



# Chiral and Charged Pion Condensate in Magnetic Field with Rotation

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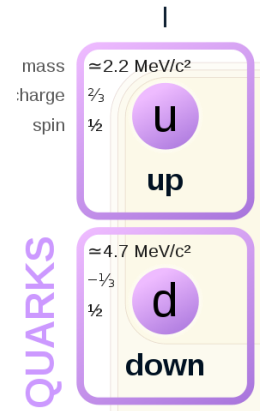
Fudan University

H-L. C, K.Fukushima, X.G.Huang, K. Mameda,  
Phys. Rev. D 93, 104052 (2016), arXiv: 1512.08974;  
Phys. Rev. D 96, 054032 (2017), arXiv: 1707.09130.  
H-L. C, X.G.Huang, K. Mameda, arXiv: 1910.02700.

# Outline

- Motivation
- Rotating frame & NJL model
- Rotational magnetic inhibition
- Charged pion condensate in rotating frame

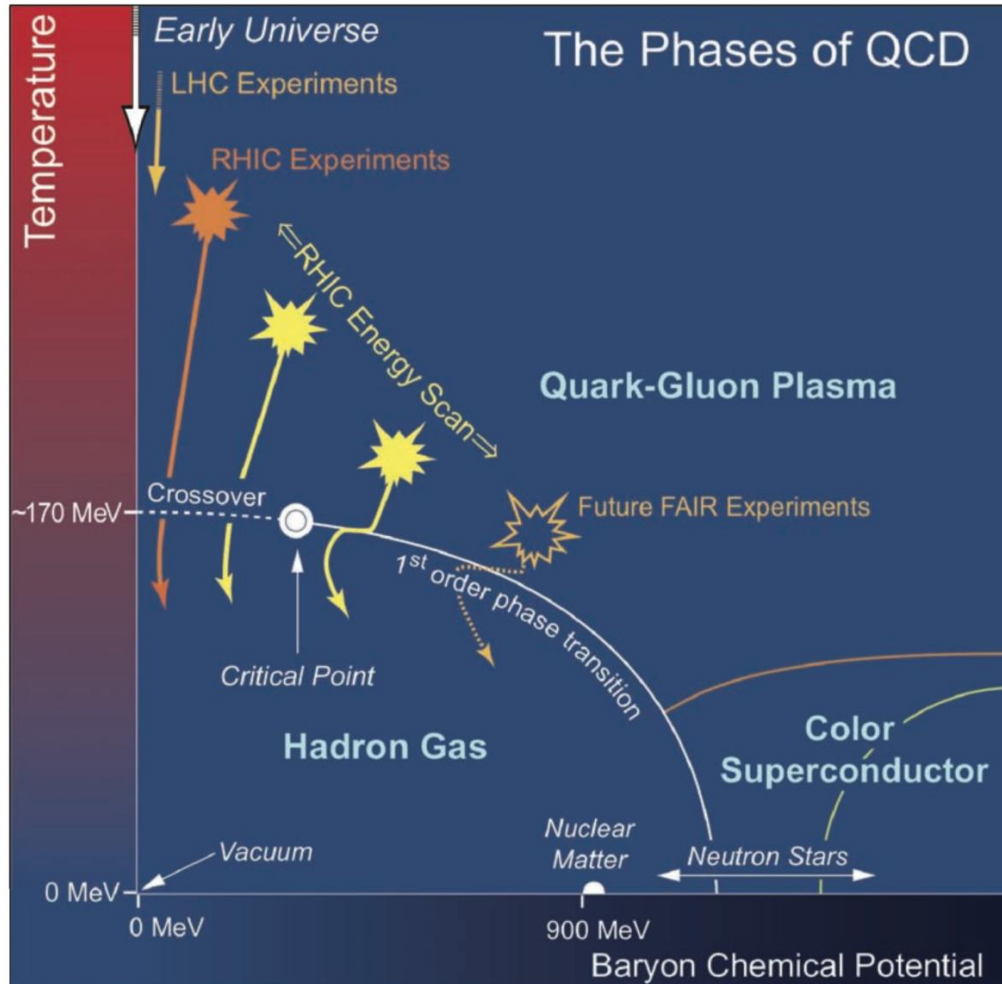
# Chiral Symmetry Breaking



**In the chiral limit**  $m_u = m_d = 0$

- The Lagrangian of QCD acquires chiral symmetry  $SU(2)_L \times SU(2)_R$
- The ground state of QCD breaks the chiral symmetry to isospin symmetry  $SU(2)_V$
- Quark condensation  $\langle \bar{q}q \rangle \neq 0$
- Chiral symmetry is not broken at sufficiently high temperature or density

# $T-\mu$ phase diagram of QCD



- Other Backgrounds:

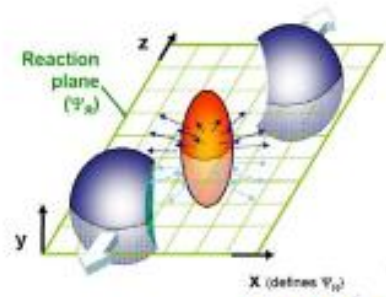
- Isospin chemical potential

- Magnetic Field

- Rotation

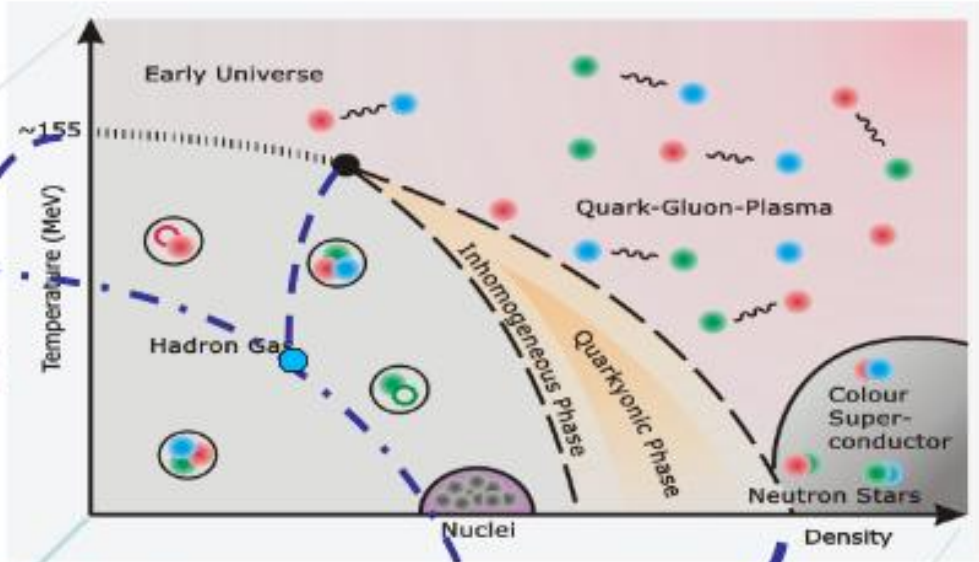
- Gravity

# QCD in strong magnetic fields



The early universe  
 $eB \sim 10^{18}$  G

**CME**  
**CVE**  
**Inverse**  
**Magnetic**  
**Catalysis**



Compact stars  
(Magnetar)

$eB \sim 10^{15}$  G

**Magnetic Catalysis**  
**Vacuum SC?**

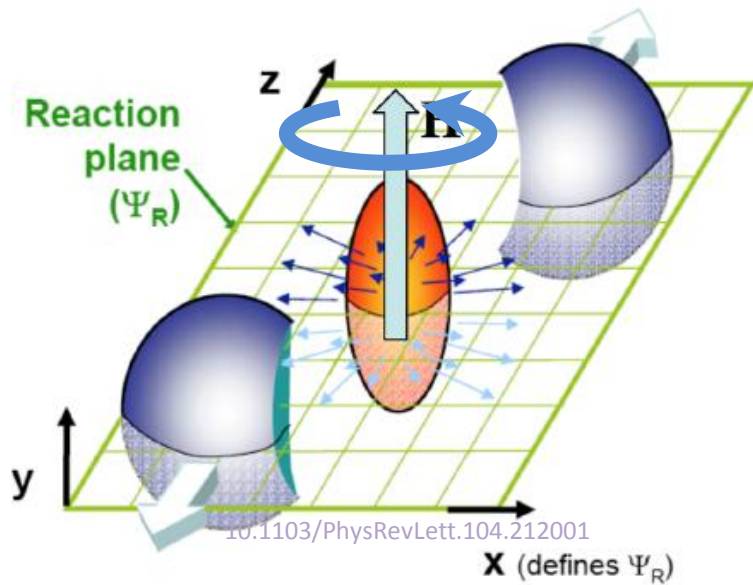
**(Inverse)**  
**Magnetic**  
**Catalysis?**

**Magnetar**

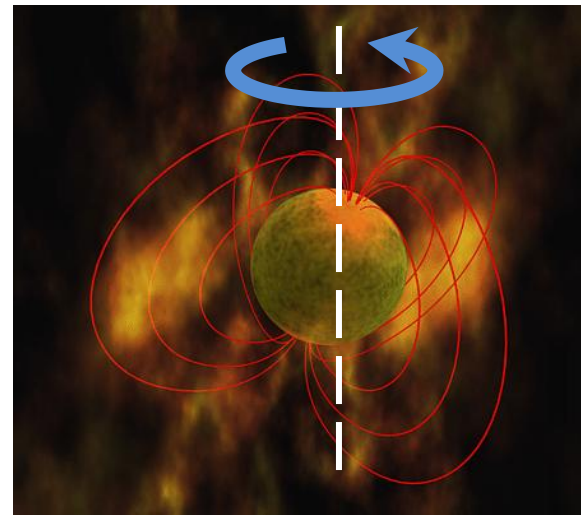
**B**



# QCD in strong magnetic fields with rotation



$$R\Omega \sim 10^{-1}$$



# The Nambu-Jona-Lasinio (NJL) model

- The two flavor effective Lagrangian:

$$\mathcal{L}_{NJL} = \bar{\psi}i\gamma^\mu\nabla_\mu\psi - m_0\bar{\psi}\psi + \mu_B\bar{\psi}\gamma^0\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma^5\vec{\tau}\psi)^2]$$

$$\nabla_\mu = \partial_\mu + iQA_\mu + \Gamma_\mu$$

- Chiral symmetry breaking
- Lack of confinement
- Nonrenormalizable field theory
- The results depend on the regularization scheme and on the UV cut-off that is used

# Mean Field Approximation

- Effective action Hard to diagonalize

$$\Gamma = - \int d^4x \frac{\sigma^2 + \vec{\pi}^2}{2G} + \frac{1}{i} \ln \det(iD - i\gamma^5 \vec{\pi} \cdot \vec{\tau}),$$

$$iD = i\gamma^\mu \nabla_\mu - \sigma + \mu_B \gamma^0$$

- Magnetic field breaks SU(2) symmetry
- Perturbative expansion in  $\pi$  field

$$\Gamma = \Gamma^{(0)} + \Gamma^{(2)} + \dots,$$

$$\Gamma^{(0)} = - \int d^4x \frac{\sigma^2}{2G} + \frac{1}{i} \ln \det iD,$$

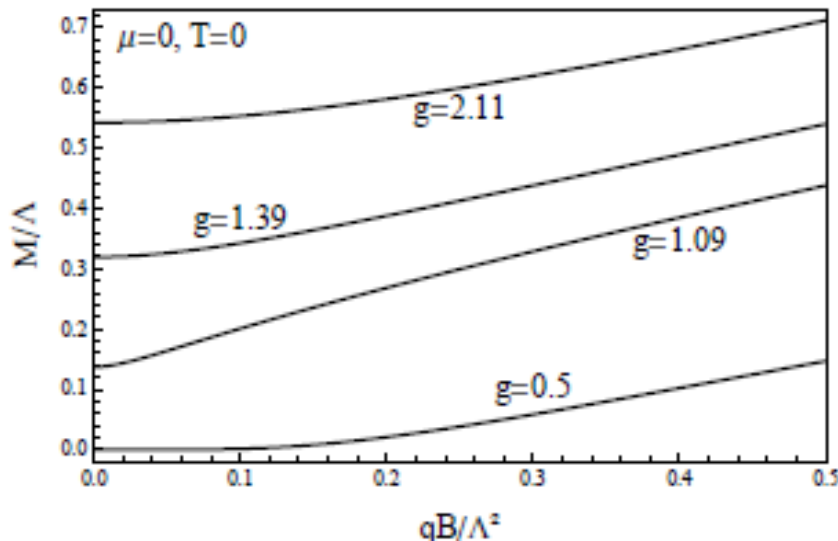
Assumption:  $\langle \pi^0 \rangle = 0$

$$\Gamma^{(2)} = - \int d^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} \text{Tr}[(iD)^{-1} \gamma^5 \vec{\pi} \cdot \vec{\tau}]^2,$$



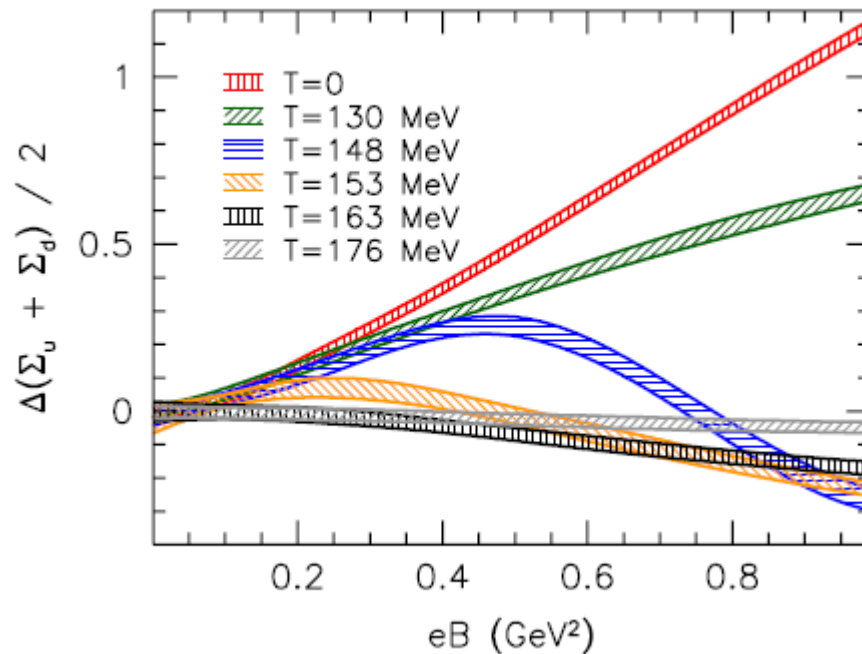
# Magnetic catalysis

- Condensation  $m$  increases with magnetic field  $eB$  increasing



# Inverse magnetic catalysis

- Condensation  $m$  decreases with magnetic field  $eB$  increasing!



# Rotating Frame

- K-G equation

$$\frac{1}{\sqrt{-g}} D_\mu (\sqrt{-g} g^{\mu\nu} D_\nu \phi) + m^2 \phi = 0$$

- Dirac equation

$$[ie_i^\mu \gamma^i (\partial_\mu + iqA_\mu + \Gamma_\mu) - m] \psi(x) = 0$$

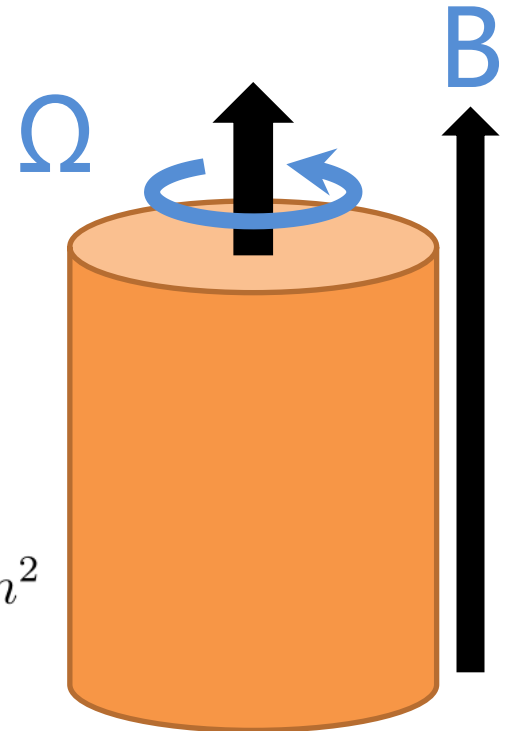
- $e_i^\mu$  is vierbein

$\Gamma_\mu$  is spin connection

- Dispersion relation

$$[E + \text{sgn}(q)\Omega(l + s_z)]^2 = p_z^2 + (2\lambda + 1 - 2s_z)eB + m^2$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - (x^2 + y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



# Schwinger Phase

Gauge independent

$$S(x, x') = e^{i\Phi(x, x')} S_{\text{inv}}(x, x')$$

$$\Gamma^{(2)} = - \int d^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} \text{Tr}[(i\gamma^\mu \nabla_\mu - \sigma + \hat{\mu}\gamma^0)^{-1} \gamma^5 \vec{\pi} \cdot \vec{\tau}]^2$$

- For spacetime with R=0

$$\Phi(x, x') = q \int_{x'}^x \left[ A_\mu(z) + \frac{1}{2} F_{\mu\rho} \nabla_z^\rho \sigma(z, x) \right] dz^\mu$$

- Synge's world function (half the squared geodesic distance between z and x)

$$\sigma(z, x) = \frac{1}{2} \int_0^1 g_{\mu\nu}(y(\tau)) \frac{dy^\mu}{d\tau} \frac{dy^\nu}{d\tau} d\tau,$$

- Zero curl (path independent)
- If we choose the integral path as the geodesic

$$\Phi(x, x') = q \int_{x'}^x A_\mu(z) dz^\mu,$$

# Schwinger Phase

- LLL  $\sum_{l=0}^{\infty} \frac{1}{l!} e^{il(\Delta\theta + \Omega\Delta t)} \left(\frac{1}{2}qBr_1r_2\right)^l e^{-\frac{1}{4}qB(r_2^2 + r_1^2)}$

$$= \exp\left[i\frac{1}{2}qBr_1r_2 \sin(\Delta\theta + \Omega\Delta t) - \frac{1}{4}qB(r_2^2 - 2r_1r_2 \cos(\Delta\theta + \Omega\Delta t) + r_1^2)\right]$$

$$= \exp\left[-iq \int_{x_1}^{x_2} A_\mu dz^\mu - \frac{1}{4}qBC_\perp^2\right].$$
- In curved spacetime: geodesic line  $\frac{d^2x^\mu}{d\sigma^2} + \Gamma_{\nu\rho}^\mu \frac{dx^\nu}{d\sigma} \frac{dx^\rho}{d\sigma} = 0$

$$-iq \int_{x_1}^{x_2} A_\mu(z) dz^\mu = i\frac{1}{2}qBr_1r_2 \sin(\Delta\theta + \Omega\Delta t)$$
- In flat spacetime: straight line

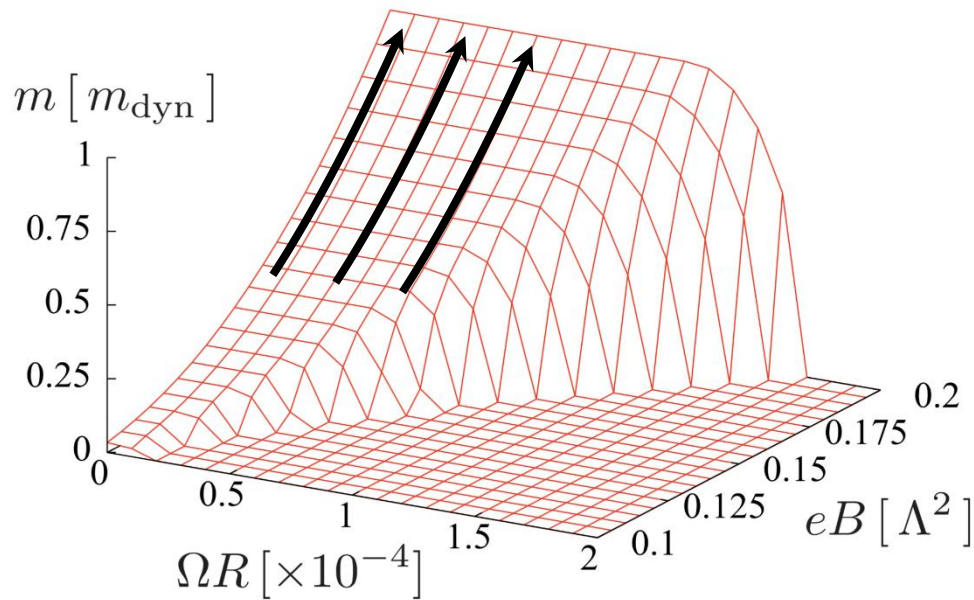
$$-iq \int_{x'_1}^{x'_2} A'_\mu(z) dz^\mu = i\frac{1}{2}qBr'_1r'_2 \sin \Delta\theta'$$

$$C'^2_\perp = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2$$

# Weak Coupling

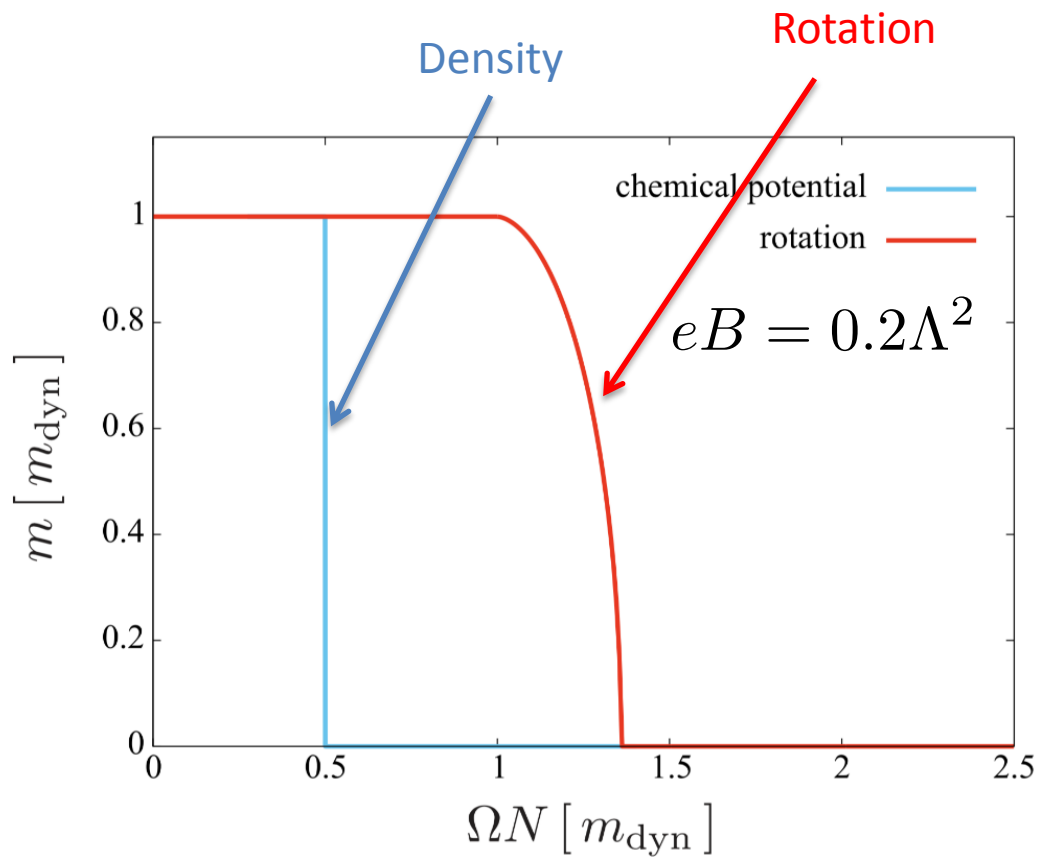
- Magnetic Catalysis

Increasing  $eB$   $\longrightarrow$  Increasing  $m$

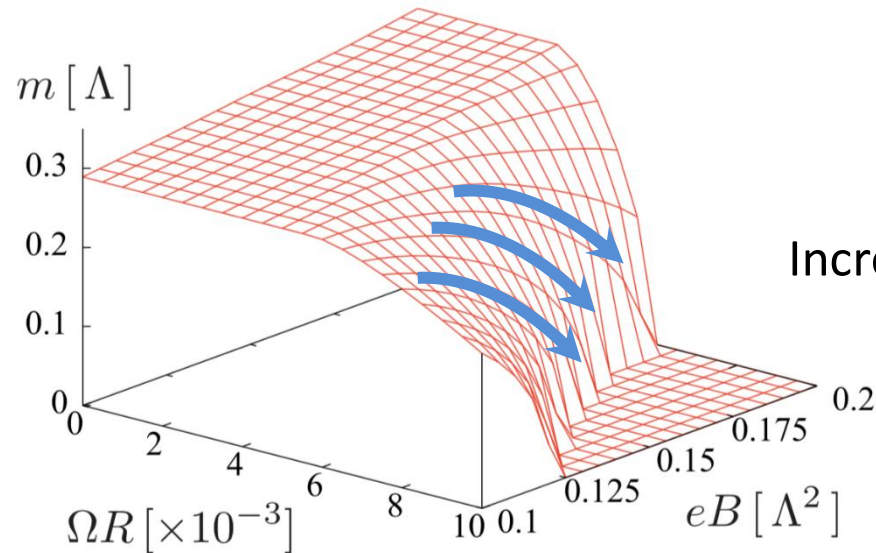


# Comparison with density

- Second Order transition



# Strong Coupling: “Rotational Magnetic Inhibition”



$$1/\Omega \geq R \gg 1/\sqrt{qB}$$

Increasing  $eB$   $\longrightarrow$  Decreasing  $m$

HLC, et al, Phys. Rev. D 93, 104052 (2016)

- Dropping start around  $\Omega N : \sqrt{eB}$
- For finite density system the inverse magnetic catalysis start around  $\mu : \sqrt{eB}$  Preis, Rebhan, Schmitt (2012)



# Boundary effect (without rotation)

- Boundary condition

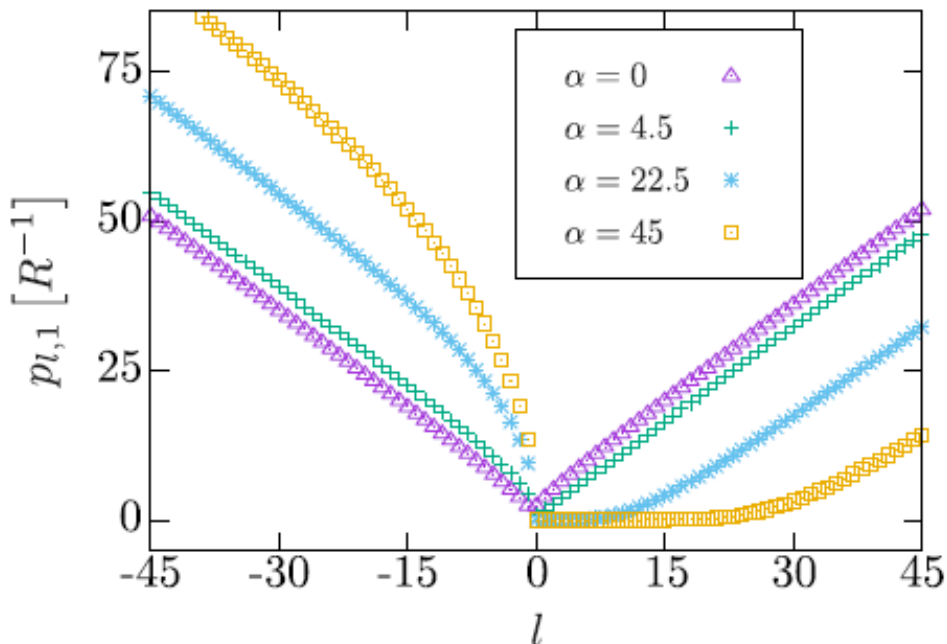
$$R \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta \bar{\psi} \gamma^r \psi \Big|_{r=R} = 0$$

HLC, et al, Phys. Rev. D 96, 054032 (2017)

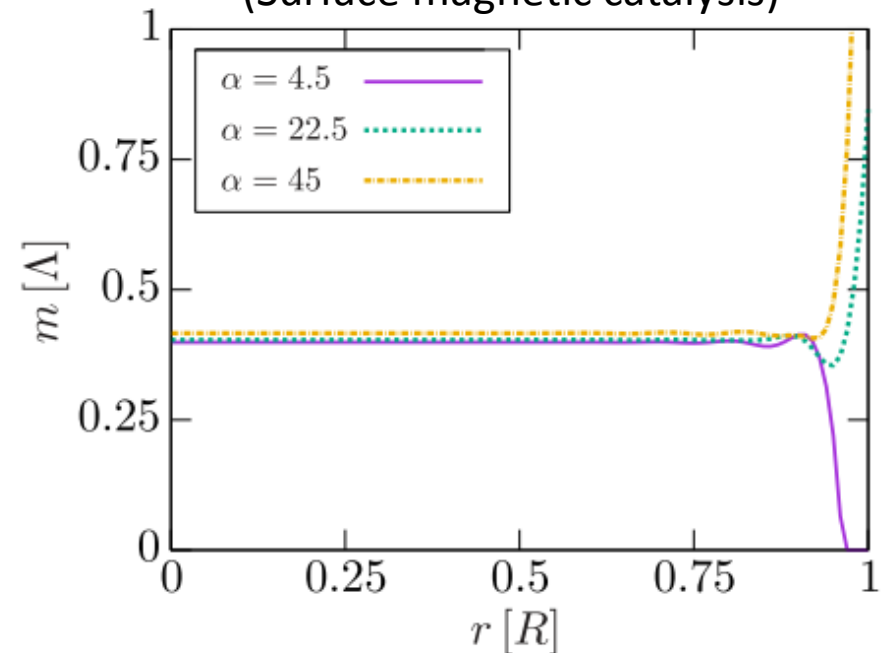
$$\alpha \equiv \frac{1}{2} e B R^2$$

Dispersion relation

$$p_{l,1} = \sqrt{2eB\lambda}$$

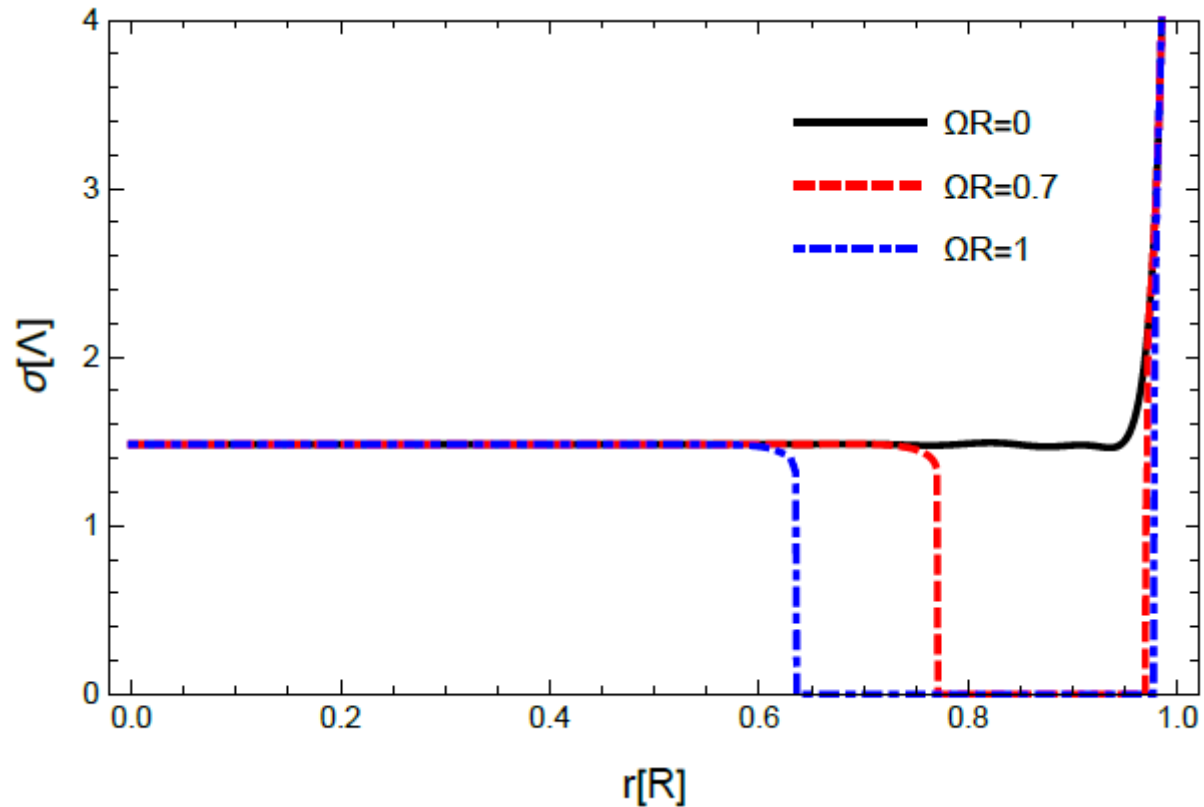


Chiral condensate  
(Surface magnetic catalysis)



# Rotational Magnetic Inhibition (with BC)

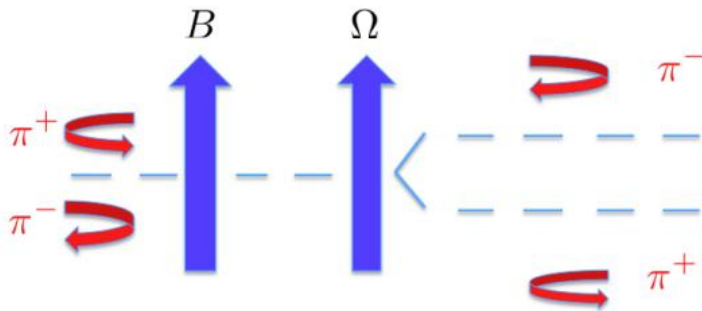
- Rotational Magnetic Inhibition & Surface magnetic catalysis



# Pion as the DoF

- Dispersion relation

$$[E + \text{sgn}(q)\Omega l]^2 = p_z^2 + (2\lambda + 1)eB + m^2$$



Y. Liu, I. Zahed, Phys.Rev.Lett. 120 (2018)

- $\Omega l \rightarrow \mu \rightarrow$  pion condensate

# Charged Pion Condensate

$$\Gamma^{(2)} = - \int d^4x \frac{\vec{\pi}^2}{2G} + \frac{1}{2i} \text{Tr} [(i\gamma^\mu \nabla_\mu - \sigma + \hat{\mu}\gamma^0)^{-1} \gamma^5 \vec{\pi} \cdot \vec{\tau}]^2$$

- Ansatz

$$\pi^+(x') \pi^-(x) = e^{ie \int_{x'}^x A_\mu dz^\mu} \tilde{\pi}^+ \tilde{\pi}^-,$$

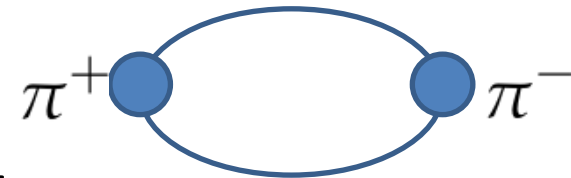
Gauge dependent part:  
Wilson line

Constant for simplicity

- 2<sup>nd</sup> order thermodynamic potential

$$V_{eff}^{(2)} = C^{(2)} \tilde{\pi}^+ \tilde{\pi}^-$$

- If  $C^{(2)} < 0 \longrightarrow$  Pion condensate



# Choice of the wilson line

$$\pi^+(x')\pi^-(x) = e^{ie \int_{x'}^x A_\mu dz^\mu} \tilde{\pi}^+ \tilde{\pi}^-,$$

- Integrate along geodesic

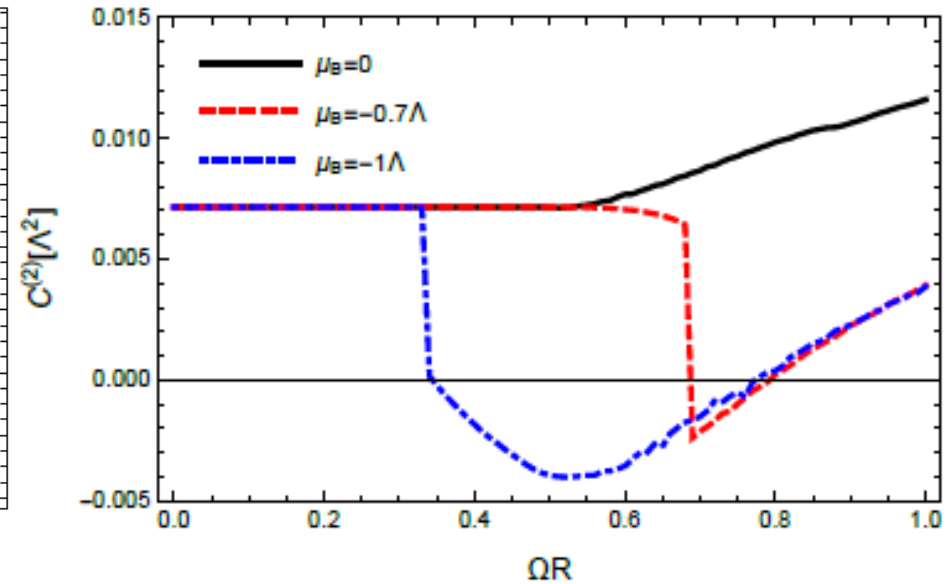
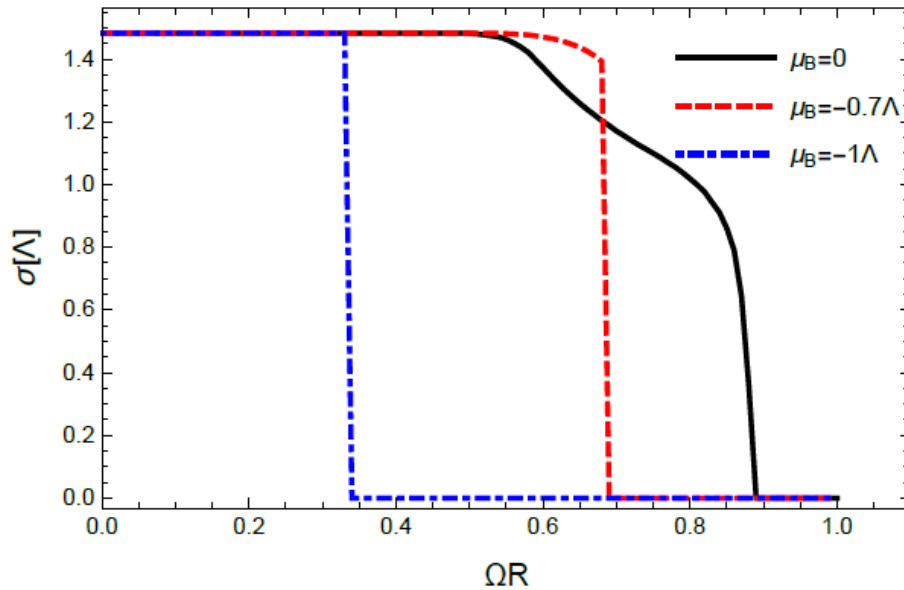
$$ie \int_{x_1}^{x_2} A_\mu dz^\mu = -i \frac{1}{2} e B r_1 r_2 \sin(\Delta\theta + \Omega\Delta t)$$

- Exactly cancel the Schwinger phase from the quark propagator
- Unfortunately, no pion condensate(  $C^{(2)} > 0$  )

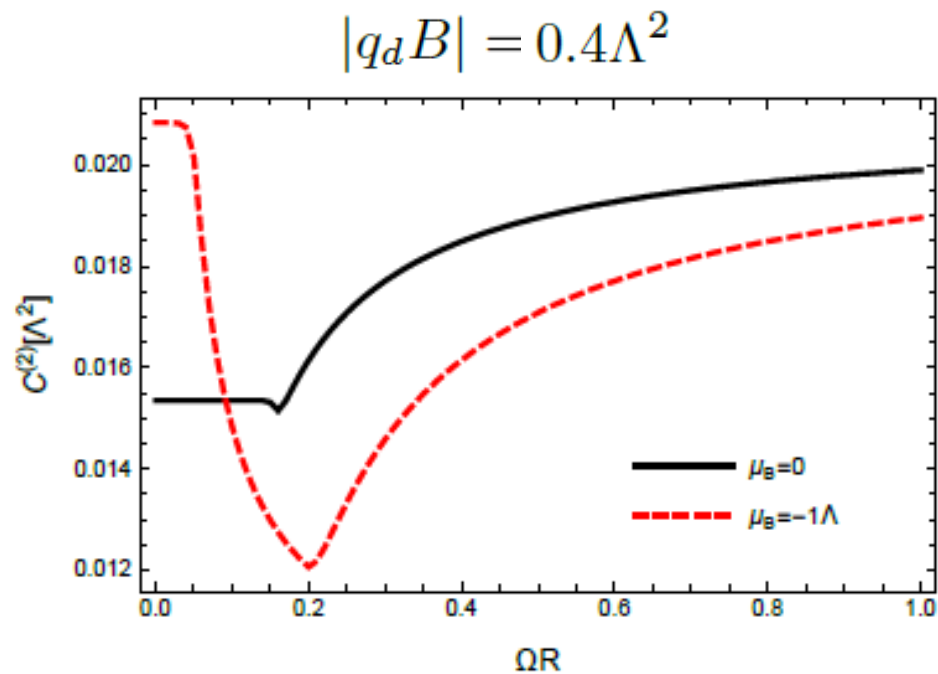
# Charged pion condensate

- Include chemical potential

$$|q_d B| = 0.1 \Lambda^2$$



# Charged pion condensate



# Charged pion condensate

- Both the magnetic field and rotation tend to destroy the charged pion condensate at quark level
- The split cause Bose-Einstein condensation at pion level
- Inhibition wins the competition

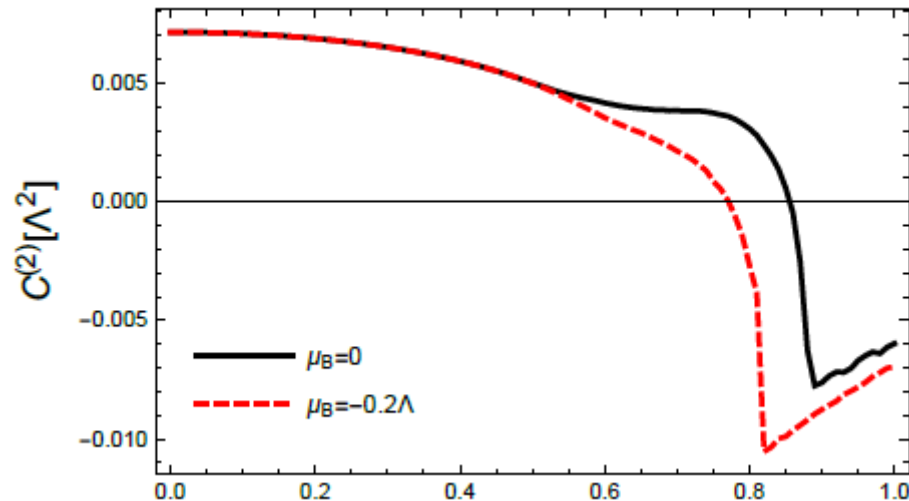


# Another choice

$$(t_1, x_1, y_1, z_1) \rightarrow (t_1, 0, 0, z_1) \rightarrow (t_2, 0, 0, z_2) \\ \rightarrow (t_2, x_1, y_1, z_2) \rightarrow (t_2, x_2, y_2, z_2),$$

- The same form as the Schwinger phase in flat spacetime

$$ie \int_{x_1}^{x_2} A_\mu dz^\mu = -i \frac{1}{2} e B r_1 r_2 \sin \Delta\theta$$



# Conclusion & Outlook

- Rotational magnetic inhibition
- Surface magnetic catalysis
- In certain region, rotation can induce pion condensate
- Analogy between rotation and(isospin) chemical potential?
- Superconductivity caused by charged pion condensate
- Rho condensation

Thank you very much!