Solving the Strong CP Problem with Horizontal Gauge Symmetry

Work in collaboration with Tsutomu T. Yanagida
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Outline

1. Problem to solve (Strong CP problem)

2. A spontaneous CP violation solution

3. Conclusion and Challenges

Problem to solve

- The quantity defined as $\theta^* = \theta_0 + \mathrm{Arg}\,[\det M^u M^d]$ is invariant under phase rotation of fermions

- Neutron electric dipole moment (NEDM) from χPT: $d_n = 3.6 \times 10^{-16}\,\theta^*$ (e cm)
- Experimental constraint on NEDM: $d_n < 3 \times 10^{-26}$ (e cm)
- Constraint on $\theta^* < 10^{-10}$

Problem to solve

- Any reasonable explanation for the smallness?

- For $p \ll 1$, if certain additional symmetries are restored or enhanced in the limit $p \to 0$, then the smallness is understood to be natural

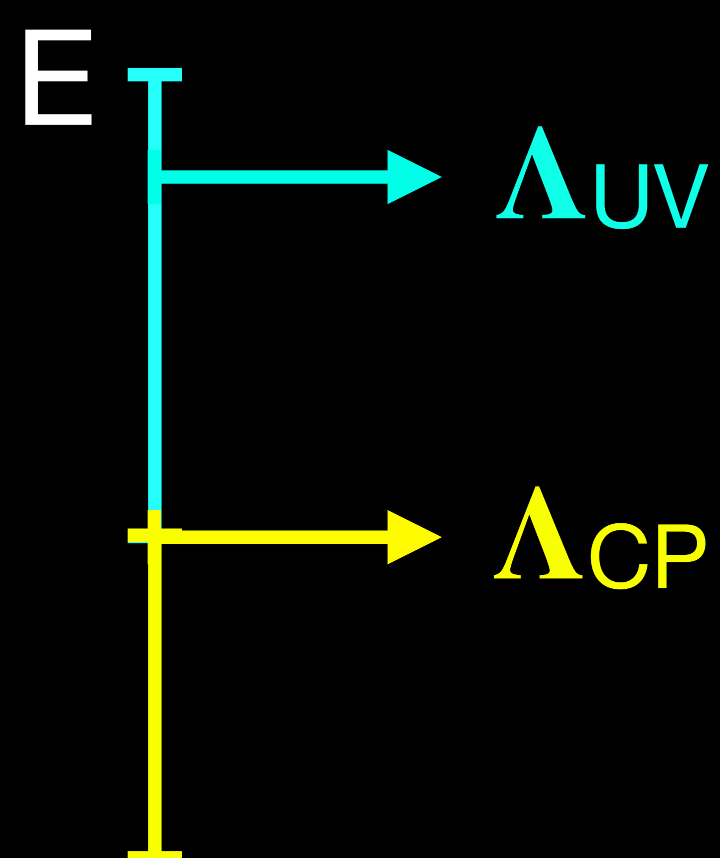
G. 't Hooft (1980)

- CP is not restored for $\theta^* \to 0$ due to $\delta_{KM} \sim O(1)$
- No reason for θ^* to be small…
- How can we understand this unnatural smallness provided it is a finite non-zero value?

Model (spontaneous CP violation)

- CP transformation → exact symmetry in a full theory
- Attribute the origin of CP violation in both strong and weak sector to complex VEVs of complex scalars

A. Nelson (1984), S. Barr (1984)



$$\mathscr{L} = a_1\mathscr{O}_1 + a_2\mathscr{O}_2 + \ldots, \quad a_i \in \mathbb{R}$$

$$\theta_0 = 0, \ \mathscr{M}_f \in \mathbb{R} \rightarrow \theta^* = 0$$

$$<\varphi> = v \in \mathbb{C}$$

$$\delta\mathscr{M}_f \in \mathbb{C} \rightarrow \theta^* \neq 0$$

Model (a suspicion)

- In a SCPV solution, non-zero θ^* and δ_{KM} are originated from the same source ($\langle\varphi\rangle = v \in \mathbb{C}$)
- δ_{KM} has something to do with quark-Higgs Yukawa
- Yukawa has the unexplained hierarchical structure
- A sensible suspicion might be that non-zero θ^* and the Yukawa structure stem from a common origin → hierarchical VEVs of complex scalars?

Model (basic framework)

- Symmetry group of the model
  : G_{SM} x $SU(3)_F$ x $Z_{2,(1)}$ x $Z_{2,(2)}$ x $Z_{2,(3)}$ x CP
- Particle contents
  : SM particles
  + $SU(3)_F$ triplet Dirac fermions ψ^u, ψ^d
  + three $SU(3)_F$ triplet complex scalars

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(3)_f$	$Z_2^{(1)}$	$Z_2^{(2)}$	$Z_2^{(3)}$
q	$\square$	$\square$	$+1/6$	$\square$	+	+	+
$\overline{u}$	$\overline{\square}$	1	$-2/3$	$\overline{\square}$	+	+	+
$\overline{d}$	$\overline{\square}$	1	$+1/3$	$\overline{\square}$	+	+	+
H	1	$\square$	$-1/2$	1	+	+	+
U	$\square$	1	$+2/3$	$\square$	+	+	+
$\overline{U}$	$\overline{\square}$	1	$-2/3$	$\overline{\square}$	+	+	+
D	$\square$	1	$-1/3$	$\square$	+	+	+
$\overline{D}$	$\overline{\square}$	1	$+1/3$	$\overline{\square}$	+	+	+
Φ_1	1	1	0	$\square$	-	+	+
Φ_2	1	1	0	$\square$	+	-	+
Φ_3	1	1	0	$\square$	+	+	-

Model (scalar vev)

- Without loss of generality, we may write down

$$\Phi_1 = \begin{bmatrix} 0 \\ 0 \\ X_1 \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} 0 \\ Y_2 \\ X_2 \end{bmatrix}, \quad \Phi_3 = \begin{bmatrix} Z_3 \\ Y_3 \\ X_3 \end{bmatrix}$$

  $X_1, Y_2 \in \mathbb{R}$ and the rest is complex
- Assume a scalar dynamics such that $|\mathbf{\Phi}_1| > |\mathbf{\Phi}_2| > |\mathbf{\Phi}_3|$ is achieved.
- Sequential breaking of $SU(3)_F$ may explain the hierarchy in Yukawa structure.

Model (tree and renormalizable level)

- Yukawa sector in the model at the tree and renormalizable level

$$\mathcal{L}_{Yuk} = \mathcal{L}_q + \mathcal{L}_Q + \mathcal{L}_{qQ}$$

$$\mathcal{L}_q = a^u H^\dagger q_\alpha \overline{u}_\alpha + a^d H q_\alpha \overline{d}_\alpha + \mathrm{h.c.}$$

$$\mathcal{L}_Q = M^U U_\alpha \overline{U}_\alpha + M^D D_\alpha \overline{D}_\alpha + \mathrm{h.c.}$$

$$\mathcal{L}_{qQ} = b^u H^\dagger q_\alpha \overline{U}_\alpha + b^d H q_\alpha \overline{D}_\alpha + \mathrm{h.c.}$$

Model (tree and renormalizable level)

- Fermion mass matrix (6 x 6)

$$\mathcal{M}^u = \begin{bmatrix} \mathcal{M}^u_{11} & \mathcal{M}^u_{12} \\ \mathcal{M}^u_{21} & \mathcal{M}^u_{22} \end{bmatrix} = \begin{bmatrix} a^u H_0^* I_{3\times 3} & b^u H_0^* I_{3\times 3} \\ 0 & M^U I_{3\times 3} \end{bmatrix}$$

$$\mathcal{M}^d = \begin{bmatrix} \mathcal{M}^d_{11} & \mathcal{M}^d_{12} \\ \mathcal{M}^d_{21} & \mathcal{M}^d_{22} \end{bmatrix} = \begin{bmatrix} a^d H_0 I_{3\times 3} & b^d H_0 I_{3\times 3} \\ 0 & M^D I_{3\times 3} \end{bmatrix}$$

- Determinant (four 3 x 3 block matrices)

$$\begin{aligned} \det\mathcal{M} &= [\det\mathcal{M}_{11}][\det(\mathcal{M}_{22} - \mathcal{M}_{21}\mathcal{M}_{11}^{-1}\mathcal{M}_{12})] \\ &= [\det\mathcal{M}_{22}][\det(\mathcal{M}_{11} - \mathcal{M}_{12}\mathcal{M}_{22}^{-1}\mathcal{M}_{21})] \end{aligned}$$

- Up to this point, θ^*= Arg[det$M^u M^d$] = 0

Model (corrections to θ^*)

- On acquisition of VEVs of $\mathbf{\Phi}_i$s, CP and $SU(3)_F$ are spontaneously broken → there arise complex corrections to θ^* (via corrections to mass matrix)
- Corrections of two kinds can be considered
  : higher dimensional operators
  + radiative corrections
- In SM, non-zero contribution to $\boldsymbol{\beta}$-function for θ^* starts from 7-loop order J. Ellis, M. Gaillard (1979)
- RG flow for θ^* may be neglected up to Λ_{UV} of the theory → apply constraint $\theta^* < 10^{-10}$ to the energy scale ~ $<\varphi>$

Model (higher dimensional operators)

- Higher dimensional operators cause $\det M^u M^d \in \mathbb{C}$ and $\delta\theta^* \neq 0$
- Dangerous contributions come from dim 7 operators

$$\mathcal{O}_{21}^{(u,7)} \ni \sum_{i,j=1}^{3} c_{1,ij}^{(u,7)} \frac{\Phi^\dagger_{\gamma i}\Phi_{\gamma i}\Phi^\dagger_{\alpha j}\Phi_{\beta j}}{M_P^3} U_\alpha \overline{u}_\beta + \sum_{i,j=1}^{3} c_{2,ij}^{(u,7)} \frac{\Phi^\dagger_{\gamma i}\Phi_{\gamma j}\Phi^\dagger_{\alpha i}\Phi_{\beta j}}{M_P^3} U_\alpha \overline{u}_\beta + \sum_{i,j=1}^{3} c_{3,ij}^{(u,7)} \frac{\Phi^\dagger_{\gamma i}\Phi_{\gamma j}\Phi^\dagger_{\alpha j}\Phi_{\beta i}}{M_P^3} U_\alpha \overline{u}_\beta$$

$$\mathcal{O}_{22}^{(u,7)} \ni \sum_{i,j=1}^{3} d_{1,ij}^{(u,7)} \frac{\Phi^\dagger_{\gamma i}\Phi_{\gamma i}\Phi^\dagger_{\alpha j}\Phi_{\beta j}}{M_P^3} U_\alpha \overline{U}_\beta + \sum_{i,j=1}^{3} d_{2,ij}^{(u,7)} \frac{\Phi^\dagger_{\gamma i}\Phi_{\gamma j}\Phi^\dagger_{\alpha i}\Phi_{\beta j}}{M_P^3} U_\alpha \overline{U}_\beta + \sum_{i,j=1}^{3} d_{3,ij}^{(u,7)} \frac{\Phi^\dagger_{\gamma i}\Phi_{\gamma j}\Phi^\dagger_{\alpha j}\Phi_{\beta i}}{M_P^3} U_\alpha \overline{U}_\beta$$

Model (higher dimensional operators)

- Mass matrix with higher dim-op's corrections

$$\mathcal{M}^u = \begin{bmatrix} \mathcal{M}^u_{11} & \mathcal{M}^u_{12} \\ \mathcal{M}^u_{21} & \mathcal{M}^u_{22} \end{bmatrix}$$
$$= \begin{bmatrix} a^u H_0^* I_{3\times 3} + \mathcal{O}^{(u,6)}_{11} & b^u H_0^* I_{3\times 3} + \mathcal{O}^{(u,6)}_{12} \\ \mathcal{O}^{(u,5)}_{21} + \mathcal{O}^{(u,7)}_{21} & M^U I_{3\times 3} + \mathcal{O}^{(u,5)}_{22} + \mathcal{O}^{(u,7)}_{22} \end{bmatrix}$$

- Evaluation of θ^*

$$\delta\bar{\theta} \sim \frac{c^{(q,7)}_{2,i=1,j=2}}{c^{(q,5)}_{2}} \frac{|X_2|^2}{M_P^2} \lesssim 10^{-10}$$

e.g., for P=0 and -2, $X_2 < 10^{13}$ and 10^{14}GeV respectively (c-ratios = 10^P)

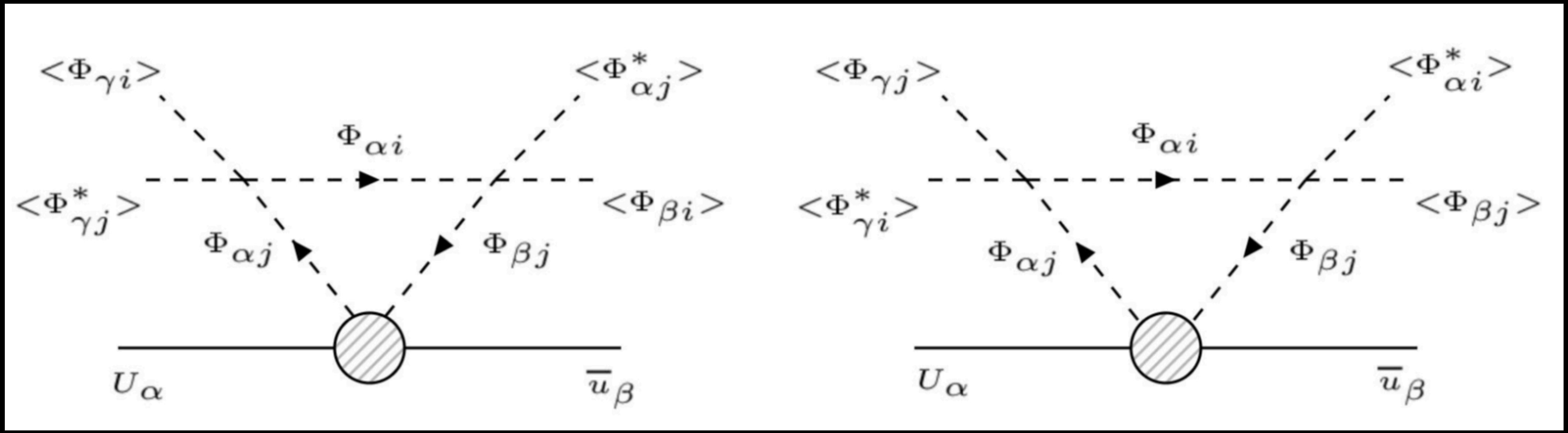
Model (radiative correction)

- Radiative corrections with external lines of $<\varphi>$ may cause $\det M^u M^d \in \mathbb{C}$, $\delta\theta^* \neq 0$
- Renormalizable scalar potential respecting $SU(3)_F \times (Z_2)^3$

$$V(\Phi) = -\sum_{i=1}^{3} \frac{1}{2} m_{\Phi_i}^2 |\Phi_{\alpha i}|^2 + \frac{\lambda_0}{4} \sum_{i,j=1}^{3} |\Phi_{\alpha i}|^2 |\Phi_{\beta j}|^2 + \frac{\lambda_+}{4} \sum_{i,j=1}^{3} \Phi_{\alpha i}^\dagger \Phi_{\alpha j} \Phi_{\beta i}^\dagger \Phi_{\beta j} + \frac{\lambda_-}{4} \sum_{i,j=1}^{3} \Phi_{\alpha i}^\dagger \Phi_{\alpha j} \Phi_{\beta j}^\dagger \Phi_{\beta i}$$

Model (radiative correction)

- We find corrections with four external lines of $<\varphi>$ can spoil reality of det[$M^u M^d$]
- e.g.,



correction to $c^{(u,7)}_{(2,ij)}$ and $c^{(u,7)}_{(2,ji)}$

- Different vertex factors: $\lambda_- \lambda_+ \neq \lambda_+ \lambda_o \rightarrow \delta\theta^* \neq 0$

Model (radiative correction)

- $\delta c^{(u,7)}{}_{(2,ij)} \neq \delta c^{(u,7)}{}_{(2,ji)}$ → hermiticity of M_{21} & M_{22} breaks down → $\det M^u M^d \in \mathbb{C}$ and $\delta\theta^* \neq 0$

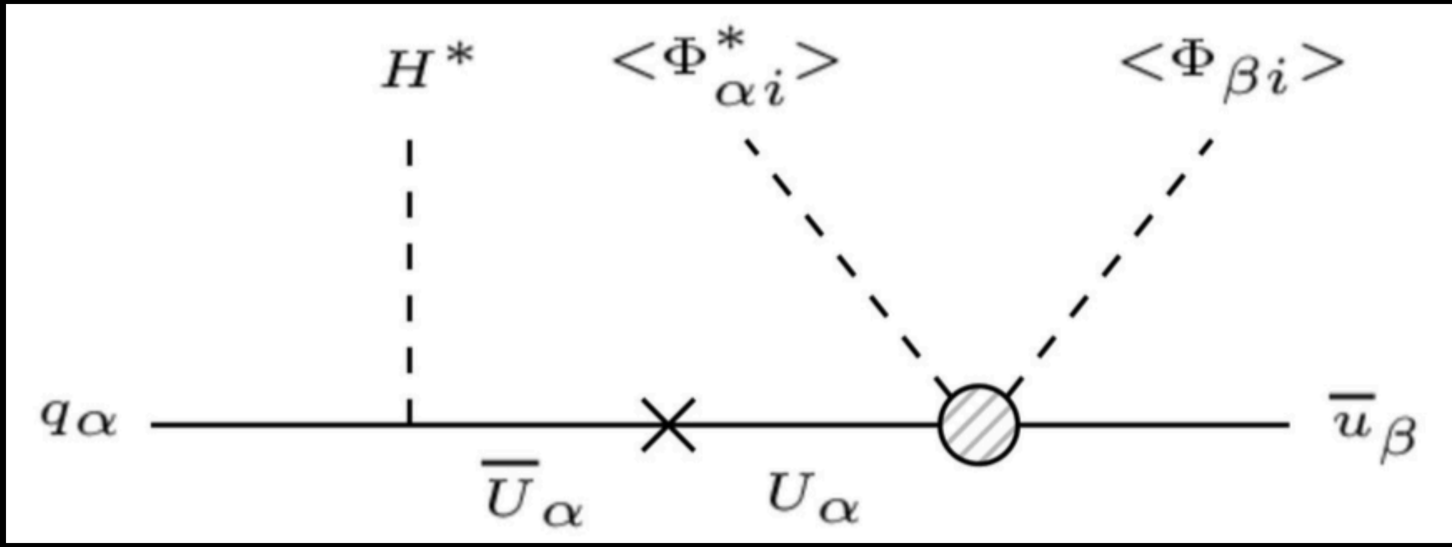
$$(\delta\bar{\theta})_{1-\mathrm{loop}} \simeq 10^{-2R} \frac{\lambda^2}{16\pi^2} \frac{|X_1|^2}{m_\Phi^2} \lesssim 10^{-10}$$

where $|X_1|=10^R|X_2|$

- $\theta^* < 10^{-10}$ → $\lambda_\pm < 10^{-8+2R}$ (e,g. R=1→ $\lambda_\pm < 10^{-6}$)
- $\lambda_\pm$ → 0: $SU(3)_{F,(1)}$ x $SU(3)_{F,(2)}$ x $SU(3)_{F,(3)}$ enhancement
- Conversion of the unnatural smallness to a natural smallness

Model (SM Yukawa)

- Below M_Q, integrating out heavy fermions induces effective SM Yukawa



- There are 14 free parameters → reproduce 3 quark mixing angles, Jarlskog invariant, 4 quark mass ratios
- For example, we obtained $|Y_2| \simeq 0.08\ |X_1|$

Conclusion

- With introduction of $SU(3)_F \times (Z_2)^3$ & complex scalars and heavy fermions, the model is able to convert $\theta^* < 10^{-10}$ (unnatural) to $\lambda_\pm < 10^{-6}$ (natural).
- The model results in a hermitian Yukawa in the SM
- Challenges: (1) construction of $V(\varphi)$

  (2) a UV physics giving a little bit of hierarchy btw Wilson coefficients (for effective SM Yukawa)