.7 GeV, E=0.6 GeV. The short-distance part of V_{si} originates from one-gluon exchange plus one- and two-loop corrections. The intermediate-distance and large-distance part of V_{si} fits well the numerical potential obtained in the lattice gauge calculations.

Spin-spin interaction

$$V_{ss}(\vec{r}) = \frac{\vec{\lambda}_a}{2} \cdot \frac{\vec{\lambda}_b}{2} \{-\frac{16\pi^2}{25}\frac{d^3}{\pi^{3/2}} e^{-d^2r^2} + \frac{4\pi}{25}\frac{1}{r}\frac{d^2v(\lambda r)}{dr^2}\} \frac{\vec{s}_a \cdot \vec{s}_b}{m_a m_b}$$

Including relativistic corrections to the gluon propagator.

X.-M. Xu, Nucl. Phys. A697 (2002) 825

Y.-P. Zhang, X.-M. Xu, H.-J. Ge, Nucl. Phys. A832 (2010) 112

J. Zhou, X.-M. Xu, Phys. Rev. C85 (2012) 064904

Transition amplitudes

$$M^{prior} = \sqrt{2E_{q_1\bar{q}_1} 2E_{q_2\bar{q}_2} 2E_{q_1\bar{q}_2} 2E_{q_2\bar{q}_1}} \int \frac{d^3 p_{q_1\bar{q}_2}}{(2\pi)^3} \frac{d^3 p_{q_2\bar{q}_1}}{(2\pi)^3}$$

$$\psi_{q_1\bar{q}_2}{}^{+} \psi_{q_2\bar{q}_1}{}^{+} (V_{q_1\bar{q}_2} + V_{\bar{q}_1 q_2} + V_{q_1 q_2} + V_{\bar{q}_1\bar{q}_2}) \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2}$$

for the prior form

$$M^{post} = \sqrt{2E_{q_1\bar{q}_1} 2E_{q_2\bar{q}_2} 2E_{q_1\bar{q}_2} 2E_{q_2\bar{q}_1}} \int \frac{d^3 p_{q_1\bar{q}_1}}{(2\pi)^3} \frac{d^3 p_{q_2\bar{q}_2}}{(2\pi)^3}$$

$$\psi_{q_1\bar{q}_2}{}^{+} \psi_{q_2\bar{q}_1}{}^{+} (V_{q_1\bar{q}_1} + V_{\bar{q}_2 q_2} + V_{q_1 q_2} + V_{\bar{q}_1\bar{q}_2}) \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2}$$

for the post form

Phase shift

$$\delta_l = -\frac{2\pi^2|\vec{P}|E_{q_1\bar{q}_1}E_{q_2\bar{q}_2}}{E_{q_1\bar{q}_1}+E_{q_2\bar{q}_2}}\int_{-1}^{1} T_{fi}P_l(x')\mathrm{d}x'$$

reduced T-matrix element

$$T_{fi} = \frac{1}{(2\pi)^3\sqrt{2E_{q_1\bar{q}_1}2E_{q_2\bar{q}_2}2E_{q_1\bar{q}_2}2E_{q_2\bar{q}_1}}}\frac{M^{prior}+M^{post}}{2}$$

The experimental data of S-wave I=2 elastic phase shifts for $\pi\pi$ scattering for $0 < \sqrt{s} < 2.4$ GeV in vacuum are reproduced.

Cross section

The unpolarised cross section for the scattering in the prior form

$$\sigma_{prior} = \frac{1}{(2S_{q_1\bar{q}_1}+1)(2S_{q_2\bar{q}_2}+1)(2L_{q_2\bar{q}_2}+1)}\frac{1}{32\pi s}$$

$$\frac{|\vec{P}'|}{|\vec{P}|}\sum_{SL_{q_2\bar{q}_2z}}(2S+1)\int_0^{\pi}d\theta|M^{prior}|^2 sin\theta$$

The unpolarised cross section for the scattering in the post form

$$\sigma_{post} = \frac{1}{(2S_{q_1\bar{q}_1}+1)(2S_{q_2\bar{q}_2}+1)(2L_{q_2\bar{q}_2}+1)}\frac{1}{32\pi s}$$

$$\frac{|\vec{P}'|}{|\vec{P}|}\sum_{SL_{q_2\bar{q}_2z}}(2S+1)\int_0^{\pi}d\theta|M^{post}|^2 sin\theta$$

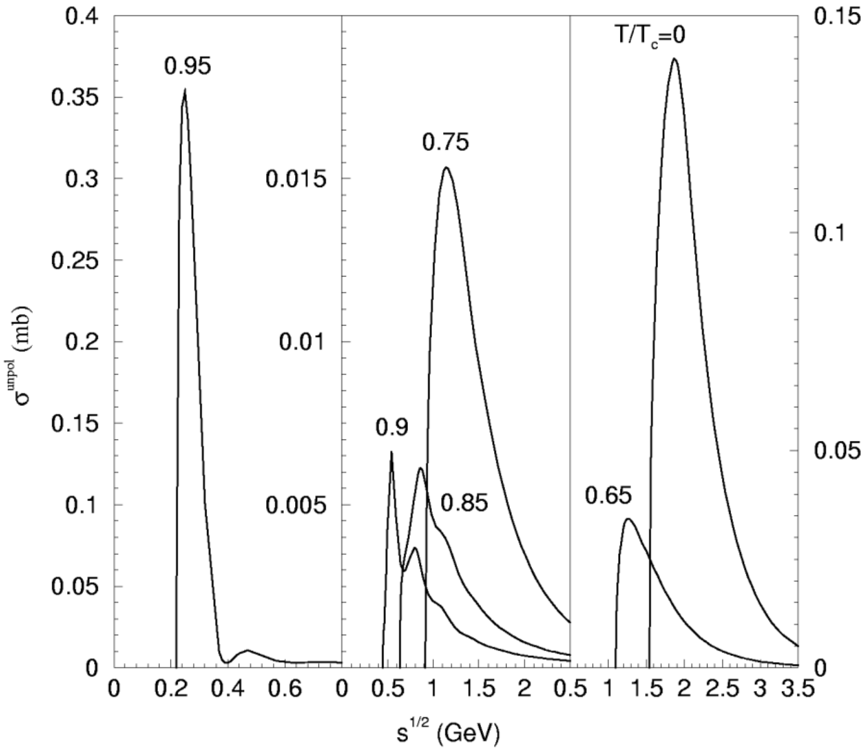
The unpolarised cross section for $A(q_1\bar{q}_1)+B(q_2\bar{q}_2)\to C(q_1\bar{q}_2)+D(q_2\bar{q}_1)$

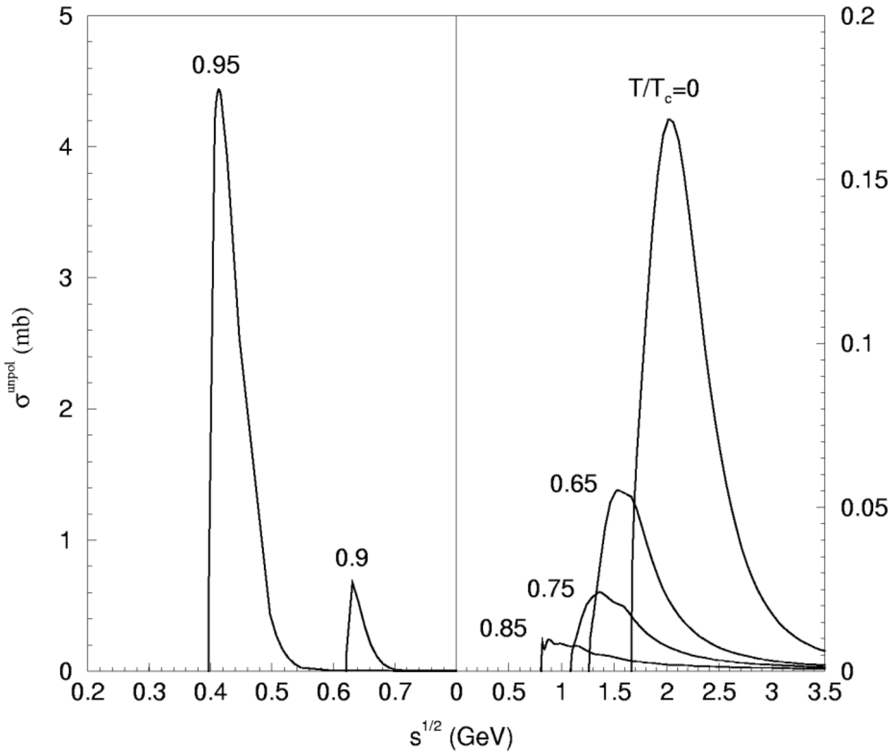
$$\sigma^{unpol} = \frac{1}{2}(\sigma_{prior}+\sigma_{post})$$

The cross section depends on temperature T and the total energy of the two initial mesons in their center-of-mass frame $\sqrt{s}$.

$I = 2\ \pi\pi \to \rho\rho$

$I = \frac{3}{2}\ \pi K^* \to \rho K^*$





Reactions governed by quark-antiquark annihilation and creation

Z.-Y. Shen, X.-M. Xu, H. J. Weber, Phys. Rev. D94 (2016) 034030

T.-T. Wang, X.-M. Xu, Chin. Phys. C43 (2019) 024102

I=1 $\pi\pi \to \rho\rho$

$$K\bar{K} \to K\bar{K}^*, K^*\bar{K}, K^*\bar{K}^*$$

$K\bar{K}^* \to K^*\bar{K}^*$ $\quad$ $K^*\bar{K} \to K^*\bar{K}^*$

$$\pi\pi \to K\bar{K}, K\bar{K}^*, K^*\bar{K}, K^*\bar{K}^*$$

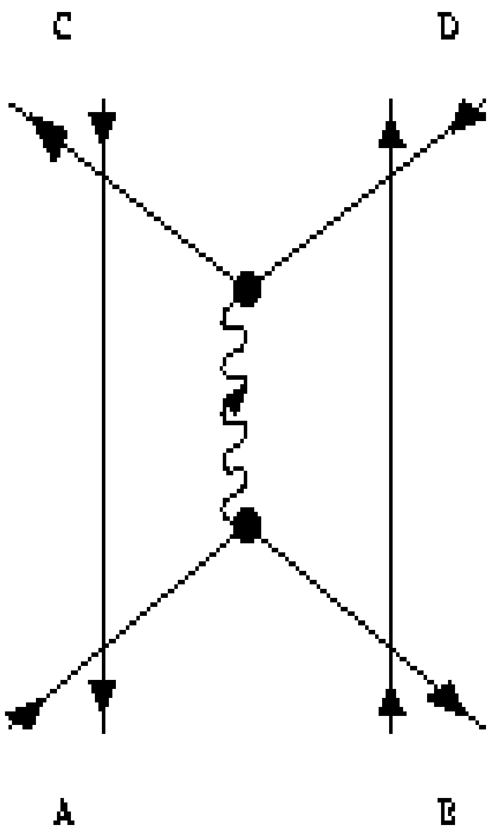
$K\bar{K} \to \rho\rho$ $\quad$ $K\bar{K}^* \to \rho\rho$ $\quad$ $K^*\bar{K} \to \rho\rho$ $\quad$ $\rho\rho \to K^*\bar{K}^*$

$$\pi\rho \to K\bar{K}, K\bar{K}^*, K^*\bar{K}, K^*\bar{K}^*$$

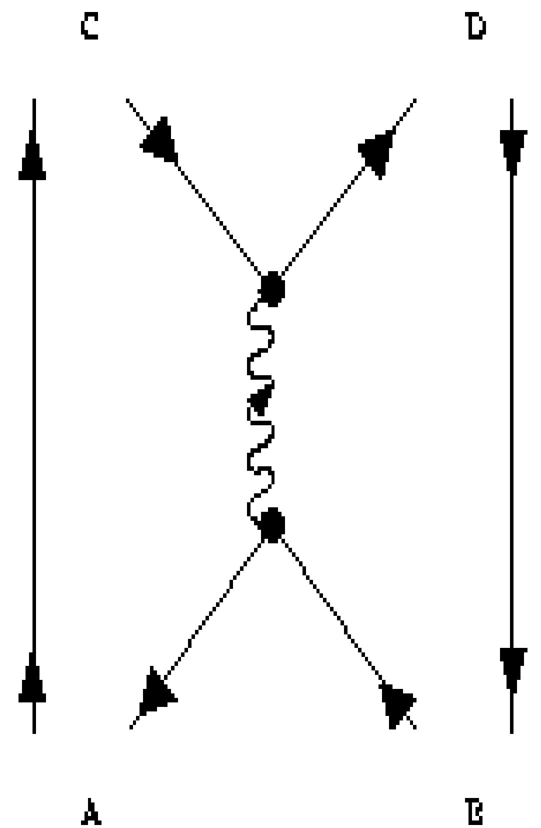
I=1/2 $\pi K \to \pi K^*$ $\quad$ I=1/2 $\pi K \to \rho K$

Quark-antiquark annihilation and creation

$q_1 + \bar{q}_2 \rightarrow q_3 + \bar{q}_4$



$\bar{q}_1 + q_2 \rightarrow q_3 + \bar{q}_4$



S-matrix element

for $A + B \rightarrow C + D$

via quark-antiquark annihilation and creation

$$S_{fi} = \delta_{fi} - 2\pi i \delta(E_f - E_i)$$
$$(< q_3\bar{q}_1, q_2\bar{q}_4 \left| V_{aq_1\bar{q}_2} \right| q_1\bar{q}_1, q_2\bar{q}_2 >$$
$$+< q_1\bar{q}_4, q_3\bar{q}_2 \left| V_{a\bar{q}_1 q_2} \right| q_1\bar{q}_1, q_2\bar{q}_2 >)$$

Transition amplitudes

$$M_{aq_1\bar{q}_2} = \sqrt{2E_A 2E_B 2E_C 2E_D} \int \frac{d^3 p_{q_1\bar{q}_1}}{(2\pi)^3} \frac{d^3 p_{q_2\bar{q}_2}}{(2\pi)^3}$$

$$\psi_{q_3\bar{q}_1}{}^{+} \psi_{q_2\bar{q}_4}{}^{+} V_{aq_1\bar{q}_2} \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2}$$

$$M_{a\bar{q}_1 q_2} = \sqrt{2E_A 2E_B 2E_C 2E_D} \int \frac{d^3 p_{q_1\bar{q}_1}}{(2\pi)^3} \frac{d^3 p_{q_2\bar{q}_2}}{(2\pi)^3}$$

$$\psi_{q_1\bar{q}_4}{}^{+} \psi_{q_3\bar{q}_2}{}^{+} V_{a\bar{q}_1 q_2} \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2}$$

Phase shift

$$e^{i\delta_l} sin\delta_l = -\frac{2\pi^2 \left|\vec{P}\right| E_A E_B}{E_A + E_B} \int_{-1}^{1} dx T_{fi} P_l(x)$$

reduced T-matrix element

$$T_{fi} = \frac{M_{aq_1\bar{q}_2} + M_{a\bar{q}_1 q_2}}{(2\pi)^3 \sqrt{2E_A 2E_B 2E_C 2E_D}}$$

The experimental data of S-wave I=0 and P-wave I=1 elastic phase shifts for $\pi\pi$ scattering near the threshold energy are reproduced.

Cross section

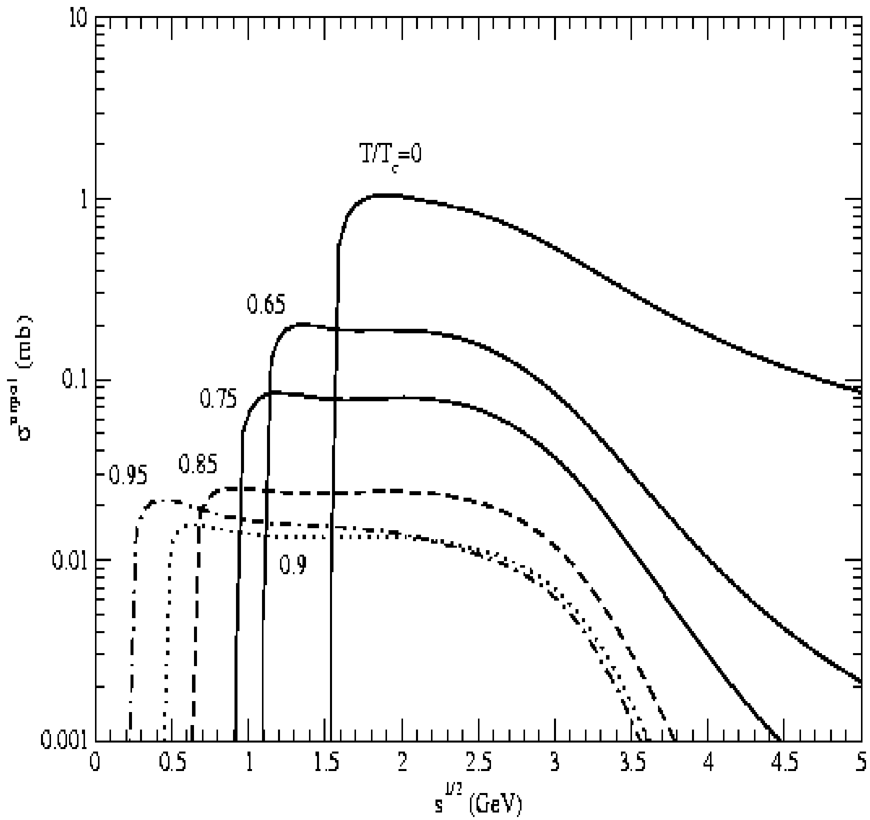
The unpolarised cross section for $A + B \rightarrow C + D$ via $q_1 + \bar{q}_2 \rightarrow q_3 + \bar{q}_4$ and $\bar{q}_1 + q_2 \rightarrow q_3 + \bar{q}_4$ is

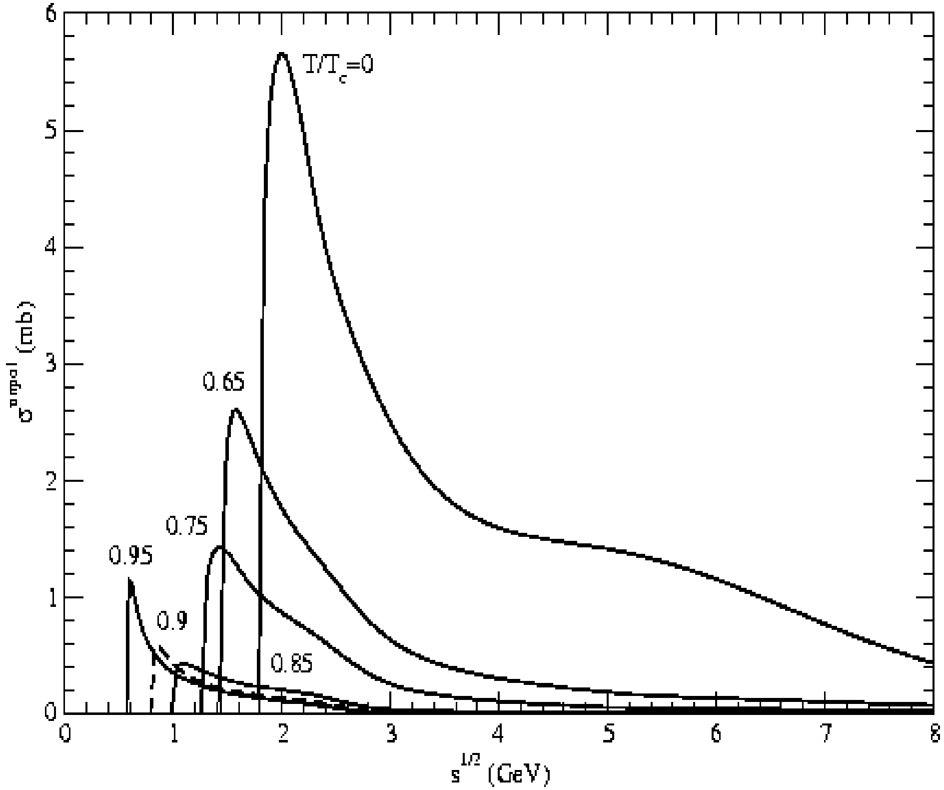
$$\sigma^{unpol} = \frac{1}{(2J_A + 1)(2J_B + 1)} \frac{1}{32\pi s} \frac{|\vec{P}'|}{|\vec{P}|}$$

$$\int_0^{\pi} d\theta \sum_{J_{Az}J_{Bz}J_{Cz}J_{Dz}} |M_{aq_1\bar{q}_2} + M_{a\bar{q}_1q_2}|^2 sin\theta$$

I=1 $\pi\pi \to \rho\rho$

I=0 $K\bar{K} \to K^*\bar{K}^*$





Reactions governed by quark interchange as well as quark-antiquark annihilation and creation

Z.-Y. Shen, X.-M. Xu, H. J. Weber, Phys. Rev. D94 (2016) 034030
K. Yang, X.-M. Xu, H. J. Weber, Phys. Rev. D96 (2017) 114025

$I = 0 \quad \pi\pi \to \rho\rho$

$I = 1/2 \;\; \pi K \to \rho K^*$ $\qquad I = 1/2 \;\; \pi K^* \to \rho K$

$I = 1/2 \;\; \pi K^* \to \rho K^*$ $\qquad I = 1/2 \;\; \rho K \to \rho K^*$

Cross section

The unpolarised cross section for $A + B \rightarrow C + D$ via quark interchange as well as quark-antiquark annihilation and creation is

$$\sigma^{unpol} = \frac{1}{2}(\sigma_{unpol}^{prior} + \sigma_{unpol}^{post})$$

$$\sigma_{unpol}^{prior} = \frac{1}{(2J_A + 1)(2J_B + 1)} \frac{1}{32\pi s} \frac{|\vec{P}'|}{|\vec{P}|}$$

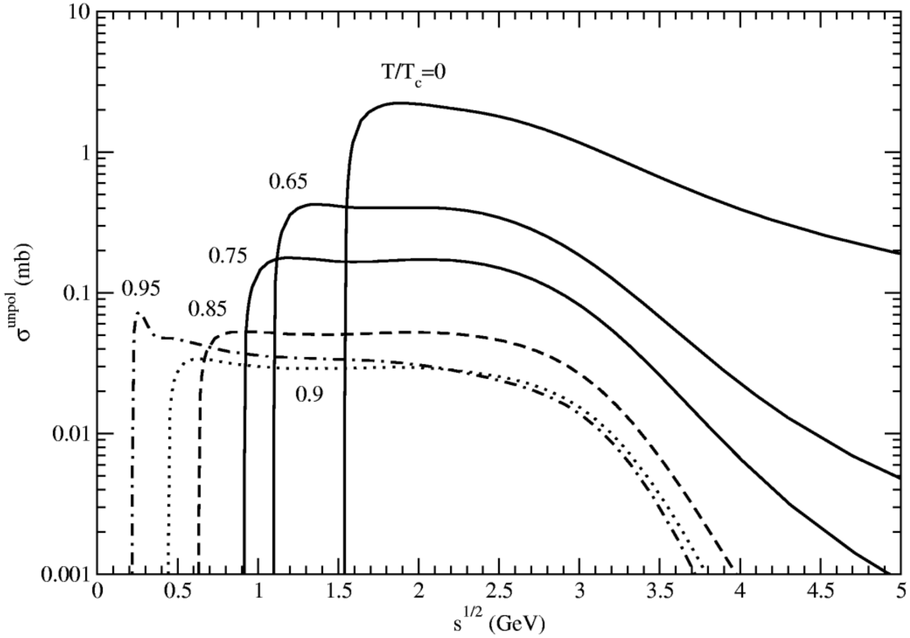
$$\int_0^{\pi} d\theta \sum_{J_{Az}J_{Bz}J_{Cz}J_{Dz}} |M_{aq_1\bar{q}_2} + M_{a\bar{q}_1q_2} + M_{fi}^{prior}|^2 sin\theta$$

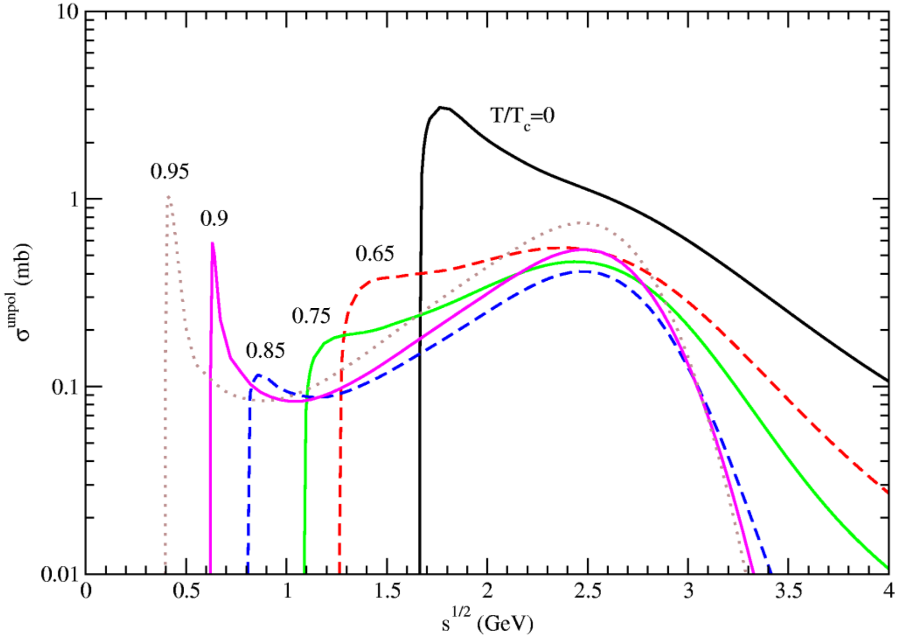
$$\sigma_{unpol}^{post} = \frac{1}{(2J_A + 1)(2J_B + 1)} \frac{1}{32\pi s} \frac{|\vec{P}'|}{|\vec{P}|}$$

$$\int_0^{\pi} d\theta \sum_{J_{Az}J_{Bz}J_{Cz}J_{Dz}} |M_{aq_1\bar{q}_2} + M_{a\bar{q}_1q_2} + M_{fi}^{post}|^2 sin\theta$$

$I = 0 \ \pi\pi \to \rho\rho$

$I = \frac{1}{2} \ \rho K \to \rho K^*$





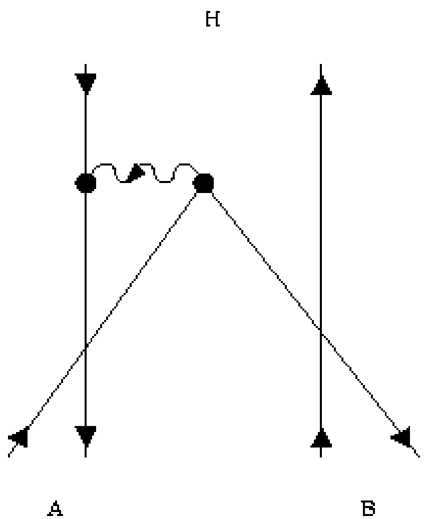
Reactions governed by quark-antiquark annihilation

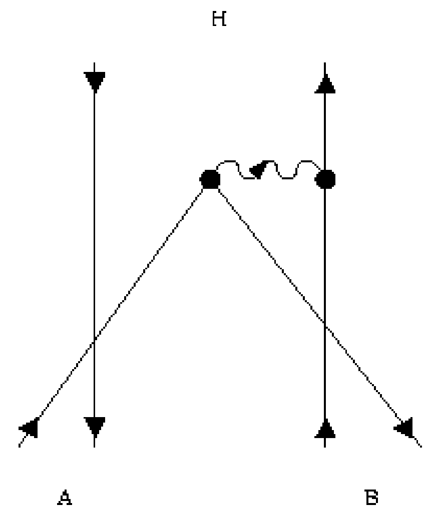
K. Yang, X.-M. Xu, H. J. Weber, Phys. Rev. D96 (2017) 114025

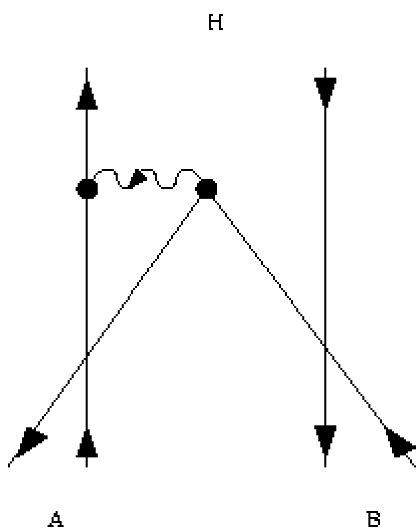
Two mesons annihilate into one meson

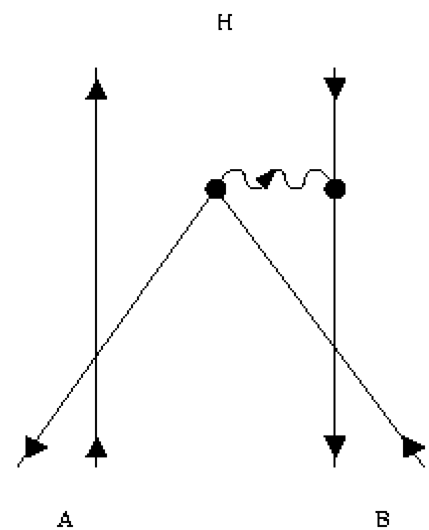
$$\pi\pi \to \rho \qquad\qquad \pi K \to K^*$$

$A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \rightarrow H(q_2\bar{q}_1)$ or $H(q_1\bar{q}_2)$









S-matrix element

for $A + B \to H$

$$S_{fi} = \delta_{fi} - 2\pi i \delta(E_f - E_i)$$
$$(< H \left| V_{rq_1\bar{q}_2\bar{q}_1} \right| A, B > + < H \left| V_{rq_1\bar{q}_2 q_2} \right| A, B >$$
$$+ < H \left| V_{rq_2\bar{q}_1 q_1} \right| A, B > + < H \left| V_{rq_2\bar{q}_1\bar{q}_2} \right| A, B >)$$

$V_{rq_1\bar{q}_2\bar{q}_1}$ is the transition potential for $q_1 + \bar{q}_2 + \bar{q}_1 \to \bar{q}_1$.

$V_{rq_1\bar{q}_2 q_2}$ is the transition potential for $q_1 + \bar{q}_2 + q_2 \to q_2$.

$V_{rq_2\bar{q}_1 q_1}$ is the transition potential for $q_2 + \bar{q}_1 + q_1 \to q_1$.

$V_{rq_2\bar{q}_1\bar{q}_2}$ is the transition potential for $q_2 + \bar{q}_1 + \bar{q}_2 \to \bar{q}_2$.

Transition amplitudes in coordinate space

$$M_{r q_1 \bar{q}_2 \bar{q}_1} = \sqrt{2E_A 2E_B 2E_H} \int d\vec{r}_{q_1\bar{q}_1} d\vec{r}_{q_2\bar{q}_2}$$

$$\psi^{+}_{q_2\bar{q}_1} V_{r q_1 \bar{q}_2 \bar{q}_1} \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2} e^{i\vec{p}_{q_1\bar{q}_1, q_2\bar{q}_2} \cdot \vec{r}_{q_1\bar{q}_1, q_2\bar{q}_2}}$$

$$M_{r q_1 \bar{q}_2 q_2} = \sqrt{2E_A 2E_B 2E_H} \int d\vec{r}_{q_1\bar{q}_1} d\vec{r}_{q_2\bar{q}_2}$$

$$\psi^{+}_{q_2\bar{q}_1} V_{r q_1 \bar{q}_2 q_2} \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2} e^{i\vec{p}_{q_1\bar{q}_1, q_2\bar{q}_2} \cdot \vec{r}_{q_1\bar{q}_1, q_2\bar{q}_2}}$$

$$M_{r q_2 \bar{q}_1 q_1} = \sqrt{2E_A 2E_B 2E_H} \int d\vec{r}_{q_1\bar{q}_1} d\vec{r}_{q_2\bar{q}_2}$$

$$\psi^{+}_{q_1\bar{q}_2} V_{r q_2 \bar{q}_1 q_1} \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2} e^{i\vec{p}_{q_1\bar{q}_1, q_2\bar{q}_2} \cdot \vec{r}_{q_1\bar{q}_1, q_2\bar{q}_2}}$$

$$M_{r q_2 \bar{q}_1 \bar{q}_2} = \sqrt{2E_A 2E_B 2E_H} \int d\vec{r}_{q_1\bar{q}_1} d\vec{r}_{q_2\bar{q}_2}$$

$$\psi^{+}_{q_1\bar{q}_2} V_{r q_2 \bar{q}_1 \bar{q}_2} \psi_{q_1\bar{q}_1} \psi_{q_2\bar{q}_2} e^{i\vec{p}_{q_1\bar{q}_1, q_2\bar{q}_2} \cdot \vec{r}_{q_1\bar{q}_1, q_2\bar{q}_2}}$$

Unpolarised cross section

The unpolarised cross section for $A + B \to H$ is

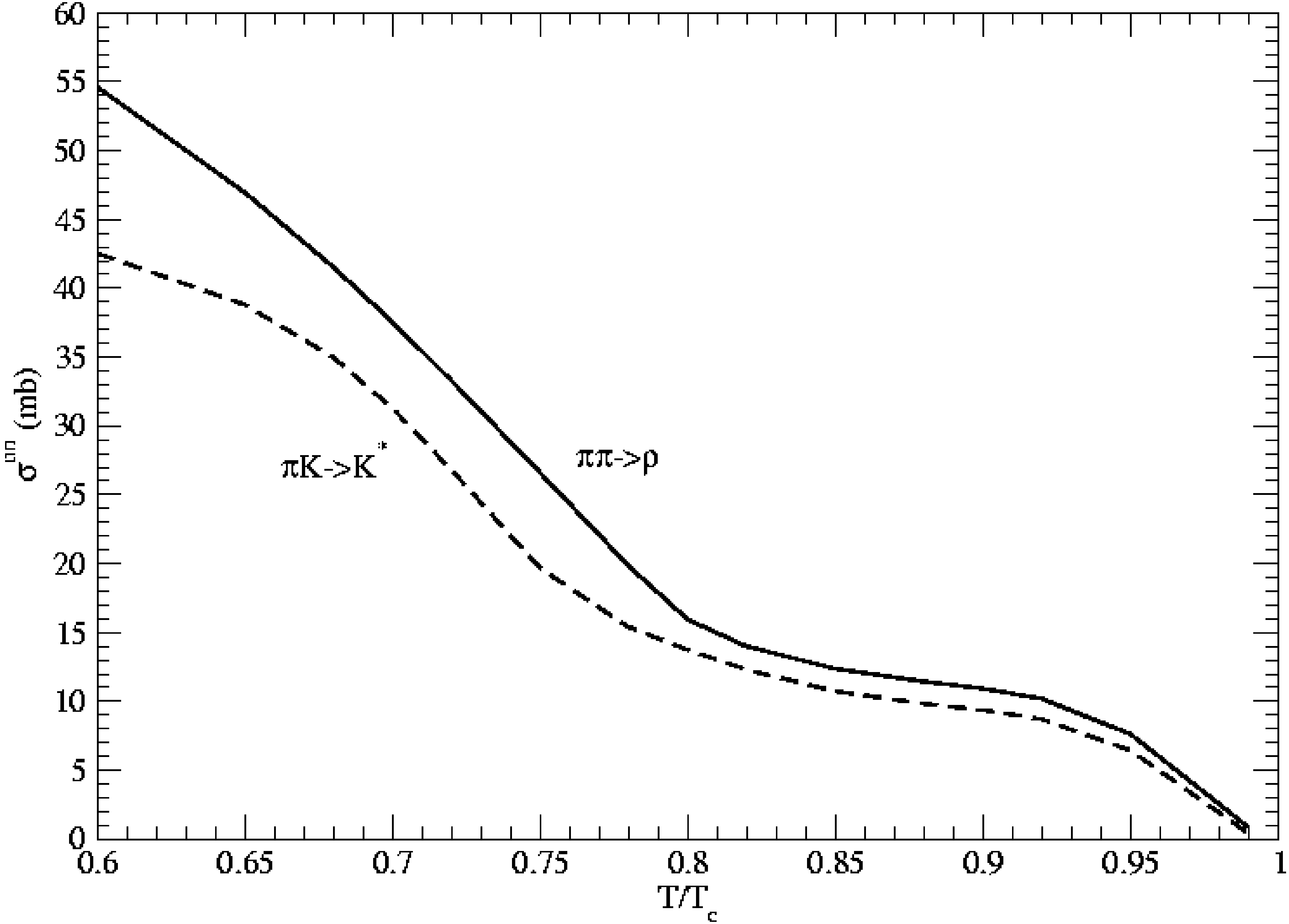
$$\sigma^{unpol} = \frac{1}{(2J_A+1)(2J_B+1)} \frac{\sqrt{\pi}\exp(-\frac{(\sqrt{s}-m_H)^2}{h^2})}{2h\sqrt{[s-(m_A+m_B)^2][s-(m_A-m_B)^2]}E_H} \sum_{J_{Az}J_{Bz}J_{Hz}} |M_{rq_1\bar{q}_2\bar{q}_1} + M_{rq_1\bar{q}_2q_2} + M_{rq_2\bar{q}_1q_1} + M_{rq_2\bar{q}_1\bar{q}_2}|^2$$

With $\sqrt{s} = m_H$

Isospin-averaged unpolarised cross section

$$\sigma^{un} = \frac{2I_H + 1}{(2I_A + 1)(2I_B + 1)} \sigma^{unpol}$$

	empirical values
For $\pi\pi \rightarrow \rho$ at $T = 0$, $\sigma^{un} = 80.07$ mb	80 mb
For $\pi K \rightarrow K^*$ at $T = 0$, $\sigma^{un} = 60.5$ mb	60 mb



σ (mb)
T/T_c
πK->K*
ππ->ρ
0
5
10
15
20
25
30
35
40
45
50
55
60
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1

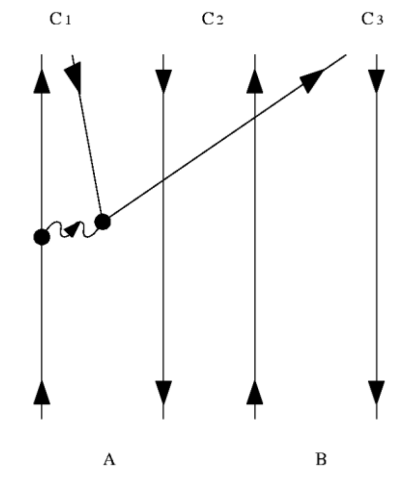
Reactions governed by quark-antiquark creation

arXiv:1907.03229

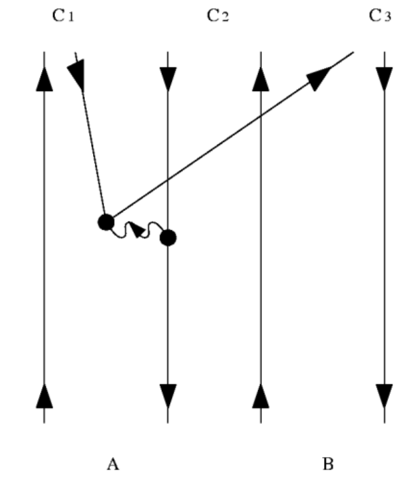
meson-meson collisions produce three mesons:

$$\pi\pi \to \pi K\bar{K},\ \pi K \to \pi\pi K,\ \pi K \to KK\bar{K},$$
$$KK \to \pi KK,\ K\bar{K} \to \pi K\bar{K}$$

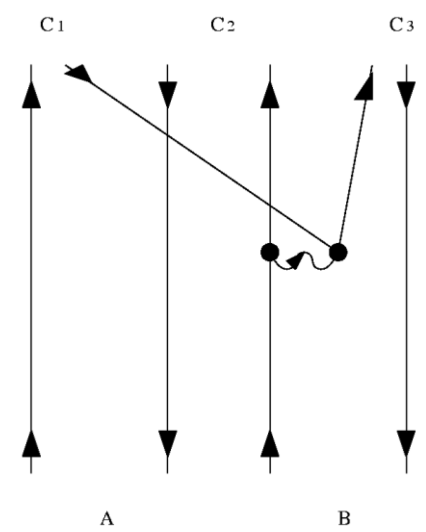
$$A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \rightarrow C_1(q_1\bar{q}_4) + C_2(q_2\bar{q}_1) + C_3(q_3\bar{q}_2)$$



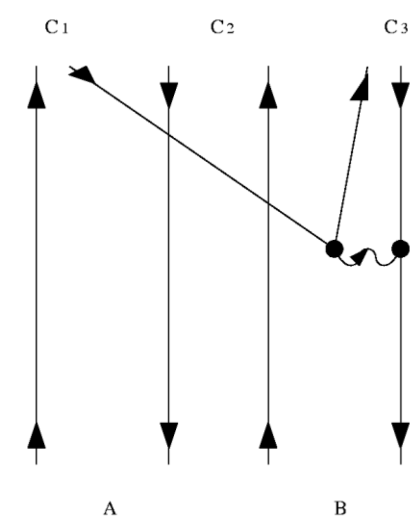
D$_1$



D$_2$

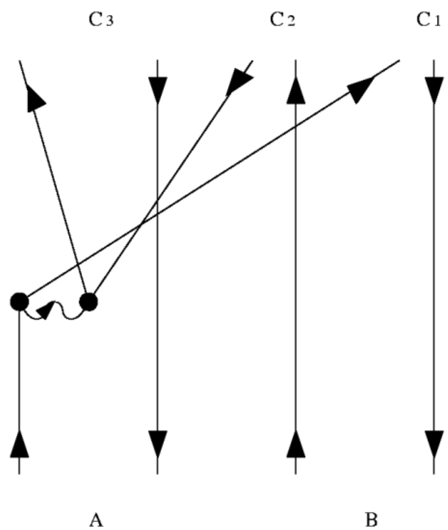


D$_3$

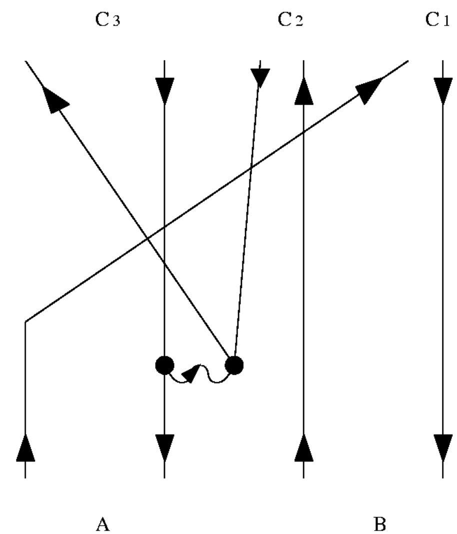


D$_4$

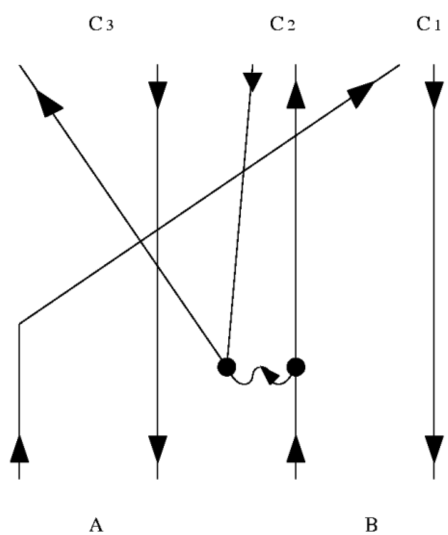
$$A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \rightarrow C_1(q_1\bar{q}_2) + C_2(q_2\bar{q}_4) + C_3(q_3\bar{q}_1)$$



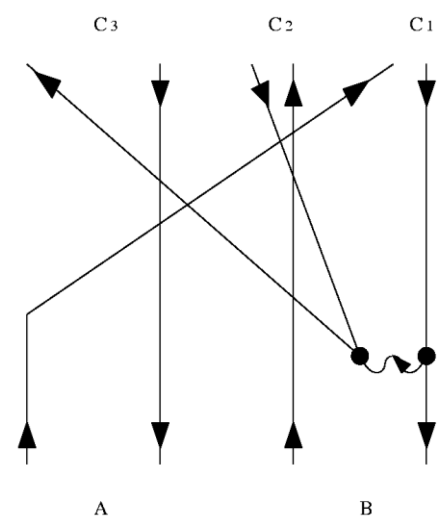
D_5



D_6



D_7



D_8

S-matrix element

for $A + B \rightarrow C_1 + C_2 + C_3$

$$S_{fi} = \delta_{fi} - 2\pi i \delta\left(E_f - E_i\right)(< C_1, C_2, C_3 \left| V_{D_1} \right| A, B > + < C_1, C_2, C_3 \left| V_{D_2} \right| A, B >$$
$$+ < C_1, C_2, C_3 \left| V_{D_3} \right| A, B > + < C_1, C_2, C_3 \left| V_{D_4} \right| A, B >$$
$$+ < C_1, C_2, C_3 \left| V_{D_5} \right| A, B > + < C_1, C_2, C_3 \left| V_{D_6} \right| A, B >$$
$$+ < C_1, C_2, C_3 \left| V_{D_7} \right| A, B > + < C_1, C_2, C_3 \left| V_{D_8} \right| A, B >)$$

V_{D_1}, V_{D_2}, V_{D_3}, V_{D_4}, V_{D_5}, V_{D_6}, V_{D_7} and V_{D_8} stand for the transition potentials corresponding to diagrams D_1, D_2, D_3, D_4, D_5, D_6, D_7, and D_8.

Transition amplitudes

$$M_{D_1} = \sqrt{2E_A 2E_B 2E_{C_1} 2E_{C_2} 2E_{C_3} \frac{3\sqrt{3}(m_{q_1}+m_{\bar{q}_4})^3(m_{q_2}+m_{\bar{q}_1})^3(m_{q_3}+m_{\bar{q}_2})^3}{(m_{q_1}m_{q_2}m_{q_3}+m_{\bar{q}_1}m_{\bar{q}_2}m_{\bar{q}_4})^3}}$$

$$\int d\vec{r}_{q_1\bar{q}_1} d\vec{r}_{q_2\bar{q}_2} d\vec{\rho}_X d\vec{\lambda}_X \psi^+_{q_1\bar{q}_4}(\vec{r}_{q_1\bar{q}_4}) \psi^+_{q_2\bar{q}_1}(\vec{r}_{q_2\bar{q}_1}) \psi^+_{q_3\bar{q}_2}(\vec{r}_{q_3\bar{q}_2}) V_{D_1}$$

$$\psi_{q_1\bar{q}_1}(\vec{r}_{q_1\bar{q}_1}) \psi_{q_2\bar{q}_2}(\vec{r}_{q_2\bar{q}_2}) \exp(-i\vec{p}_{\rho_X}\cdot\vec{\rho}_X - i\vec{p}_{\lambda_X}\cdot\vec{\lambda}_X + i\vec{p}_{q_1\bar{q}_1,q_2\bar{q}_2}\cdot\vec{r}_{q_1\bar{q}_1,q_2\bar{q}_2})$$

$$M_{D_5} = \sqrt{2E_A 2E_B 2E_{C_1} 2E_{C_2} 2E_{C_3} \frac{3\sqrt{3}(m_{q_1}+m_{\bar{q}_2})^3(m_{q_2}+m_{\bar{q}_4})^3(m_{q_3}+m_{\bar{q}_1})^3}{(m_{q_1}m_{q_2}m_{q_3}+m_{\bar{q}_1}m_{\bar{q}_2}m_{\bar{q}_4})^3}}$$

$$\int d\vec{r}_{q_1\bar{q}_1} d\vec{r}_{q_2\bar{q}_2} d\vec{\rho}_Y d\vec{\lambda}_Y \psi^+_{q_1\bar{q}_2}(\vec{r}_{q_1\bar{q}_2}) \psi^+_{q_2\bar{q}_4}(\vec{r}_{q_2\bar{q}_4}) \psi^+_{q_3\bar{q}_1}(\vec{r}_{q_3\bar{q}_1}) V_{D_5}$$

$$\psi_{q_1\bar{q}_1}(\vec{r}_{q_1\bar{q}_1}) \psi_{q_2\bar{q}_2}(\vec{r}_{q_2\bar{q}_2}) \exp(-i\vec{p}_{\rho_Y}\cdot\vec{\rho}_Y - i\vec{p}_{\lambda_Y}\cdot\vec{\lambda}_Y + i\vec{p}_{q_1\bar{q}_1,q_2\bar{q}_2}\cdot\vec{r}_{q_1\bar{q}_1,q_2\bar{q}_2})$$

Cross section

The unpolarised cross section for $A + B \to C_1 + C_2 + C_3$ is

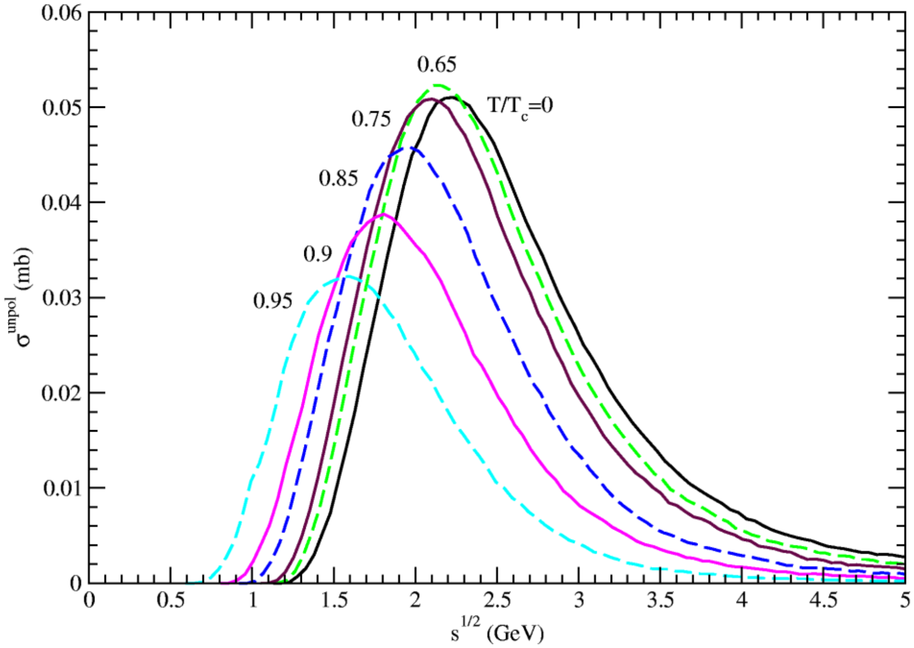
$$\sigma^{unpol} = \frac{1}{(2J_A+1)(2J_B+1)} \frac{1}{16(2\pi)^5\sqrt{[s-(m_A+m_B)^2][s-(m_A-m_B)^2]}}$$

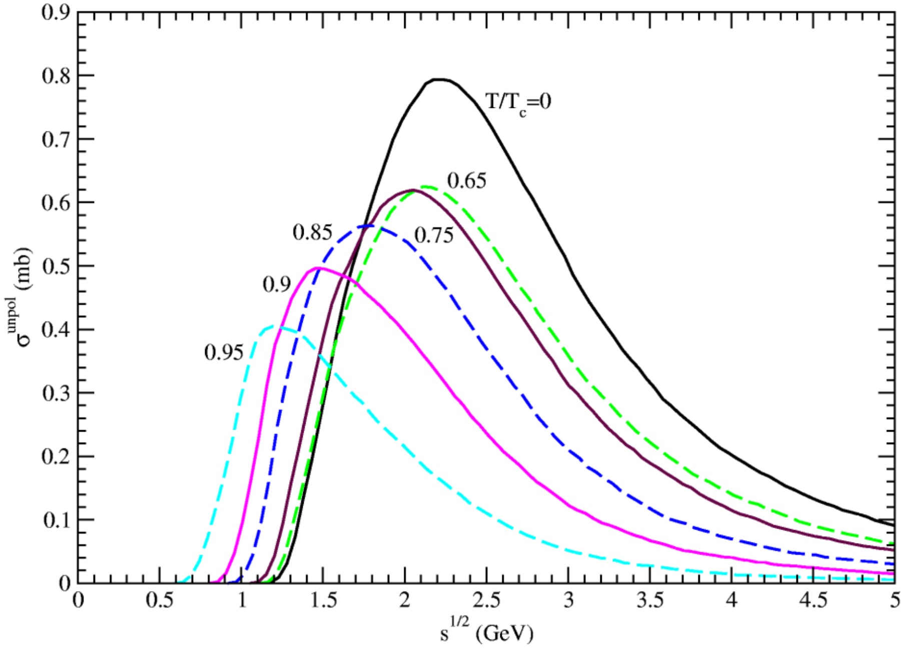
$$\int \frac{d^3P_{C_3}}{E_{C_3}} d\Omega_{C_1} \frac{|\vec{P}_{C_1}|_0^2}{||\vec{P}_{C_1}|_0 E_{C_2} + (|\vec{P}_{C_1}|_0 - \left|\vec{P}_A + \vec{P}_B - \vec{P}_{C_3}\right| \cos\Theta) E_{C_1}|}$$

$$\sum_{J_{Az}J_{Bz}J_{C_1z}J_{C_2z}J_{C_3z}} |M_{D_1} + M_{D_2} + M_{D_3} + M_{D_4} + M_{D_5} + M_{D_6} + M_{D_7} + M_{D_8}|^2$$

$$I = 1 \;\; I^f_{\pi K} = \tfrac{3}{2} \;\; \pi\pi \to \pi K \bar{K}$$

$$I = 1 \;\; I^f_{\pi \bar{K}} = \tfrac{3}{2} \;\; K\bar{K} \to \pi K \bar{K}$$





summary

- We have proposed a model to study 2-to-2 meson-meson scattering.
- We have proposed a model to study 2-to-1 meson-meson scattering.
- We have proposed a model to study 2-to-3 meson-meson scattering.
- We have done the first calculations of cross sections for many meson-meson reactions.
- We find remarkable temperature dependence of the cross sections.