Spin polarizations in a covariant angular momentum conserved chiral transport model

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New development of hydrodynamics and its applications in Heavy-Ion Collisions

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Based on work, Liu, Sun, Ko, arXiv:1910.06774

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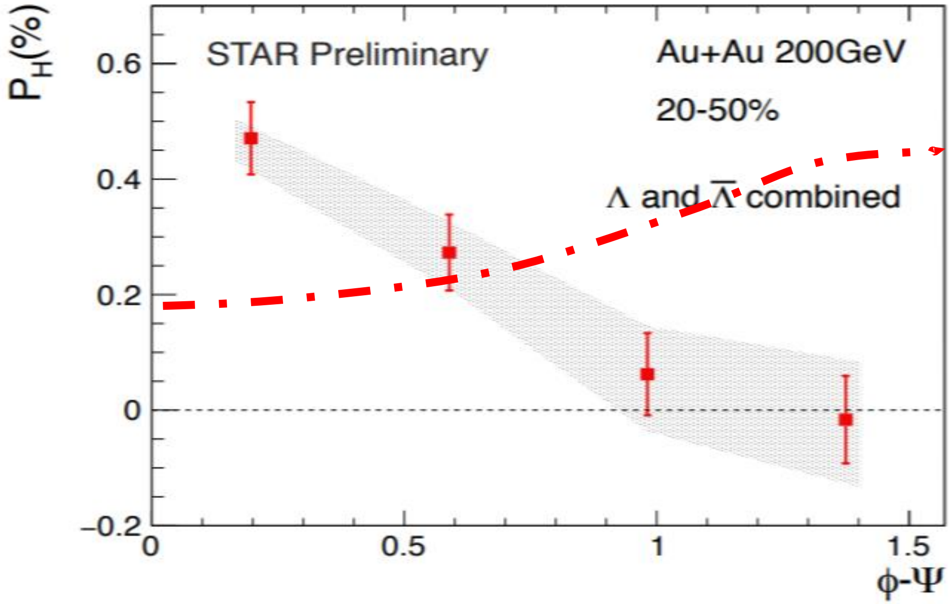
Outline

1) Background and motivation
2) The side-jump formalism
3) Benchmark calculation
4) Simulation for heavy-ion collision
5) Conclusion and perspective

The Spin Puzzle

$-\mathcal{P}_y$: Polarization along the orbital angular momentum direction

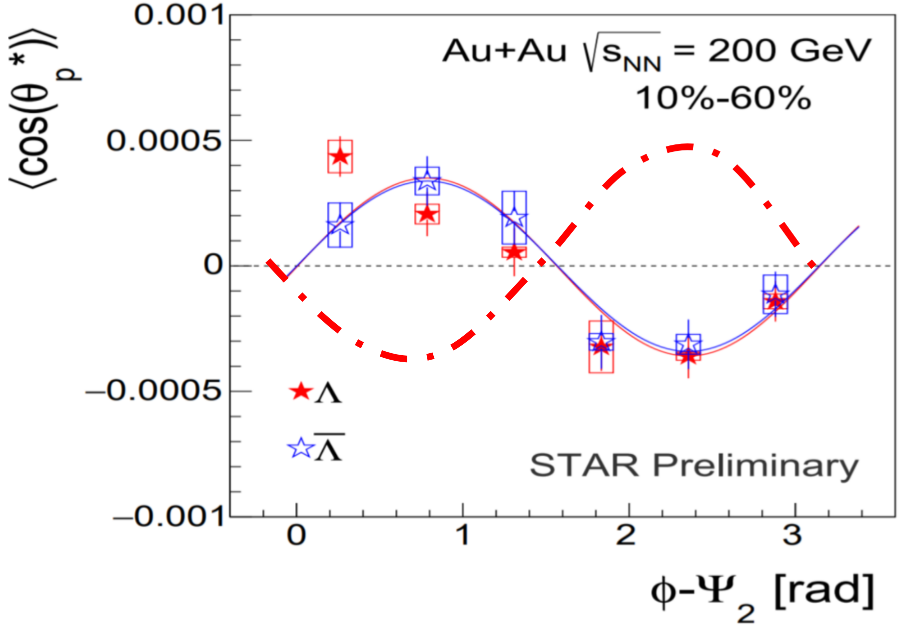


Schematic Sketch

Thermal model prediction

$$\mathcal{P}_z = \langle \cos(\theta) \rangle / [\alpha_H \langle \cos^2(\theta) \rangle]$$

Polarization along the beam direction



- ❖ Some physics beyond the thermal model?
- ❖ Ambiguity in local thermal equilibrium?

Becattini ,Florkowski ,Speranza, 2018

The Spin Puzzles

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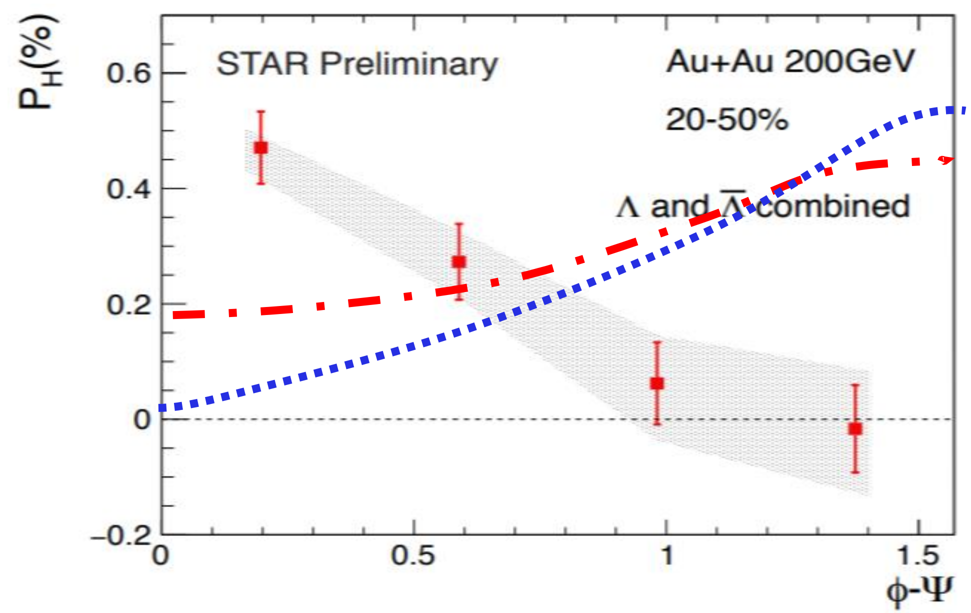


Schematic Sketch

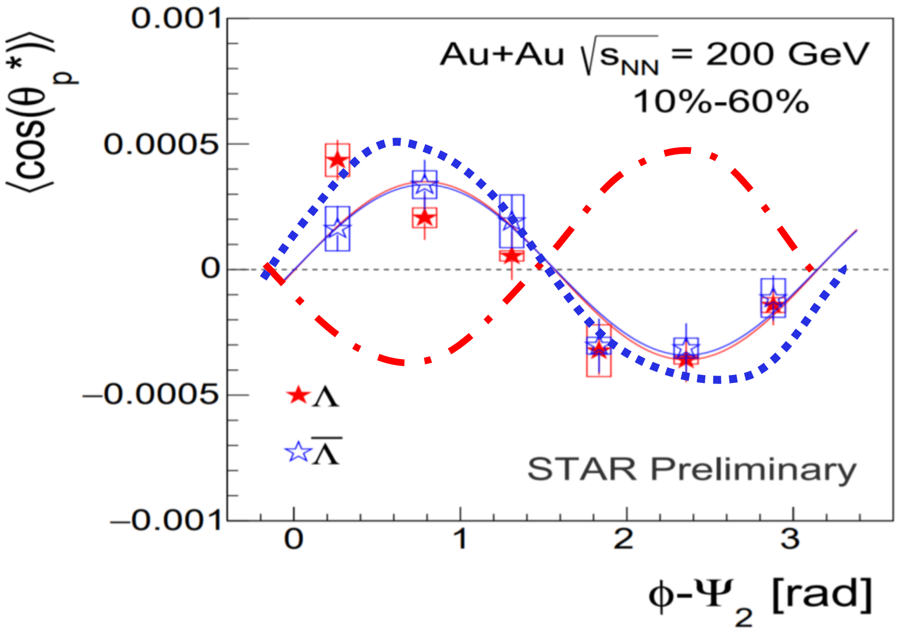
Thermal model prediction

Previous chiral kinetic transport

Sun, Ko, 2019

$\mathcal{P}_z = \langle\cos(\theta)\rangle/[\alpha_H\langle\cos^2(\theta)\rangle]$

Polarization along the beam direction



- ❖ Some physics beyond the thermal model?
- ❖ Ambiguity in local thermal equilibrium?
- ❖ Previous chiral kinetic transport seems indicates some new features, but failed for $\mathcal{P}_y$
- ❖ Several conceptually important questions in chiral kinetic framework remain to be solved

Paradox: Chiral Kinetic Equations or Newton's first Law?

❖Chiral kinetic equation:

$$\dot{\mathbf{r}}' = \frac{\hat{\mathbf{p}}' + 2\lambda p'(\hat{\mathbf{p}}' \cdot \mathbf{b}')\boldsymbol{\omega}}{1 + 2\lambda p'(\boldsymbol{\omega} \cdot \mathbf{b}')}$$

- $v_z \neq 0$ due to anomalous velocity along the $\boldsymbol{\omega}$

❖Newton's First Law:

- $v_z = 0$ since particle should move straight line

Contradictions!

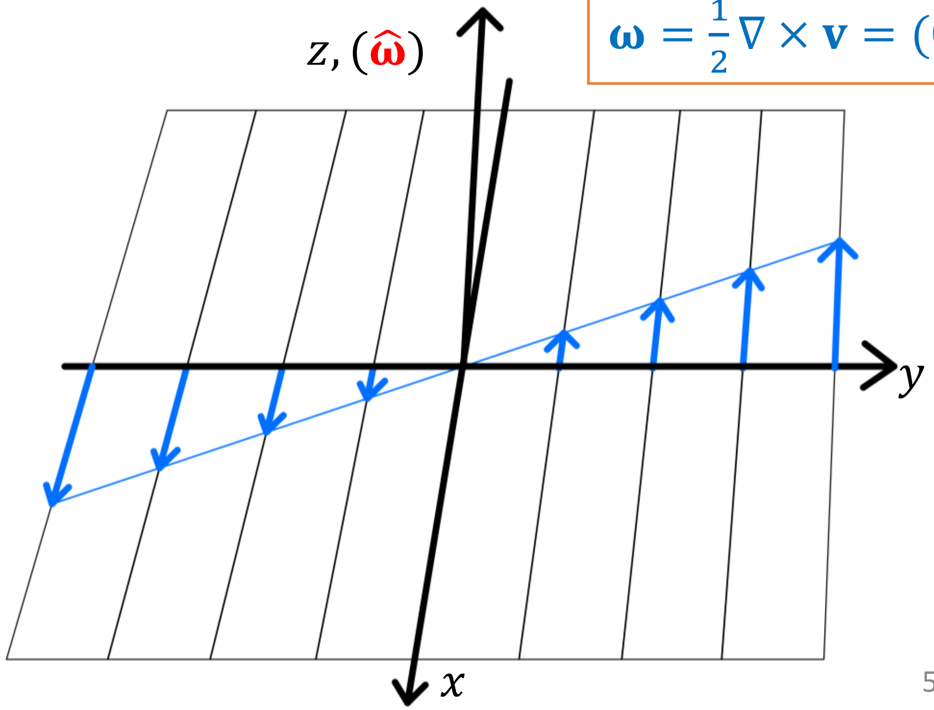
How to construct a theoretical formalism for CVE to be consistent with newton's first law?

❖A thought experiment

- Prepare a system of particles with vorticity
- **Turn off all interactions!**
- Inject a test particle with $\mathbf{v}$ along the x-direction into the system
- **How should the particle move?**

$$\mathbf{v} = (0, -2\omega y, 0)$$

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \mathbf{v} = (0,0,\omega)$$



Challenge: total angular momentum conservation

❖ Power of total angular momentum conservation

- Chiral kinetic equation by $\mathbf{J} = \mathbf{L} + \mathbf{S}$ conservation in external force:

$$\frac{d\mathbf{J}}{dt} = \frac{d\left(\mathbf{r}\times\mathbf{p}\pm\frac{\hat{\mathbf{p}}}{2}\right)}{dt} = \mathbf{r}\times\mathbf{F}$$

$$\dot{\mathbf{r}} = \hat{\mathbf{p}} \pm \dot{\mathbf{p}}\times\frac{\mathbf{p}}{2p^3} = \hat{\mathbf{p}} + \dot{\mathbf{p}}\times\mathbf{b}$$

Sun, Ko, Li, 2016

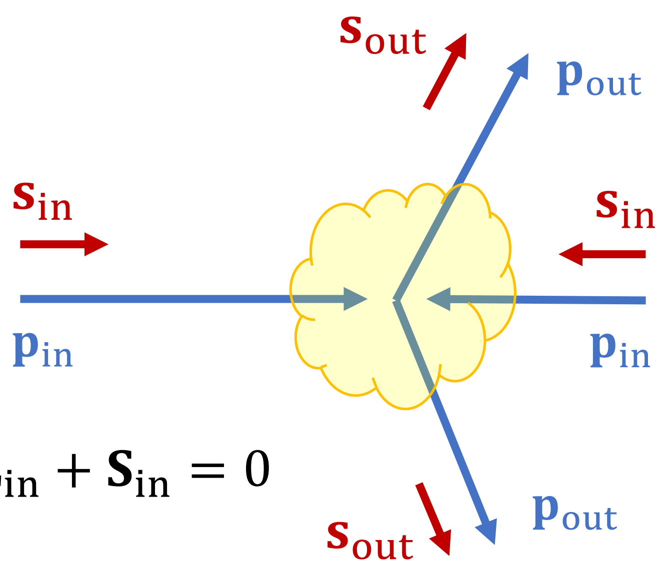
- Statistical mechanics from conserved quantities:

$$\exp[-[E - \mathbf{v}\cdot\mathbf{P} - \boldsymbol{\omega}\cdot(\mathbf{L}+\mathbf{S})]/T]$$

Polarization: $\langle S\rangle = \omega/(4T)$

❖ Interactions between partons in the QGP : scattering process

❖ Does it conserve $\mathbf{J}$?



$\mathbf{L}_{\text{in}} + \mathbf{S}_{\text{in}} = 0$

$\mathbf{L}_{\text{out}} + \boldsymbol{S}_{\text{out}} \neq 0$, Since $\boldsymbol{S}_{\text{out}} \neq 0$

J is not conserved !

$\mathbf{J}$ Conservation By Side-Jump

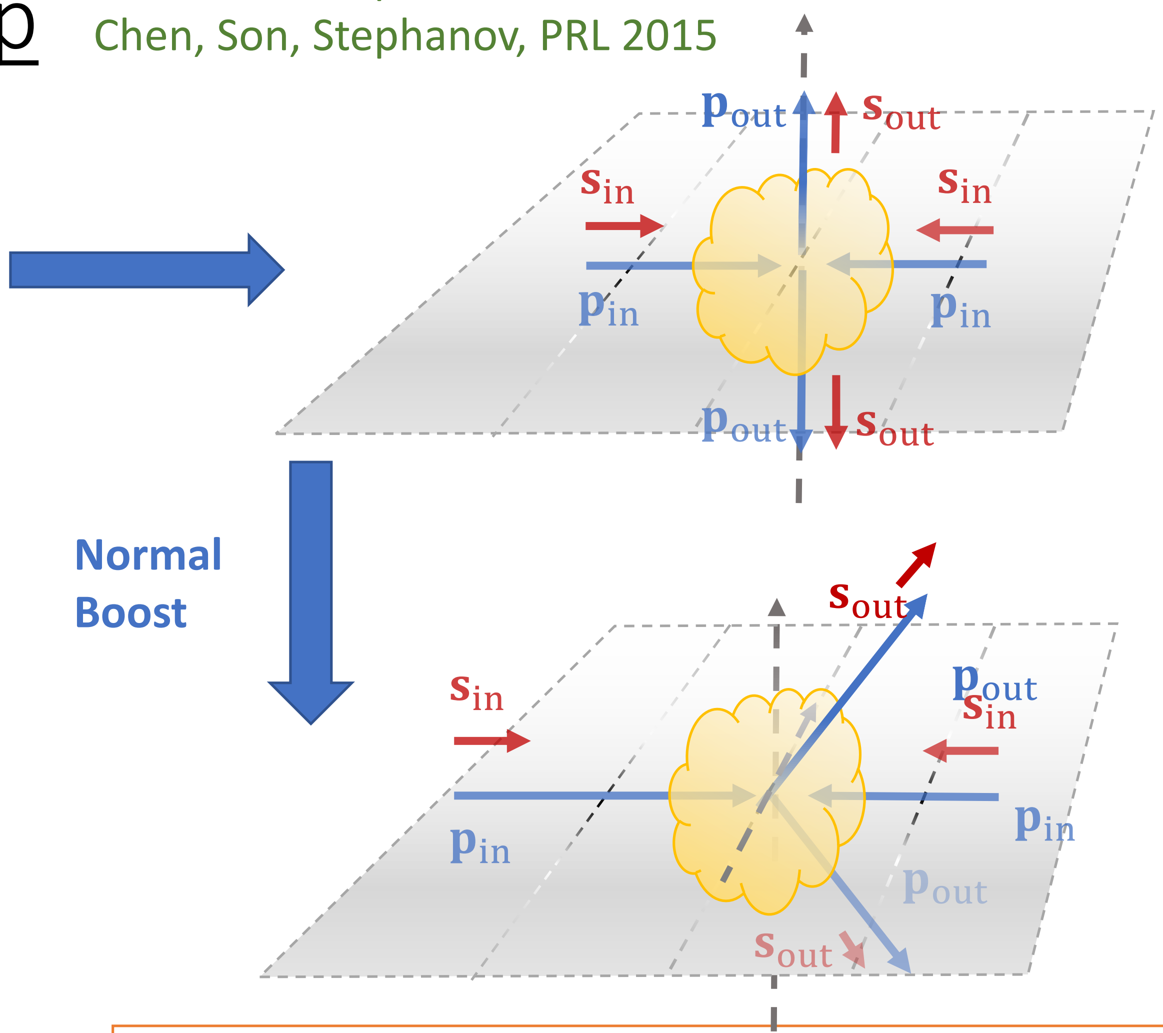
Chen, Son, Stephanov, Yee, Yin, PRL, 2014
Chen, Son, Stephanov, PRL 2015

- ❖Conservation of $\mathbf{J}$ in CM frame

  Relatively Simple

- ❖How to boost this results to the general Lab frame?

  Non-trivial for chiral fermion



➢J does not conserve in the LAB frame

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❖ Conservation of $\mathbf{J}$ in CM frame

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Non-trivial for chiral fermion

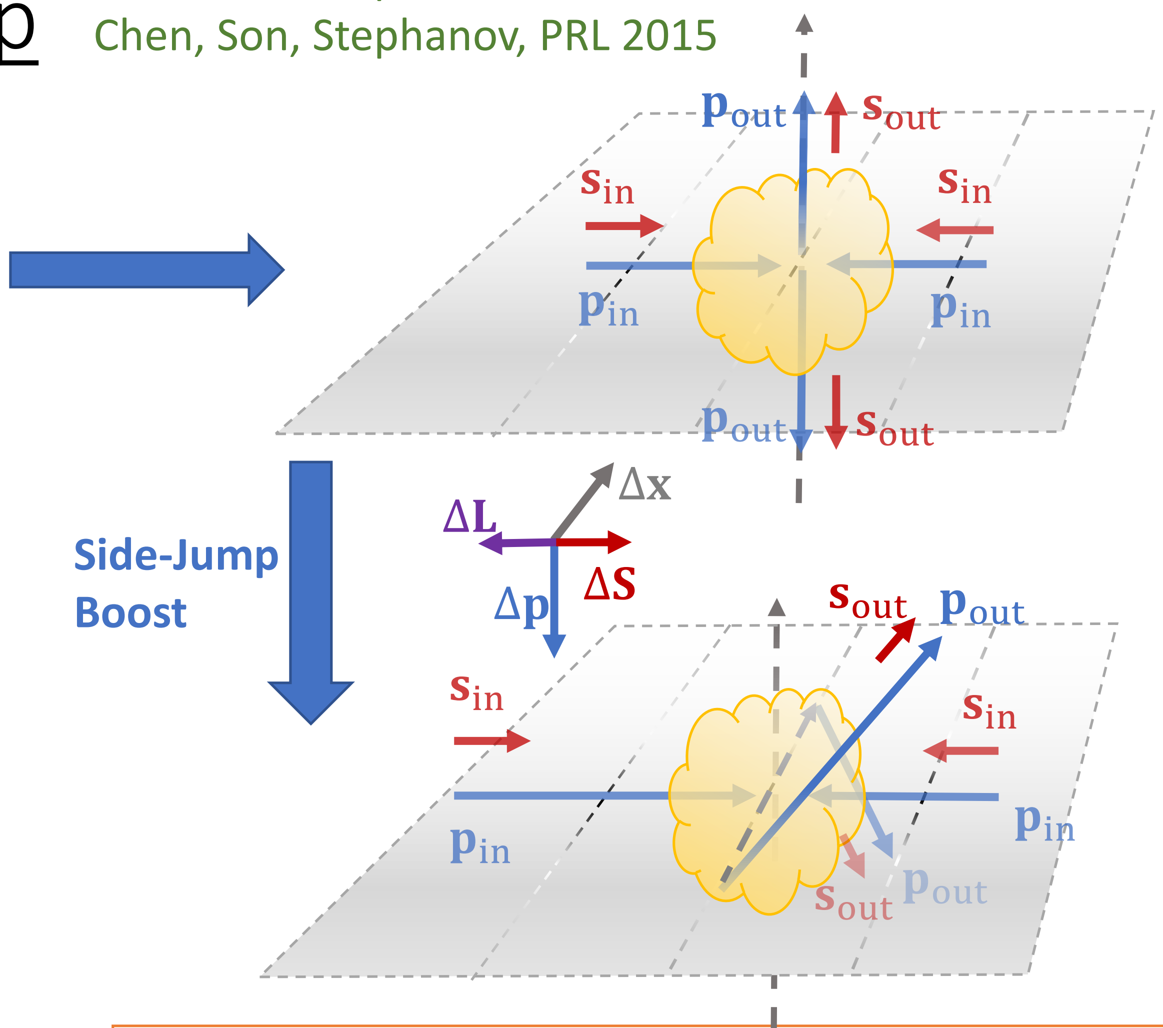
❖ Requiring the $J' = \Lambda^T J \Lambda$ covariant and $\boldsymbol{s} = \lambda\, \hat{\mathbf{p}}$ in all frames leads to **side-jump boost:**

$$x'^{\mu} = \Lambda^{\mu}{}_{\alpha} x^{\alpha} + \Delta^{\mu}_{\tilde{n}n'}$$

❖ A side-jump term $\Delta_{\tilde{n}n'}$ appear:

$$\Delta^{\mu}_{\tilde{n}n'} = \lambda \frac{\epsilon^{\mu\alpha\beta\gamma} p'_{\alpha} \tilde{n}_{\beta} n'_{\gamma}}{(p' \cdot \tilde{n})(p' \cdot n')}$$

❖ $\Delta_{\tilde{n}n'} \perp \mathbf{p}'$, and $\Delta_{\tilde{n}n'} \perp \mathbf{P}_t{}'$ in the lab frame



- As shown, $\Delta\mathbf{J} = \Delta\mathbf{L} + \Delta\mathbf{S} = \mathbf{0}$
- $\mathbf{J}$ is conserved by a side-jump

Generalization Required for Transport Simulation

❖ Idealized zero impact parameter
- Permittable phase space for $\mathbf{p}_{out}$ is a 3D sphere
- Non-jump in position in CM frame

❖ Collision can happen between partons with the same helicity

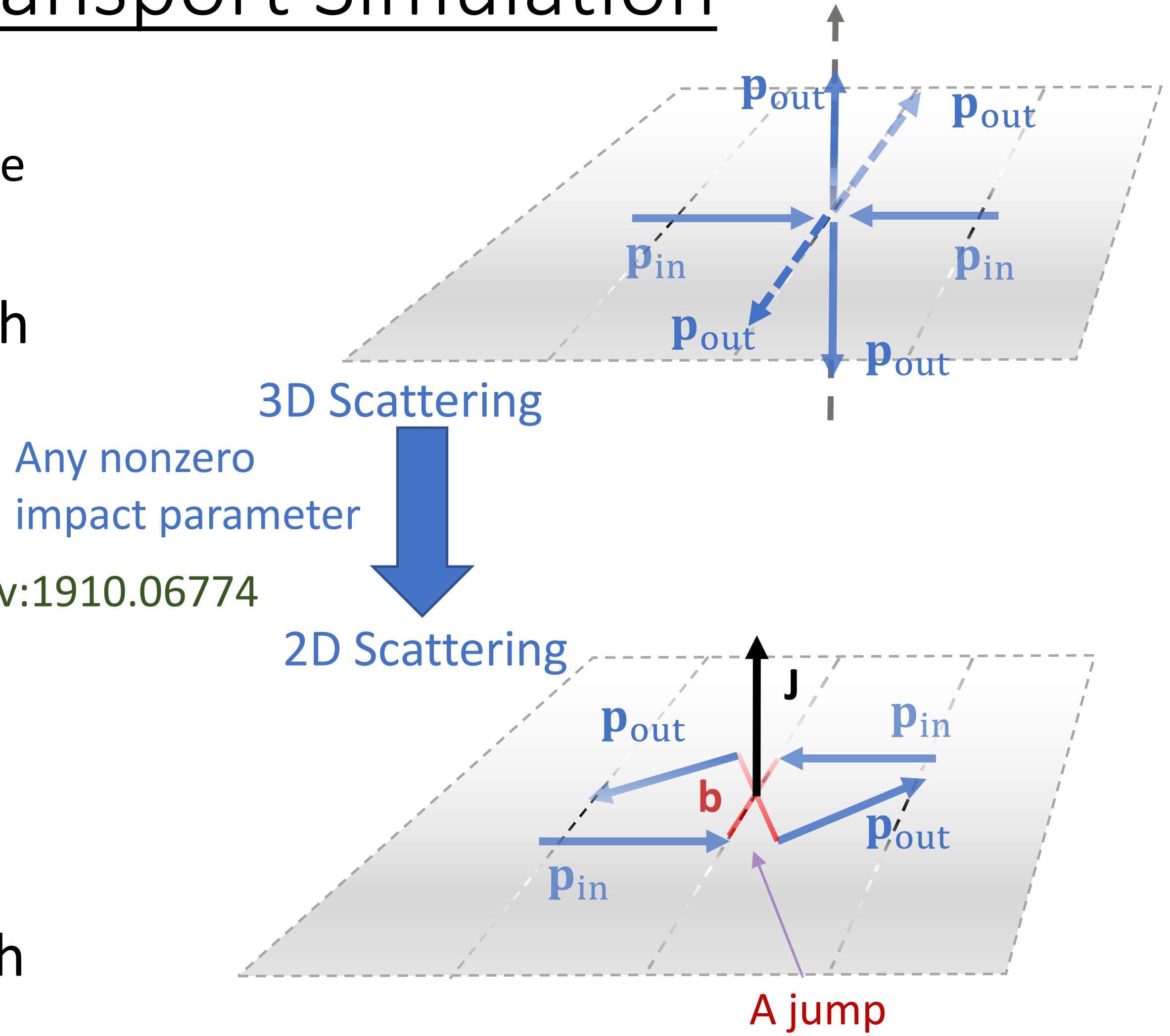
❖ Finite impact parameter **b**

Liu, Sun, Ko, arXiv:1910.06774

- Permittable phase space for $\mathbf{p}_{\text{out}}$ is a 2D circle lie in the plane perpendicular to **J**.
- Jump in position in CM frame
- Other new features, still numerical feasible

❖ Collision can happen between partons with same or different helicity

❖ Using the same side-jump boost to obtain results in the LAB frame.



➢ $J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu + S^{\mu\nu}$ conserved in realistic simulation!

How we address the paradox and go beyond

❖The kinetic equation in vortical flows in our approach (No B field):

$$\dot{\mathbf{r}}' = \frac{\hat{\mathbf{p}}' + 2\lambda p'(\hat{\mathbf{p}}' \cdot \mathbf{b}')\boldsymbol{\omega}}{1 + 2\lambda p'(\boldsymbol{\omega} \cdot \mathbf{b}')} \quad \Longrightarrow \quad \dot{\boldsymbol{r}} = \widehat{\mathbf{p}}$$

Usual cross section ⟹ Generalized side-jump J conserved collision

❖Without interaction (no collision), all particles move in straight line, newton's first law recovered

❖With collisions, side-jump collisions will transport the axial charge along the ω direction and the anomalous currents can reproduce chiral vortical effects.

The paradox is solved by J conserved scattering

Chen, Son, Stephanov, Yee, Yin, PRL, 2014
Chen, Son, Stephanov, PRL 2015

A Box Calculation as Benchmark

- Box initally at $5 \times 5 \times 5$ fm, $\omega = 0.012/\text{fm}$ (z direction), T=0.3GeV, then, free expand
- Check conservation angular $J = \sum_i r_i \times p_i + \lambda_i \hat{p}_i$
- Define covariant current as:

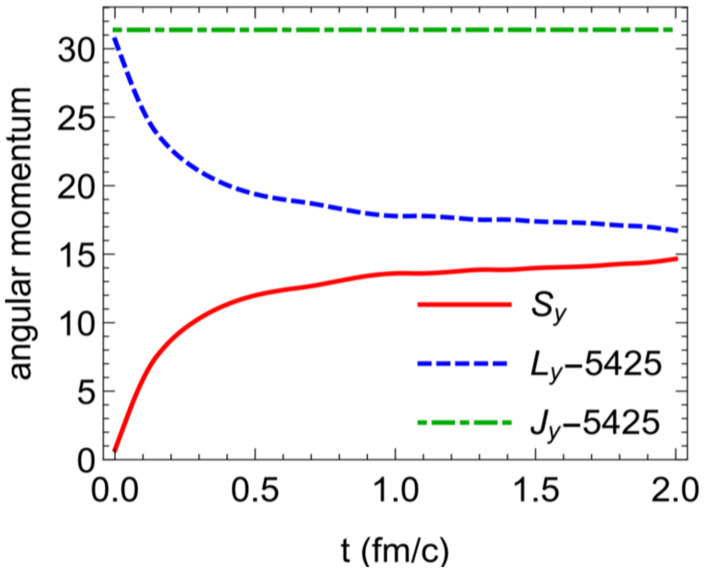
Chen, Son, Stephanov, Yee, Yin, PRL, 2014
Chen, Son, Stephanov, PRL 2015

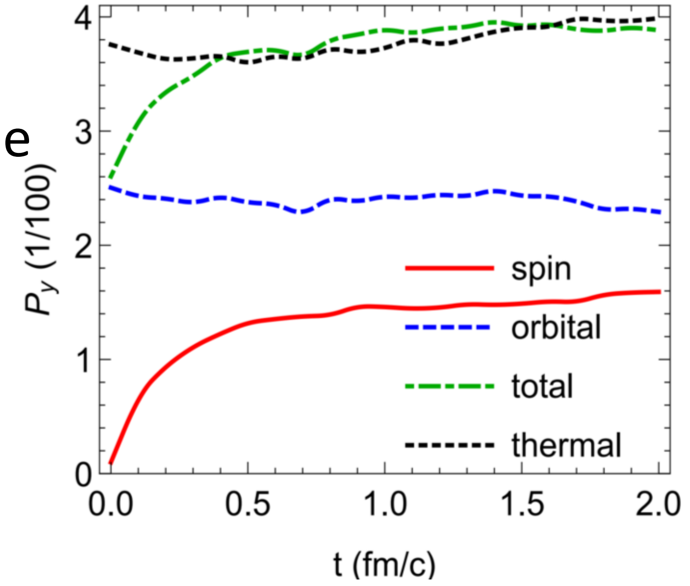
$$j^{\mu}_{R/L}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3 p}\left(p^{\mu} f_{R/L} + S^{\mu\nu}\partial_{\nu} f_{R/L}\right).$$

- In addition to the normal spin term, there is an additional magnetization term (orbital term) required by the covariance of the $j^{\mu}_{L/R}$ of chiral fermion.
- The space component of the $j^{\mu}_{R/L}$ is defined as the **total "spin"** so that polarization can be related to $\mathbf{j_5}$ as

$$\mathcal{P} = \int d^3x \mathbf{j}_5(x) / \int d^3x\, n(x)$$

- Recover thermal benchmark, well defined Lorentz transformation





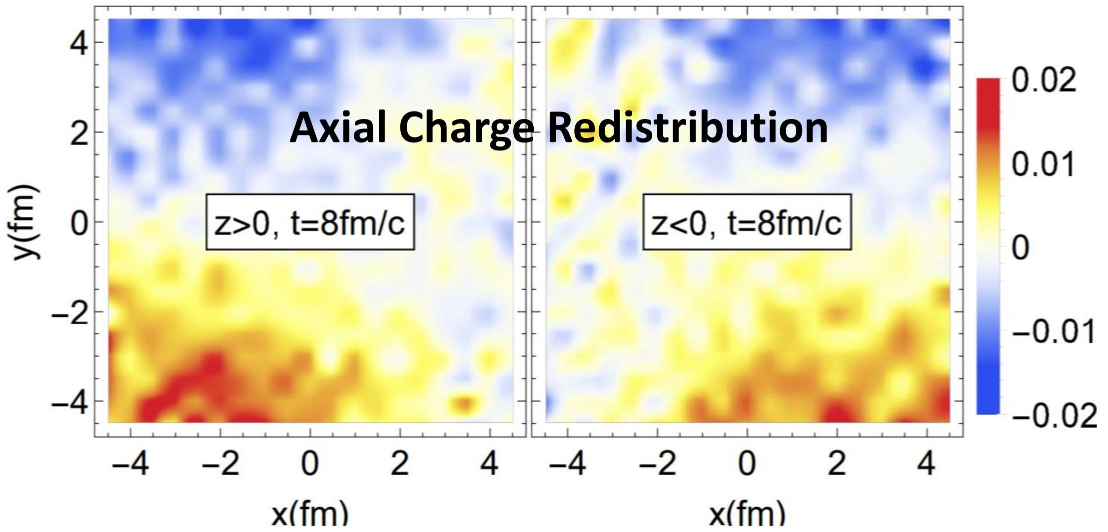
Spin in proton also has an orbital contribution

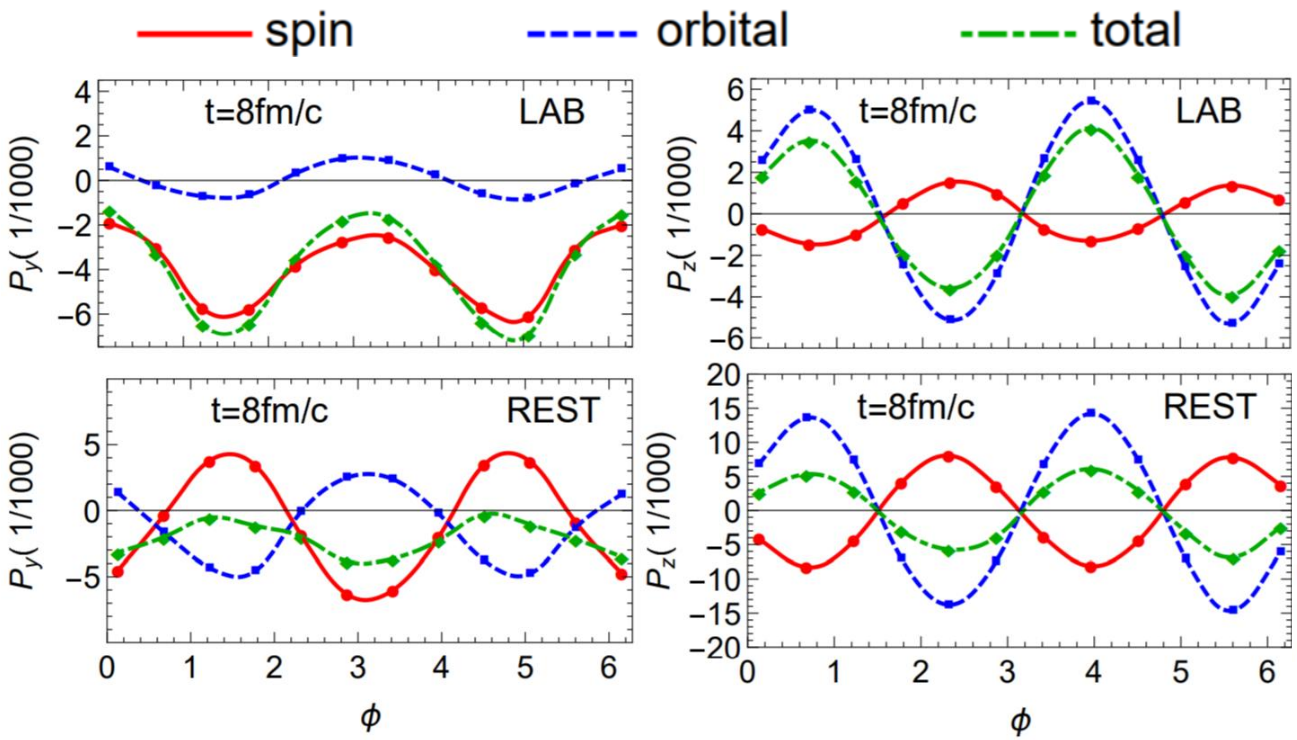
J conservation dynamically leads to polarization

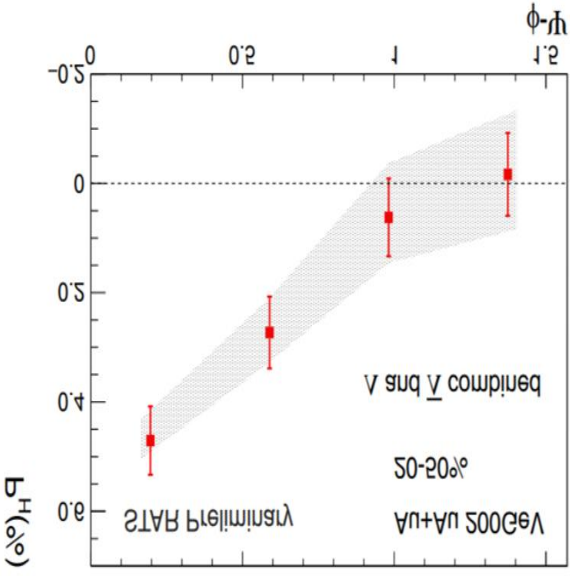
Transport Simulation for Heavy-ion Collision

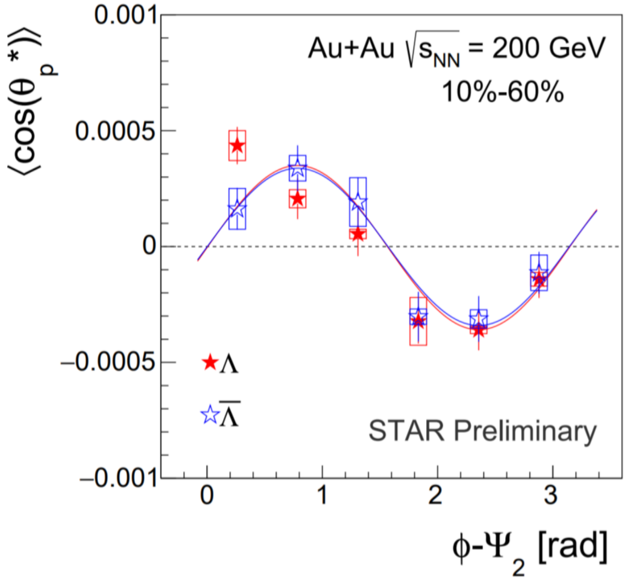
$\int dz\, n_5/n$

- ❖Large axial charge redistribution according to the vorticity through side-jump collisions
- ❖Both spin part and orbital part are important for total polarization
- ❖Boost affects the result









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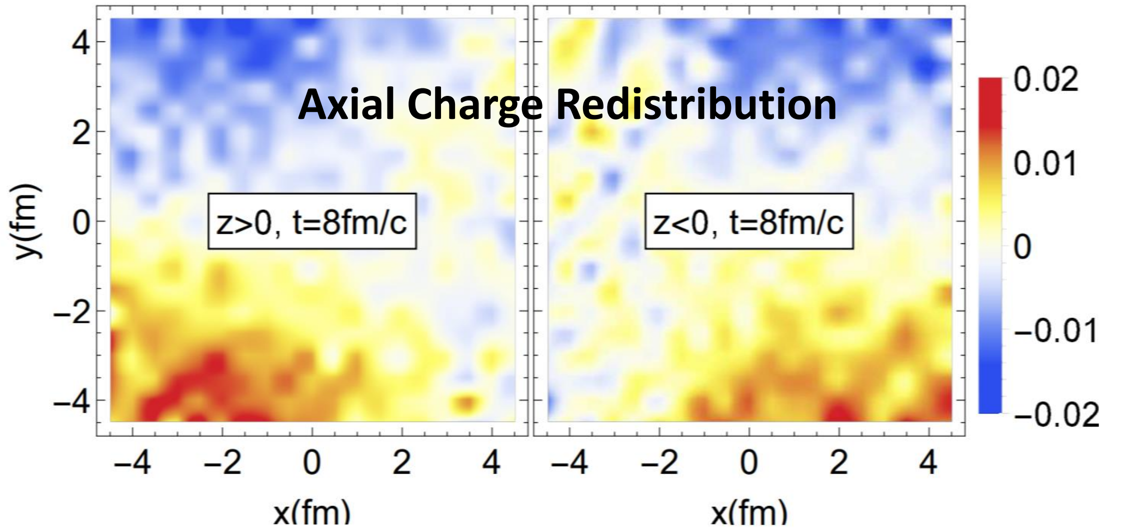
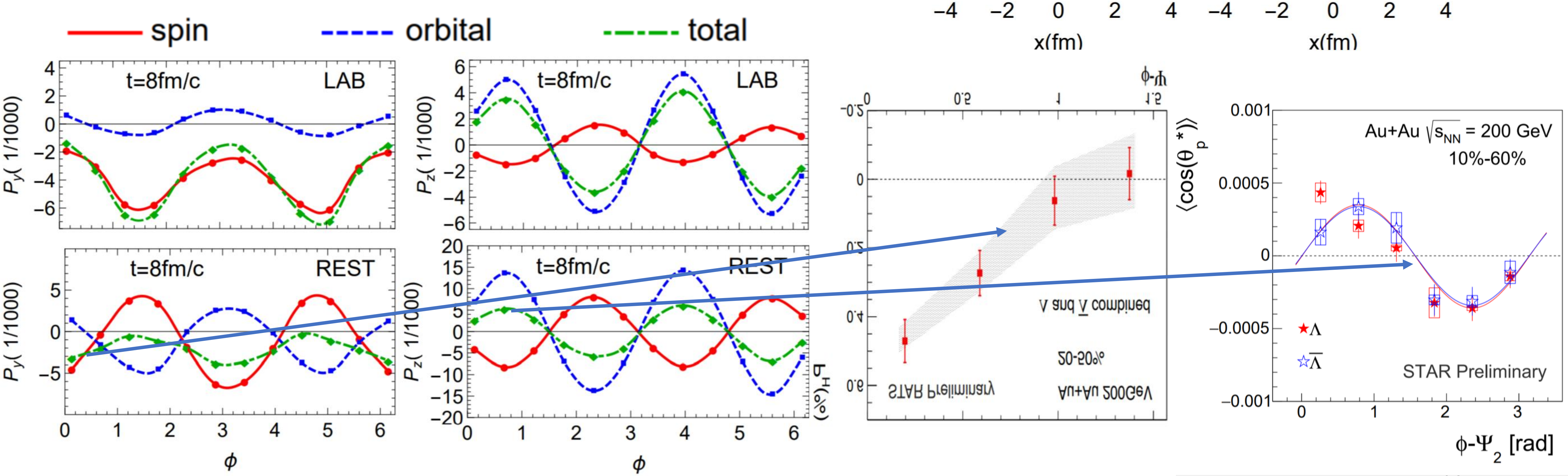
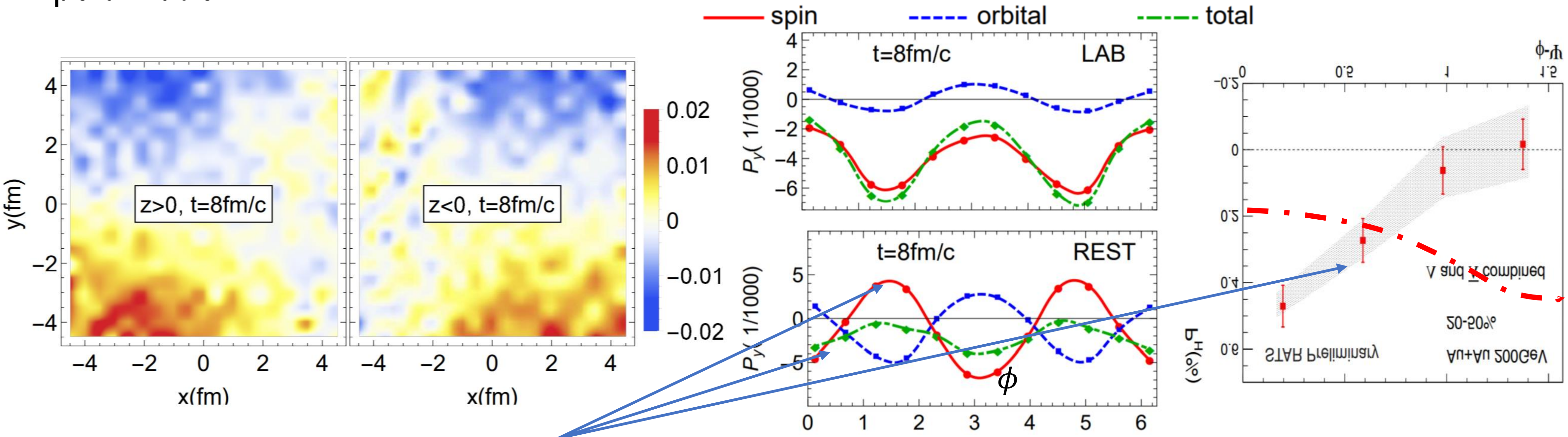




How Axial charge Redistribution and Boost Affect Polarization?

- ❖Polarization is: $\mathcal{P} = \int d^3x \mathbf{j}_5(x) / \int d^3x\, n(x)$
- ❖$j_5^\mu = (n_5, \mathbf{j}_5)$ is a well defined four-vector with the time component
- ❖$(\mathbf{j_5})_{||} = \gamma \left((\mathbf{j_5})_{||} - v\, n_5 \right)$, $(\mathbf{j_5})_{\perp} = (\mathbf{j_5})_{\perp}$
- ❖ With the nontrivial distribution of n_5, it affects the space part of $\mathbf{j_5}$, thus the polarization



- ➢ Without n_5, we do not get this trend.
- ➢ Does this trend provide us indications for the **axial charge redistribution** ?

Conclusion and perspective

❖Conclusion

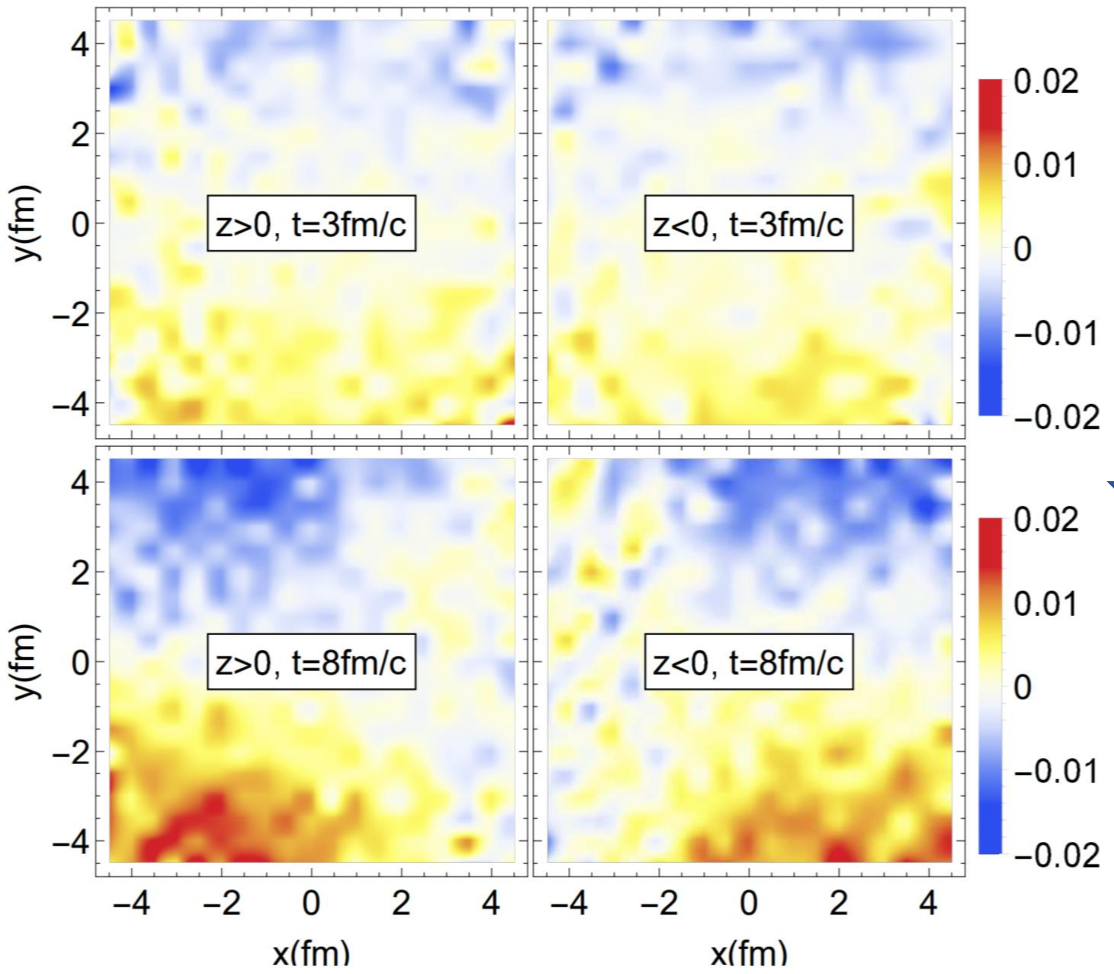
- Construct a chiral transport approach that respects the Newtown’s first law and total angular momentum, which also can recover thermal limit
- There is “orbital” contribution to polarization in addition to spin contribution, which plays an important role
- Axial charge redistribution and boost are also essential

❖Perspective

- How to perform angular momentum conserved hadronization to convert parton spin to Lambda spin
- Mass effects for the side-jump approach
- More sophisticated medium evolution that can recover lattice EoS

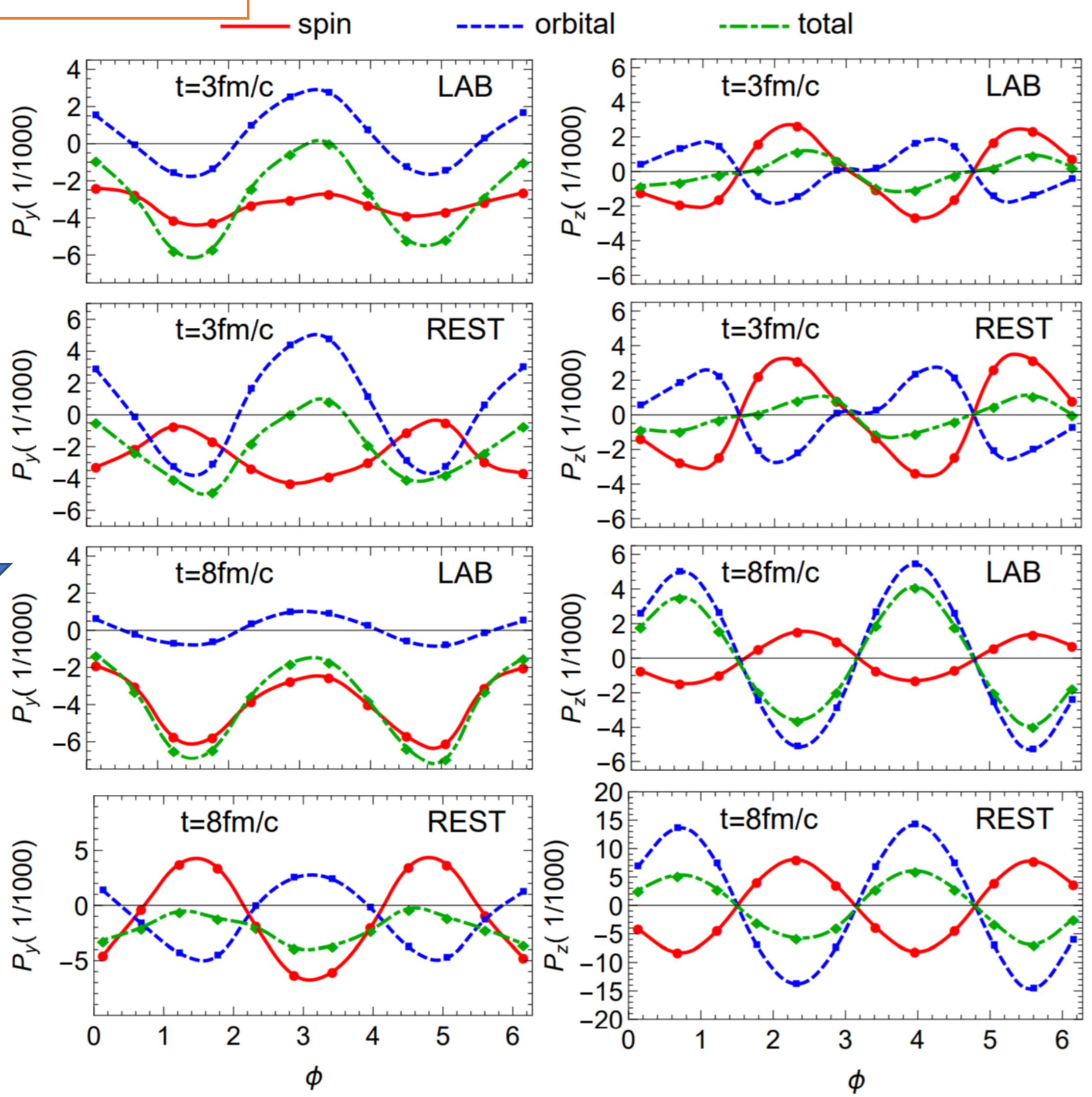
Transport Simulation for Heavy-ion Collision

Axial charge redistribution



Time

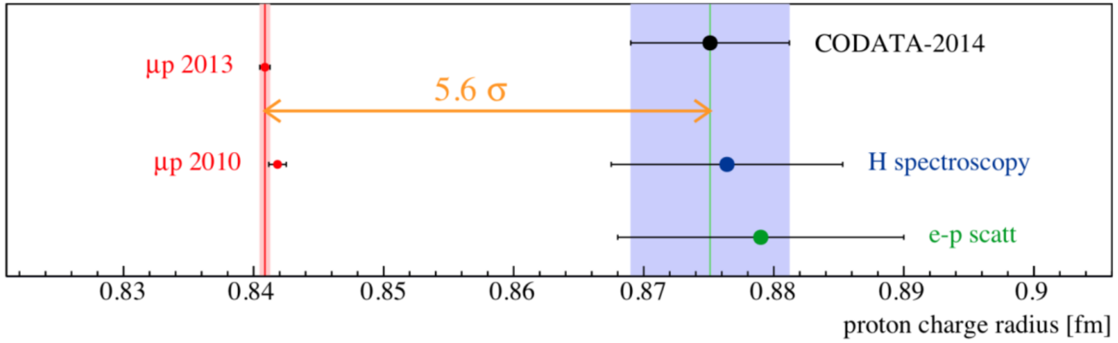
Polarization



Background

Slides From Xingbo Zhao and Siqi Xu at IMP working on proton strucutres.

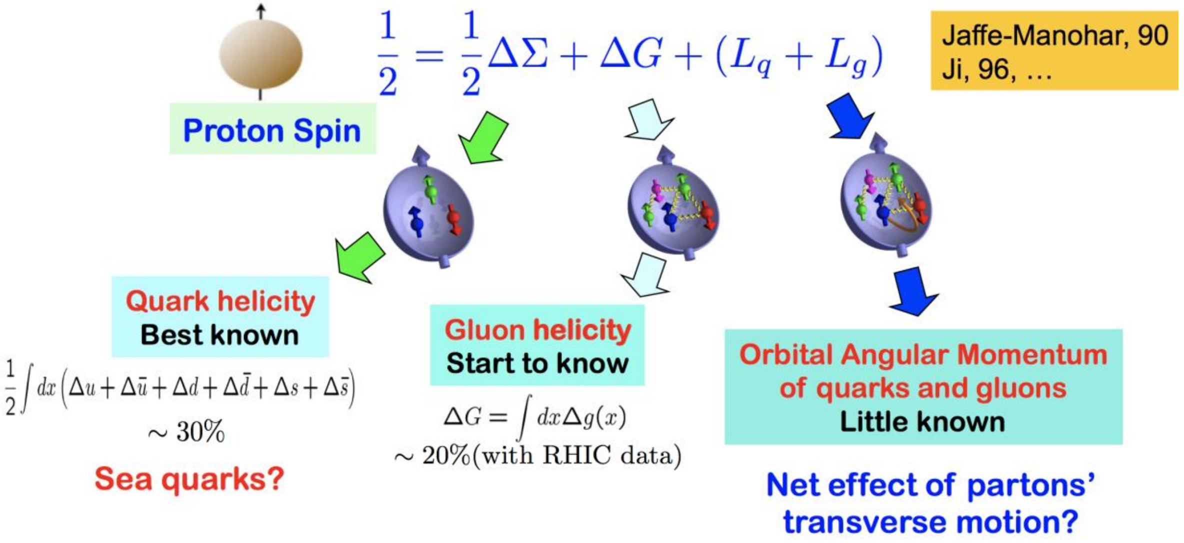
➢Proton radius puzzle



- Elastic electron scattering established the extended nature of the proton, [*R. Hofstadter, Nobel Prize 1961*]
- Different experiments give the different ***radius***.

➢ Spin crisis

- In 1988s, **EMC**(European Muon Collaboration) found the contribution of ***spin of quark*** is **smaller** than ***expected***.



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  - Non-jump in position in CM frame
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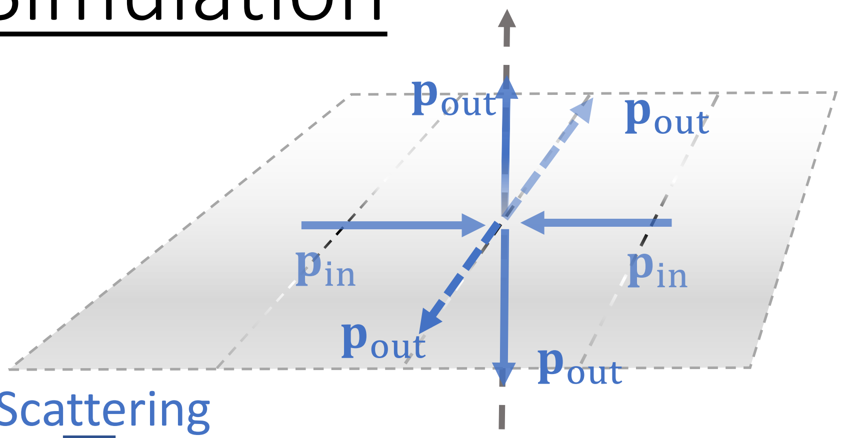


3D Scattering

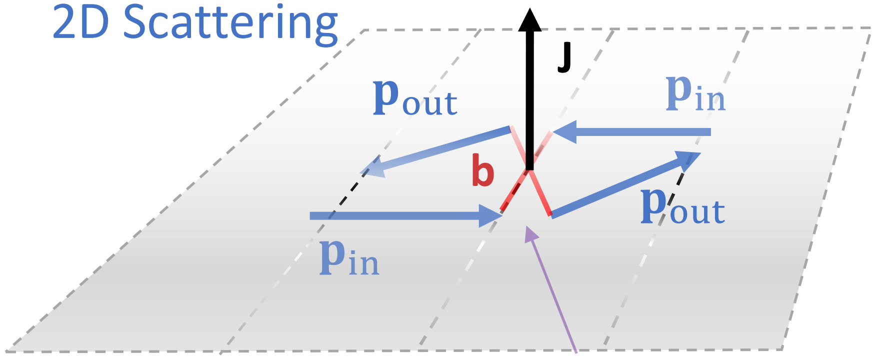
Any nonzero impact parameter

- Finite impact parameter **b**

Liu, Sun, Ko, ArXiv:1910.06774

  - Permittable phase space for $\mathbf{p}_{out}$ is a 2D circle lie in the plane perpendicular to **J**.
  - Jump in position in CM frame
  - Other new features, still numerical feasible

2D Scattering



A jump

- Collision can happen between partons with same or different helicity
- Using the same side-jump boost to obtain results in LAB frame.

> $J^{\mu\nu} = x^{\mu}p^{\nu} - x^{\nu}p^{\mu}$ conservation realistic

$$S^{\mu\nu} = \lambda \frac{\epsilon^{\mu\nu\alpha\beta} p_{\alpha} n_{\beta}}{p \cdot n}$$