

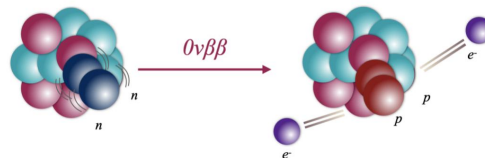
# Impact of different nuclear matrix element calculations on the interpretation of current and future $0\nu\beta\beta$ experiments

**Jing-Yu Zhu (朱景宇)**

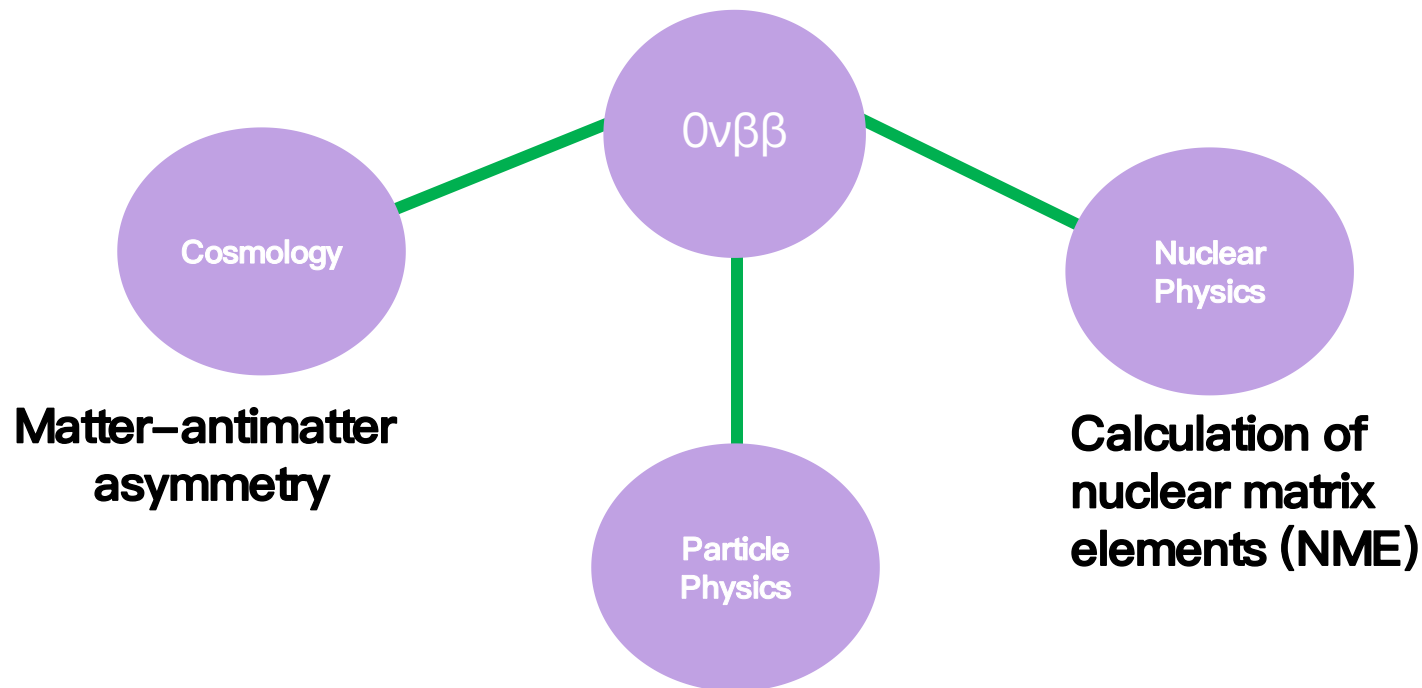
**Institute of Modern Physics**

**Based on the work with Prof. Thomas Schwetz and Federica Popma,**

**JHEP 06 (2023) 104**



# Big picture

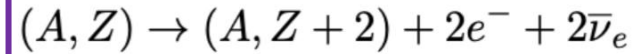
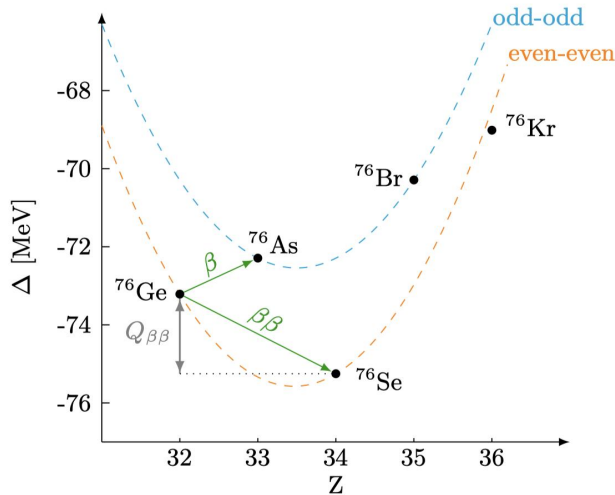


The production mechanism BSM

## Experiments:

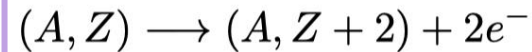
- Find compromises between nature abundance, Q-value, priced enrichment and detector techniques
- Key parameter: background, exposure, energy resolution

# Brief background



Mayer, 1935; first detected in 1987 by Moe

$$\nu_i^c = \nu_i \quad \text{Majorana, 1937}$$



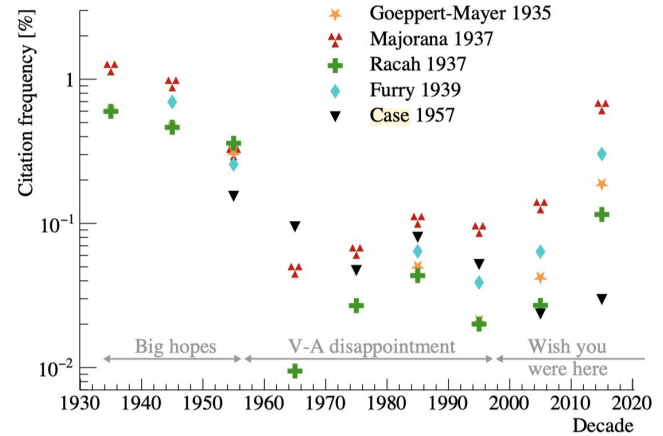
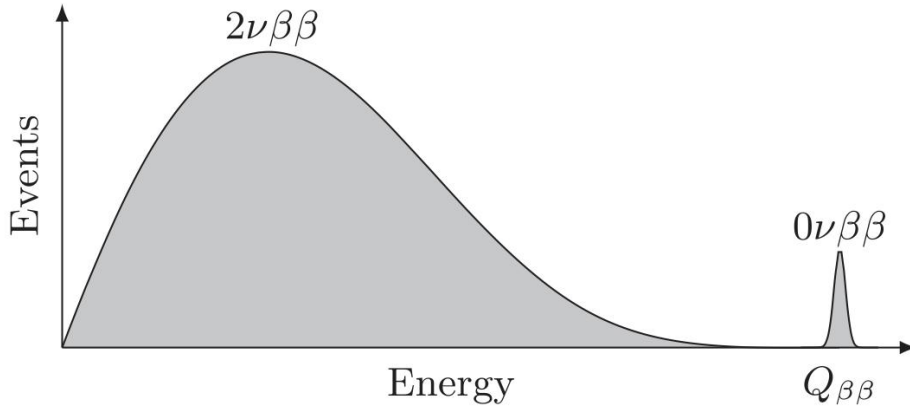
Furry, 1939

Isotope	Daughter	$Q_{\beta\beta}$ (keV) <sup>a</sup>	$f_{\text{nat}}$ (%) <sup>b</sup>	$f_{\text{enr}}$ (%) <sup>c</sup>	$T_{1/2}^{2\nu\beta\beta}$ (yr) <sup>d</sup>	$T_{1/2}^{0\nu\beta\beta}$ (yr) <sup>e</sup>
<sup>48</sup> Ca	<sup>48</sup> Ti	4267.98(32)	30.187(21)	16	$[6.4^{+0.7}_{-0.6}(\text{stat})^{+1.2}_{-0.9}(\text{syst})] \times 10^{19}$	$> 5.8 \times 10^{22}$
<sup>76</sup> Ge	<sup>76</sup> Se	2039.061(7)	37.75(12)	92	$(1.926 \pm 94) \times 10^{21}$	$> 1.8 \times 10^{26}$
<sup>82</sup> Se	<sup>82</sup> Kr	2997.9(3)	38.82(15)	96.3	$[8.60 \pm 0.03(\text{stat})^{+0.19}_{-0.13}(\text{syst})] \times 10^{19}$	$> 3.5 \times 10^{24}$
<sup>96</sup> Zr	<sup>96</sup> Mo	3356.097(86)	32.80(2)	86	$[2.35 \pm 0.14(\text{stat}) \pm 0.16(\text{syst})] \times 10^{19}$	$> 9.2 \times 10^{21}$
<sup>100</sup> Mo	<sup>100</sup> Ru	3034.40(17)	39.744(65)	99.5	$[7.12^{+0.18}_{-0.14}(\text{stat}) \pm 0.10(\text{syst})] \times 10^{18}$	$> 1.5 \times 10^{24}$
<sup>116</sup> Cd	<sup>116</sup> Sn	2813.50(13)	37.512(54)	82	$2.63^{+0.11}_{-0.12} \times 10^{19}$	$> 2.2 \times 10^{23}$
<sup>130</sup> Te	<sup>130</sup> Xe	2527.518(13)	34.08(62)	92	$[7.71^{+0.08}_{-0.06}(\text{stat})^{+0.12}_{-0.15}(\text{syst})] \times 10^{20}$	$> 2.2 \times 10^{25}$
<sup>136</sup> Xe	<sup>136</sup> Ba	2457.83(37)	38.857(72)	90	$[2.165 \pm 0.016(\text{stat}) \pm 0.059(\text{syst})] \times 10^{21}$	$> 1.1 \times 10^{26}$
<sup>150</sup> Nd	<sup>150</sup> Sm	3371.38(20)	35.638(28)	91	$[9.34 \pm 0.22(\text{stat})^{+0.62}_{-0.60}(\text{syst})] \times 10^{18}$	$> 2.0 \times 10^{22}$

**10<sup>18</sup> yr – 10<sup>21</sup> yr**

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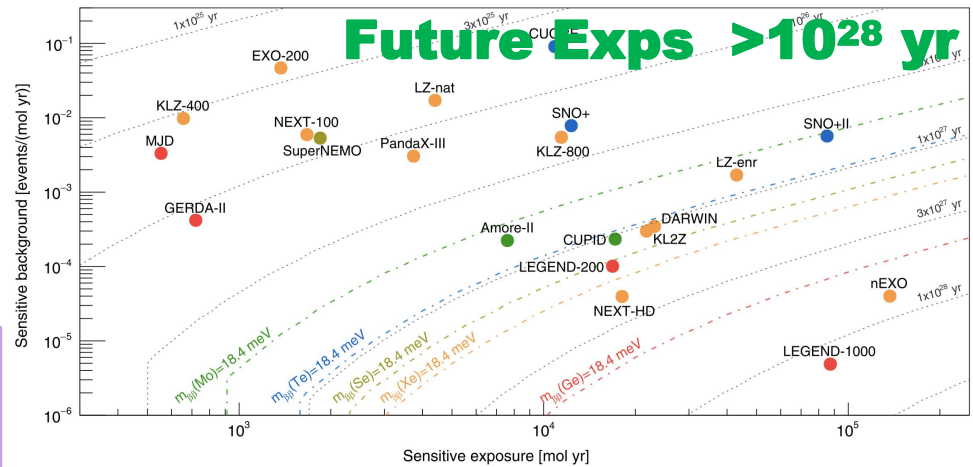
# Brief background



PHYSICAL REVIEW LETTERS 130, 051801 (2023)  
 Editors' Suggestion Featured in Physics  
 **$^{136}\text{Xe} > 10^{26}$  yr**  
 Search for the Majorana Nature of Neutrinos in the Inverted Mass Ordering Region with KamLAND-Zen  
 S. Abe,<sup>1</sup> S. Asami,<sup>1</sup> M. Eizuka,<sup>1</sup> S. Futagi,<sup>1</sup> A. Gando,<sup>1</sup> Y. Gando,<sup>1</sup> T. Gima,<sup>1</sup> A. Goto,<sup>1</sup> T. Hachiya,<sup>1</sup> K. Hata,<sup>1</sup>

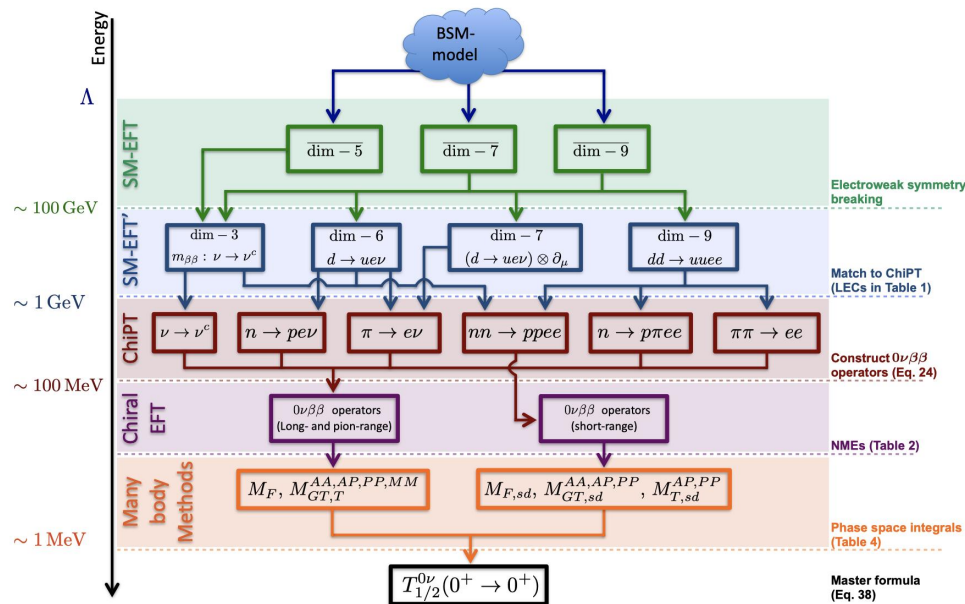
Article  
**Search for Majorana neutrinos exploiting millikelvin cryogenics with CUORE**  
 **$^{130}\text{Te} > 10^{26}$  yr**  
<https://doi.org/10.1038/s41586-022-04497-4> The CUORE Collaboration\*  
 Received: 14 April 2021

PHYSICAL REVIEW LETTERS 125, 252502 (2020)  
 Editors' Suggestion Featured in Physics  
 **$^{76}\text{Ge} > 10^{26}$  yr**  
 Final Results of GERDA on the Search for Neutrinoless Double- $\beta$  Decay  
 M. Agostini,<sup>9,17</sup> G. R. Araujo,<sup>21</sup> A. M. Bakalyarov,<sup>15</sup> M. Balata,<sup>1</sup> I. Barabanov,<sup>13</sup> L. Baudis,<sup>21</sup> C. Bauer,<sup>8</sup> E. Bellotti,<sup>10,11</sup>  
 S. Belogurov,<sup>14,13,1</sup> A. Bettini,<sup>18,19</sup> L. Bezrukov,<sup>13</sup> V. Biancacci,<sup>18,19</sup> D. Borowicz,<sup>6</sup> E. Bossio,<sup>17</sup> V. Bothe,<sup>8</sup> V. Brudanin,<sup>6</sup>



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# A general theoretical framework



Cirigliano et al., JHEP 2018

Naive Dimensional Analysis(NDA) problem

- D. B. Kaplan, M. J. Savage, and M. B. Wise, *Nucl. Phys.* **B478**, 629 (1996).
- S. R. Beane, P. F. Bedaque, M. J. Savage, and U. van Kolck, *Nucl. Phys.* **A700**, 377 (2002).
- A. Nogga, R. G. E. Timmermans, and U. van Kolck, *Phys. Rev. C* **72**, 054006 (2005).
- B. Long and C.-J. Yang, *Phys. Rev. C* **86**, 024001 (2012).
- M. Pavón Valderrama and D. R. Phillips, *Phys. Rev. Lett.* **114**, 082502 (2015).

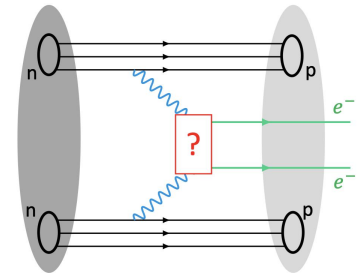
and  $F_\pi = 92.2$  MeV is the pion decay constant. However, it is known that Weinberg's power counting leads to inconsistent results in nucleon-nucleon scattering [34–37] and nuclear processes mediated by external currents [38], due to a conflict between naive dimensional analysis and nonperturbative renormalization. We therefore investigate the scaling of  $g_\nu^{NN}$  by studying the amplitude  $\mathcal{A}(nn \rightarrow ppe e) \equiv \mathcal{A}_{\Delta L=2}$  with strong interactions  $H_{\text{strong}}$  included nonperturbatively.

Cirigliano et al, Phys.Rev.Lett. 120 (2018) 20, 202001

# Theoretical mechanism → which one dominates?

mechanism	amplitude and particle physics parameter	current limit	test
light neutrino exchange	$\frac{G_F^2}{q^2}  U_{ei}^2 m_i $	0.5 eV	oscillations, cosmology, neutrino mass
heavy neutrino exchange	$G_F^2 \frac{S_{ei}^2}{M_i^2}$	$2 \times 10^{-8} \text{ GeV}^{-1}$	LFV, collider
heavy neutrino and RHC	$G_F^2 m_W^4 \frac{V_{ei}^2}{M_i M_{WR}^4}$	$4 \times 10^{-16} \text{ GeV}^{-5}$	flavor, collider
Higgs triplet and RHC	$G_F^2 m_W^4 \left  \frac{(M_R)_{ee}}{m_{\Delta_R}^2 M_{WR}^4} \right $	$10^{-15} \text{ GeV}^{-1}$	flavor, collider $e^-$ distribution
$\lambda$ -mechanism with RHC	$G_F^2 \frac{m_W^2}{q} \left  \frac{U_{ei} \tilde{S}_{ei}}{M_{WR}^2} \right $	$1.4 \times 10^{-10} \text{ GeV}^{-2}$	flavor, collider, $e^-$ distribution
$\eta$ -mechanism with RHC	$G_F^2 \frac{1}{q} \tan \zeta \left  U_{ei} \tilde{S}_{ei} \right $	$6 \times 10^{-9}$	flavor, collider, $e^-$ distribution
short-range $\mathcal{R}$	$\frac{ \lambda'_{111} }{\Lambda_{\text{SUSY}}^5}$ $\Lambda_{\text{SUSY}} = f(m_{\tilde{g}}, m_{\tilde{u}_L}, m_{\tilde{d}_R}, m_{\chi_i})$	$7 \times 10^{-18} \text{ GeV}^{-5}$	collider, flavor
long-range $\mathcal{R}$	$\frac{G_F}{q} \left  \sin 2\theta^b \lambda'_{131} \lambda'_{113} \left( \frac{1}{m_{\tilde{b}_1}^2} - \frac{1}{m_{\tilde{b}_2}^2} \right) \right $ $\sim \frac{G_F}{q} m_b \frac{ \lambda'_{131} \lambda'_{113} }{\Lambda_{\text{SUSY}}^3}$	$2 \times 10^{-13} \text{ GeV}^{-2}$ $1 \times 10^{-14} \text{ GeV}^{-3}$	flavor, collider
Majorons	$\propto  \langle g_\chi \rangle  \text{ or }  \langle g_\chi \rangle ^2$	$10^{-4} \dots 1$	spectrum, cosmology

Rodejohann, *Int.J.Mod.Phys.E* 20 (2011)

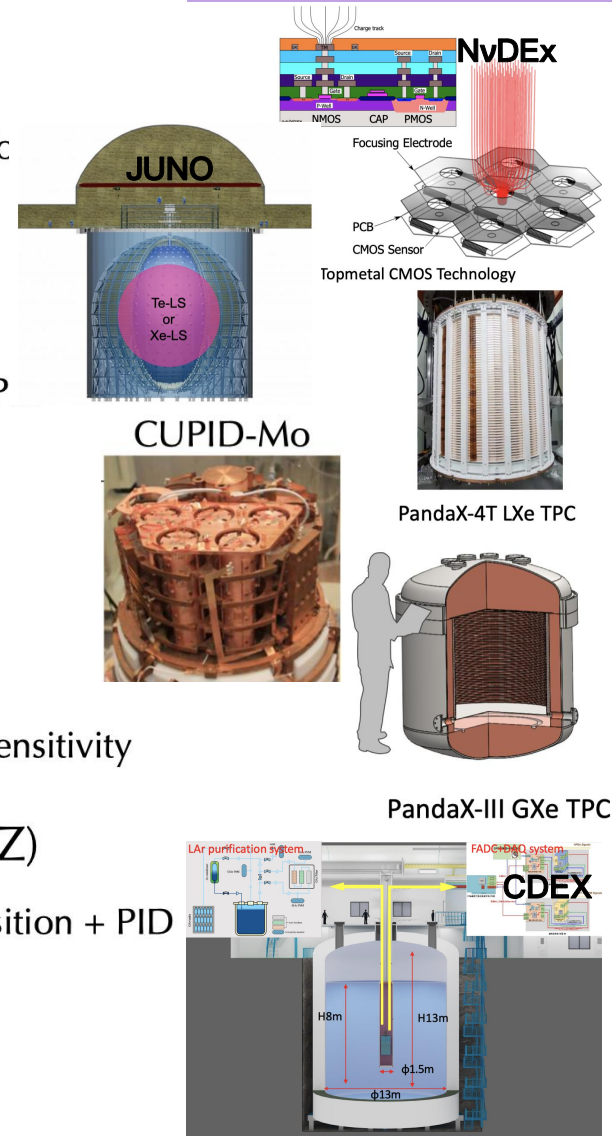


Phase factor + nuclear matrix element ? + new physics parameter ? (effective neutrino mass)

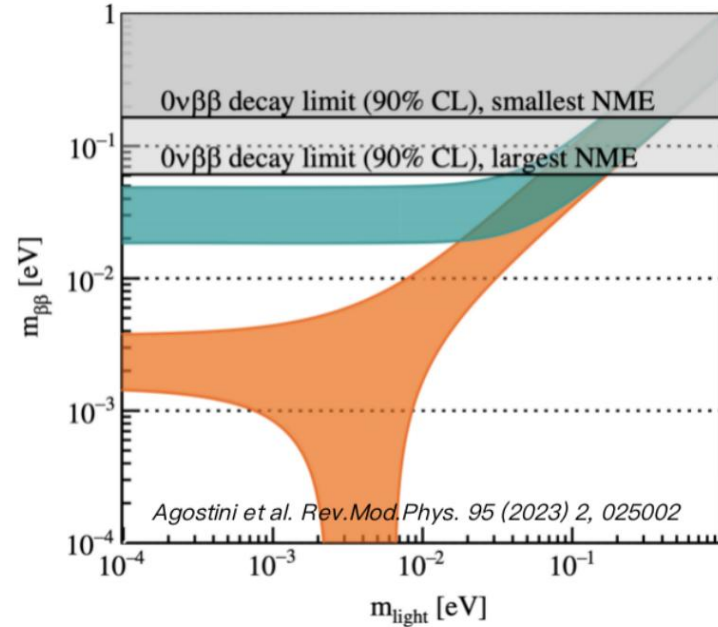
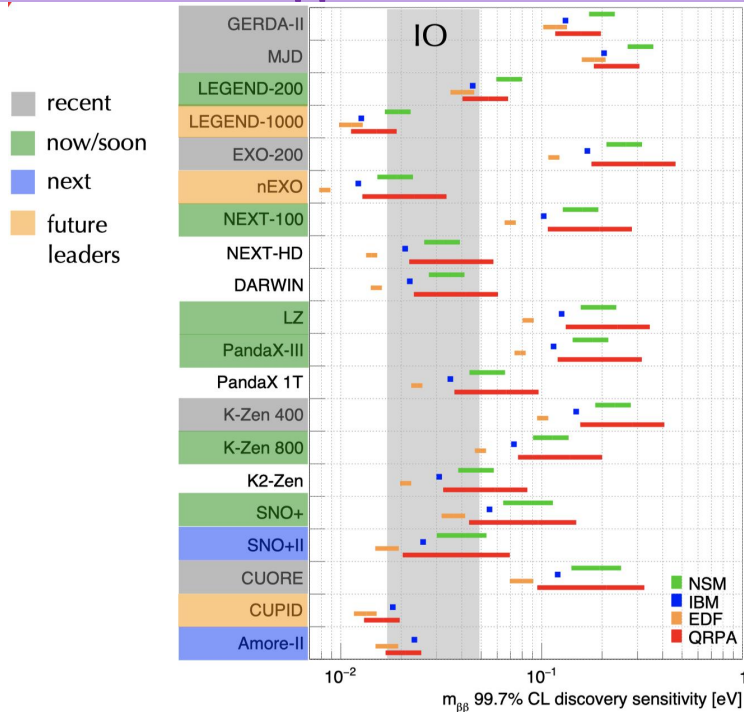


# Experimental techniques @CJPL

- Bolometers (CUPID, AMoRE, CANDLES IV)
  - Measure  $E$  ( $\sigma \sim 0.1-0.3\%$ ) from phonons; granularity gives position info
  - Instrumenting with photon detectors for background rejection
- External trackers (SuperNEMO)
  - Trackers + calorimeters, measure  $E$  ( $\sigma \sim 3-10\%$ ) + tracks / positions + P
- Scintillators (KamLAND2-Zen, SNO+, Theia, ZICOS)
  - Measure  $E$  ( $\sigma \sim 3-10\%$ ) + position from scintillation light; some PID
- Semiconductors (LEGEND, SELENA) CDEX
  - Measure  $E$  ( $\sigma \sim 0.05-0.3\%$ ) from ionization; some tracking / position sensitivity
- TPCs (nEXO, NEXT PandaX, AXEL, NvDEx, DARWIN, LZ)
  - Collect scintillation + ionization: measure  $E$  ( $\sigma \sim 0.4-3\%$ ) + tracks / position + PID



# NME $\rightarrow$ $m_{\beta\beta}$



New techniques and more exposure are being pursued to take us beyond the IO. Discovery could come at any time!

**Motivations:** Schwetz, Popma, Zhu, JHEP 06 (2023) 104

- Interpreting the constraints/sensitivities on  $m_{\beta\beta}$  of current/future  $0\nu\beta\beta$  experiments
- Checking the possibilities of discriminating NME models in future  $0\nu\beta\beta$  experiments



# Formula (light neutrino exchange mechanism)

$$(T_{1/2}^{-1})_{\alpha} = \tilde{\Gamma}_{\alpha}(m_{\beta\beta}, M_{\alpha i}) = \frac{\Gamma_{\alpha}(m_{\beta\beta}, M_{\alpha i})}{\ln 2} = G_{\alpha} |M_{\alpha i}|^2 m_{\beta\beta}^2$$

$$m_{\beta\beta} = \left| \sum_j U_{ej}^2 m_j \right|$$

$$M_{\alpha i} = M_{\alpha i}^{\text{long}} + M_{\alpha i}^{\text{short}} = M_{\alpha i}^{\text{long}}(1 + n_{\alpha i}) \quad n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}}$$

$$g_A^{\text{eff}} = q g_A^{\text{free}} \quad g_A^{\text{free}} = 1.27$$

- **Quenching effect: correct the NME by  $q^2$  and the decay rate by  $q^4$**   
(Ab initio many-body theory, see **Yao's talk** later)
- Short-range NME: Contact operator suggested to contribute to light-neutrino exchange, *Cirigliano et al. PRL2018*
- We do not know neither the value or the sign of short-range NME well.
- Unknown value of the hadronic coupling  $g_{\nu}^{\text{NN}}$ , to be determined experimentally or Lattice QCD calculations

# Long-range NME

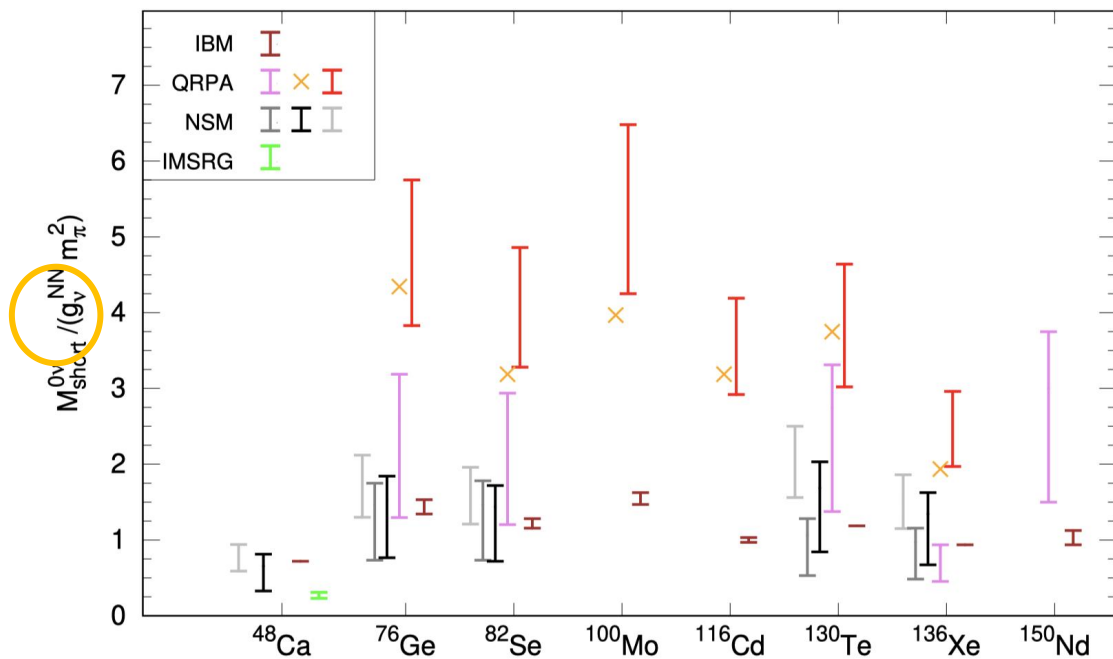
Nuclear Model	Index [Ref.]	$^{76}\text{Ge}$	$^{82}\text{Se}$	$^{100}\text{Mo}$	$^{130}\text{Te}$	$^{136}\text{Xe}$	
NSM	N1 [25]	2.89	2.73	-	2.76	2.28	
	N2 [25]	3.07	2.90	-	2.96	2.45	
	N3 [26]	3.37	3.19	-	1.79	1.63	
	N4 [26]	3.57	3.39	-	1.93	1.76	
	N5 [27, 28]	2.66	2.72	2.24	3.16	2.39	
QRPA	Q1 [29]	5.09	-	-	1.37	1.55	
	Q2 [30]	5.26	3.73	3.90	4.00	2.91	
	Q3 [31]	4.85	4.61	5.87	4.67	2.72	
	Q4 [32]	3.12	2.86	-	2.90	1.11	<b>Fang, Faessler, Simkovic, PRC2018</b>
	Q5 [32]	3.40	3.13	-	3.22	1.18	<b>Fang, Faessler, Simkovic, PRC2018</b>
	Q6 [33]	-	-	-	4.05	3.38	
EDF	E1 [34]	4.60	4.22	5.08	5.13	4.20	
	E2 [35]	5.55	4.67	6.59	6.41	4.77	
	E3 [36]	6.04	5.30	6.48	4.89	4.24	<b>Song, Yao, Ring, Meng, PRC2017</b>
IBM	I1 [37]	5.14	4.19	3.84	3.96	3.25	
	I2 [13]	6.34	5.21	5.08	4.15	3.40	

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# Short-range NME

$$n_{\alpha i} = \frac{M_{\alpha i}^{\text{short}}}{M_{\alpha i}^{\text{long}}}$$

Isotope	NSM %	QRPA %
$^{76}\text{Ge}$	15–42	32–73
$^{82}\text{Se}$	15–41	30–70
$^{100}\text{Mo}$	-	49–108
$^{130}\text{Te}$	17–47	34–77
$^{136}\text{Xe}$	17–47	30–70

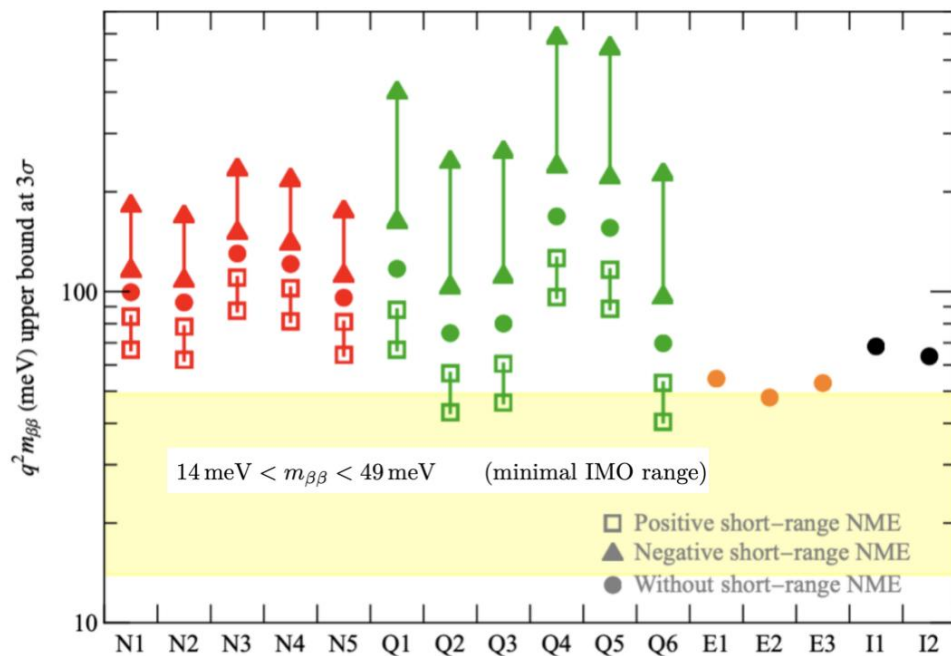
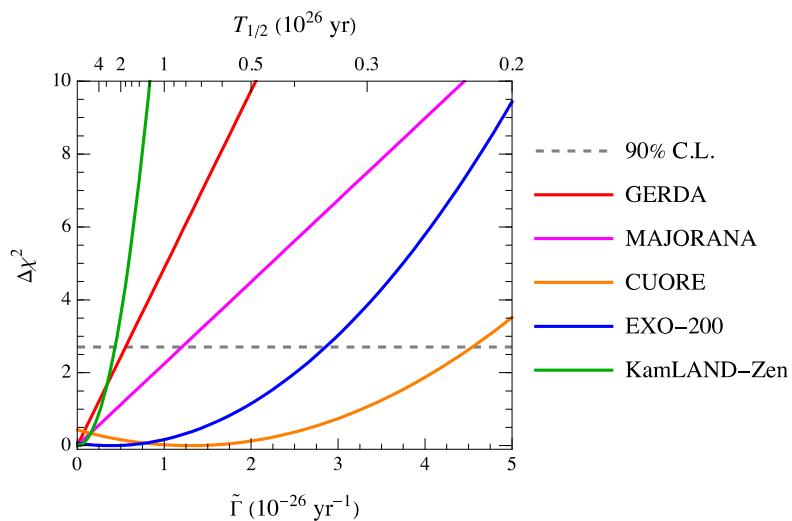


*Phys. Lett. B 823 (2021) 136720*

*Agostini et al. Rev.Mod.Phys. 95 (2023) 2, 025002*

# Current constraints

$$\Delta\chi_r^2(\tilde{\Gamma}_\alpha) = a_r (\tilde{\Gamma}_\alpha)^2 + b_r \tilde{\Gamma}_\alpha + c_r$$



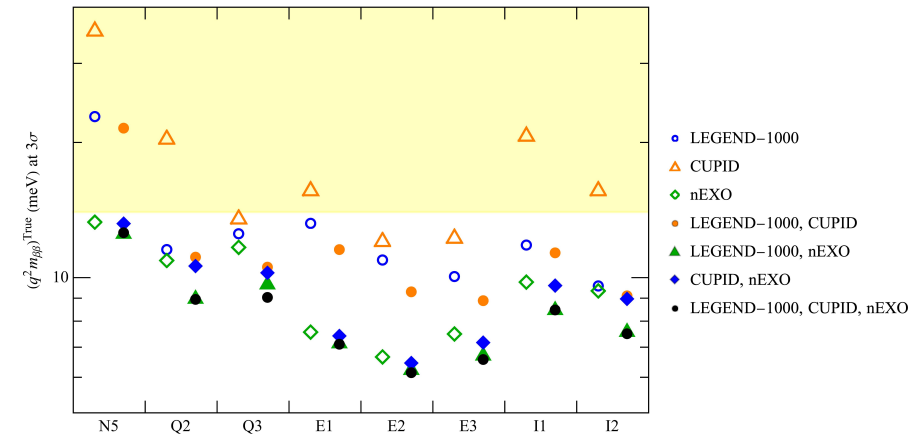
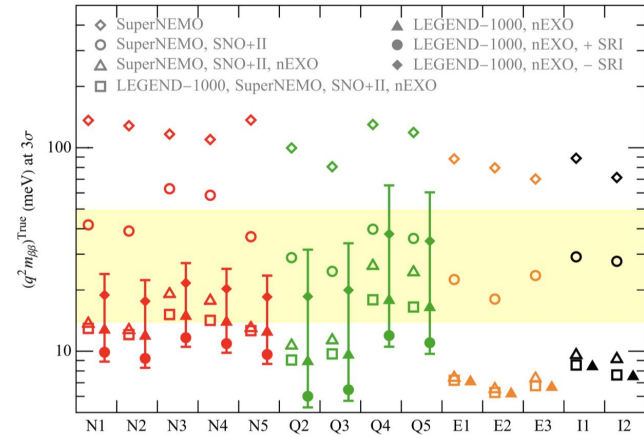
# Sensitivities to $(q^2 m_{\beta\beta})^{\text{True}}$ at $3\sigma$

Experiment	Isotope	$\varepsilon$ [mol·yr]	$b$ [events/(mol·y)]	PSF [yr <sup>-1</sup> eV <sup>-2</sup> ]
LEGEND-1000	<sup>76</sup> Ge	8736	$4.9 \cdot 10^{-6}$	$2.36 \cdot 10^{-26}$
SuperNEMO	<sup>82</sup> Se	185	$5.4 \cdot 10^{-3}$	$10.19 \cdot 10^{-26}$
CUPID	<sup>100</sup> Mo	1717	$2.3 \cdot 10^{-4}$	$15.91 \cdot 10^{-26}$
SNO-II	<sup>130</sup> Te	8521	$5.7 \cdot 10^{-3}$	$14.2 \cdot 10^{-26}$
nEXO	<sup>136</sup> Xe	13700	$4.0 \cdot 10^{-5}$	$14.56 \cdot 10^{-26}$

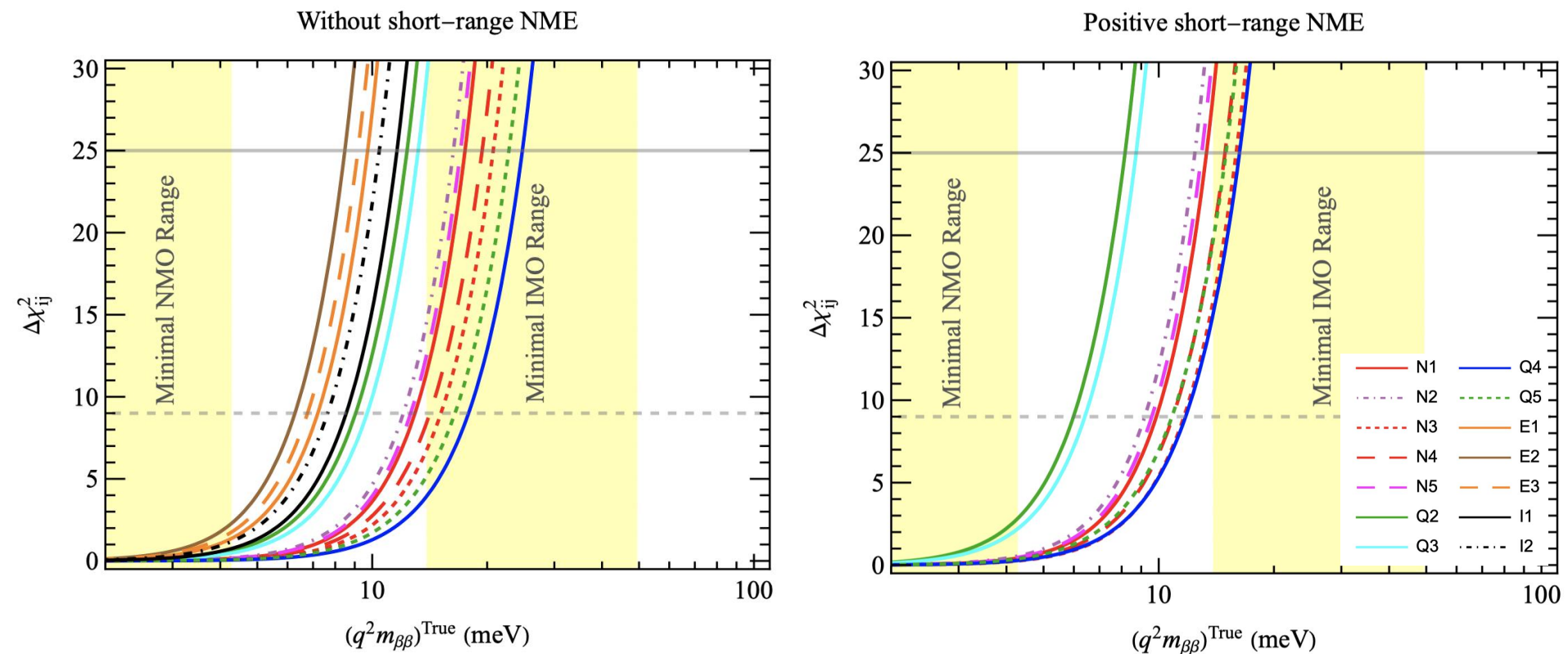
$$N_{\text{LEGEND-1000}} = \left\{ 0.97 \times \left[ \frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left( \frac{M_{\text{Ge}}^{\text{long}}}{2.66} \right)^2 + 0.04 \right\} \times \frac{T}{1 \text{ yr}}$$

$$N_{\text{SuperNEMO}} = \left\{ 0.09 \times \left[ \frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left( \frac{M_{\text{Se}}^{\text{long}}}{2.72} \right)^2 + 1.0 \right\} \times \frac{T}{1 \text{ yr}}$$

$$N_{\text{nEXO}} = \left\{ 1.64 \times \left[ \frac{(q^2 m_{\beta\beta})^{\text{True}}}{40 \text{ meV}} \right]^2 \left( \frac{M_{\text{Xe}}^{\text{long}}}{1.11} \right)^2 + 0.5 \right\} \times \frac{T}{1 \text{ yr}}$$



# The significance of observing one positive signal:



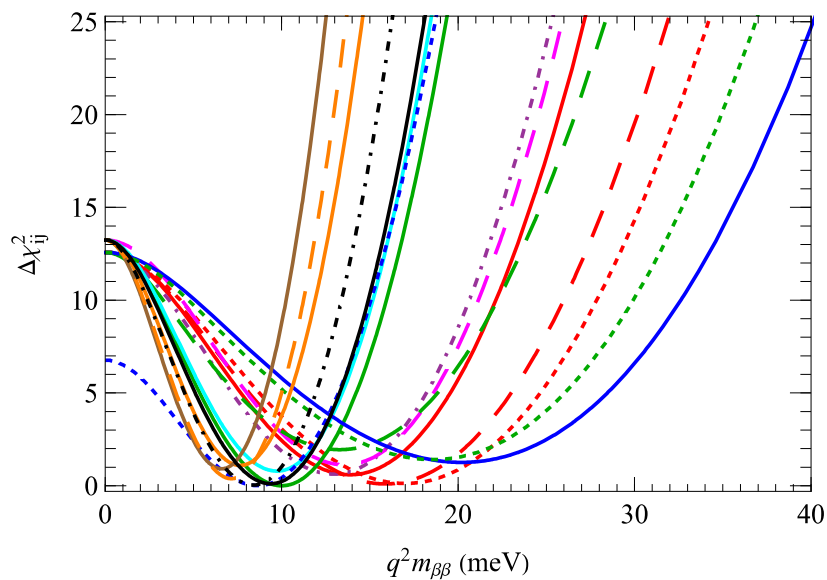
$m_{\beta\beta}=0, T=10 \text{ yr}$



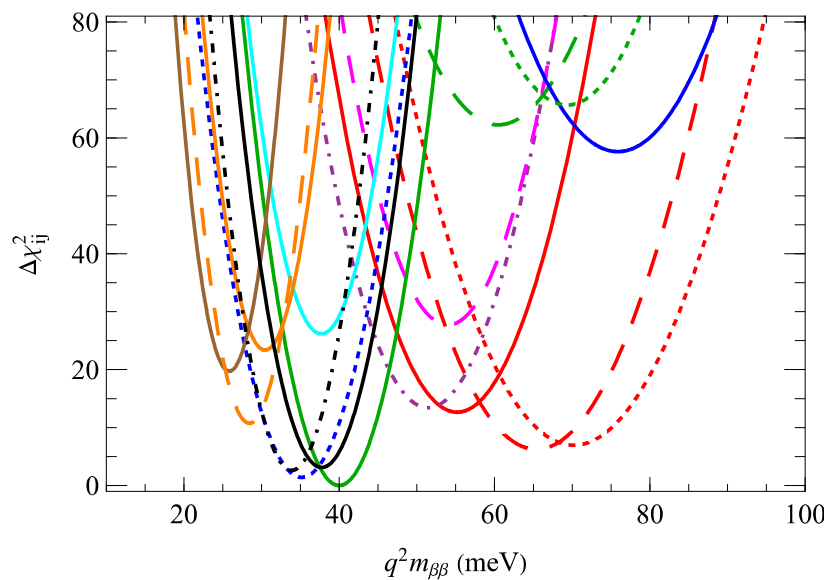
# $(\Delta\chi_{ij}^2)$ as function of $q^2 m_{\beta\beta}$

$i=Q2$

— N1    - - - N2    - - - N3    - - - N4    - - - N5    - - - Q1    — Q2    — Q3  
— Q4    - - - Q5    - - - Q6    — E1    — E2    - - - E3    — I1    - - - I2



$(q^2 m_{\beta\beta})_{True} = 10 \text{ meV}$



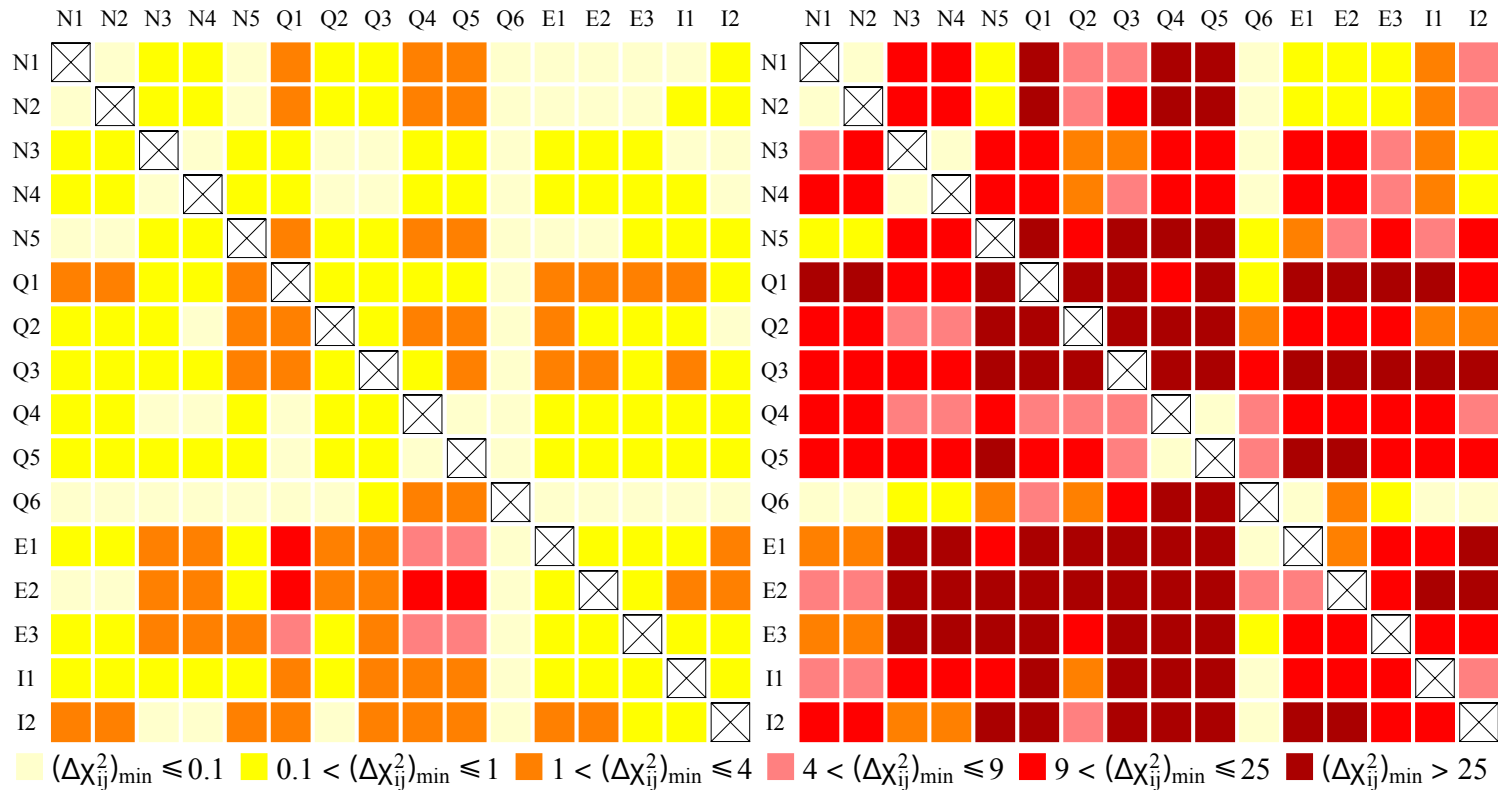
$(q^2 m_{\beta\beta})_{True} = 40 \text{ meV}$

# Discrimination without short-range NME

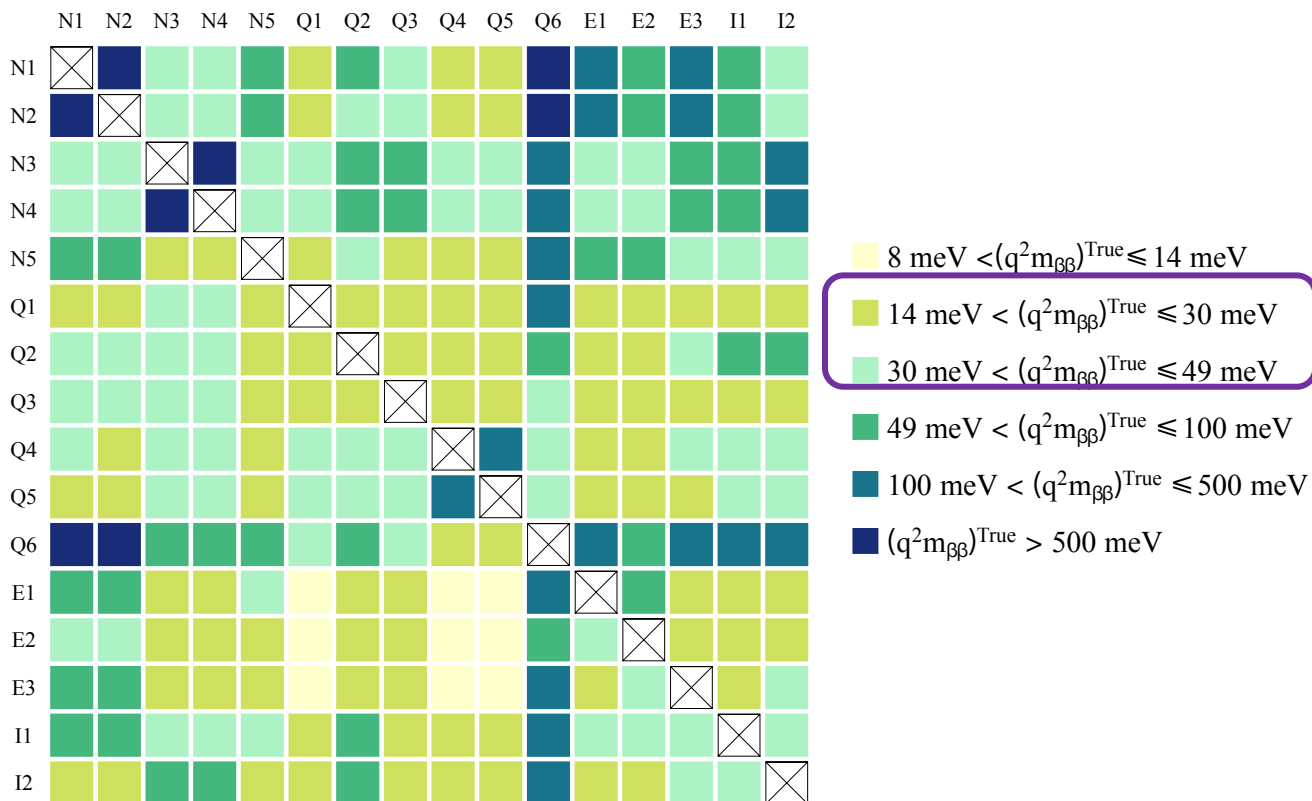
$$(\Delta\chi_{ij}^2)_{\min} = \min_{m_{\beta\beta}} \Delta\chi_{ij}^2(m_{\beta\beta}, M_{\alpha j}; (q^2 m_{\beta\beta})^{\text{True}}, M_{\alpha i}^{\text{True}})$$

$(q^2 m_{\beta\beta})^{\text{True}} = 10 \text{ meV}$

$(q^2 m_{\beta\beta})^{\text{True}} = 40 \text{ meV}$

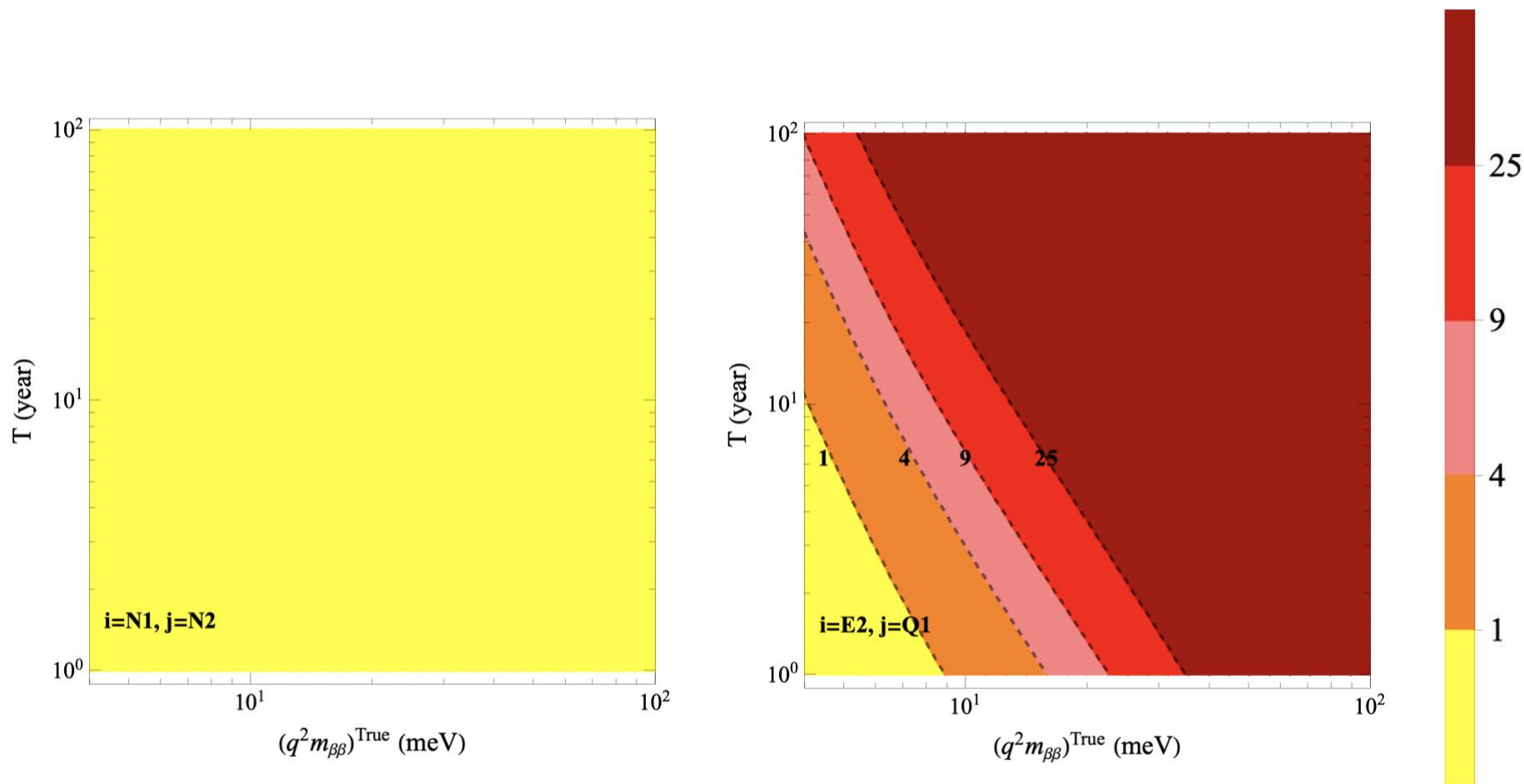


# $m_{\beta\beta}^{\text{True}}$ corresponding to discrimination at $3\sigma$



**(without short- range NME)**

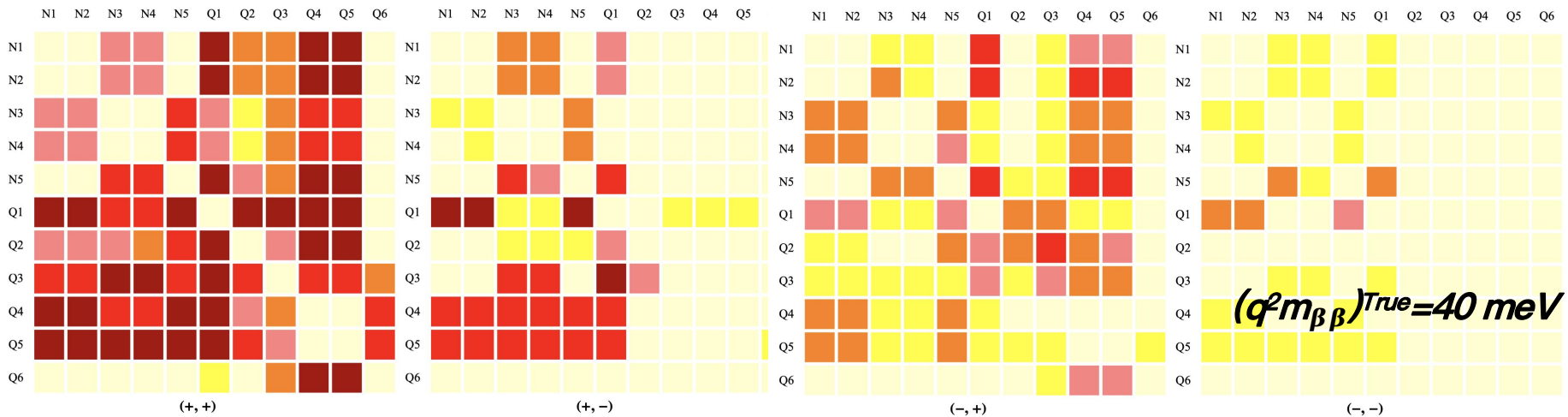
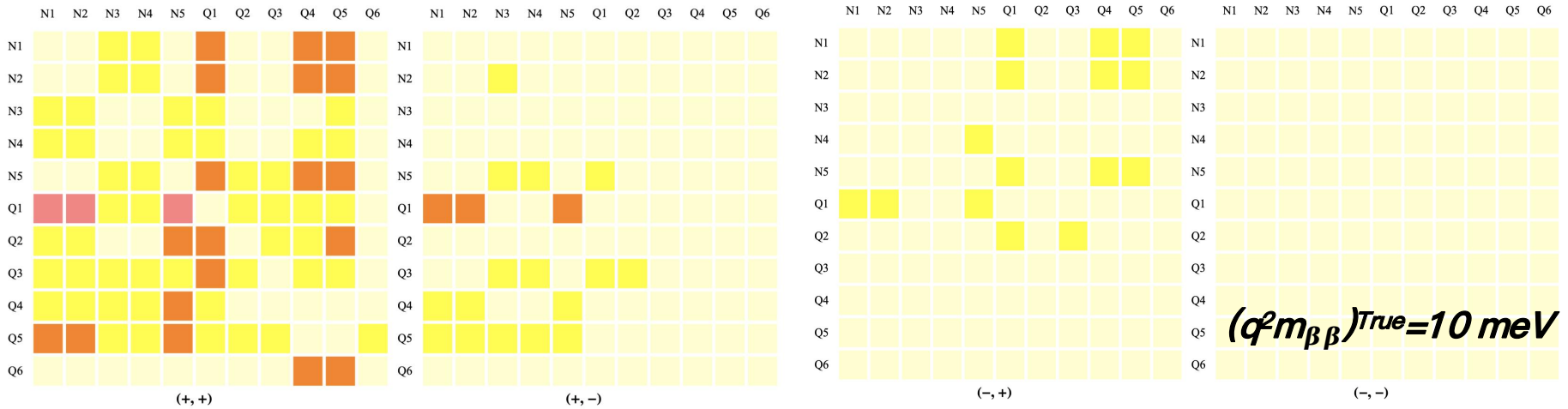
# Contours of $(\Delta\chi_{ij}^2)_{\min}$ as function of $T$ and $(q^2m_{\beta\beta})^{\text{True}}$



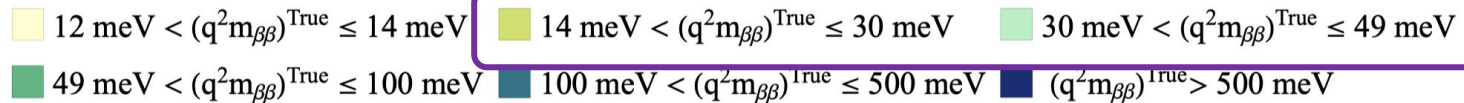
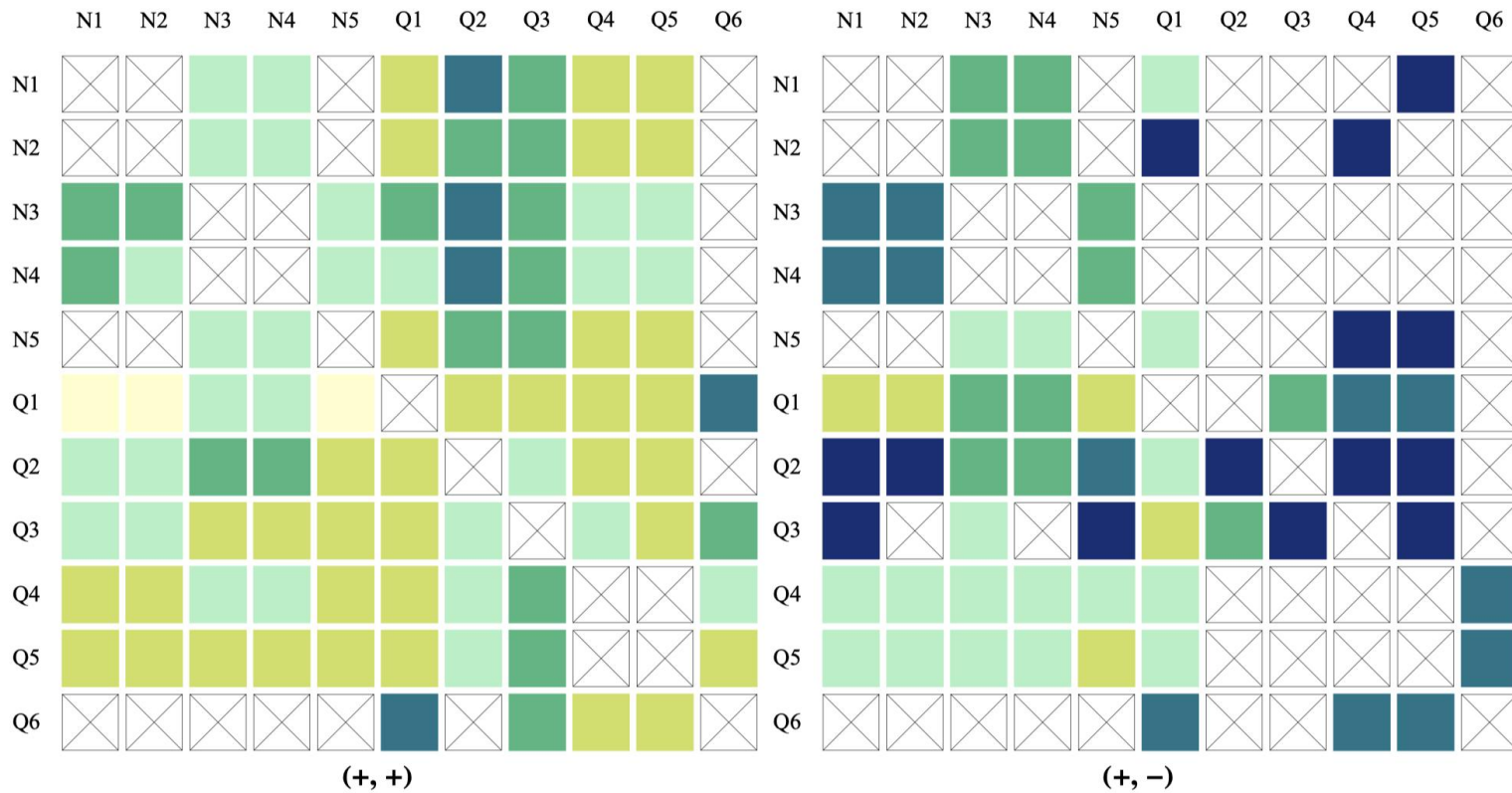
**(without short- range NME)**

# Discrimination with short-range NME, $T=10$ yr

$(\Delta\chi_{ij}^2)_{\min} \leq 0.1$ 
  $0.1 < (\Delta\chi_{ij}^2)_{\min} \leq 1$ 
  $1 < (\Delta\chi_{ij}^2)_{\min} \leq 4$ 
  $4 < (\Delta\chi_{ij}^2)_{\min} \leq 9$ 
  $9 < (\Delta\chi_{ij}^2)_{\min} \leq 25$ 
  $(\Delta\chi_{ij}^2)_{\min} > 25$



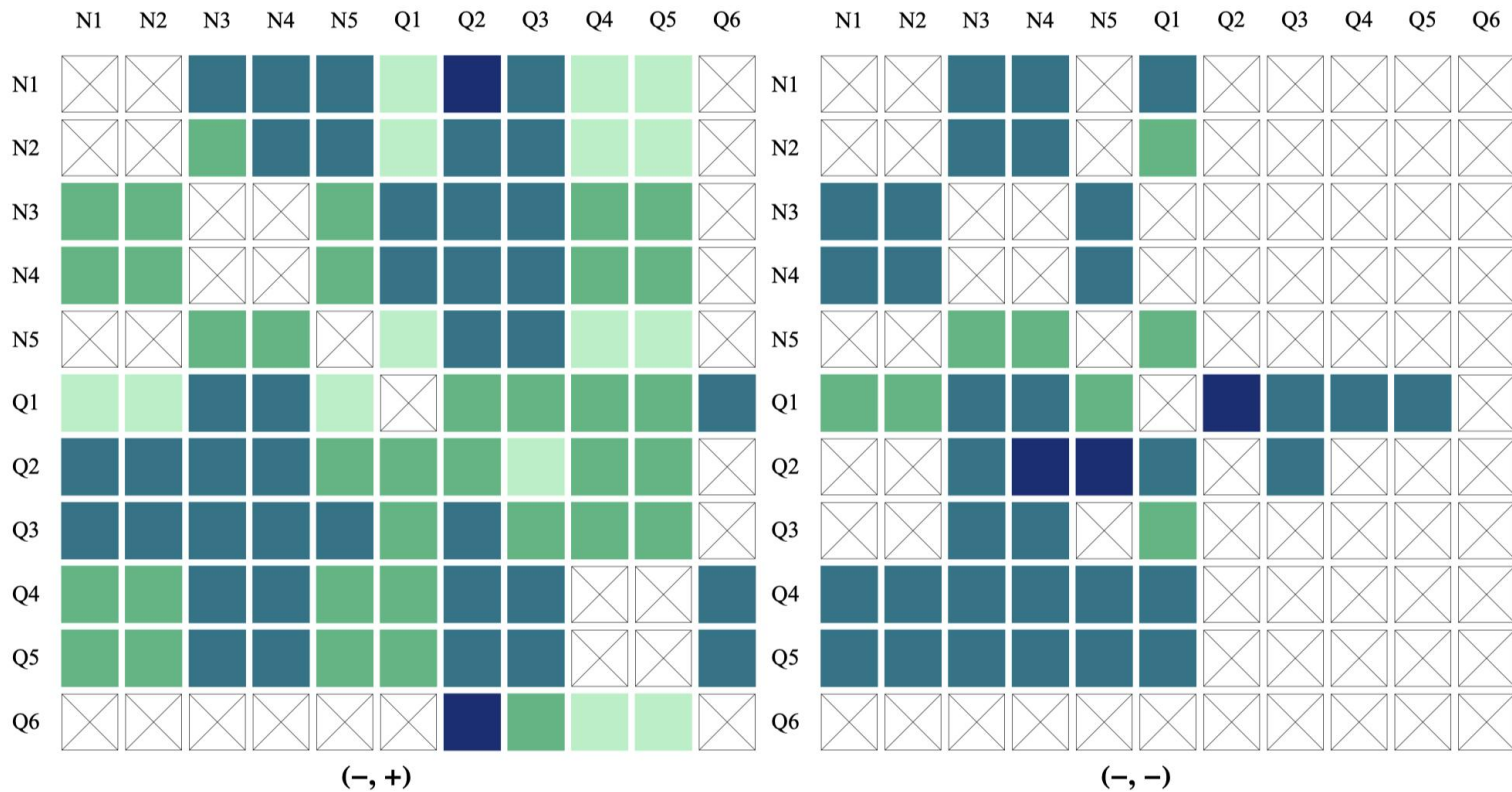
# $m_{\beta\beta}^{\text{True}}$ corresponding to discrimination at $3\sigma$



**(with short-range NME)**



# $m_{\beta\beta}^{\text{True}}$ corresponding to discrimination at $3\sigma$



**(with short-range NME)**

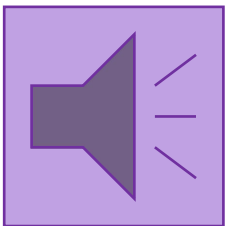
# Conclusions

- **NME uncertainties due to the SRI may lead to the bound on  $q^2 m_{\beta\beta}$  varying by a factor of order 10**
- **Promising discrimination of different NMEs if  $(q^2 m_{\beta\beta})^{\text{True}} > 40 \text{ meV}$ , positive SRI and 10 year exposure**
- **Similar analysis can be performed in the case of other  $0\nu\beta\beta$  production mechanism** (*Phys.Rev.D 108 (2023) 5, 055023*)

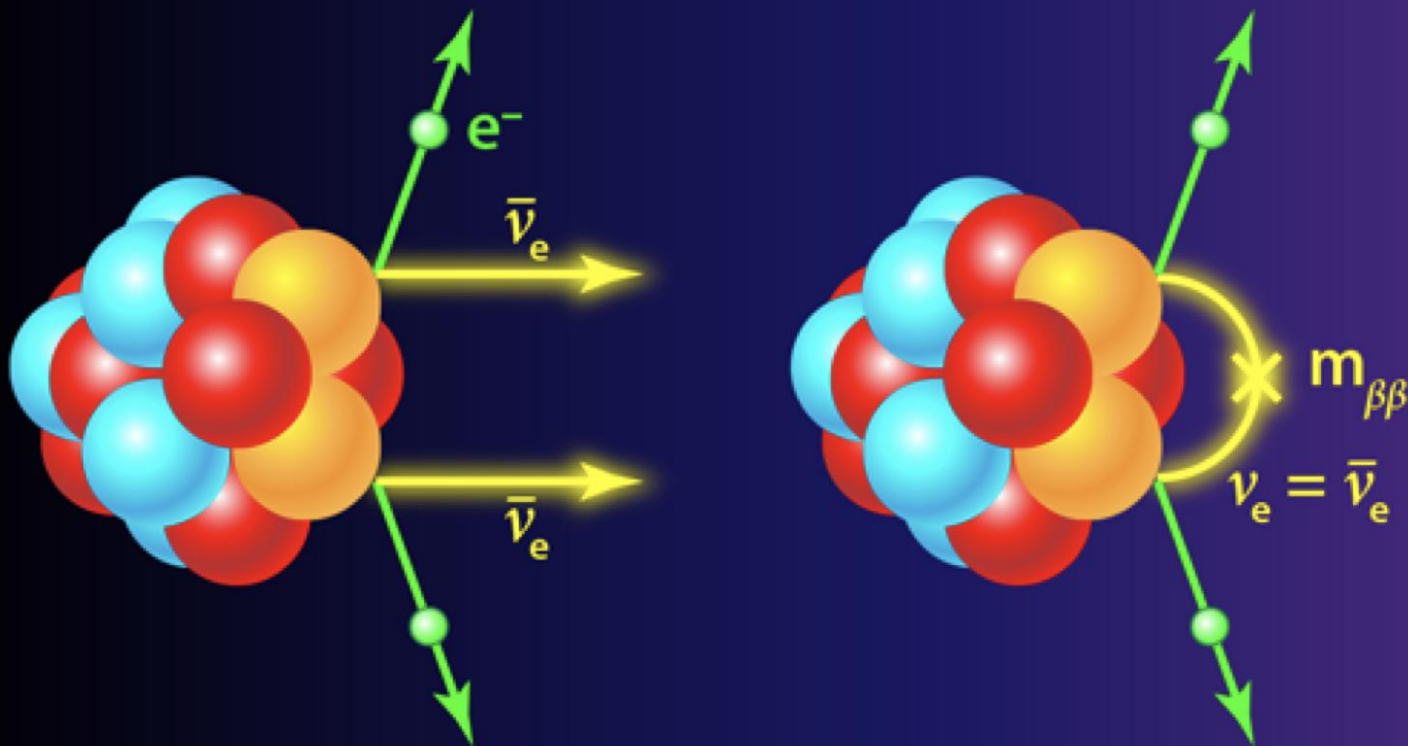
# Outlook



- **Better understanding the short-range NME in  $0\nu\beta\beta$**
- **Better understanding the nuclear structure**
- **The quenching problem**
- **NME statistical uncertainties** *More talks on this soon*
- **LEC from lattice calculations**
- **From  $0\nu\beta\beta$  to  $m_{\beta\beta}$ : improving the calculations of NME**
- **From  $0\nu\beta\beta$  to discriminating NME models: more information on  $m_{\beta\beta}$**



**Which direction do you bet will get through first?  
(the former one?)**



**Where  
are  
you?**





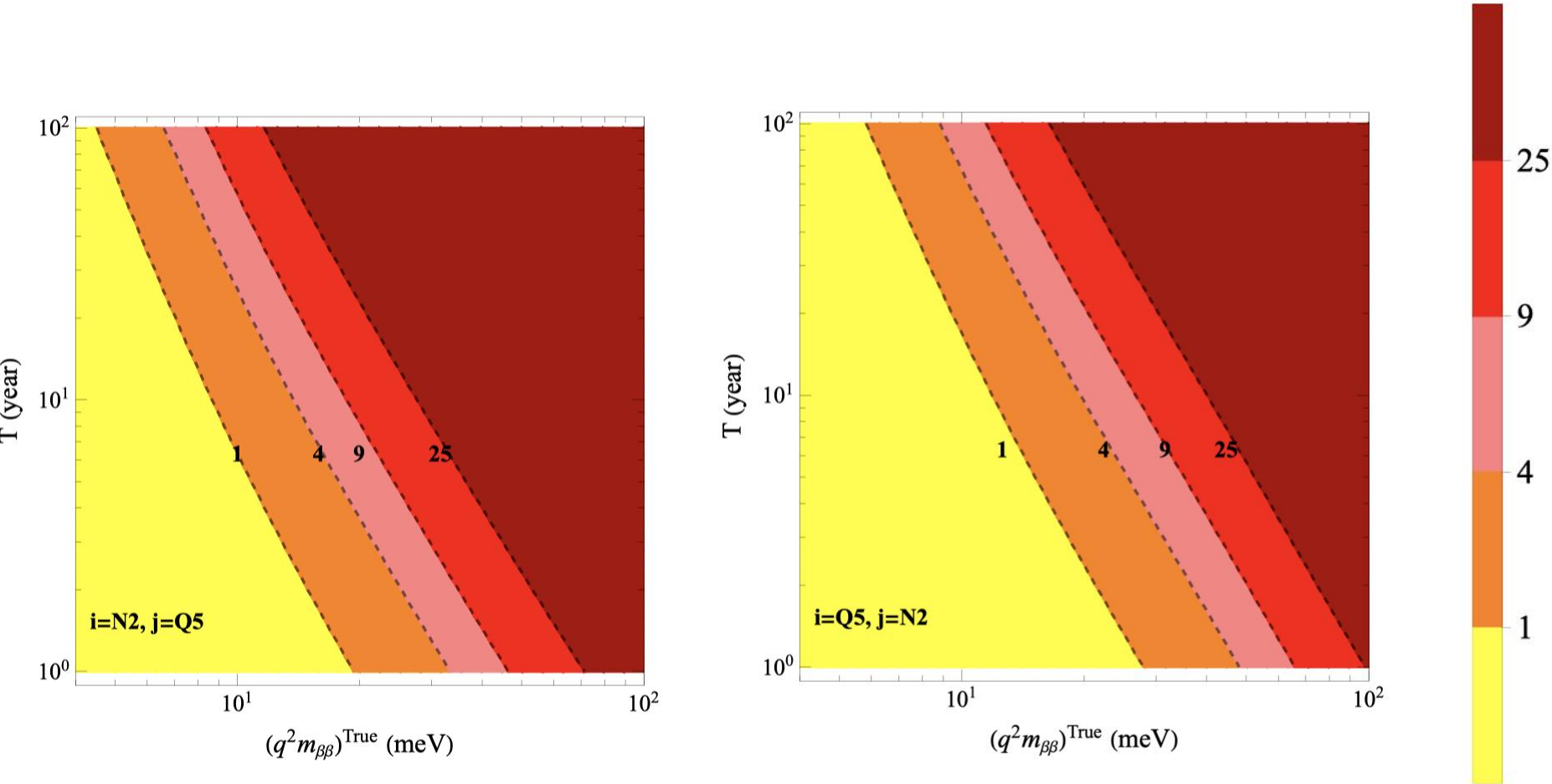
**Majorana neutrino,  
to be or not to be  
This is a question!**

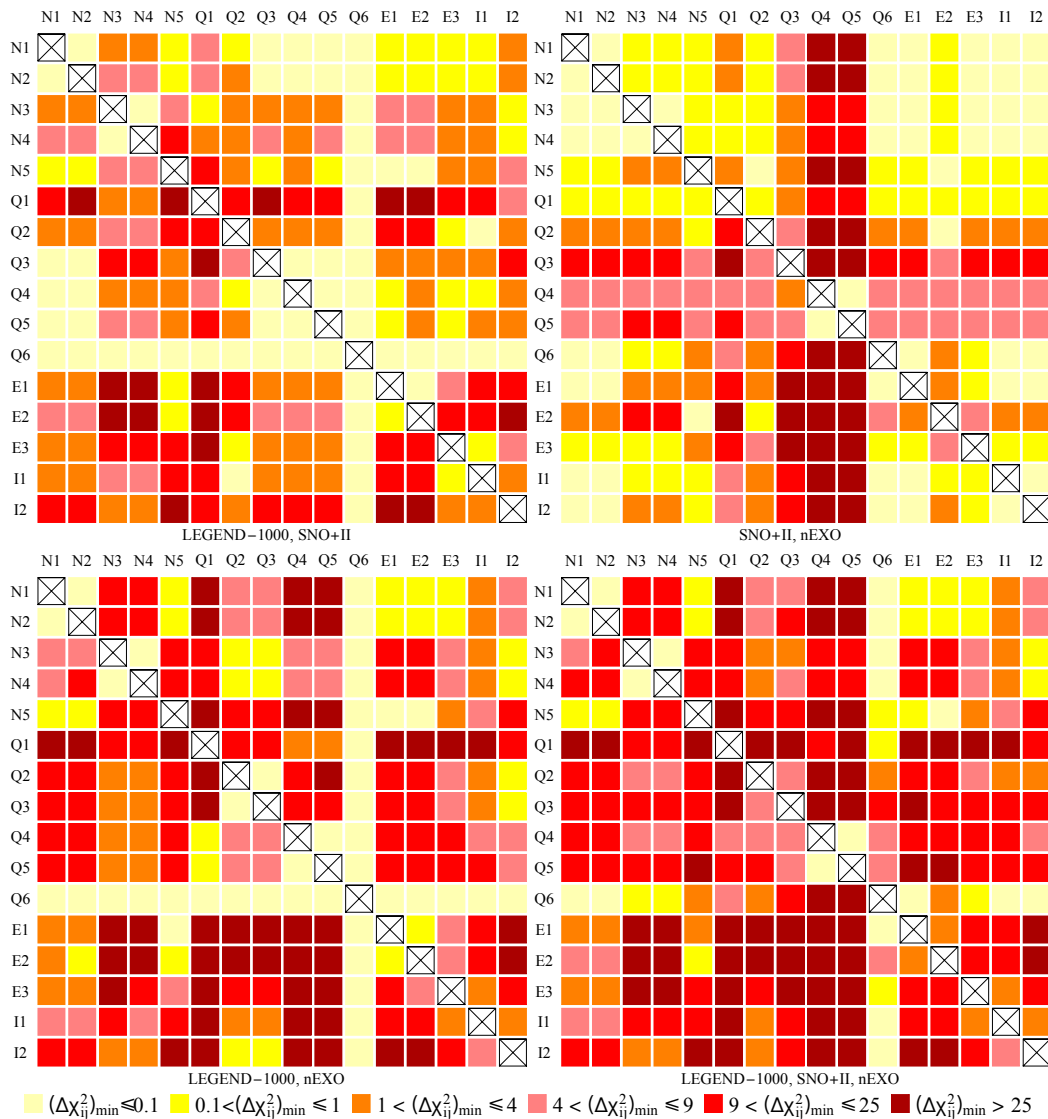
**Thank you!**

# Backups



# The contours of $(\Delta \chi^2_{ij})_{min}$ as function of the exposure time T and $(q^2 m_{\beta\beta})^{True}$





**nEXO and LEGEND-1000 dominate**

$$m_{\beta\beta}^{True} = 40 \text{ meV}$$

