

# Hot neutron stars with microscopic equations of state

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# Outline

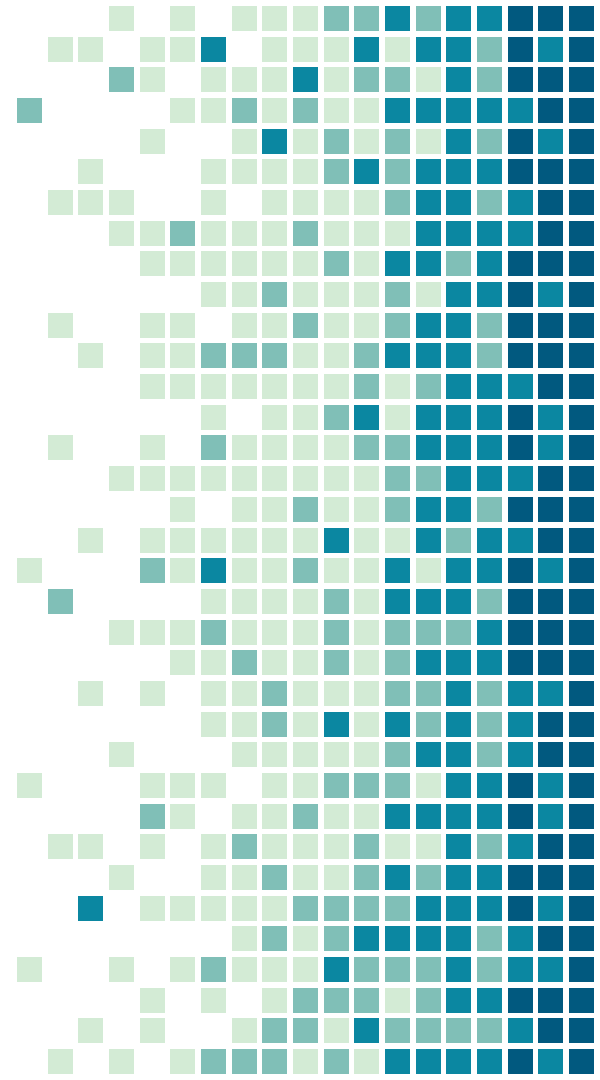
- Introduction
- Theory
  - BHF theory with Frozen Correlation Approx.
  - Beta Equilibrium and TOV Equations
- Results
  - Composition of stellar matter
  - Neutron star structure
  - Temperature dependence of max mass
- Conclusion



1.

# Introduction

Gravitational waves and EOSs...

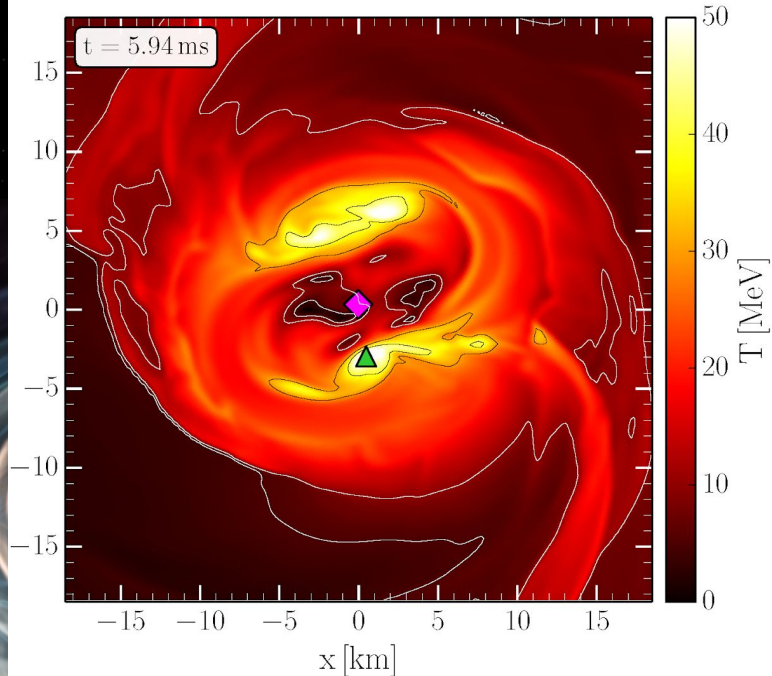


# GW170817

FIRST COSMIC EVENT OBSERVED IN GRAVITATIONAL WAVES AND LIGHT

- FT-EOS works as a critical input to BNS simulation;
- Analysis of BNS merger events provides constraints to EOS.

\* Right figure shows the snapshot of temperature profile at time = 5.94 ms after merger. [Hanuske+2019]



LDM: LS [Lattimer&Swesty 1991]

RMF: HShen [HShen+1998,2011]

GShen-NL3 [GShen+2010]

GShen-FSU2.1 [GShen+2010]

SFHo [Steiner+2013]

BHF+Chiral: BL [Bombaci&Logoteta 2018]

Ranges of baryon number density  $n$ , temperature  $T$ , net electron fraction  $Y_e = n_e/n$ , and entropy per baryon  $S$  encountered in the indicated astrophysical phenomena.

	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
$n/n_s$	$10^{-8}$ –10	$10^{-8}$ –10	$10^{-8}$ –10
$T$ (MeV)	0–30	0–50	0–100
$Y_e$	0.35–0.45	0.01–0.3	0.01–0.6
$S$ ( $k_B$ )	0.5–10	0–10	0–100

# Temp dependence of max mass

[Kaplan+2014]

[Burgio&Schulze 2010] isentropic **BHF** **RMF**

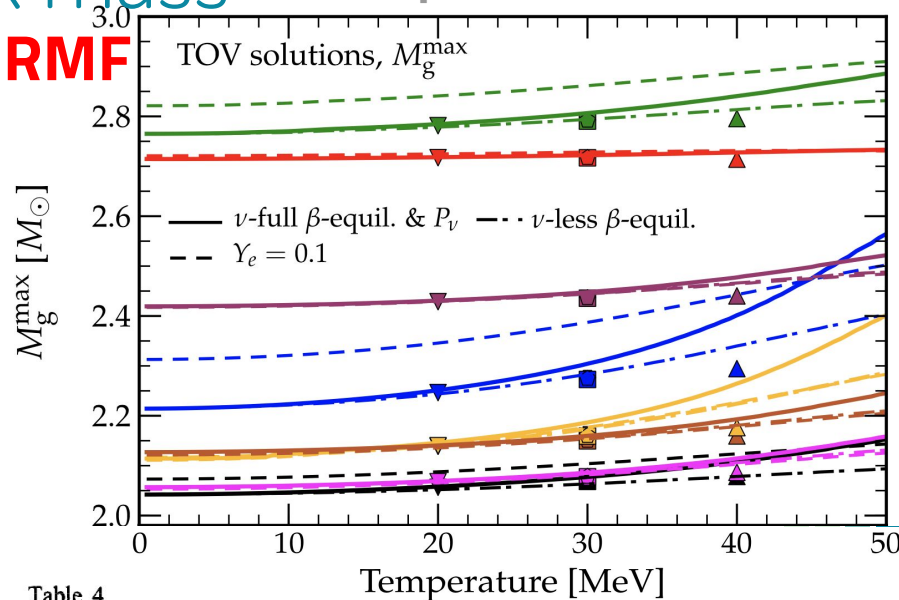
**Table 2.** Properties of (P)NS minimum and maximum mass configurations.

		Minimum mass			Maximum mass		
		$M/M_\odot$	$R$ (km)	$\rho_c/\rho_0$	$M/M_\odot$	$R$ (km)	$\rho_c/\rho_0$
untrapped $T = 0$	LS				2.03	9.86	10.55
	SKa				2.03	9.86	10.42
	Shen				2.03	9.93	10.42
trapped $S/A = 1$	LS	0.58	40	1.02	1.95	10.2	11.34
	SKa	0.60	38	1.08	1.95	10.2	11.20
	Shen	0.58	44	1.02	1.95	10.3	11.20
trapped $S/A = 2$	LS	0.70	44	0.90	1.95	10.7	10.85
	SKa	0.77	42	0.90	1.95	10.8	10.70
	Shen	0.75	52	0.77	1.95	10.8	10.80

**Table 1.** Characteristics of the maximum mass configurations for different stellar compositions and temperatures.

Composition	$T$ (MeV)	$M/M_\odot$	$R$ (km)	$\rho_c/\rho_0$
$N, l$	0	1.86	9.5	8.2
	10	1.82	9.5	8.1
	30	1.73	9.7	7.7

[Nicotra+2006] isothermal

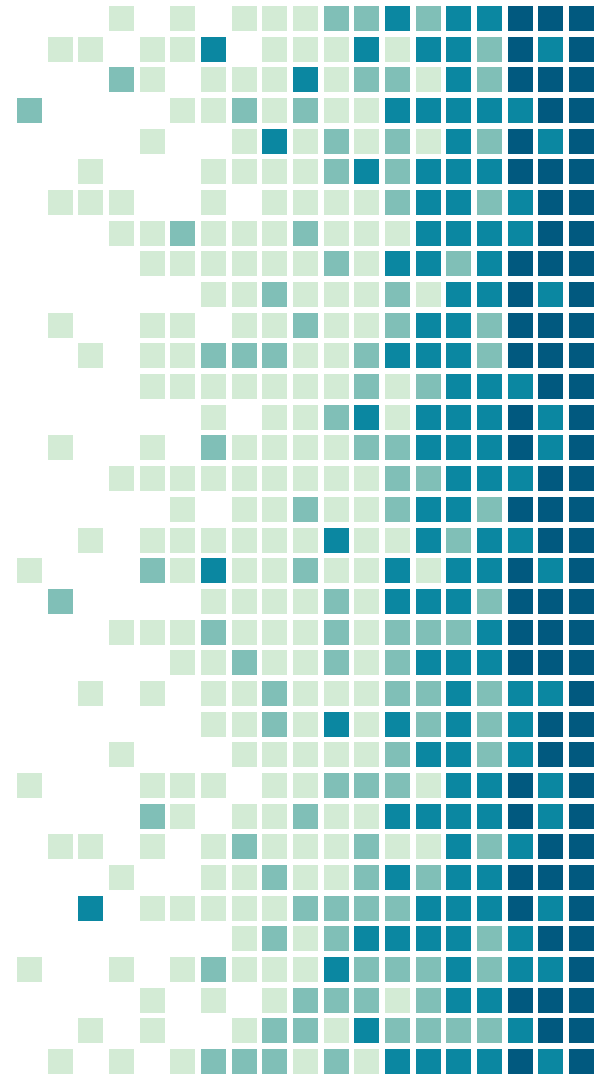


**Table 4**  
Star properties for matter in beta equilibrium at finite entropy in the BPAL potential model

EOS	$S$	$\frac{M_{\max}}{M_\odot}$	$R$ (km)	$\frac{n_c}{n_0}$	[Prakash+1997]	
BPAL 32	0	1.933	10.420	7.343	590.2	0.0
	1	1.943	10.589	7.138	577.7	36.7
	2	1.974	11.136	6.506	482.8	71.5
BPAL 33	0	1.955	10.797	7.000	532.0	0.0
	1	1.966	11.020	6.719	507.0	33.3
	2	1.994	11.518	6.198	454.3	66.0

# 2. Theory

BHF, FCA, BETA-EQ, and TOV...



# Brueckner-Hartree-Fock Theory

$$K(\rho, \beta; \omega) = v_{NN} + v_{NN} \mathbf{Re} \sum_{k_1, k_2} \frac{|k_1 k_2\rangle Q(1,2) \langle k_1 k_2|}{\omega - \epsilon(k_1) - \epsilon(k_2)} K(\rho, \beta; \omega) \quad (1)$$

- Nucleon-Nucleon Interaction

$$V_{NN} = v_2 + V_3^{\text{eff}}$$

We consider here: BOB, V18, N93, and UIX

- Single Particle Energy

$$\epsilon(k) = \frac{\hbar^2 k^2}{2m} + U(k) \quad (2)$$

- Single Particle Potential(Non-locality), cont. choice: for all  $k$

$$U(k) = \sum_{k' < k_F} \mathbf{Re} \langle k k' | K(\rho, \beta; \omega = e + e') | k k' \rangle \quad (3)$$

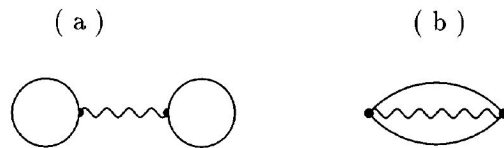


Fig. Two terms in the BHF level of BBG expansion. [Song+1998]

# Frozen Correlation Approximation

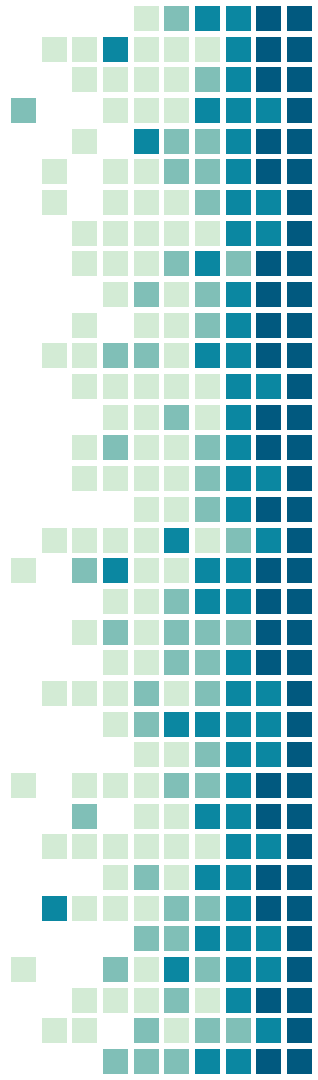
The finite temperature BHF should be done under Block & De Dominicis formalism [Block & Dominicis 1958,1959] and the framework is constructed within BHF theory by our group [Baldo & Ferreira 1999].

Ignore the effects of finite temperature on the single particle potential, we introduce a simplified way called "FCA":

$$f = \sum_i [\sum_k n_i(k) (\frac{k^2}{2m_i} + \frac{1}{2} U_i(k)) - T s_i] \quad (1)$$

where

$$s_i = - \sum_k (n_i(k) \ln n_i(k) + [1 - n_i(k)] \ln [1 - n_i(k)]) \quad (2)$$





# Beta Equilibrium



We impose following requirements with  $n, p, e, \mu$ :

- Charge neutrality  $x_p = x_e + e_\mu$
- Chemical equilibrium condition between leptons

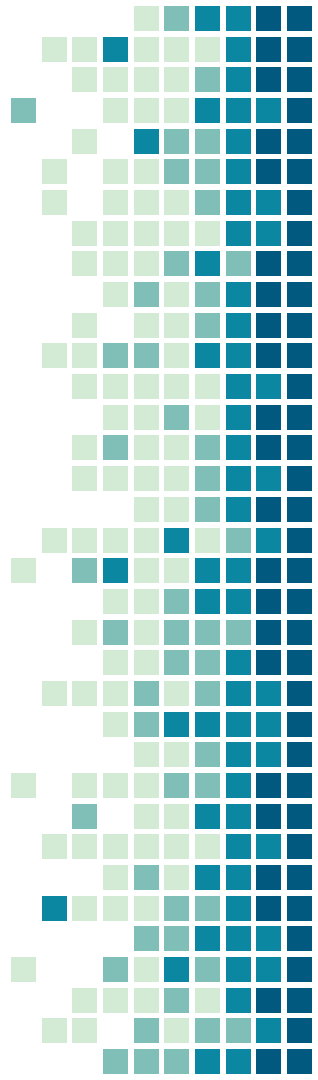
$$\mu_e = \mu_\mu$$

- Chemical equilibrium condition between leptons and baryons

$$\mu_n - \mu_p = \mu_e$$

We assume a cold crust ( $T=0$ ) and then attach NV EOS in the medium-density range and BPS EOS for outer crust.

Fixed phase transition point at  $\rho = 0.08\text{fm}^{-3}$



# Tolman-Oppenheimer-Volkoff Equations

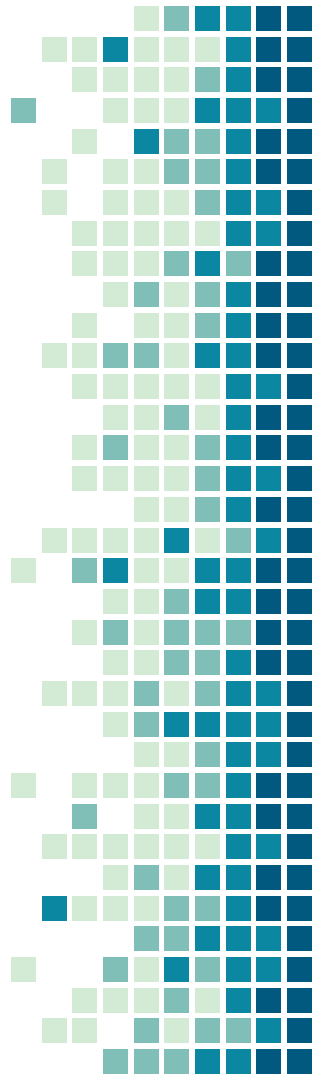
$$\frac{dp(r)}{dr} = - \frac{Gm(r)\epsilon(r)}{r^2} \frac{\left[1 + \frac{p(r)}{\epsilon(r)}\right] \left[1 + \frac{4\pi r^3 p(r)}{m(r)}\right]}{1 - \frac{2Gm(r)}{r}}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r)$$

$$\frac{dm_B}{dr} = 4\pi r^2 \frac{\rho m_N}{\sqrt{1 - 2Gm/r}}$$

Once the relation of internal energy density and pressure  $p(\epsilon)$ , i.e. the EOS, is given.

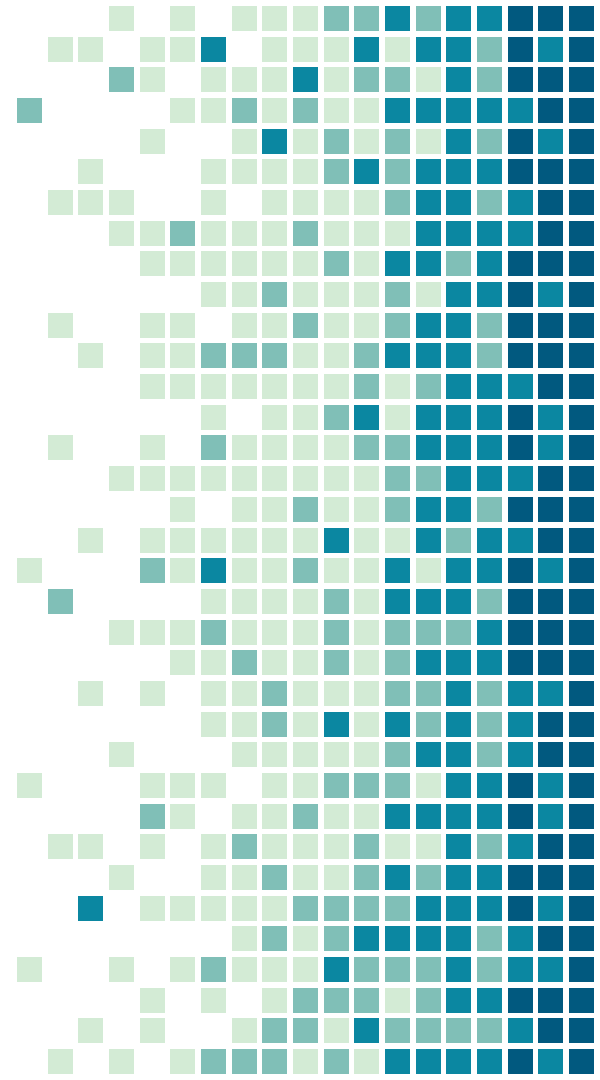
For a chosen central value of energy density, the numerical integration of the above equations provides the mass-radius relation.



3.

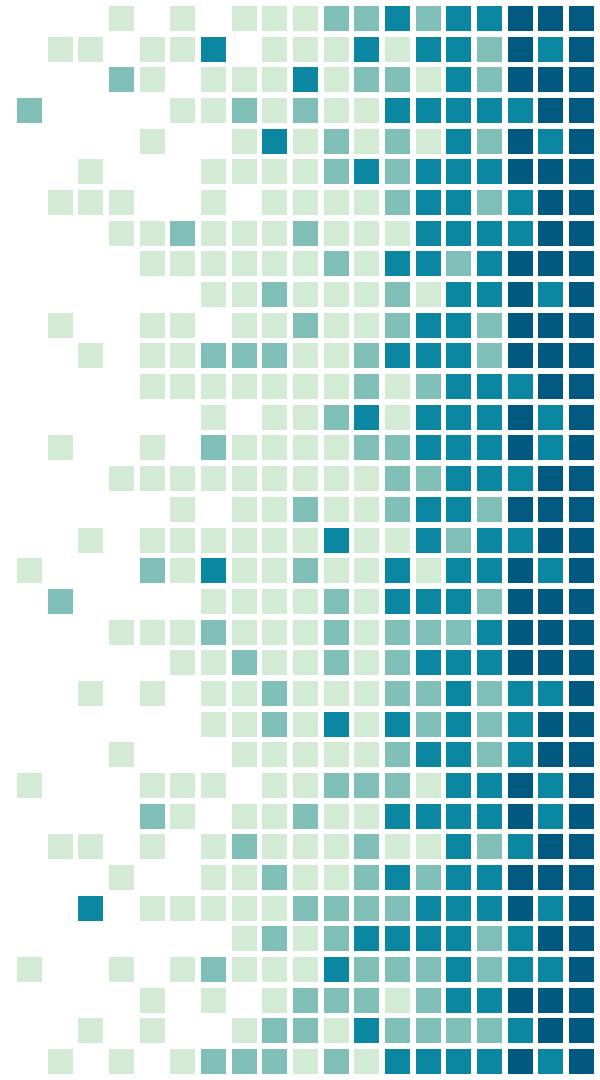
# Results

Free Energy, NS Structure, M-R Relation...

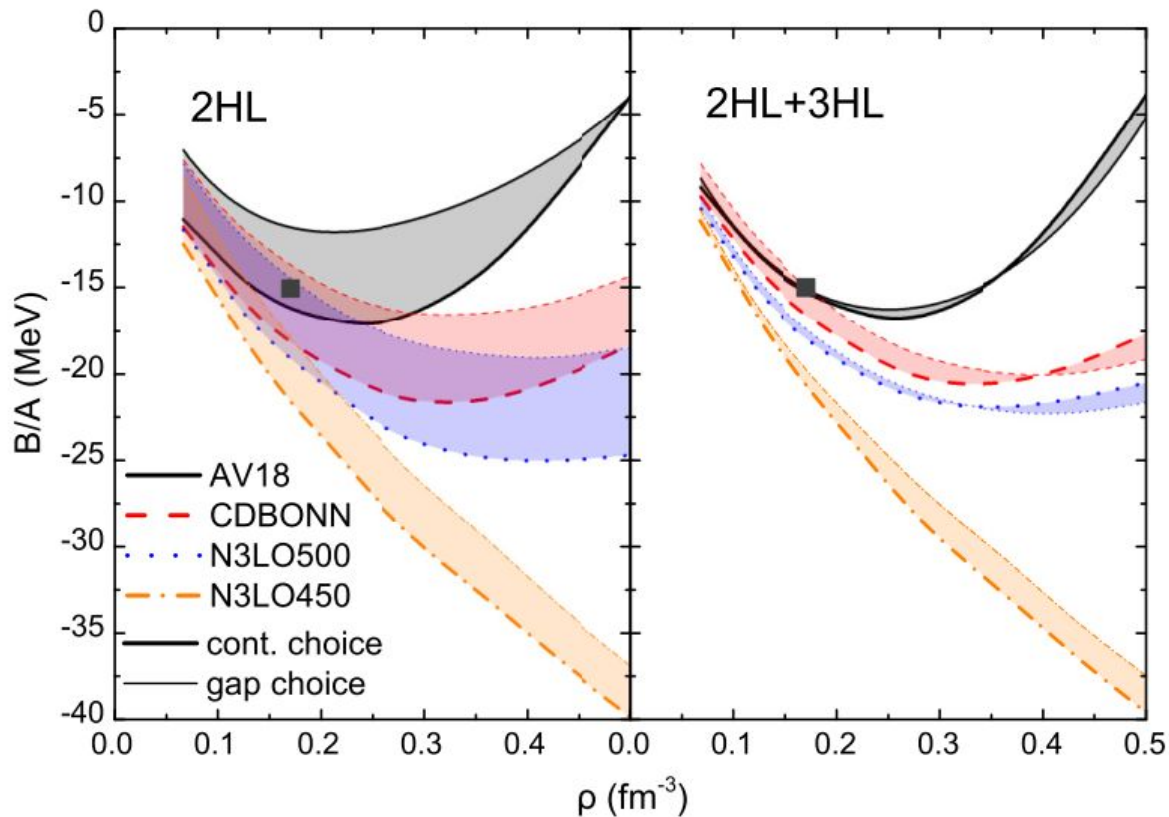
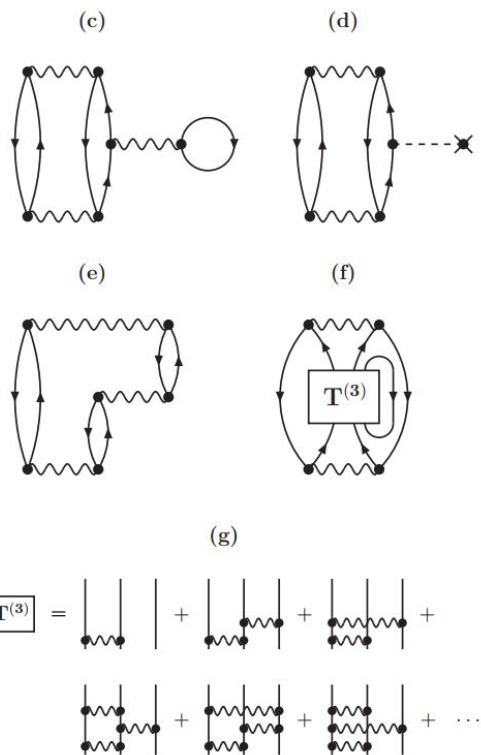


# 3.1

Convergence of  
Brueckner-Bethe-Goldstone expansion

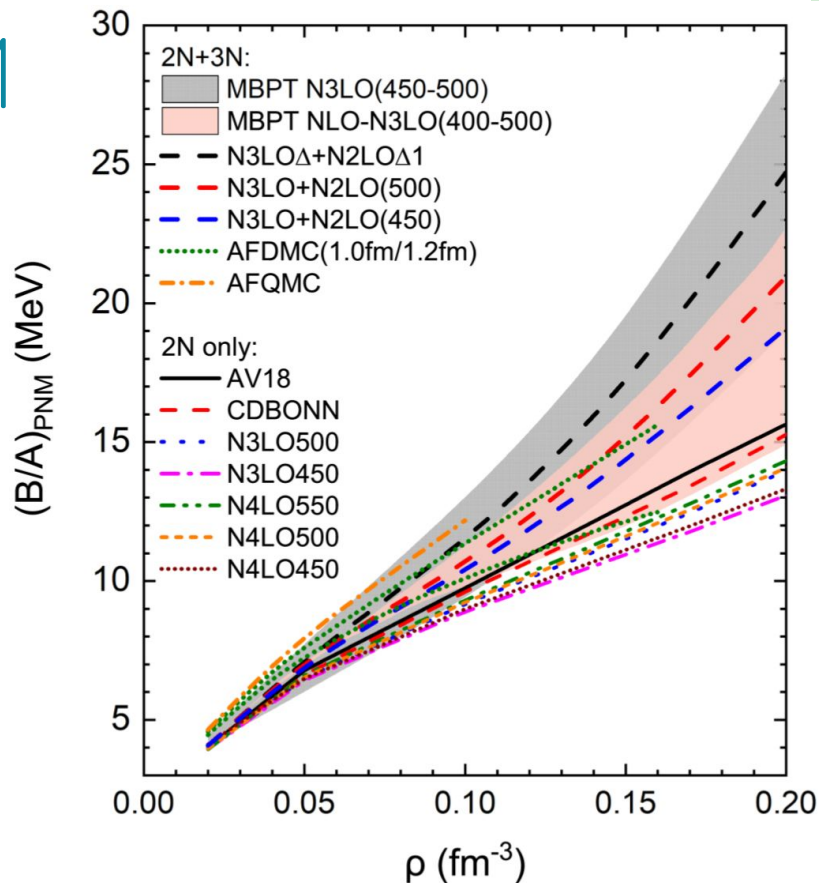
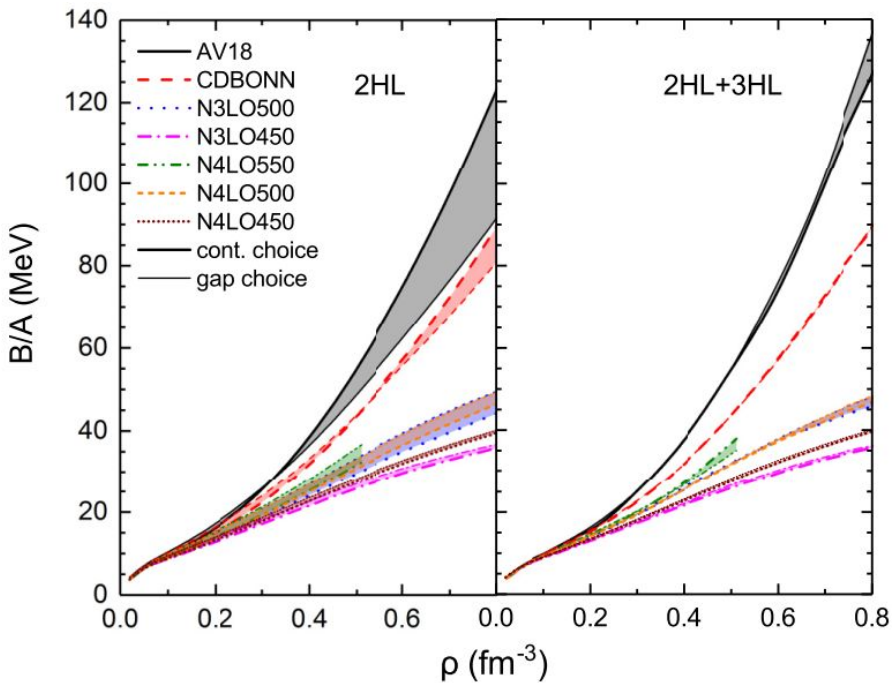


# 3HL Convergence - SNM



(Editors' Suggestion) 陆家靖, 李增花 et al. Phys. Rev. C 96, 044309 (2017)

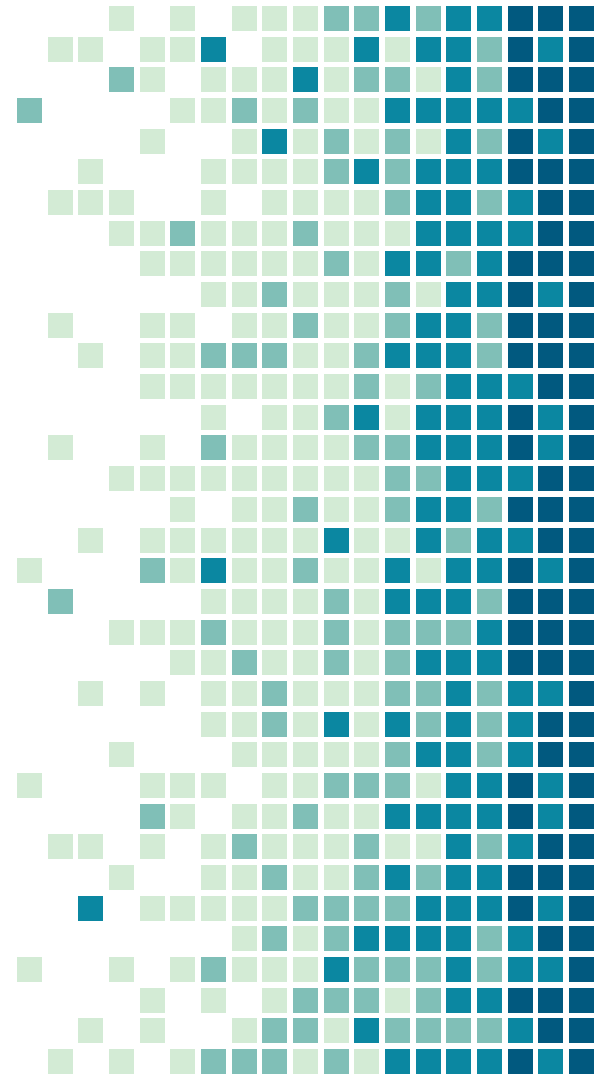
# 3HL Convergence - PNM



陆家靖, 李增花 et al. Phys. Rev. C 98, 064322 (2018)

# 3.2

Build the finite temperature EOSs



# Free Energy of nuclear matter

We use the parametrization introduced in [Burgio & Schulze 2010]

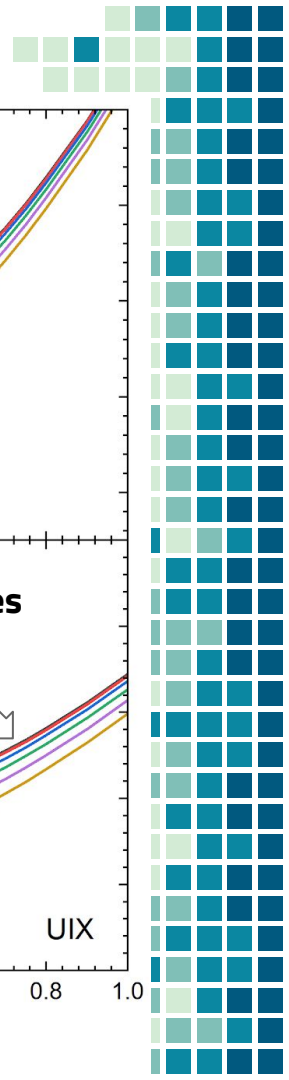
$$\frac{F}{A}(\rho, T) = (A_0 + A_2 t^2)\rho + B_0 \rho^{B_1} + C t^2 \ln(\rho) + (D_0 t^2 + D_1 t^{D_2})/\rho + E$$

where  $t \equiv T/(100 \text{ MeV})$  and is dimensionless.

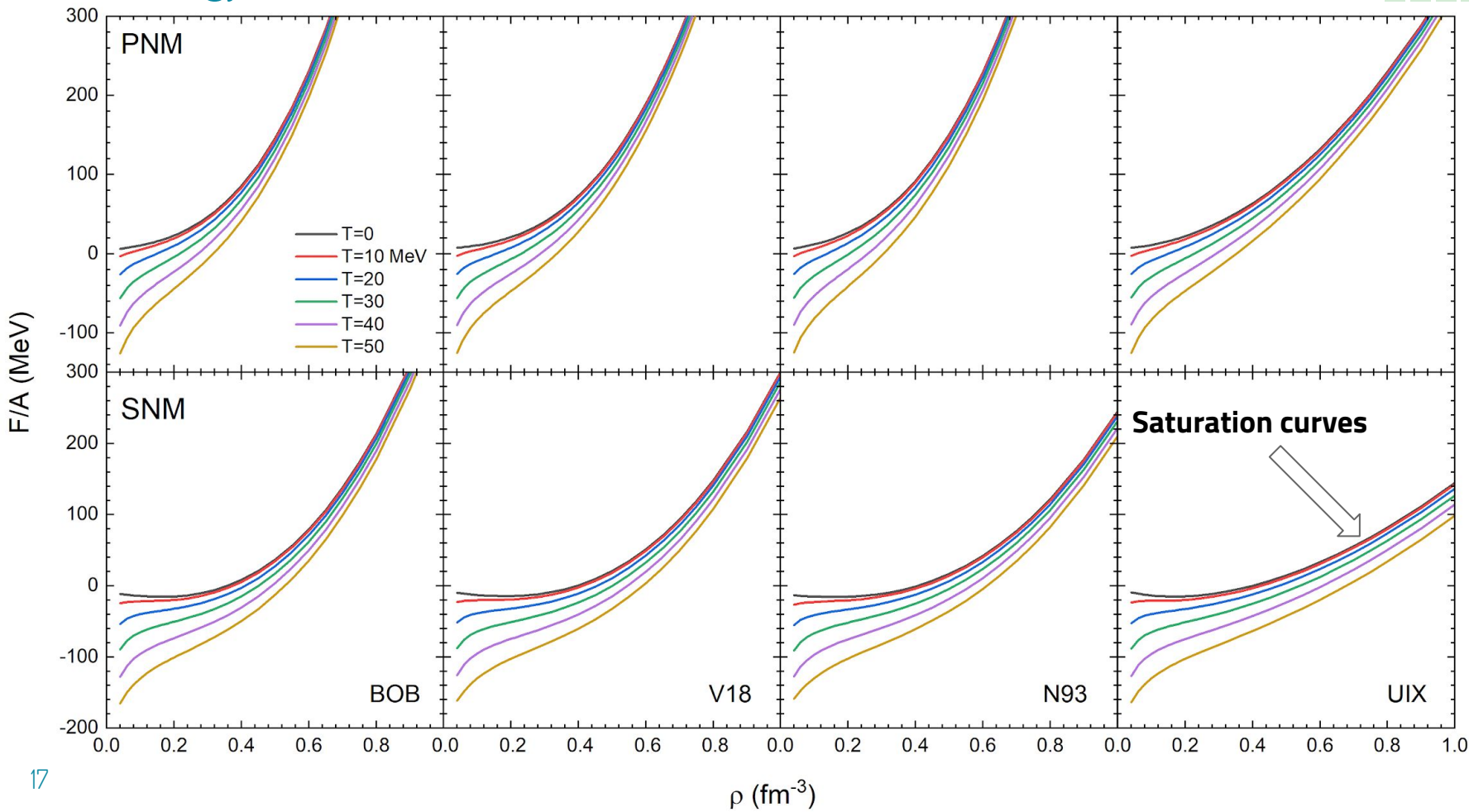
	$A_0$	$A_2$	$B_0$	$B_1$	C	$D_0$	$D_1$	$D_2$	E
BOB SNM	-65	-124	498	2.67	203	-105	122	2.20	-9
BOB PNM	57	-85	856	2.91	152	-32	43	2.47	4
V18 SNM	-60	-147	369	2.66	209	-66	85	2.32	-8
V18 PNM	37	-91	667	2.78	154	-52	62	2.28	6
UIX SNM	-174	-186	323	1.61	199	-136	153	2.16	-4
UIX PNM	24	-117	326	2.09	153	-85	94	2.16	6
N93 SNM	-42	-142	298	2.61	211	-64	87	2.35	-12
N93 PNM	67	-95	743	2.71	154	-35	46	2.44	4

Table 1. Parameterizations of free energy at finite temperature

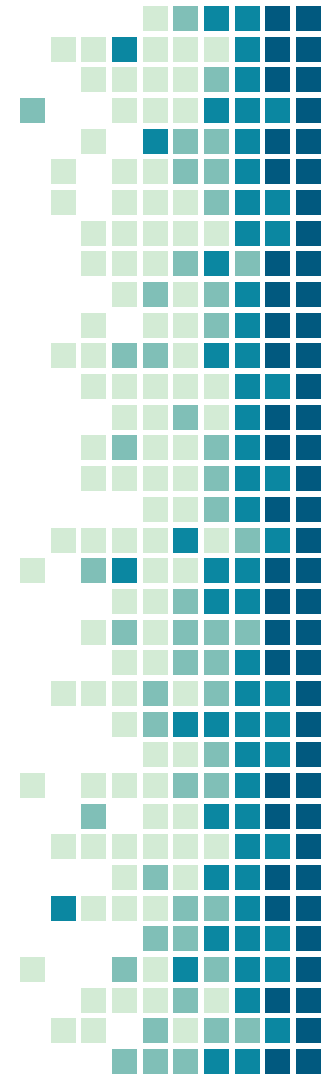
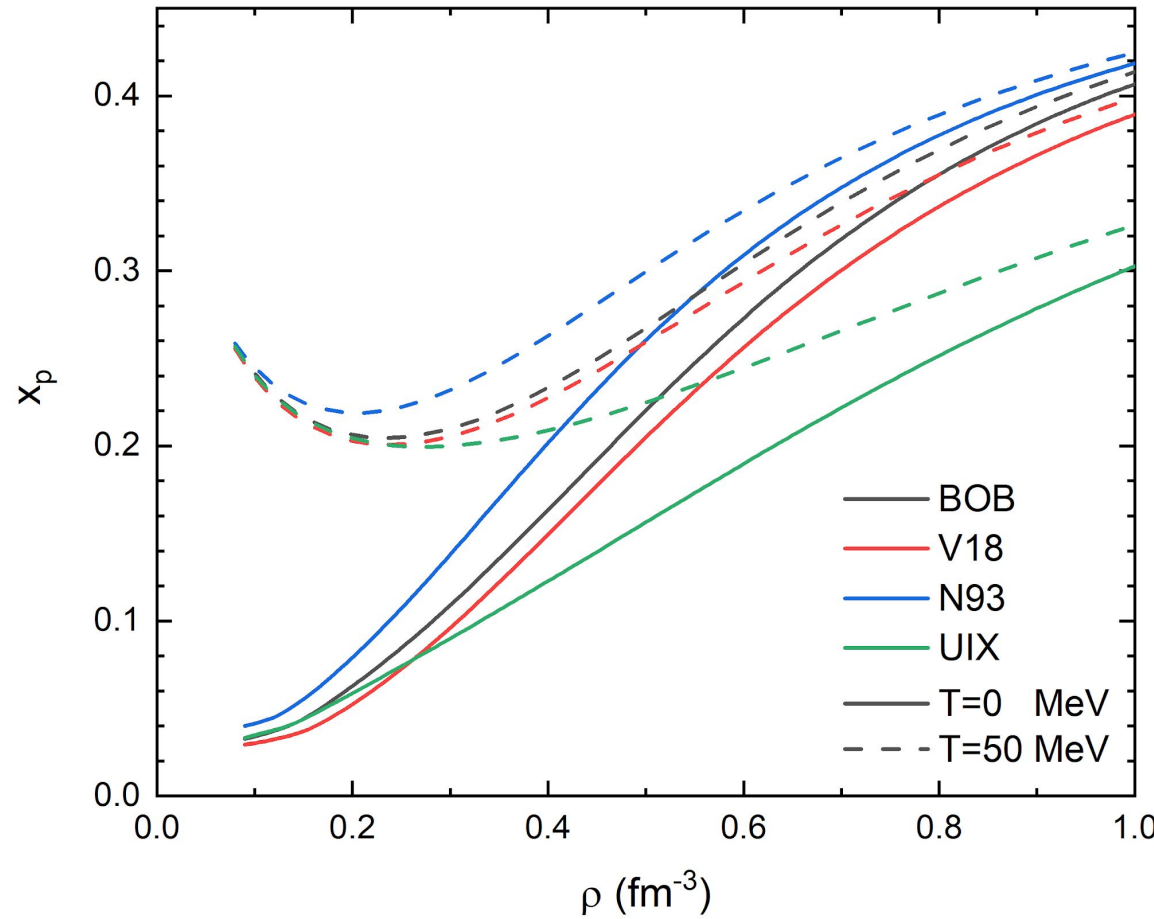




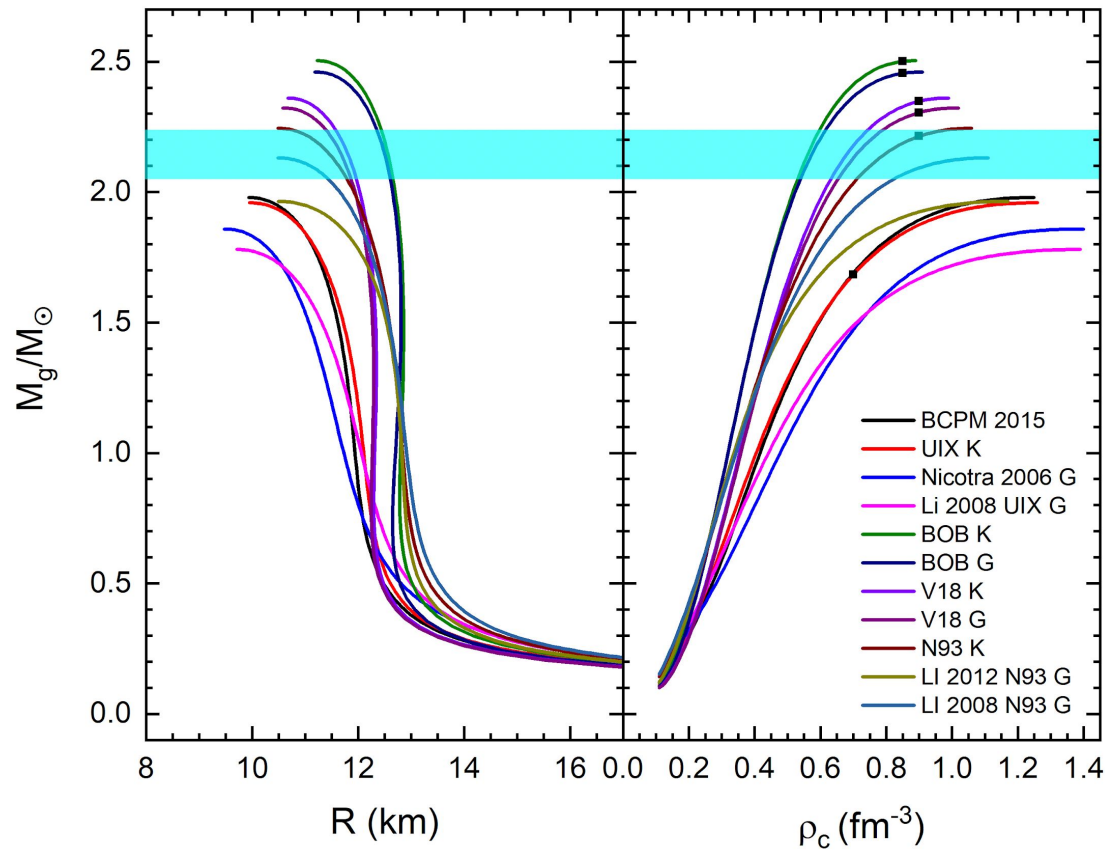
# Free Energy



# Composition of beta-stable matter at different T



# Mass - Radius relation at T=0 MeV



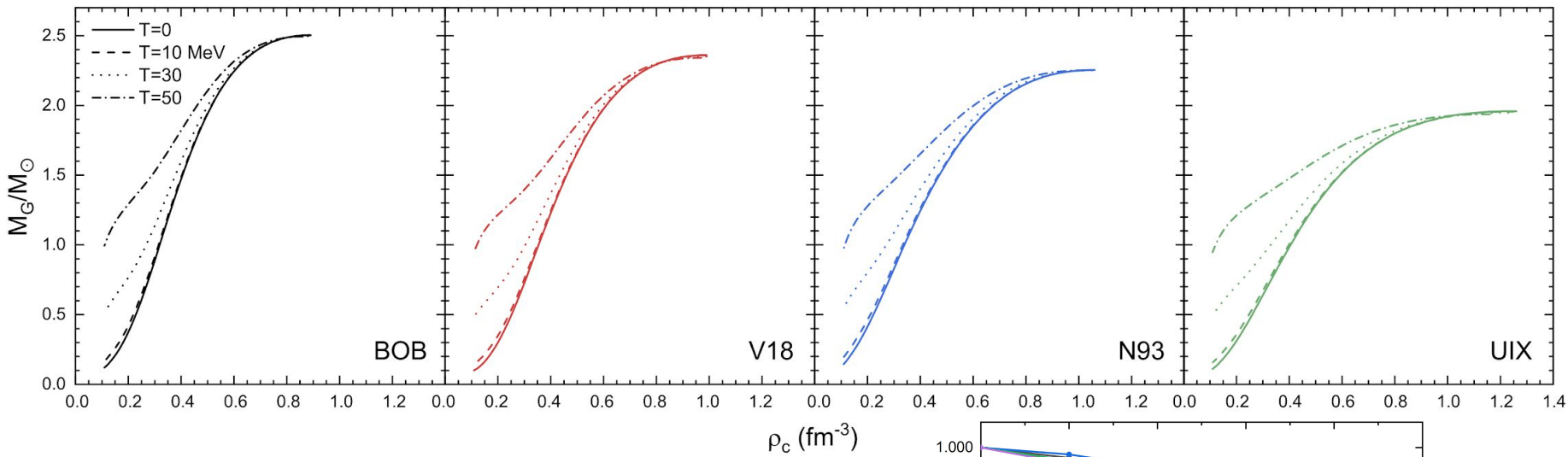
68.3% credibility interval  
 [Cromartie+2019; Nature Astronomy]

←  $2.14^{+0.10}_{-0.09} M_{\odot}$

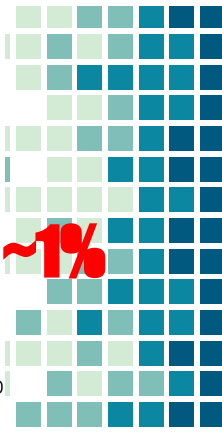
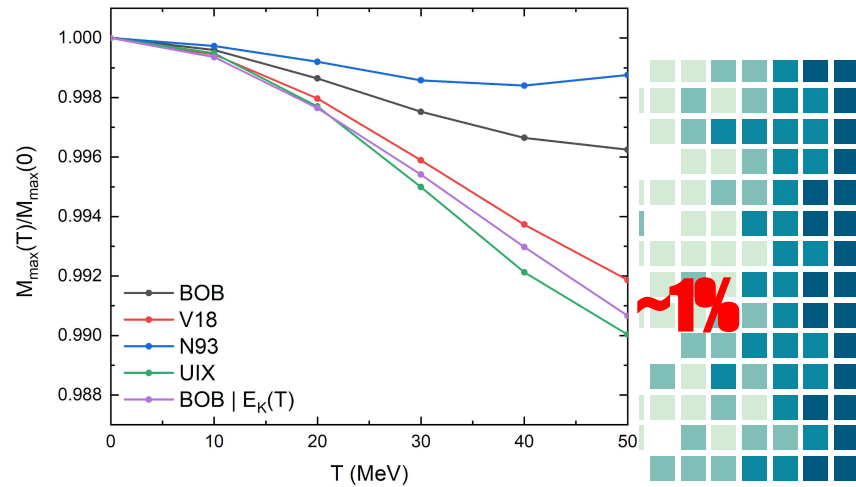
- BCPM [Sharma+2015]
- Nicotra 2006 G [Nicotra+2006]
- Li 2008 N93/UIX G [Li+2008]
- Li 2012 N93 G [Li+2012]



# Neutron star structure



- The max mass itself is mainly dependent on different EOSs.
- The decrease of max mass is almost negligible, at most **1%**.
- **Density** is still the dominant feature in neutron star mergers. [arXiv:1907.12760]



# Adiabatic index

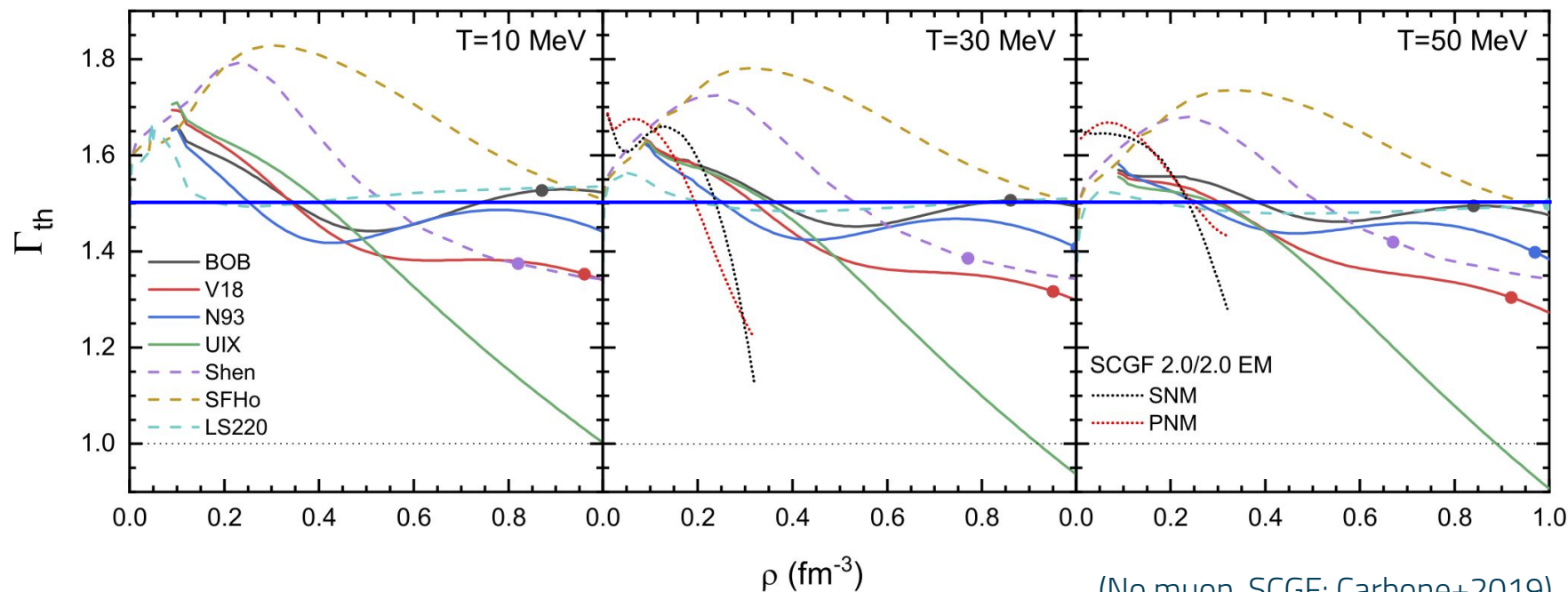
The gamma-law EOS [Endrizzi+2018]:

$$p(\rho, \epsilon) = p_{\text{cold}}(\rho) + (\Gamma_{\text{th}} - 1)(\epsilon - \epsilon_{\text{cold}})\rho$$

Definition:  $\Gamma_{\text{th}} = 1 + p_{\text{th}} / \epsilon_{\text{th}}$

Originally chosen as  $\sim 1.5$  [Kanta+1993]

**Strong violations** were found in the post-merger phase [Bauswein+2010]



(No muon, SCGF: Carbone+2019)

# Tabulation EOSs in HShen Format

Finite temperature equation of states based on BHF theory

equation-of-state nuclear-physics many-body-theory Manage topics

## Crust: HShen EOS 2011 version

4 commits 1 branch 0 releases 1 contributor

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lujiajing1126 add affil		Latest commit #645c85 on Jun 8
.gitattributes	add EOSs version1	4 months ago
README.md	add affil	4 months ago
eos_LU2019_BOB_HShen_v20190326.tab.gz	add EOSs version1	4 months ago
eos_LU2019_N93_HShen_v20190509.tab.gz	add EOSs version1	4 months ago
eos_LU2019_UIX_HShen_v20190510.tab.gz	add EOSs version1	4 months ago
eos_LU2019_V18_HShen_v20190508.tab.gz	add EOSs version1	4 months ago

README.md

### FT-EOS

this repository provides finite temperature equation of states (EOS) in HShen format

### Contributors

- Jia-Jing Lu<sup>1,2</sup>
- Zeng-Hua Li<sup>1</sup>
- Fiorella Burgio<sup>2</sup>
- Hans-Josef Schulze<sup>2</sup>

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<sup>2</sup>INFN sezione di Catania

陆家靖, 李增花 et al. Phys. Rev. C 100, 054335

Available on <https://github.com/bhfeos/FTEOS>

Will be soon available on compOSE

# CompOSE

CompStar Online  
Supernovae Equations of State

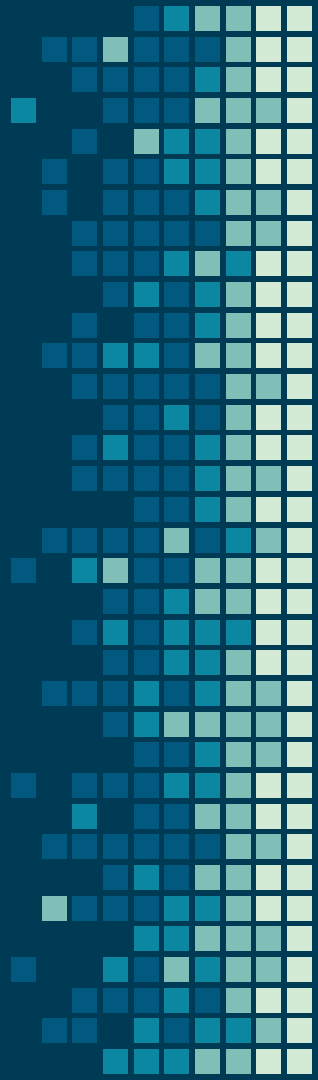


# Conclusion

- Present convenient parameterizations of FT-EOSs within Brueckner theory
- Discuss the density dependence of the adiabatic index
- Temperature dependence of the maximum NS mass
- A set of EOS tabulations is also provided.



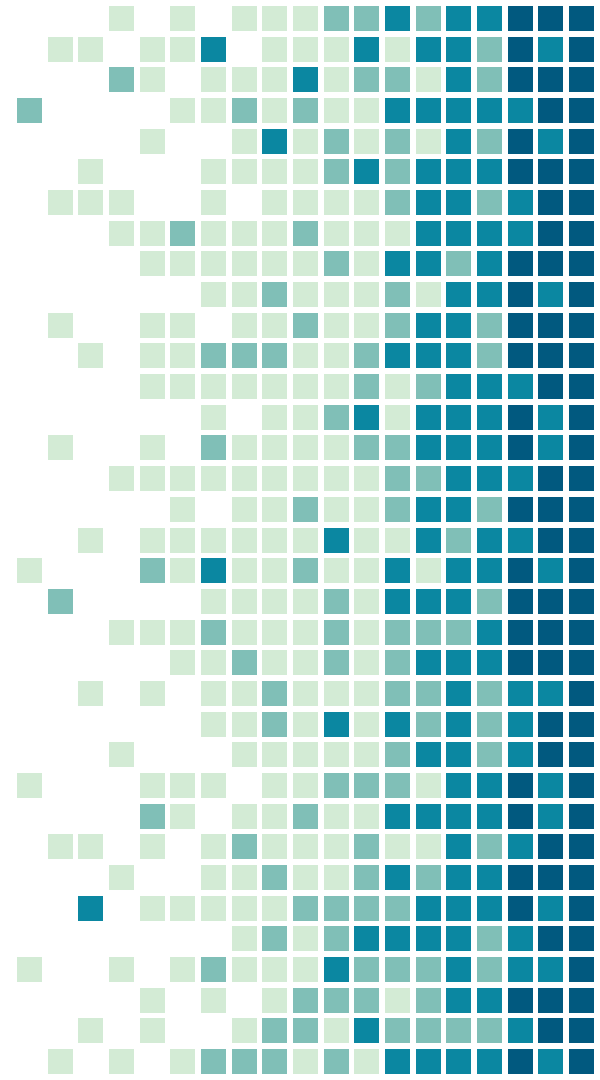
Thanks for your attention





# Appendix

Numerical methods



# Beta Equilibrium

- Chemical Potentials

- Bayron [Baldo+1998, 2000; Baldo & Ferreira 1999]

$$\mu_{p,n}(\rho_N, \beta) = \mu_{p,n}(\rho_N, 0) - (\beta^2 \pm 2\beta - \beta^2 \rho_N \frac{\partial}{\partial \rho_N}) E_{\text{sym}}(\rho_N) \quad (1)$$

where  $\mu_{p,n}(\rho_N, 0)$  is the chemical potential of a nucleon in symmetric matter (+ for  $p$ , - for  $n$ ), and in particular

$$[\mu_n - \mu_p](\rho_N, \beta) = 4\beta E_{\text{sym}}(\rho_N)$$

$$\mu_{p,n}(\rho_N, 0) = f + p/\rho = f + \rho \frac{\partial f}{\partial \rho}$$

At finite temperature,  $E_{\text{sym}}$  should be replaced by  $F_{\text{sym}}$ .



# Beta Equilibrium

- Chemical Potentials
  - Leptons [Shapiro & Teukolsky 2008]
  - in the natural units:  $c = \hbar = k_B = 1$

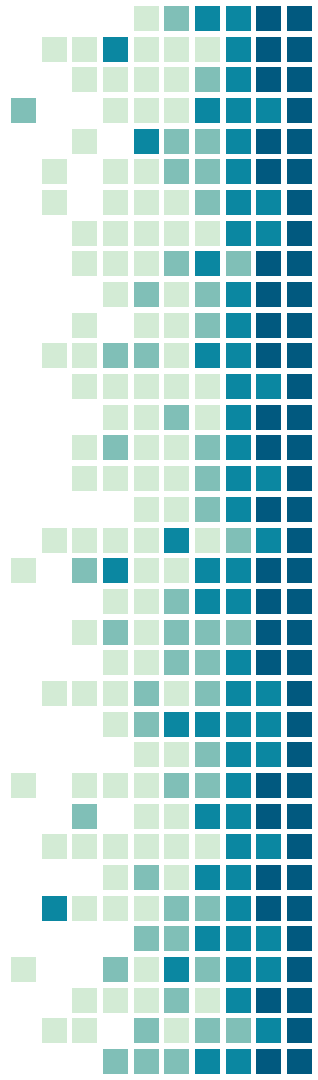
$$\mu_{e,\mu} = \frac{2}{h^3} \int_0^{p_{\text{cutoff}}} \frac{E_{e,\mu}}{1+e^{(E(k)-\mu)/T}} dp^3 = \frac{1}{\pi^2} \int_0^{p_{\text{cutoff}}} p^2 \frac{E_{e,\mu}}{1+e^{(E(k)-\mu)/T}} dp$$

For electron (ultrarel.):

$$E_e(k) = \hbar k = 197.33(\text{MeV}/\text{fm})k$$

For muon (rel.):

$$E_\mu(k) = \sqrt{m_\mu^2 + (\hbar k)^2} = \sqrt{(m_\mu^2) + (197.33(\text{MeV}/\text{fm})k)^2}$$





# Temperature Dependence

Prakash+1997 got increasing max masses with temperature in the RMF framework. Also confirmed by Kaplan+2014.

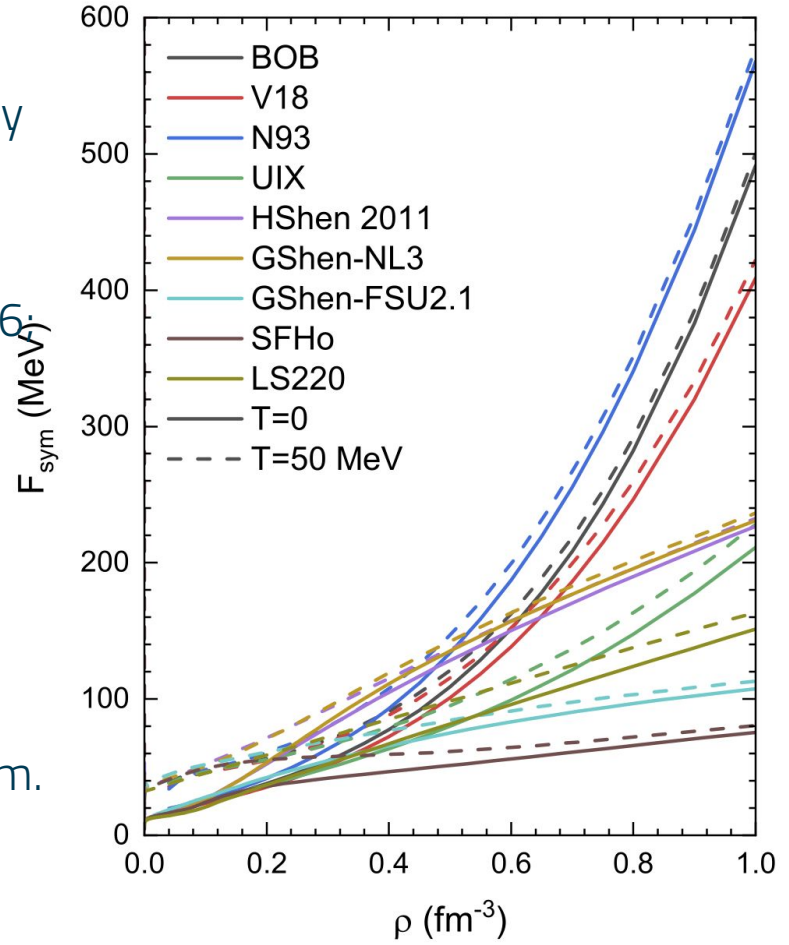
Possible reasons of decreasing max mass:

- **Isothermal/Isentropic** profile? No. [Nicotra+2006, Burgio & Schulze 2009,2010]
- The **locality** and **temperature independence** of interaction part in RMF? [Nicotra+2006]

$$f = \sum_i [\sum_k n_i(k) \frac{k^2}{2m_i} + \frac{n_i^{T=0}(k)}{2} U_i(k) - T s_i]$$

where  $n_i^{T=0}(k)$  is a step function.

- **Three-Nucleon Force** (TNF)
  - Free symmetry energy -> larger isospin asym.
  - Free energy of SNM







# Neutron star structure

Thermal internal energy

$$\epsilon_{th} = \frac{E}{V} = \frac{E}{N} \frac{N}{V} = \frac{3}{2} T \rho$$

significantly **overestimates** the thermal effects.

## Competitions

- 1. Thermal pressure 
- 2. Baryonic pressure  ISOSPIN ASYM. 
- 3. Leptonic pressure 

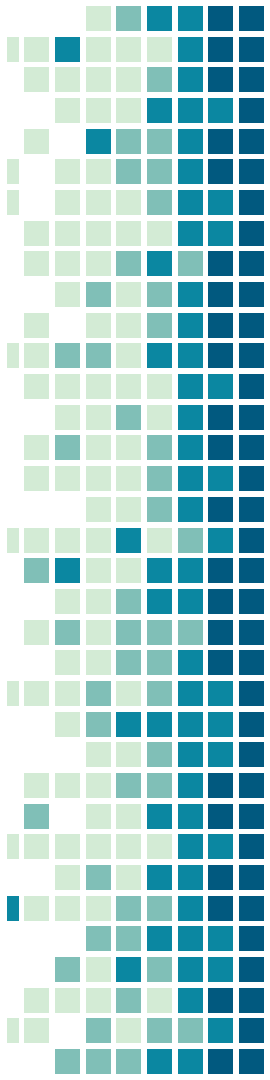
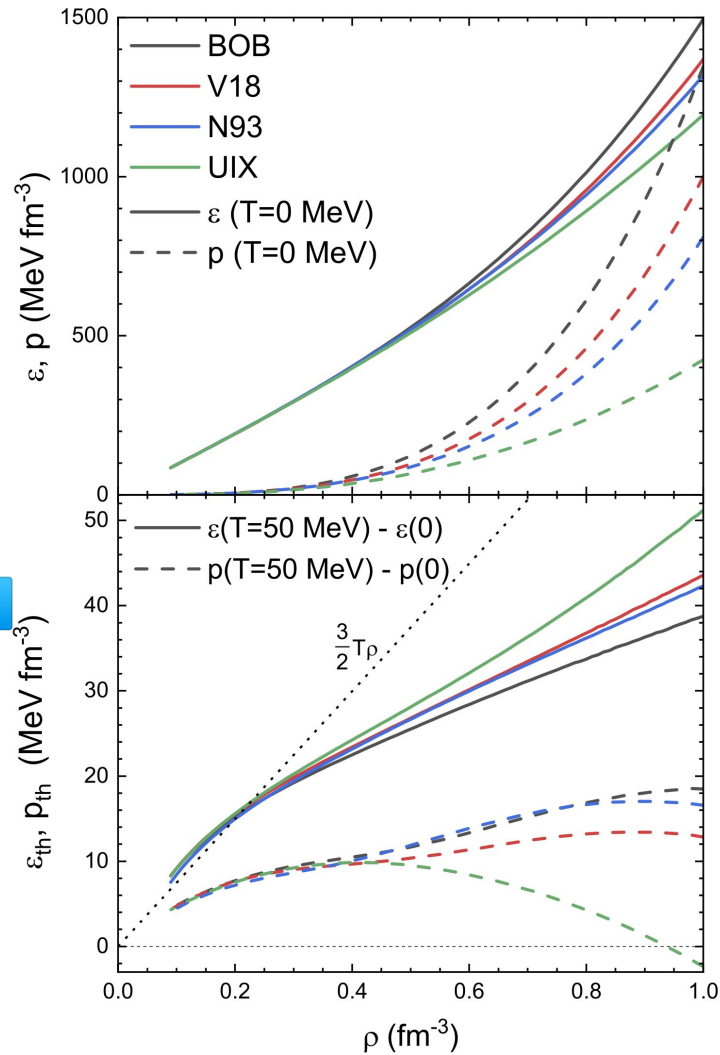
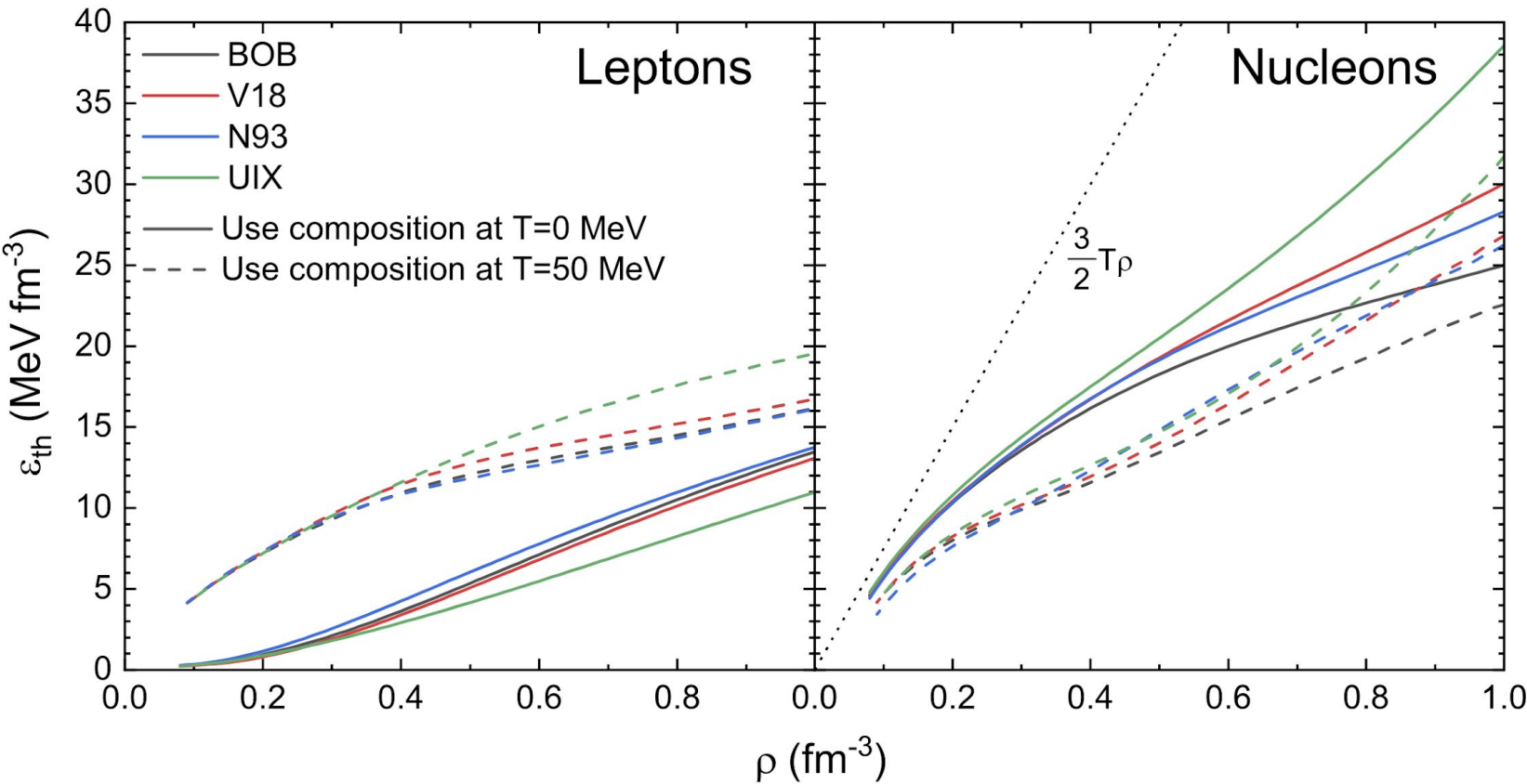
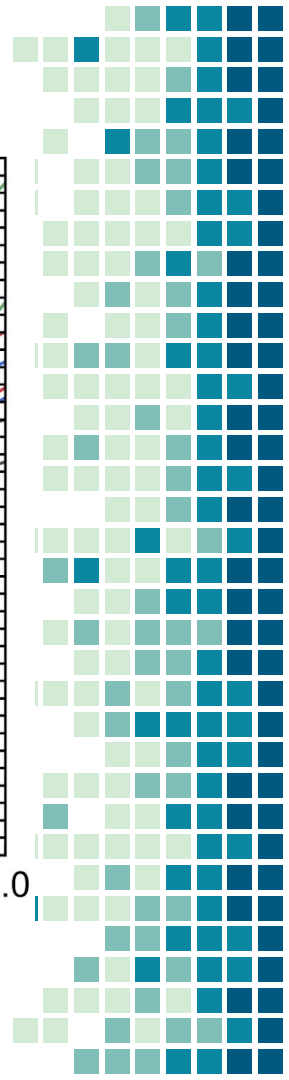


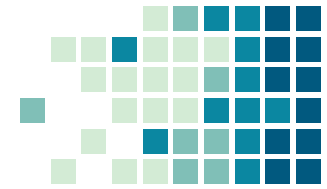
Fig. Internal energy density and pressure of beta-stable matter at  $T=0$  and the change of these thermal quantities at  $T=50 \text{ MeV}$ .

# Neutron star structure

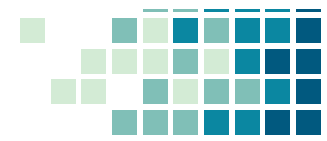
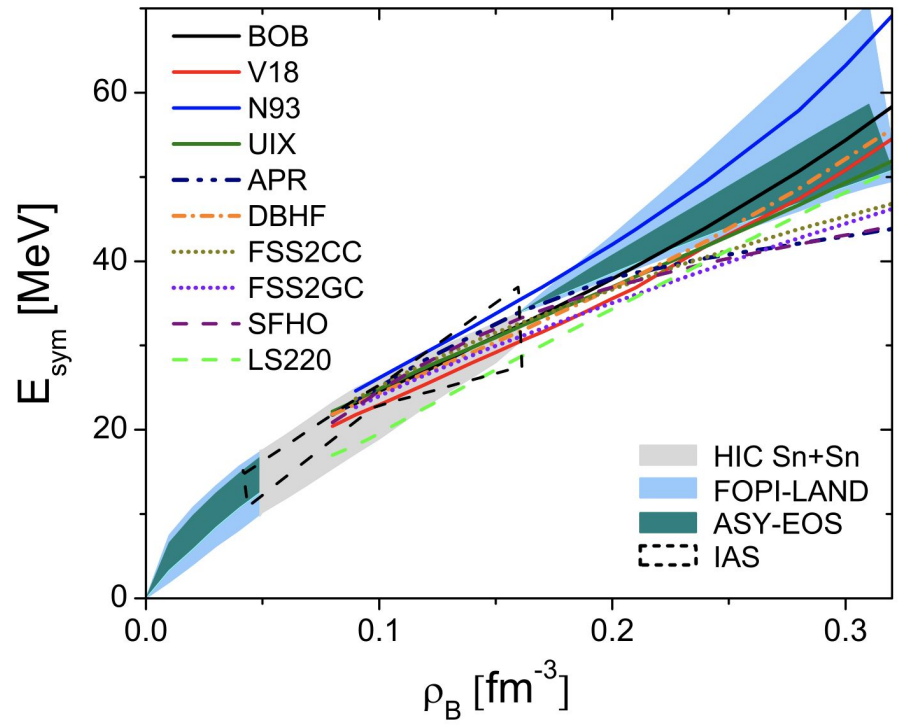
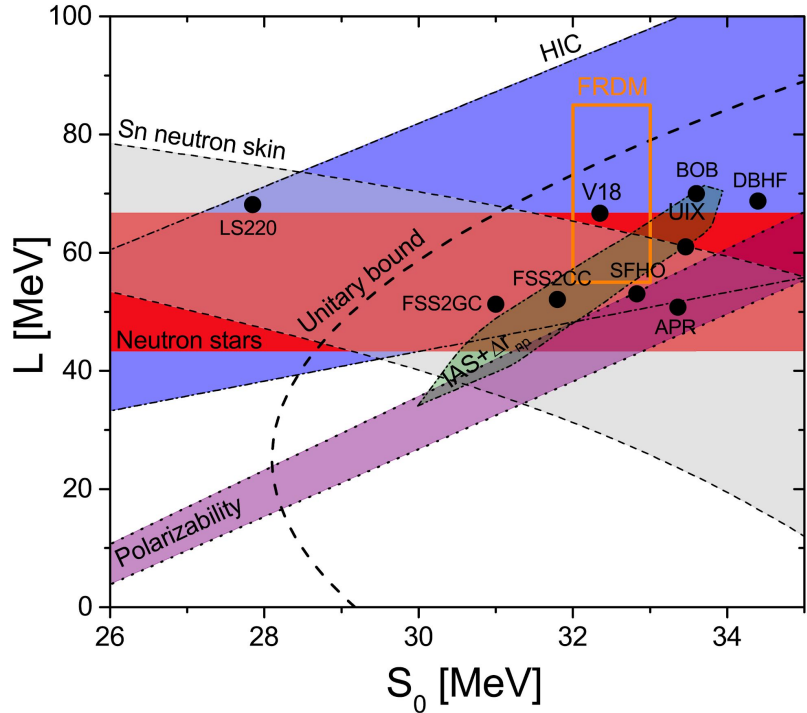


30 Fig. Lepton and nucleon contributions to the thermal internal energy density of beta-stable matter at  $T = 50 \text{ MeV}$  for the different EoSs.





# Constraints on the EOS from HIC and GW



# Constraints on the EOS from HIC and GW

