Constraining the nuclear matrix elements of $0\nu\beta\beta$ decay by double Gamow-Teller transitions

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Outline

- Introduction
- Theoretical framework
- Results and discussion
- Summary

Neutrinoless $\beta\beta$ decay

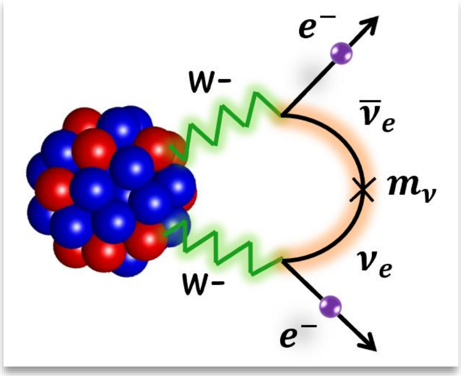
- Neutrinoless $\beta\beta$ decay ($0\nu\beta\beta$) $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

✓ Violation of **lepton number**

✓ **Majorana nature** of neutrinos

✓ Neutrino **mass scale** and **hierarchy**

✓ **Matter dominance** in the universe



Avignone, Elliott, Engel, Rev. Mod. Phys. 80, 481 (2008)

- Experimental progress on $0\nu\beta\beta$ decay

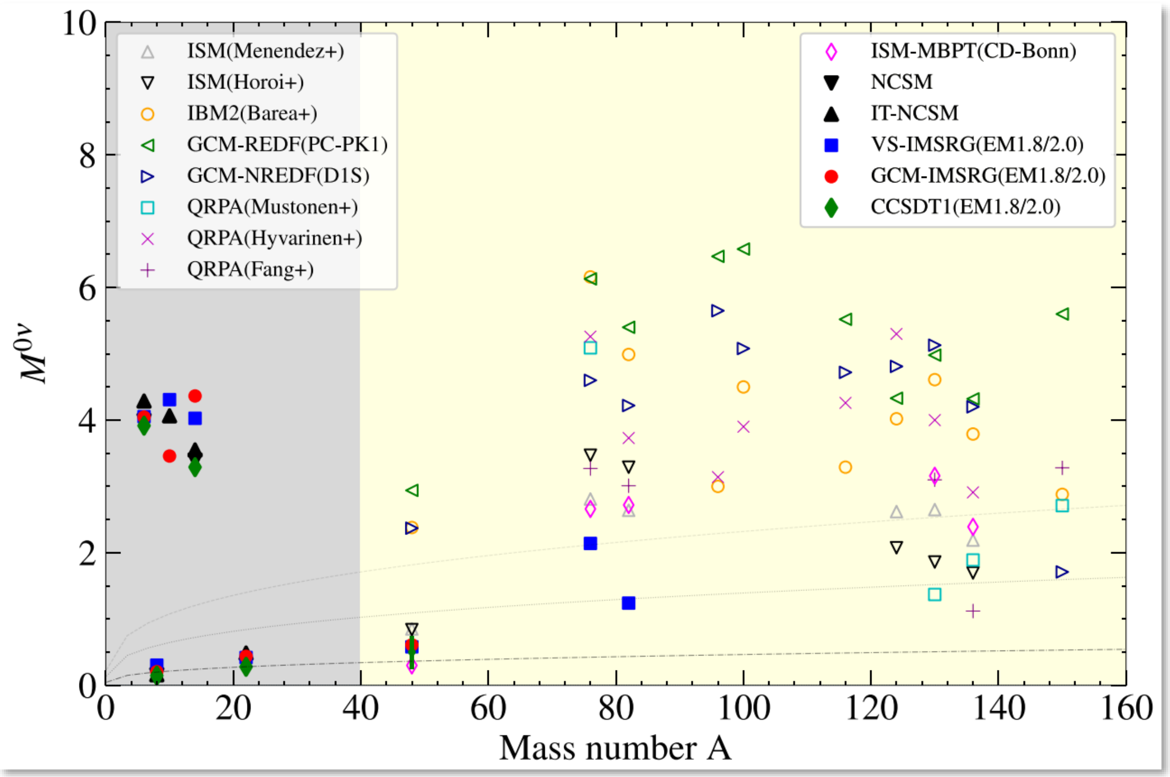
Isotopes	$T^{0\nu}_{1/2}$ (yr)	Collaborations
^{48}Ca	$> 5.8 \times 10^{22}$	ELEGANT VI
^{76}Ge	$> 1.8 \times 10^{26}$	GERDA, MAJORANA, CDEX
^{82}Se	$> 3.5 \times 10^{24}$	CUPID-0, NνDEx
^{100}Mo	$> 1.5 \times 10^{24}$	CUPID-Mo
^{130}Te	$> 3.2 \times 10^{25}$	CUORE
^{136}Xe	$> 2.3 \times 10^{26}$	KamLAND-Zen, EXO-200, PandaX
^{150}Nd	$> 2.0 \times 10^{22}$	NEMO-3

✓ No $0\nu\beta\beta$ -decay signal has been observed so far.

✓ Current limit on the decay half-life ranges from 10^{22} yr to 10^{26} yr.

Nuclear matrix elements

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}(Q,Z)|M^{0\nu}|^2\left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2$$



Yao, Meng, Niu, Ring, Prog. Part. Nucl. Phys. 126, 103965 (2022)

Nuclear structure:

Pairing correlations
Shapes and fluctuations
Noncollective collections
Valence space …

Decay operator:

Relativistic/non-relativistic
Nucleon size effects
Short-range correlation
Closure approximation
Two-body currents
Contact operator …

Double Gamow-Teller transition

- Double charge-exchange reaction $(A,Z)_T + (a,z)_p \rightarrow (A,Z+2)_T + (a,z-2)_p$
  - ✓ Double Fermi transition
  - ✓ Double Gamow-Teller transition (DGT) ⇒ Dominant decay process

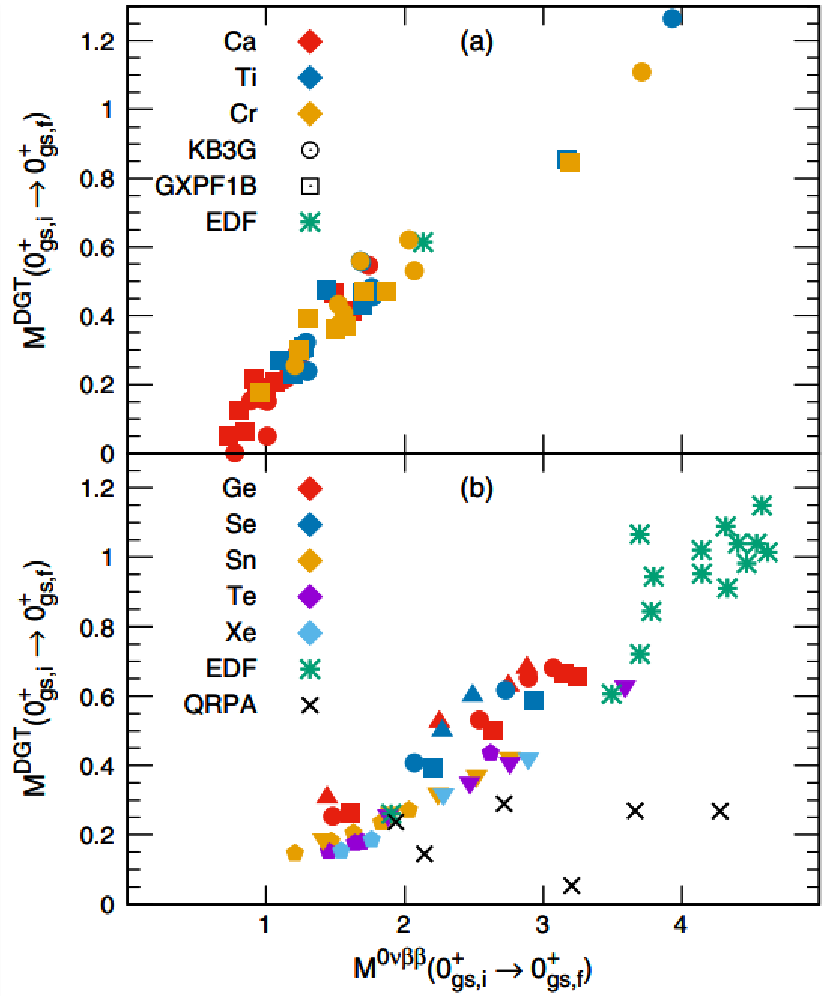
- Differential cross section of DGT transition can be factorized into a nuclear part and a reaction factor Santopinto et al., PRC 98, 061601(R) (2018)
  - ✓ Determining the DGT NMEs from the cross section of DGT transition

- $0\nu\beta\beta$ decay vs DGT transition: same initial and final nuclear wavefunctions, similar spin-dependent parts in the decay operators

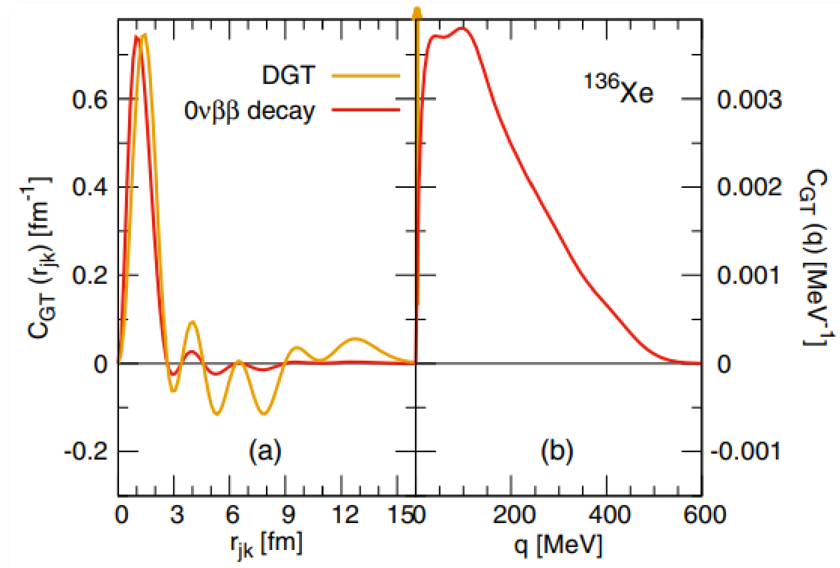
Rodríguez et al., PLB 719, 174 (2013), Cappuzzello et al., EPJA 51, 145 (2015)

Constraining the $0\nu\beta\beta$-decay NMEs from the DGT transitions?

Correlation between $0\nu\beta\beta$ decay and DGT transition



A good linear correlation is observed

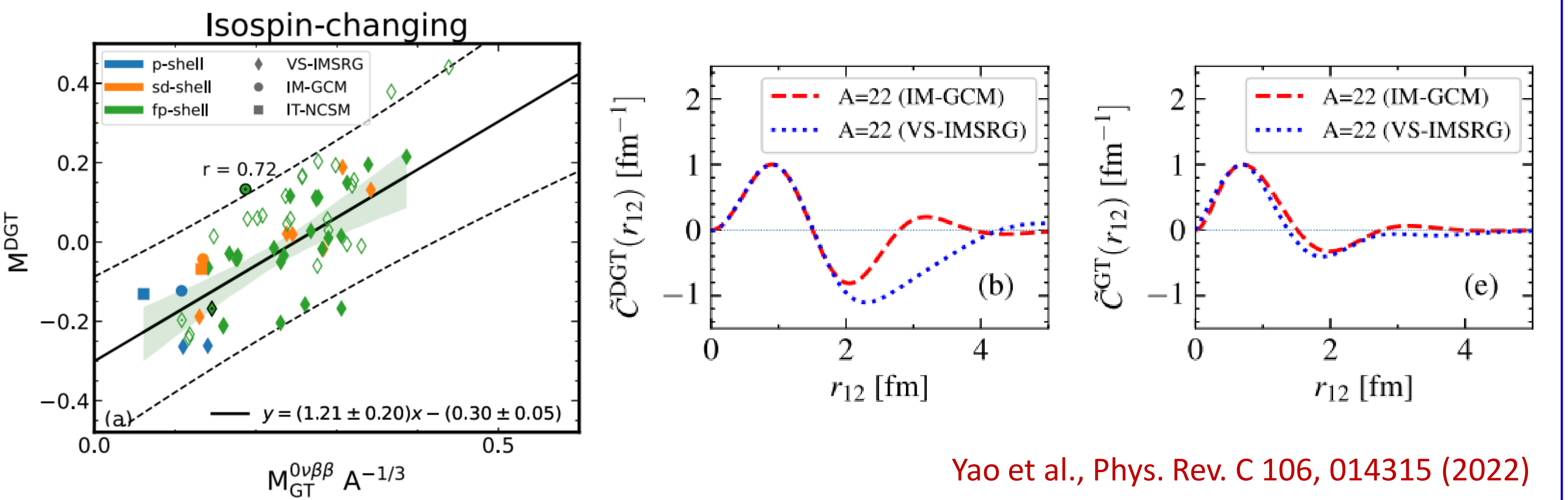


$$M^{\alpha} = \int dr_{12} C^{\alpha}(r_{12}), r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$$

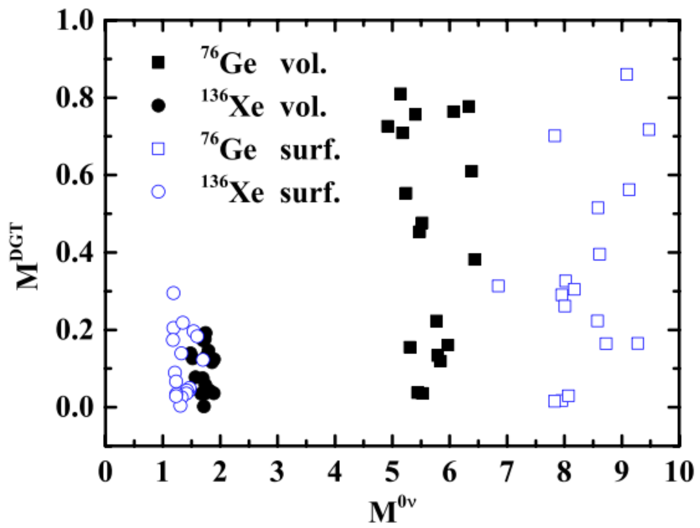
The short-range character of both DGT and $0\nu\beta\beta$ decay matrix elements can explain the simple linear relation between them. References [72,73] showed that if an operator only probes the short-range physics of low-energy states, the corresponding matrix elements factorize into a universal operator-dependent constant times a state-dependent number common to all short-range operators.

Shimizu, Menéndez, Yako, Phys. Rev. Lett. 120, 142502 (2018)

Correlation between $0\nu\beta\beta$ decay and DGT transition



Yao et al., Phys. Rev. C 106, 014315 (2022)



Lv et al., Phys. Rev. C 108, L051304 (2023)

- The linear correlation between $0\nu\beta\beta$ decay and DGT transition is much weaker in IMSRG, IMGCM, and QRPA calculations
  - ✓ IMSRG and IMGCM ⇒ very light nuclei
  - ✓ QRPA ⇒ spherical symmetry

In this work

- The correlation between $0\nu\beta\beta$ decay and DGT transition is investigated within the framework of Relativistic Configuration-interaction Density functional (ReCD) theory.
  - ✓ The NMEs of $0\nu\beta\beta$ decay and DGT transition in ^{48}Ca, ^{76}Se, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{128}Te, ^{130}Te, and ^{136}Xe, which are must relevant to the current $0\nu\beta\beta$ decay experiments, are evaluated.
  - ✓ The axial and triaxial deformations, which are important for describing the $0\nu\beta\beta$ decay, are included.
  - ✓ The origin of the linear correlation is analyzed.

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Relativistic density functional theory

- Lagrangian density and Hamiltonian operator:

$$(\bar{\psi}\mathcal{O}\Gamma\psi), \quad \mathcal{O}\in\{1,\tau\}$$

$$\Gamma\in\{1,\gamma_\mu,\cancel{\gamma_5},\cancel{\gamma_\mu\gamma_5},\cancel{\sigma_{\mu\nu}}\}$$

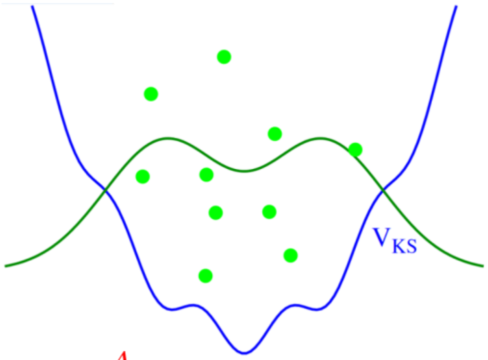
$$\Longrightarrow$$

$$\mathcal{L}=\mathcal{L}^{\text{free}}+\mathcal{L}^{4\text{f}}+\mathcal{L}^{\text{der}}+\mathcal{L}^{\text{hot}}+\mathcal{L}^{\text{em}}$$

$$\hat{H}=\int d\boldsymbol{r}\mathcal{H}(\boldsymbol{r})$$

- Relativistic energy density functional:

$$\begin{aligned}E\equiv\langle\Phi|\hat{H}|\Phi\rangle=\int d\boldsymbol{r}\Bigg\{&\sum_{i=1}^{A}\psi_i^\dagger(\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta m)\psi_i\\&+\frac{1}{2}\alpha_S\rho_s^2+\frac{1}{3}\beta_S\rho_s^3+\frac{1}{4}\gamma_S\rho_s^4+\frac{1}{2}\delta_S\rho_s\Delta\rho_s\\&+\frac{1}{2}\alpha_V j_\mu j^\mu+\frac{1}{4}\gamma_V(j_\mu j^\mu)^2+\frac{1}{2}\delta_V j_\mu\Delta j^\mu\\&+\frac{1}{2}\alpha_{TV}\vec{j}_\mu\vec{j}^\mu+\frac{1}{2}\delta_{TV}\vec{j}_\mu\Delta\vec{j}^\mu+\frac{1}{2}e^2A_\mu j_p^\mu\Bigg\}\end{aligned}$$



$$|\Phi\rangle=\prod_{k=1}^{A}\hat{a}_i^\dagger|0\rangle$$ Slater Det.

- Single-particle Dirac equation:

$$[\boldsymbol{\alpha}\cdot(\boldsymbol{p}-\boldsymbol{V})+V^0+\beta(m+S)]\psi_k=\varepsilon_k\psi_k$$

$$\rho_s=\sum_{i=1}^{A}\bar{\psi}_k\psi_k \quad j^\mu=\sum_{i=1}^{A}\bar{\psi}_k\gamma^\mu\psi_k$$

$$\vec{j}^\mu=\sum_{i=1}^{A}\bar{\psi}_k\gamma^\mu\tau_3\psi_k$$

ReCD theory

Trial wavefunction:

$$|\Psi_\alpha\rangle = \sum_\kappa f_\kappa^\alpha |\Phi_\kappa\rangle, \quad |\Phi_0\rangle = \prod_k \hat\beta_k |-\rangle$$

$$|\Phi_\kappa\rangle \in \{|\Phi_0\rangle, \hat\beta^\dagger_{\pi_i}\hat\beta^\dagger_{\pi_j}|\Phi_0\rangle, \hat\beta^\dagger_{\nu_i}\hat\beta^\dagger_{\nu_j}|\Phi_0\rangle, \hat\beta^\dagger_{\pi_i}\hat\beta^\dagger_{\pi_j}\hat\beta^\dagger_{\nu_i}\hat\beta^\dagger_{\nu_j}|\Phi_0\rangle, \cdots\}$$

Symmetry restoration:

$$|\Psi_\alpha^{JNZ}\rangle = \sum_{K=-J}^{J}\sum_\kappa f_{K\kappa}^{J\alpha}|JMK,\kappa\rangle$$

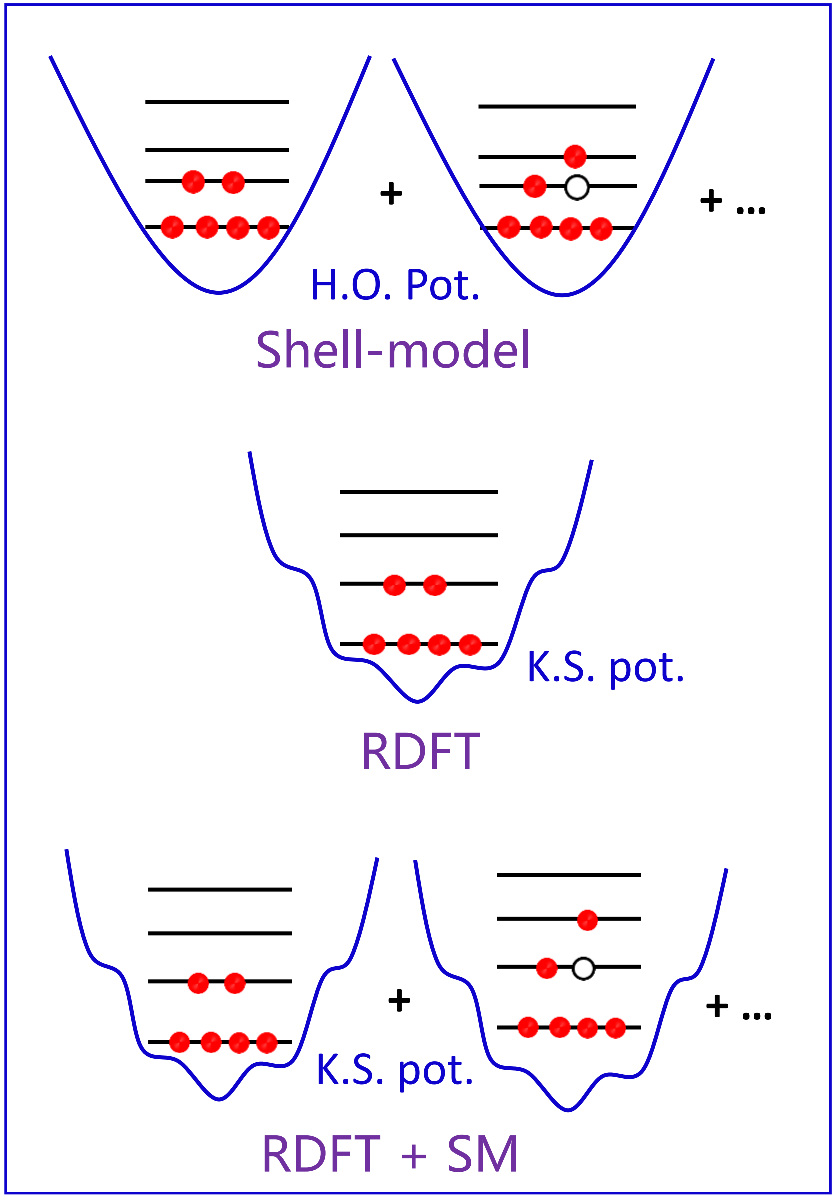
$$\sum_{K'\kappa'}\{\mathcal{H}^J_{KK';\kappa\kappa'} - E^J_\alpha N^{J\alpha}_{KK';\kappa\kappa'}\} f^{J\alpha}_{K'\kappa'} = 0$$

P. W. Zhao, P. Ring, J. Meng, PRC 94, 041301(R) (2016)

Y. K. Wang, P. W. Zhao, J. Meng, PRC 105, 054311 (2022)

Y. K. Wang, P. W. Zhao, J. Meng, arXiv: 2304.12009

Y. K. Wang, P. W. Zhao, J. Meng, PLB 848, 138346 (2024)



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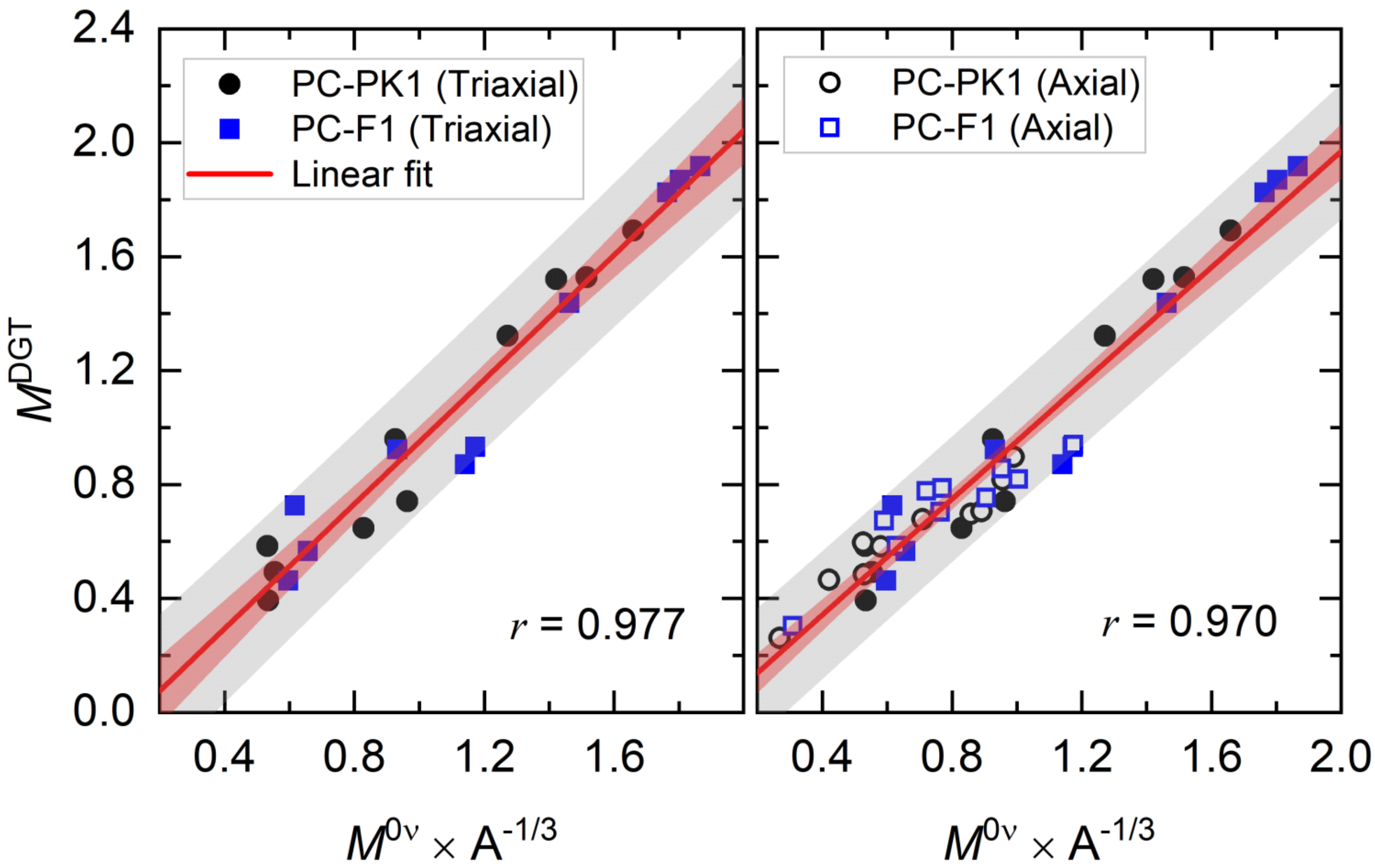
Correlation between DGT and $0\nu\beta\beta$ decay



10 nuclei:

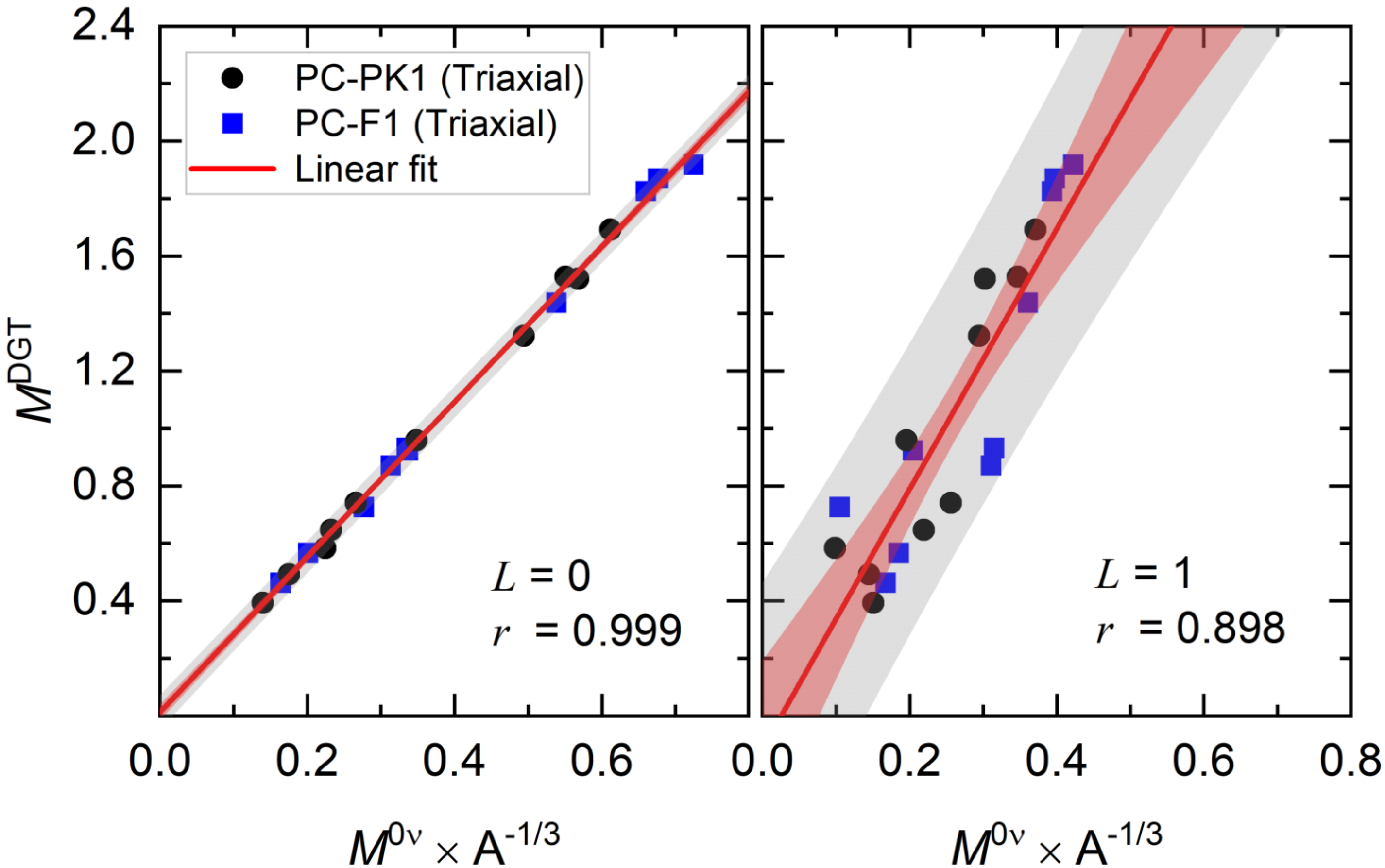
^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{128}Te, ^{130}Te, ^{136}Xe

Axial and triaxial deformations

Full model space

- A strong linear correlation between $0\nu\beta\beta$ decay and DGT transition is demonstrated
- The linear correlation is robust against nuclear deformations

Decomposition of the NMEs

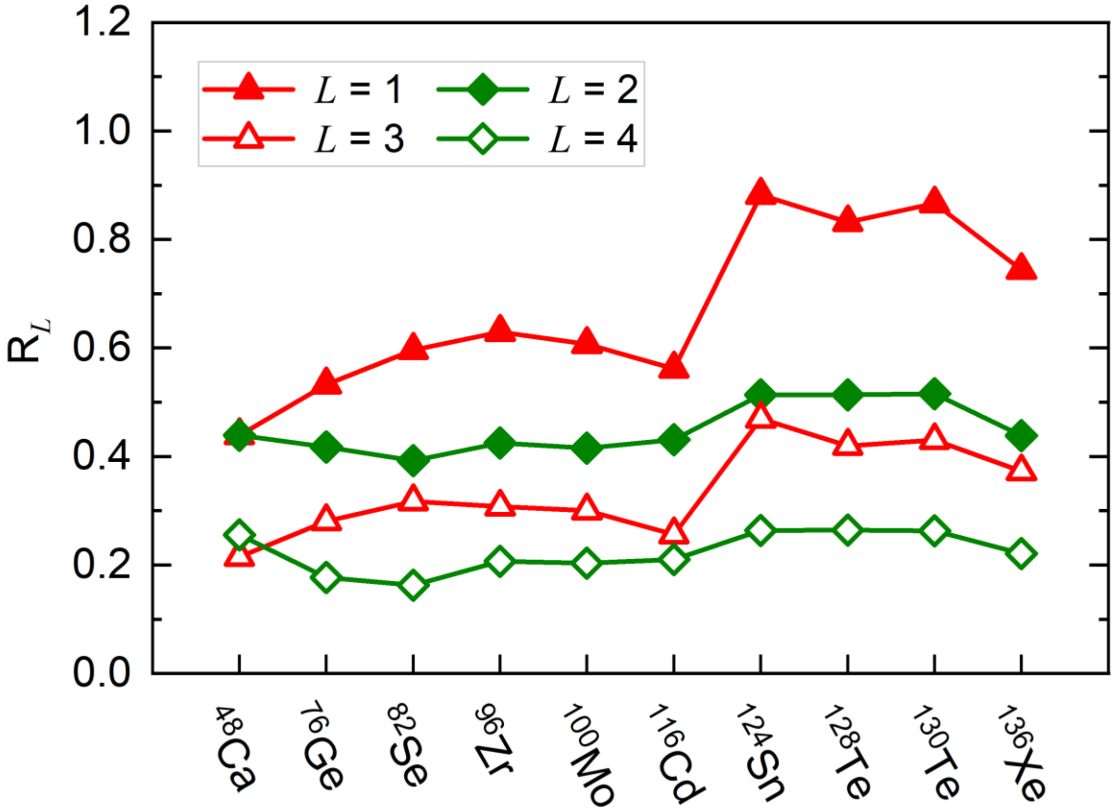


$$\hat{\mathcal{O}}^{0\nu} = \sum_L \hat{\mathcal{O}}_L^{0\nu}$$

$$M^{0\nu} = \sum_L M_L^{0\nu}$$

- The **leading order term** $M_{L=0}^{0\nu}$ correlated strongly with M^{DGT}, while the correlation between $M_{L=1}^{0\nu}$ and M^{DGT} is much weaker.

Contributions from higher-order terms



$$R_L = M_L^{0\nu} / M_{L=0}^{0\nu}$$

- $R_2 \approx 0.45, R_4 \approx 0.20$, and they are independent on the decay candidates
- Consideration of higher-order terms with even L would not worsen the correlation
- Contributions from NMEs with odd L are generally smaller than those from the NMEs with even L

The origin of the linear correlation

- The $0\nu\beta\beta$-decay operator contains five terms

$$\hat{O}^{0\nu} = \hat{O}^{0\nu}_{VV} + \hat{O}^{0\nu}_{AA} + \hat{O}^{0\nu}_{AP} + \hat{O}^{0\nu}_{PP} + \hat{O}^{0\nu}_{MM}$$

- Decay operator in *AA* coupling channel

$$\hat{O}^{0\nu}_{AA} = \sum_{1234} \langle 13|\mathcal{O}^{AA}(\boldsymbol{r}_1, \boldsymbol{r}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2|24\rangle \hat{d}^\dagger_1 \hat{d}^\dagger_3 \hat{c}_4 \hat{c}_2, \quad |1\rangle \equiv |n_1 l_1 j_1 m_1\rangle$$

Neutrino potential in coordinate space

$$\mathcal{O}^{AA}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \int \frac{d\boldsymbol{q}}{(2\pi)^3} H(\boldsymbol{q}) e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}$$

Multipole expansion for plane waves $e^{\pm i\boldsymbol{p}\cdot\boldsymbol{r}}$ by spherical harmonics

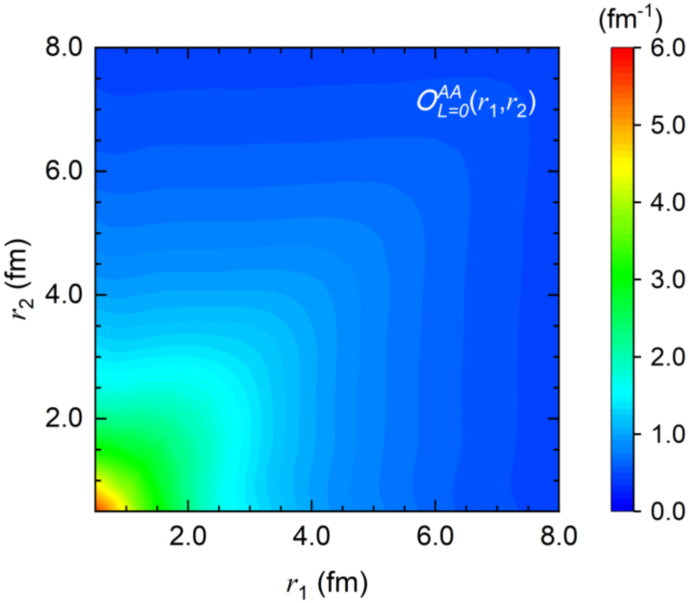
$$e^{i\boldsymbol{q}\cdot\boldsymbol{r}} = 4\pi \sum_{LM} i^L j_L(qr) Y^*_{LM}(\hat{\boldsymbol{q}}) Y_{LM}(\hat{\boldsymbol{r}})$$

$$\mathcal{O}^{AA}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{2}{\pi} \int q^2 dq H(q) \sum_{LM} [j_L(qr_1) Y_{LM}(\hat{\boldsymbol{r}}_1)][j_L(qr_2) Y^*_{LM}(\hat{\boldsymbol{r}}_2)] = \sum_L \mathcal{O}^{AA}_L(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

The origin of the linear correlation

- The neutrino potential with $L = 0$

$$\mathcal{O}^{AA}_{L=0}(r_1, r_2) = \frac{1}{2\pi^2}\int q^2 dq H(q) j_0(qr_1) j_0(qr_2)$$



$$\mathcal{O}^{AA}_{L=0}(r_1, r_2) = \sum_{ij} a_{ij} X_i(r_1) Y_j(r_2)$$

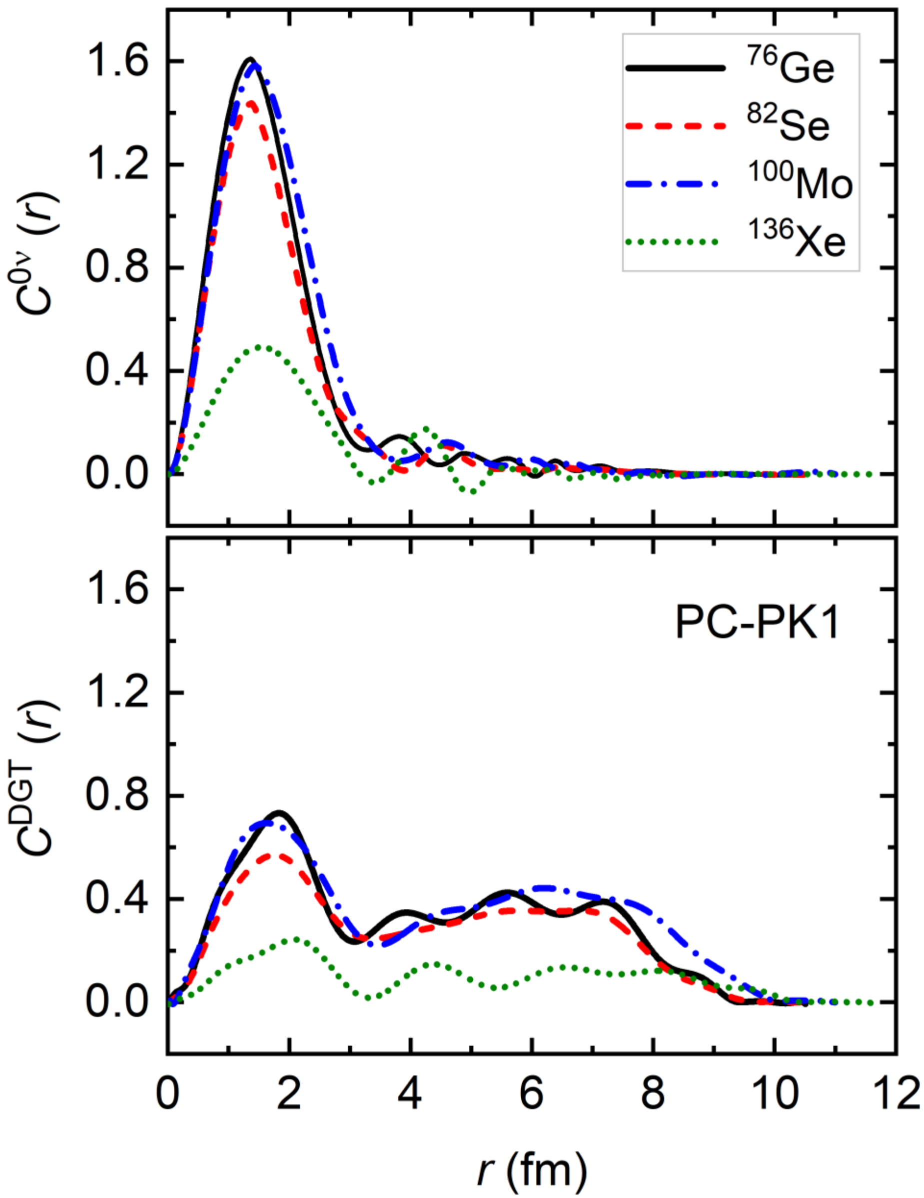
$$\mathcal{O}^{AA}_{L=0}(r_1, r_2) \approx \frac{1}{2}\left[X_1(r_1)Y_1(r_2) + X_1(r_2)Y_1(r_1)\right]$$

$X_1(r)$ and $Y_1(r)$ are smoothly decreasing functions that are larger than zero

$L = 0$ term: $\langle 13|\mathcal{O}^{AA}_{L=0}(r_1, r_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2|24\rangle \approx \langle n_1 l_1|X_1(r_1)|n_2 l_2\rangle\langle n_3 l_3|Y_1(r_2)|n_4 l_4\rangle$
$\times \langle j_1 m_1|\boldsymbol{\sigma}_1|j_2 m_2\rangle\langle j_3 m_3|\boldsymbol{\sigma}_2|j_4 m_4\rangle \delta_{n_1 n_2}\delta_{n_3 n_4}\delta_{l_1 l_2}\delta_{l_3 l_4}$

DGT transition: $\langle 13|\mathcal{O}^{\mathrm{DGT}}|24\rangle = \frac{1}{\sqrt{3}}\langle j_1 m_1|\boldsymbol{\sigma}_1|j_2 m_2\rangle\langle j_3 m_3|\boldsymbol{\sigma}_2|j_4 m_4\rangle \delta_{n_1 n_2}\delta_{n_3 n_4}\delta_{l_1 l_2}\delta_{l_3 l_4}$

NME distributions in coordinate space



$$M^{\alpha} = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 C^{\alpha}(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

$\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2)$; $\boldsymbol{r} = (\boldsymbol{r}_1 - \boldsymbol{r}_2)$

$$M^{\alpha} = \int dr C^{\alpha}(r)$$

- The short-range character is observed for $0\nu\beta\beta$ decay, but not for DGT transition
- The explanation that the linear correlation originates from the dominant short-range character in both transitions is thus not support

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Summary

- The NMEs of $0\nu\beta\beta$ decay and DGT transition in ten nuclei that are most relevant to the $0\nu\beta\beta$ decay experiments are investigated with the ReCD theory:
  - ✓ A strong linear correlation between $0\nu\beta\beta$ decay and DGT transition is demonstrated
  - ✓ The leading-order term of $0\nu\beta\beta$-decay operator is very similar to DGT-transition one
  - ✓ The short-range dominant character is observed in $0\nu\beta\beta$ decay but not in DGT transition
  - ✓ The resent results provides a strong support to the forthcoming experiments aiming to constrain the $0\nu\beta\beta$-decay NMEs from the double charge-exchange reactions

Summary

Predicted decay half-life for $m_{\beta\beta} = 10$ meV

Isotopes	$G_{0\nu}(\times 10^{-15}\ \mathrm{yr}^{-1})$	$M^{0\nu}$	Half-life (yr)
^{48}Ca	24.81	**1.45**	1.93×10^{28}
^{76}Ge	2.363	**5.96**	1.19×10^{28}
^{82}Se	10.16	**4.81**	4.26×10^{27}
^{96}Zr	20.58	**6.61**	1.12×10^{27}
^{100}Mo	15.92	**7.11**	1.25×10^{27}
^{116}Cd	16.70	**4.91**	2.49×10^{27}
^{128}Te	0.5878	**3.28**	1.59×10^{29}
^{130}Te	14.22	**3.85**	4.78×10^{27}
^{136}Xe	14.58	**3.34**	6.16×10^{27}

Perspective

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无中微子双贝塔衰变的
相对论手征有效场论研究

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