Fate of false vacuum in a singlet-doublet dark matter model with renormalization-group improved effective action

Yu Cheng

East China University of Science and Technology

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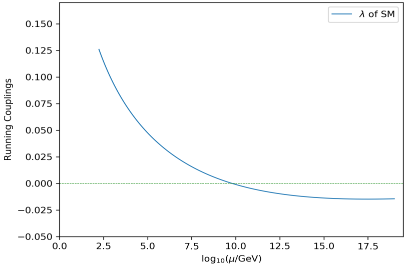
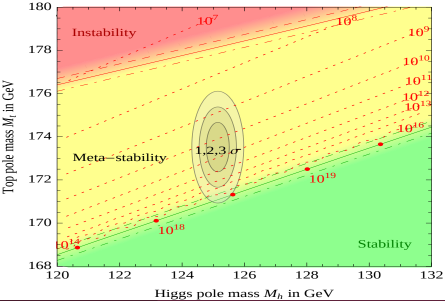
Overview

It's well-known that SM vacuum is unstable

- $\lambda < 0$ at about $10^{10} GeV$. $U(\phi \gg v) \approx \frac{\lambda(\mu)}{4}\phi^4$ and become unbounded from below.
- electroweak vacuum become a false vacuum,and will decay through quantum tunnelling.
- space-time bubble of true vacuum form and expand ,similar to liquid-gas phase transition in boiling water.



Buttazzo,Derassi (2013)



Overview

Lifetime of the SM vacuum is longer than the life of present universe.We call it metastable.

- To avoid unbounded potential,there must be new physics.

For new physics model:

- new particles will affects the RG running of λ.
- extra bosons may stabilize the EW vacuum by running effects.
- extra fermions destabilize the EW vacuum in some models.

Quantum effects affect not only the RG running,but also the vacuum structure.

- quantum corrections of new particles have effect on vacuum stability,which can be described by V_{eff}.
- quantum corrections affect both the kinetic term and non-derivative term in the action,described by S_{eff}.

Overview

Our work focus on the singlet-doublet fermionic dark matter(SDFDM) model.We study:

- the effective potential and the RG improved effective potential in this model.
- vacuum stability using the RG improved effective potential.
- false vacuum decay by using RG improved effective action.
- constraints on the parameter space.

The model

the SDFDM model has $\mathrm{SU}(2)$ doublet fermions $\psi_{L,R} = \left(\psi^0_{L,R}, \psi^-_{L,R}\right)^T$ with Y = -1/2 and singlet fermions $S_{L,R}$.

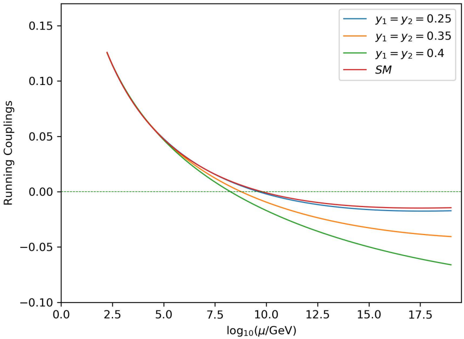
- the singlet S can be either a Dirac or a Majorana fermion.
- the neutral fermion can be a dark matter candidate.
- we mainly work on Dirac type dark matter.

The relevant terms of ψ and S in the Lagrangian are:

$$\begin{aligned}\mathcal{L}_{\mathrm{SDFDM}} &= \overline{\psi} i \not{D} \psi + \overline{S} i \not{\partial} S \\ &- M_D \overline{\psi}_L \psi_R - M_S \overline{S}_L S_R - y_1 \overline{\psi}_L \tilde{H} S_R - y_2 \overline{\psi}_R \tilde{H} S_L + \mathrm{H.c.}\end{aligned} \tag{2.1}$$

- H is the SM Higgs doublet with $Y = 1/2$, and $\tilde{H} = i\sigma_2 H^*$.
- There is a Z_2 symmetry in the Lagrangian with the new fermions ψ and S odd and SM fermions even under the Z_2 operation.
- Z_2 symmetry makes the lightest of these new fermions to be stable,and makes it a dark matter candidate if it is neutral.

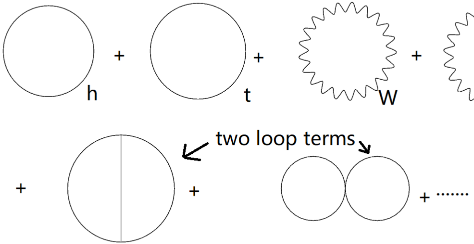
Vacuum stability in SDFDM model



- λ_{min} in the SDFDM model is more negative than in the SM.
- In the SDFDM model the EW vacuum is unstable.
- The greater the Yukawa couplings,the greater the destabilization effects.

effective potential

- To take into account the quamtum effects,we need the effective potential. propagators,vertices generated by $\mathcal{L}(\phi + \hat{\phi})$.(jackiw 1973)



- In the vacuum stability analysis, we must consider the behavior of the effective potential for large external field.
- To summing over large logarithms,we need RGE. V_{eff} satisfies the RGE:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i} - \gamma\phi\frac{\partial}{\partial\phi}\right)V_{\text{eff}} = 0 \tag{2.2}$$

The one-loop RG improved effective potential for SDFDM can be written as:

$$V_{eff}(\phi,t) = V_0^{\rm SM}(\phi,t) + V_1^{\rm SM}(\phi,t) + V_1^{\rm Ext}(\phi,t) \tag{2.3}$$

with

$$\begin{aligned} V_0^{\rm SM}(\phi,t) &= -\frac{m_\phi^2(t)}{2}\phi^2(t) + \frac{1}{4}\lambda(t)\phi^4(t) \\ V_1^{\rm SM}(\phi,t) &= \sum_i \frac{(-1)^i n_i}{64\pi^2} M_i^4(\phi,t)\left[\ln\frac{M_i^2(\phi,t)}{\mu^2(t)} - c_i\right] \\ V_1^{\rm Ext}(\phi,t) &= \sum_i \frac{(-1)^i n_i}{64\pi^2} M_{\chi_i}^4(\phi,t)\left[\ln\frac{M_{\chi_i}^2(\phi,t)}{\mu^2(t)} - 3/2\right] \end{aligned} \tag{2.4}$$

where

$$\mu(t) = \mu e^t, \phi(t) = e^{\Gamma(t)}\phi, \Gamma(t) = -\int_0^t \gamma\left(\lambda\left(t'\right)\right)dt' \tag{2.5}$$

- In the limit $\phi \gg v$, Eq. (2.3) can be written approximately as follows

$$V_{\rm eff}(\phi,t) \approx \frac{\lambda_{\rm eff}(\phi,t)}{4}\phi^4, \tag{2.6}$$

- In vacuum stability analysis, we generally take $\mu(t) = \phi$,

$$\lambda_{\rm eff}(\phi,t) \approx e^{4\Gamma(t)}\left[\lambda(t) + \frac{1}{(4\pi)^2}\sum_i N_i\kappa_i^2(t)\left(\log\kappa_i(t)e^{2\Gamma(t)} - c_i\right)\right] \tag{2.7}$$

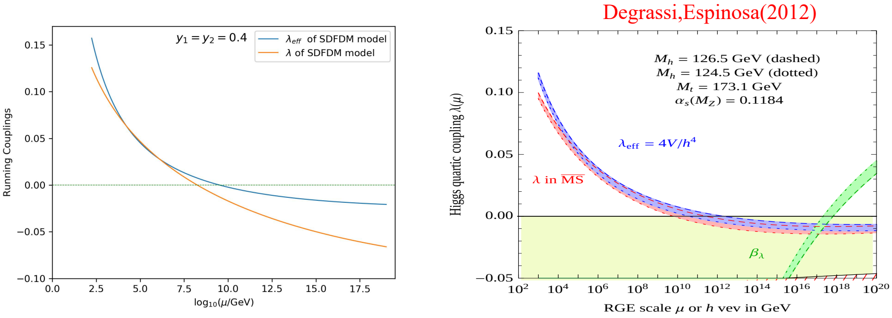


Figure 1: a:λ and $\lambda_{\rm eff}$ in SDFDM model for $y_1 = y_2 = 0.4$, b:λ and $\lambda_{\rm eff}$ in SM.

- In the SM, the difference $\lambda_{\rm eff} - \lambda$ is always positive and is negligible near the Planck scale.(right figure)
- $\lambda_{\rm eff} - \lambda$ is not negligible in the SDFDM model.(left figure)
- $\lambda_{\rm eff}$ is suppressed by the $e^{4\Gamma}$ factor.
- The instability scale Λ_I is larger when determined by $\lambda_{\rm eff}$,both in the SM and in the SDFDM model.

Decay rate compution:two ways

WKB approximation.Banks,Bender and Wu(1973)

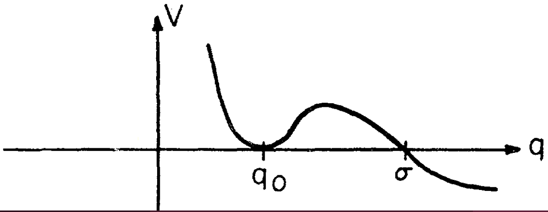
$$\Gamma = Ae^{-B/\hbar}[1 + O(\hbar)], \tag{2.8}$$

where

$$B = 2\int_{\vec{q}_0}^{\vec{\sigma}} ds[2(V-E)]^{1/2} \tag{2.9}$$

the integral is over that path for which B is minimum:

$$\delta\int_{\vec{q}_0}^{\vec{\sigma}} ds[2(V-E)]^{1/2} = 0 \tag{2.10}$$



Decay rate compution:two ways

Method of Euclidean field theory.
Coleman(1977)
Bounce solution is the solution of the Lagrange equation:

$$\delta \int d\tau L_E = 0 \qquad (2.11)$$

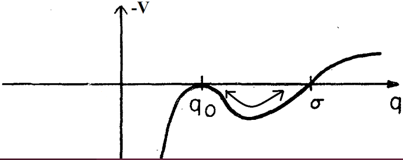
L_E is the Euclidean Lagrangian

$$L_E = \frac{1}{2}\frac{d\overrightarrow{q}}{d\tau} \cdot \frac{d\overrightarrow{q}}{d\tau} + V \qquad (2.12)$$

B factor is the Euclidean action for the bounce

$$B = \int_{-\infty}^{\infty} d\tau L_E \equiv S_E \qquad (2.13)$$

- The two methods should be equivalent.
- The decay rate calculated by these two methods are equal in 1+1 dimension field theory. Langer(1967).
- The second method are easy to be extended to field theroy in higher dimension .



decay rate in the field theory

Recalling that for a potential $U(\phi)$, the decay rate per unit time per unit volume, Γ_t, can be expressed as:

$$\Gamma_t = A_t e^{-S_{cl}} \tag{3.1}$$

- S_{cl} is the Euclidean action of bounce solution
- Bounce is the solution of the Euclidean equation of motion:

$$-\partial^2 \phi_B + U'(\phi_B) = -\frac{d^2\phi_B}{dr^2} - \frac{3}{r}\frac{d\phi_B}{dr} + U'(\phi_B) = 0 \tag{3.2}$$

  ϕ_B refers to the O(4) symmetric solution which lead to the minimum Euclidean action.

- A_t is the one-loop quantum correction:

$$A_t = \frac{S_{cl}^2}{4\pi^2}\left|\frac{\det'\left[-\partial^2 + U''(\phi_B)\right]}{\det\left[-\partial^2 + U''(\phi_0)\right]}\right|^{-1/2} \tag{3.3}$$

  $S_{cl}^2/4\pi^2$ is zero mode contribution, $\det'$ is the determinant omitting the zero mode contribution. ϕ_0 is the field value in the false vacuum.

decay rate in the field theory

For a SM like potential $U(\phi) = \frac{\lambda}{4}\phi^4$ with a negative λ:

$$\phi_B(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{r^2 + R^2}, S_{cl} = \frac{8\pi^2}{3|\lambda|} \tag{3.4}$$

- The one-loop corrections will cancel the μ dependence in S_{cl},we always choose $\mu \sim 1/R$.

To take into account quantum corrections,$U(\phi) \to U_{eff}(\phi)$,so $\lambda \to \lambda_{\text{eff}}$?

- For SM,$\lambda \approx \lambda_{\text{eff}}$ at large field value,For SDFDM model,$\lambda_{\text{eff}} - \lambda$ is not negligible.
- It's puzzling that if we calculate the decay rate by λ_{eff},it will be much smaller than calculated by λ.
- The right way to understand this problem is to compute by using the effective action.

Effective action

We can compute the effective action using derivative expansion. Neglecting terms with higher derivative, we can write the effective action in Euclidean space for external field ϕ as

$$S_{\text{eff}}[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu\phi)^2 Z_2(\phi) + V_{\text{eff}}(\phi)\right] \tag{3.5}$$

- V_{eff} is the effective potential.
- Z_2 factor can be obtained by p^2 terms of self-energy diagrams.
- propagators and vertices are generated by $\mathcal{L}(\phi + \hat{\phi})$.

RG improvement of the kinetic term can be studied similar to the effective potential.

- all parameters λ_i in $Z_2(\phi)$ substituted by $\lambda_i(t)$,all $\phi \to \phi(t)$.
- by choosing $\mu(t) = \phi$,Z_2 is ϕ independent.

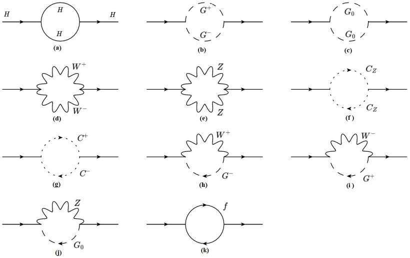


Figure 2: SM model.

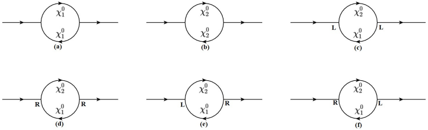


Figure 3: SDFDM model.

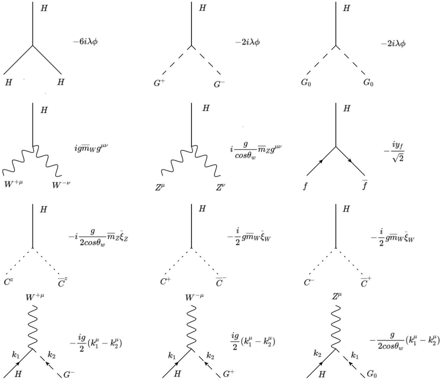


Figure 4: vertices with external field ϕ

- all the propagators are shifted propagators.

Effective action

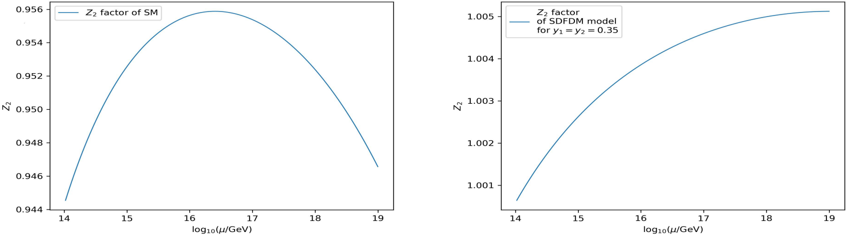


Figure 5: (a):The behavior of Z_2 at large energy scale in the SM; (b) The behavior of Z_2 at large energy scale with $y_1 = y_2 = 0.35$ in the SDFDM model.

We arrive at the Euclidean action:

$$S = \int d^4x \left[e^{2\Gamma(t)} Z_2(\phi, t) \frac{1}{2} \left(\partial_\mu \phi\right)^2 + e^{4\Gamma(t)} \frac{\tilde{\lambda}(t)}{4} \phi^4 \right] \tag{3.6}$$

where $\tilde{\lambda}$ is only different from Eq. 2.7 by a factor $e^{4\Gamma(t)}$.

Computing decay rate by effective action

- The Euclidean equation of bounce solution becomes:

$$-Z_2\partial^2\tilde{\phi}_B + \widetilde{\lambda}\tilde{\phi}_B^3 e^{2\Gamma(t)} = 0 \tag{3.7}$$

- bounce action becomes:

$$S_{cl} = e^{2\Gamma} Z_2 \times \frac{8\pi^2}{3|\tilde{\lambda}|e^{2\Gamma}/Z_2} = (Z_2)^2\,\frac{8\pi^2}{3|\tilde{\lambda}|} \tag{3.8}$$

S_{cl} depends on Z_2 but is independent of the e^{Γ_t} factor.

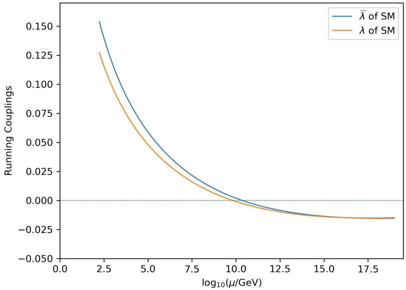
Consider the A_t factor:

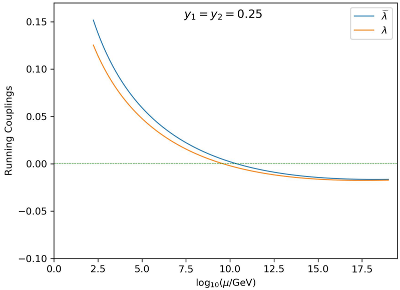
$$A_t = \frac{S_{cl}^2}{4\pi^2}\left|\frac{\det'[-\partial^2 + 3(\tilde{\lambda}/Z_2)\phi_B^2]}{\det[-\partial^2]}\right|^{-1/2} \tag{3.9}$$

- where $\phi_B = e^{\Gamma}\tilde{\phi}_B$, ϕ_B satisfying $-Z_2\ \partial^2\phi_B + \widetilde{\lambda}\ \phi_B^3 = 0$.
  $-\partial^2 + 3\left(\tilde{\lambda}/Z_2\right)e^{2\Gamma}\tilde{\phi}_B^2 \to -\partial^2 + 3\left(\tilde{\lambda}/Z_2\right)\phi_B^2$
- the $e^{2\Gamma}$ term in the det is canceled by the terms which comes from the zero mode contribution.

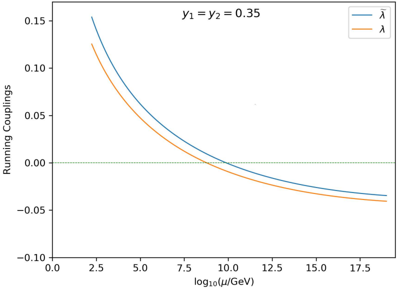
Computing decay rate by effective action

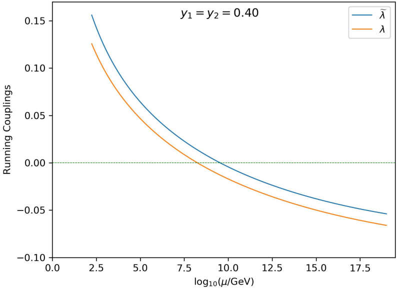
- We should use $\tilde{\lambda}$ but not λ_{eff} in the tunelling rate calculation.
- Since $Z_2 \sim 1$,the decay rate is mainly controlled by the behavior of $\tilde{\lambda}$.





Computing decay rate by effective action





- $\tilde{\lambda}$ and λ are very close at high energy scale in the SM.
- the difference betweene $\tilde{\lambda}$ and λ is more significant in SDFDM model. The larger the Yukawa coupling,the larger the difference.
- the lifetime calculated using $\tilde{\lambda}$ is longer than that computed using λ in SDFDM model.

Threshold effect

- To determine the initial value for RGE runing of the parameters at low energy scale, the threshold corrections must be taken into account.
- we work with the modified minimal subtraction($\overline{MS}$) scheme and evaluate one-loop threshold corrections.

General strategy:

- A parameter in MS scheme,$\theta(\bar{\mu})$,can be obtained from renormalized parameter in physical scheme which is directly related to physical observables.

$$\theta_0 = \theta - \delta\theta = \theta(\bar{\mu}) - \delta\theta_{\mathrm{MS}} \tag{3.10}$$

- Since the divergent parts in the $\delta\theta$ and $\delta\theta_{\mathrm{MS}}$ counterterms are of the same form, $\theta(\bar{\mu})$ can be rewritten as:

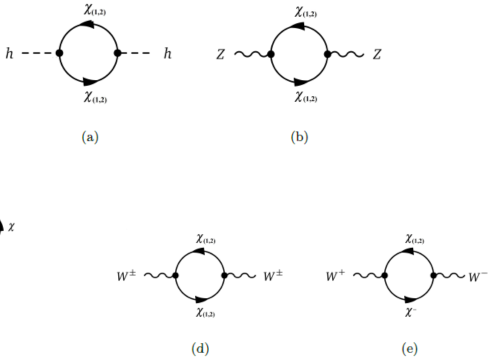
$$\theta(\bar{\mu}) = \theta - \delta\theta|_{\mathrm{fin}} \tag{3.11}$$

For example:

- $\lambda(\bar{\mu}) = \frac{G_\mu}{\sqrt{2}} M_h^2 - \delta^{(1)}\lambda\big|_{\text{fin}}$

  with $\delta^{(1)}\lambda = \frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \Delta r_0^{(1)} + \frac{1}{M_h^2} \left[\frac{T^{(1)}}{v} + \operatorname{Re}\Pi_{hh}\left(M_h^2\right) \right] \right\}$
- Here $\Pi_{hh}\left(M_h^2\right)$ is the Higgs boson self-energy evaluated on shell.

All the relevant Feynman diagrams for computing $\delta^{(1)}\theta\big|_{\text{fin}}$ with extra fermions are listed in Figure.



lifetime of the vacuum

the vacuum decay probability P_0 in our universe up to the present time can be expressed as:

$$\mathcal{P}_0 = 0.15 \frac{\Lambda_B^4}{H_0^4} e^{-S(\Lambda_\mathrm{B})} \tag{3.12}$$

where $H_0 = 67.4 \mathrm{~km~sec}^{-1} \mathrm{~Mpc}$ is the Hubble constant at the present time.

- $S(\Lambda_\mathrm{B})$ is the action of the bounce of size $R = \Lambda_\mathrm{B}^{-1}$.
- quantum corrections break scale invariance,Λ_B is he scale at which the negative $\tilde{\lambda}(\Lambda_B)$ is at the minimum.
- If $\Lambda_\mathrm{B} > M_\mathrm{Pl}$,we choose $\tilde{\lambda}(\Lambda_\mathrm{B}) = \tilde{\lambda}(M_\mathrm{Pl})$.

The stability of vacuum can be judged by the following conditions:

- Stable: $\widetilde{\lambda} > 0$ for $\mu < M_\mathrm{Pl}$;
- Metastable: $\widetilde{\lambda}(\Lambda_\mathrm{B}) < 0$ and $\mathcal{P}_0 < 1$;
- Unstable: $\widetilde{\lambda}(\Lambda_\mathrm{B}) < 0$ and $\mathcal{P}_0 > 1$;
- Non-perturbative: $|\lambda| > 4\pi$ before the Planck scale

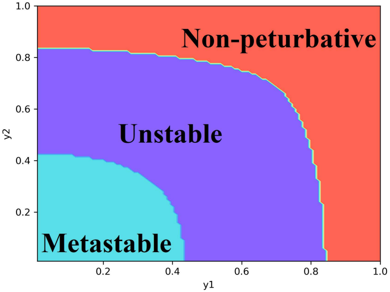
We list numerical results for the SM and for some benchmark points in the SDFDM model.

- For SM,the lifetime of EW vacuum computed using effective action is shorter than that just using λ.
- For SDFDM model,in the region that y_1 and y_2 are less than about 0.3, the situation is similar to the SM case.
- When y_1 and y_2 are larger,the lifetime calculated using effective action longer than that computed just using λ.

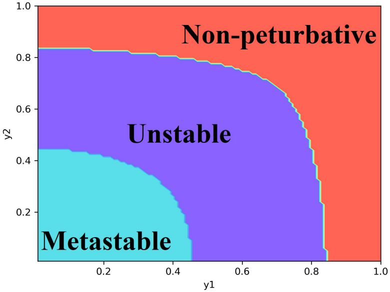
	Result with $\lambda(t)$			Result with $\tilde{\lambda}(t)$ in effective potential			
	λ_{min}	$\log_{10}(\mu_{min}/\mathrm{GeV})$	$\log_{10}(\mathcal{P}_0)$	$\widetilde{\lambda}_{min}$	$Z_2(\Lambda_B)$	$\log_{10}(\mu_{min}/\mathrm{GeV})$	$\log_{10}(\mathcal{P}_0)$
SM	-0.0148	17.46	-535.34	-0.0150	0.9521	18.07	-454.11
BMP1	-0.0176	17.60	-413.72	-0.0165	0.9557	18.23	-393.41
BMP2	-0.0406	M_{Pl}	-38.98	-0.0346	1.0051	M_{Pl}	-91.01
BMP3	-0.0661	M_{Pl}	unstable	-0.0539	1.0157	M_{Pl}	unstable

Table 1: The results computed by using $\lambda(t)$ and $\widetilde{\lambda}(t)$ are presented. Three benchmark models are BPM1($y_1 = y_2 = 0.25,$),BPM2($y_1 = y_2 = 0.35$),BPM3($y_1 = y_2 = 0.4$).

- We obtain the parameter space for the vacuum to be metastabe.
- We compare the two ways of obtaining the tunneling probability.
- the one-loop effect on effective potential slightly enlarges the parameter space for the vacuum to be metastable.



(e)



(f)

Figure 6: Status of the EW vacuum in the $y_1 - y_2$ plane with $M_S = M_D = 1000$ GeV . The left panel is given by using the ϕ^4 potential and the running $\lambda(t)$, the right panel is computed using effective action and $\tilde{\lambda}(t)$.

Conclusion

We study the vacuum stability using the RG improved effective potential.

- In SDFDM model the RG improved effective potential at high energy scale can be quite different from the potential just using $\lambda(t)$.
- The instability scale Λ_I is larger when determined by λ_{eff},both in the SM and in the SDFDM model.

Using the method of derivative expansion, we have studied the quantum correction to the effective action.
Using the RG improved kinetic term and the RG improved effective potential, we calculate the decay rate of the false vacuum.

- The factor arising from the anomalous dimension which appears in the kinetic term and the effective potential cancels in the decay rate.
- The one-loop effect on effective potential slightly enlarges the parameter space for the vacuum to be metastable.

Thanks!

Threshold effect

- changing the mass scale of dark matter particles does not give rise to change of the initial parameters as significant as changing Yukawa couplings.

Effects of different masses on initial values

$\mu = M_t$	λ	y_t	g_2	g_Y
$M_S = M_D = 800$ GeV	0.12564	0.93402*	0.64599	0.35650
$M_S = M_D = 1000$ GeV	0.12554	0.93368*	0.64574	0.35630
$M_S = M_D = 1200$ GeV	0.12546	0.93340*	0.64552	0.35613

Table 2: $y_1 = y_2 = 0.35$ for all three cases with different masses of the new particles.

Initial values in $\overline{MS}$ scheme for RGE running

$\mu = M_t$	λ	y_t	g_2	g_Y
$\mathrm{SM_{LO}}$	0.12917	0.99561	0.65294	0.34972
$\mathrm{SM_{NNLO}}$	0.12604	0.93690*	0.64779	0.35830
$\mathrm{SDFDM_{NLO}^{BMP1}}$	0.12549	0.93526*	0.64573	0.35752
$\mathrm{SDFDM_{NLO}^{BMP2}}$	0.12554	0.93368*	0.64574	0.35630
$\mathrm{SDFDM_{NLO}^{BMP3}}$	0.12586	0.93269*	0.64573	0.35553
$\mathrm{SDFDM_{NLO}^{BMP4}}$	0.13126	0.92744*	0.64573	0.35144

Table 3: All the parameters are renormalized at the top pole mass(M_t) scale in the $\overline{MS}$ scheme. BMP1: $y_1 = y_2 = 0.25$, $M_S = 1000$ GeV, $M_D = 1000$ GeV; BMP2: $y_1 = y_2 = 0.35$, $M_S = 1000$ GeV, $M_D = 1000$ GeV; BMP3: $y_1 = y_2 = 0.4$, $M_S = 1000$ GeV, $M_D = 1000$ GeV; BMP4: $y_1 = y_2 = 0.6$, $M_S = 1000$ GeV, $M_D = 1000$ GeV; The superscript $*$ indicates that the NNNLO pure QCD effects are also included. BMPs means benchmark points.

The values of coefficients N_i , κ_i , and c_i appearing in Eq. (2.7) are listed in Table. 4.

p	t	W	Z	h	G^+	G_0	$C^\pm$	C_Z
N_i	-12	6	3	1	2	1	-2	-1
c_i	$\frac{3}{2}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
κ_i	$\frac{y_t^2}{2}$	$\frac{g^2}{4}$	$\frac{g^2+g'^2}{4}$	3λ	$\lambda + \frac{\bar{\xi}_W g^2}{4}$	$\lambda + \frac{\bar{\xi}_Z (g^2+g')}{4}$	$\frac{\bar{\xi}_W g^2}{4}$	$\frac{\bar{\xi}_W (g^2+g'}{4}$

Table 4: The coefficients in Eq. (2.7) for the background R_ξ gauge . $\bar{\xi}_W$ and $\bar{\xi}_Z$ are the gauge-fixing parameters in the background R_ξ gauge, G^+ and G^0 the goldstone bosons, $C^\pm$ and C_Z the ghost fields, For $\bar{\xi}_W = \bar{\xi}_Z = 0$, Eq. (2.7) reproduces the one-loop result in the Landau gauge, and for $\bar{\xi}_W = \bar{\xi}_Z = 1$, we get the result in the 't Hooft-Feynman gauge.

Quantum corrections the tunnelling rate

- In Eq. (3.4),R is an arbitrary scale,bacause the potential is scale-invariant at tree-level.There is an infinite set of bounces of different size R that lead to the same action.
- The λ in Eq. (3.4) is a running parameter which depend on the RGE scale μ.

Which value of R and of the RGE scale μ should we use?

- Both ambiguities are solved by performing a complete one-loop calculation of the tunnelling rate.
- The one-loop corrections has been calculated first by Gino Isidori then by So Chigusa,Takeo Moroi,and Yutaro Shoji.

$$\gamma^{(\text{one-loop})} \propto \int d\ln R \frac{1}{R^4} \exp\left[-\frac{8\pi^2}{3|\lambda(\mu)|} - \frac{8\pi^2\beta_\lambda^{(1)}}{3|\lambda(\mu)|^2}\ln \mu R\right] \tag{4.1}$$

where $\beta_\lambda^{(1)}$ is the one-loop β function of λ.

Quantum corrections the tunnelling rate

- the main effect of quantum fluctuations is the breaking of scale invariance of the tree-level potential.
- As shown in Eq. (4.1), the one-loop terms cancel the μ dependence of λ in the leading term.
- The μ dependence shows up in the form of $ln\mu R$ in the one-loop result.
- In order to make our oneloop result reliable, we should take $\mu \sim O(1/R)$.
- With such a choice of μ, the integration over the size of the bounce is dominated only by the region where $|\lambda(1/R)|$ becomes largest.

The transition amplitude from point x_i to point x_f can be expressed as:

$$\left\langle x_f \left| e^{-HT/\hbar} \right| x_i \right\rangle = N \int [dx] e^{-S/\hbar} \tag{4.2}$$

we can expanding the path about the classical solution $\bar{x}(t)$.

$$x(t) = \bar{x}(t) + \sum_n c_n x_n(t) \tag{4.3}$$

x_n are eigenfunctions of the second variational derivative of S at $\bar{x}$,

$$-\frac{d^2 x_n}{dt^2} + V''(\bar{x}) x_n = \lambda_n x_n \tag{4.4}$$

keep only the terms in the action which are quadratic in x_n and the integral is Gaussians,we have

$$\begin{aligned} \left\langle x_f \left| e^{-HT/\hbar} \right| x_i \right\rangle =& N e^{-S(\bar{x})/\hbar} \prod_n \lambda_n^{-1/2} [1 + O(\hbar)] \\ =& N e^{-S(\bar{x})/\hbar} \left\{ \det \left[-\partial_t^2 + V''(\bar{x}) \right] \right\}^{-1/2} \end{aligned} \tag{4.5}$$

However one of the modes has a zero eigenvalue($c_i(t) \propto \partial_t \bar{x}$).
The zero mode corresponds to the time translation invariant of the center of the bounce.

- replace the coefficient of the zero mode by a collective coordinate.

$$x(t) = \bar{x}\left(t - t_0\right) + \sum_{i=1}^{\infty} c_i x_i \left(t - t_0\right) \tag{4.6}$$

- include only the nonzero modes in the determinant.
- Instead of a Gaussian integral over c_0,we integral over center of the bounce t_0
- the intergratal introduces a Jacobian factor:$(B/2\pi)^{1/2}$

Quantum corrections the tunnelling rate,cancellation

the A factor in the expression of the decay rate becomes:

$$A = \frac{S_c^2 l}{4\pi^2} \left| \frac{\det'[-e^{2\Gamma} Z_2 \partial^2 + 3\tilde{\lambda} e^{4\Gamma} \tilde{\phi}_B^2]}{\det[-e^{2\Gamma} Z_2 \partial^2]} \right|^{-1/2} \tag{4.7}$$

where the $(S_c l)^2$ factor in Eq. (4.7),which comes from the zero mode contribution,becomes $[8\pi^2/(3|\tilde{\lambda}|e^{2\Gamma}/Z_2)]^2$. The ratio of determinant in Eq. (4.7) equals to $\left|\det'[-\partial^2 + 3(\tilde{\lambda}/Z_2)e^{2\Gamma}\tilde{\phi}_B^2]/\det[-\partial^2]\right|^{-1/2} \times (e^{2\Gamma} Z_2)^2$ when including effects omitting four zero modes. It's easy to see that if taking $\phi_B = e^{\Gamma}\tilde{\phi}_B$ the non-zero eigenvalues of operator $-\partial^2 + 3(\tilde{\lambda}/Z_2)e^{2\Gamma}\tilde{\phi}_B^2$ for $\tilde{\phi}_B$ satisfying Eq. (**??**) would be the same of the operator $-\partial^2 + 3(\tilde{\lambda}/Z_2)\phi_B^2$ for ϕ_B satisfying

$$-Z_2\, \partial^2 \phi_B + \widetilde{\lambda}\, \phi_B^3 = 0. \tag{4.8}$$

So eventually we find that the decay rate is again expressed by Eq. (**??**) but with S_{cl} expressed by Eq. (**??**) and with

$$A = \frac{S_{cl}^2}{4\pi^2} \left| \frac{\det'[-\partial^2 + 3(\tilde{\lambda}/Z_2)\phi_B^2]}{\det[-\partial^2]} \right|^{-1/2}, \tag{4.9}$$

in which ϕ_B satisfies Eq. (4.8).

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